3 Quality Engineering: Strategy in Research and Development

3.1. Research and Development Cycle Time Reduction

Functionality evaluation, which facilitates all research and development activities, is an issue for R&D managers. Their major task is not as specialists but as, those who develop new technologies or products. Streamlining of R&D tasks is called *generic technology* in Japan and *technological strategy* in Europe and the United States.

The main job of top management in an R&D or engineering department is to plan strategy for technological development, classified into four general groupings.

1. *Selection of technical themes.* Fundamental research for creative products prior to product planning is desirable. Testing of current and new products, downsizing, and simulations without prototyping are included in this process.
2. *Creation of concepts and systems.* Parameter design is conducted through a complex system to enhance reliability. The more complicated a system becomes, the more effectively robust design must be implemented.
3. *Evaluation for parameter design.* This procedure involves functional evaluation and checkup. The former rests on the SN ratio and the latter is a checkup of additivity based on an orthogonal array.
4. *Preparation of miscellaneous tools.* The finite element and circuit calculation methods should be used in addition to computers. The difference calculation method by orthogonal array, which we introduced in the United States, is applied in numerous fields. An orthogonal array is a generic tool for difference calculation.

Quality engineering is related to all four of the strategic items above. Although item 2 does not seem to be concerned with quality engineering, it must be included because in quality engineering we do not take measures once problems occur but design a complex system and parameters to prevent problems. Quality engineering gives guidelines only; it does not include detailed technical measures.

3.2. Stage Optimization

Orthogonal Expansion for Standard SN Ratios and Tuning Robustness

In a manufacturing industry, product and process designs affect a company's future significantly, but the design of production systems to produce services is an essential role in a service industry such as telecommunications, traffic, or finance. The design process comprises the following two stages: (1) synthesis and (2) analysis.

Quality engineering classifies the stage of synthesis into another two phases: (1) system (concept) selection and (2) selection of nominal system parameters (design constants). For the former, designer creativity is desirable, and if a designer invents a new method, it can be protected with a patent. The latter is related to development cycle time and to quality engineering. How a designer determines levels of design constants (nominal system parameters), which can be selected at a designer's discretion, changes the functional stability of a product under various conditions of use. A method of minimizing deviation of a product function from an ideal function under various conditions by altering nominal design constants is called *parameter design*. To balance functional robustness and cost and to enhance productivity (minimize quality and cost) by taking advantage of the loss function after robustness is improved is also an issue in quality engineering. Here, only the strategy for parameter design is illustrated.

Research and design consist primarily of system selection and parameter design. Although creation of new systems and concepts is quite important, until parameter design is complete, it is unclear whether a system selected can become competitive enough in the market. Therefore, it is essential in the short term to design parameters effectively for the system selected. In case of a large-scale or feedback control system, we should divide a total system into subsystems and develop them concurrently to streamline whole system design. Proper division of a system is the design leader's responsibility.

To improve functional robustness (reliability) in such a way that a product can function well in the market is the basis of product design. This means that the research and design parameters should be drawn up so that a product can work under various conditions in the marketplace over the product life span. Current simulation design approaches an objective function. After improving robustness (the standard SN ratio), we should approach the objective function using only standard conditions. Reducing variability is crucial. Parameters should be dispersed around standard values in simulations. All parameters in a system are control and noise factors. If simulation calculation takes a long time, all noise factors can be compounded into two levels. In this case we need to check whether compounded noises have an original qualitative tendency. To do this we investigate compounded noise effects in an orthogonal array to which only noises are assigned under initial design conditions, in most cases the second levels of control factors. (A design example in Chapter 4 shows only compounded noise effects.)

Not all noise factors need to be compounded. It is sufficient to use only the three largest. After improving robustness, a function curve is quite often adjusted to a target curve based on a coefficient of linear term β_1 and a coefficient of quadratic term β_2. (See the example in Chapter 4.) Since it is impossible to conduct the life test of a product under varied marketplace conditions, the strategy of using an SN ratio combined with noise, as illustrated in this chapter, has fundamentally changed the traditional approaches. The following strategies are employed:

1. Noise factors are assigned for each level of each design parameter and compounded, because what happens due to the environment and deterioration is unpredictable. For each noise factor, only two levels are sufficient. As a first step, we evaluate robustness using two noise levels.
2. The robust level of a control factor (having a higher SN ratio) does not change at different signal factor levels. This is advantageous in the case of a time-consuming simulation, such as using the finite element method by reducing the number of combinations in the study and improving robustness by using only the first two or three levels of the signal factor.
3. Tuning to the objective function can be made after robustness is improved using one or two control or signal factors. Tuning is done under the standard condition. To do so, orthogonal expansion is performed to find the candidates that affect linear coefficient β_1 and quadratic coefficient β_2.

Variability (Standard SN Ratio) Improvement: First Step in Quality Engineering

In quality engineering, all conditions of use in the market can be categorized into either a signal or a noise. In addition, a signal can be classified as active or passive. The former is a variable that an engineer can use actively, and it changes output characteristics. The latter is, like a measurement instrument or receiver, a system that measures change in a true value or a signal and calculates output. Since we can alter true values or oscillated signals in research, active and passive signals do not need to be differentiated.

What is most important in two-stage optimization is that we not use or look for cause-and-effect relationships between control and noise factors. Suppose that we test circuits (e.g., logic circuits) under standard conditions and design a circuit with an objective function as conducted at Bell Labs. Afterward, under 16 different conditions of use, we perform a functional test. Quality engineering usually recommends that only two conditions be tested; however, this is only because of the cost. The key point is that we should not adjust (tune) a design condition in order to meet the target when the noise condition changes. For tuning, a cause-and-effect relationship under the standard condition needs to be used.

We should not adjust the deviation caused by the usage condition change of the model or regression relationship because it is impossible to dispatch operators in manufacturing to make adjustments for uncountable conditions of use. Quality engineering proposes that adjustments be made only in production processes that are considered standard conditions and that countermeasures for noises should utilize the interactions between noise and control factors.

TESTING METHOD AND DATA ANALYSIS

In designing hardware (including a system), let the signal factor used by customers be M and its ideal output (target) be m. Their relationship is written as $m - f(M)$. In two-stage design, control factors (or indicative factors) are assigned to an orthogonal array. For each run of the orthogonal array, three types of outputs are obtained either from experimentation or from simulation: under standard conditions N_0, under negative-side compounded noise conditions N_1, and under positive-side compounded noise conditions, N_1. The output under N_0 at each level of the signal factor (used by customers) are now "redefined" as signal factor levels: $M_1, M_2, \ldots, M_k$. Table 3.1 shows the results.

First stage: To reduce variability, we calculate SN ratios according to Table 3.1. However, in this stage, *sensitivities are not computed.* $M_1, M_2, \ldots$, and M_k are equal to output values under N_0 for each experiment. From the inputs and outputs of the table, an SN ratio is calculated. Such an SN ratio is called a *standard SN ratio.* The control factor combination that maximizes a standard SN ratio is the optimal condition. However, it is useless to compute sensitivities because we attempt to maximize standard SN ratios in the first stage. Under the optimal conditions, N_0, N_1, and N_2 are obtained to calculate the SN ratio. This SN ratio is used to check the reproducibility of gain.

Second stage: After robustness is optimized in the first stage, the output is tuned to the target. The signal used by customers (original signal, M) is tuned so that the output, Y, may meet the target, m. $y_1, y_2, \ldots, y_k$ represent the outputs under N_0 of optimal conditions, and $m_1, m_2, \ldots, m_k$ represent the targets under the conditions of the original signal, M. Objective function design is to bring output value y's close to target value m's by adjusting the signal used by the user, M. In most cases, the proportionality

$$y = \beta M \tag{3.1}$$

is regarded as an ideal function except for its coefficient. Nevertheless, the ideal function is not necessarily a proportional equation, such as equation (3.1), and can be expressed in various types of equations. Under an optimal SN ratio condition, we collect output values under standard conditions N_0 and obtain the following:

M (*signal factor*)	M_1	M_2	...	M_k
m (*target value*):	m_1	m_2	...	m_k
y (*output value*):	y_1	y_2	...	y_k

Although the data in N_1 and N_2 are available, they are not needed for adjustment because they are used only for examining reproducibility.

If a target value m is proportional to a signal M for any value of β, it is analyzed with level values of a signal factor M. An analysis procedure with a signal M is the same as that with a target value m. Therefore, we show the common method of adjustment calculation using a target value m, which is considered the Taguchi method for adjustment.

Table 3.1
Data for SN ratio analysis

N_0	$M_1, M_2, \ldots, M_k$	Linear Equation
N_1	$y_{11}, y_{12} \ldots, y_{1k}$	L_1
N_2	$y_{21}, y_{22}, \ldots, y_{2k}$	L_2

If outputs $y_1, y_2, \ldots, y_k$ under the standard conditions N_0 match the target value m's, we do not need any adjustment. In addition, if y is proportional to m with sufficient accuracy, there are various ways of adjusting β to the target value of 1. Some of the methods are the following:

1. We use the signal M. By changing signal M, we attempt to match the target value m with y. A typical case is proportionality between M and m. For example, if y is constantly 5% larger than m, we can calibrate M by multiplying it by 0.95. In general, by adjusting M's levels, we can match m with y.
2. By tuning up one level of control factor levels (sometimes, more than two control factor levels) in a proper manner, we attempt to match the linear coefficient β_1 with 1.
3. Design constants other than the control factors selected in simulation may be used.

The difficult part in adjustment is not for coefficients but for deviations from a proportional equation. Below we detail a procedure of adjustment including deviations from a proportional equation. To achieve this, we make use of orthogonal expansion, which is addressed in the next section. Adjustment for other than proportional terms is regarded as a significant technical issue to be solved in the future.

FORMULA OF ORTHOGONAL EXPANSION

The formula of orthogonal expansion for adjustment usually begins with a proportional term. If we show up to the third-order term, the formula is:

$$y = \beta_1 m + \beta_2\left(m^2 - \frac{K_3}{K_2}m\right) + \beta_3\left(m^3 + \frac{K_3K_4 - K_2K_5}{K_2K_4 - K_3^2}m + \frac{K_3K_5 - K_4^2}{K_2K_4 - K_3^2}\right) + \cdots \tag{3.2}$$

Now, $K_1, K_2, \ldots$ are expressed by

$$K_i = \frac{1}{k}(m_1^i + m_2^i + \cdots + m_k^i) \qquad (i = 2, 3, \ldots) \tag{3.3}$$

$K_2, K_3, \ldots$ are constant because m's are given. In most cases, third- and higher-order terms are unnecessary; that is, it is sufficient to calculate up to the second-order term. We do not derive this formula here.

Accordingly, after making orthogonal expansion of the linear, quadratic, and cubic terms, we create an ANOVA (analysis of variance) table as shown in Table 3.2, which is not for an SN ratio but for tuning. If we can tune up to the cubic term, the error variance becomes V_e. The loss before tuning,

$$L_0 = \frac{A_0}{\Delta_0^2} V_T$$

is reduced to

$$L = \frac{A_0}{\Delta_0^2} V_e$$

Table 3.2
ANOVA table for tuning

Source	f	S	V
β_1	1	$S_{\beta 1}$	
β_2	1	$S_{\beta 2}$	
β_3	1	$S_{\beta 3}$	
e	$k - 3$	S_e	V_e
Total	k	S_T	V_T

after tuning. When only the linear term is used for tuning, the error variance is expressed as follows:

$$V_e = \frac{1}{k-1}(S_{\beta 2} + S_{\beta 3} + S_e)$$

Since the orthogonal expansion procedure in Table 3.2 represents a common ANOVA calculation, no further explanation is necessary.

❒ Example

This design example, introduced by Oki Electric Industry at the Eighth Taguchi Symposium held in the United States in 1990, had an enormous impact on research and design in the United States. However, here we analyzed the average values of N_1 and N_2 by replacing them with data of N_0 and the standard SN ratio based on dynamic characteristics. Our analysis is different from that of the original report, but the result is the same.

A function of the color shift mechanism of a printer is to guide four-color ribbons to a proper position for a printhead. A mechanism developed by Oki Electric Industry in the 1980s guides a ribbon coming from a ribbon cartridge to a correct location for a printhead. Ideally, rotational displacement of a ribbon cartridge's tip should match linear displacement of a ribbon guide. However, since there are two intermediate links for movement conversion, both displacements have a difference. If this difference exceeds 1.45 mm, misfeeds such as ribbon fray (a ribbon frays as a printhead hits the edge of a ribbon) or mixed color (a printhead hits a wrong color band next to a target band) happen, which can damage the function. In this case, the functional limit Δ_0 is 1.45 mm. In this case, a signal factor is set to a rotational angle M, whose corresponding target values are as follows:

Rotational angle (*rad*):	1.3	2.6	3.9	5.2	6.5
Target value (*mm*):	2.685	5.385	8.097	10.821	13.555

For parameter design, 13 control factors are selected in Table 3.3 and assigned to an L_{27} orthogonal array. It is preferable that they should be assigned to an L_{36} array. In addition, noise factors should be compounded together with control factors other than design constants.

Since variability caused by conditions regarding manufacturing or use exists in all control factors, if we set up noise factors for each control factor, the number of noise factors grows, and accordingly, the number of experiments also increases. Therefore, we compound noise factors. Before compounding, to understand noise factor effects, we check how each factor affects an objective characteristic of displacement by fixing all control factor levels at level 2. The result is illustrated in Table 3.4.

By taking into account an effect direction for each noise factor on a target position, we establish two compounded noise factor levels, N_1 and N_2, by selecting the worst configuration on both the negative and positive sides (Table 3.5). For some cases we do not compound noise factors, and assign them directly to an orthogonal array.

According to Oki research released in 1990, after establishing stability for each rotational angle, the target value for each angle was adjusted using simultaneous equations. This caused extremely cumbersome calculations. By "resetting" an output value for each rotational angle under standard conditions as a new signal M, we can improve the SN ratio for output under noises. Yet since we cannot obtain a program for the original calculation, we substitute an average value for each angle at N_1 and N_2 for a standard output value N_0. This approach holds true only for a case in which each output value at N_1 and N_2 mutually has the same absolute value with a different sign around a standard value. An SN ratio that uses an output value under a standard condition as a signal is called a *standard SN ratio*, and one substituting an average value is termed a *substitutional standard SN ratio* or *average standard SN ratio.*

Table 3.3
Factors and levels[a]

	Level				Level		
Factor	**1**	**2**	**3**	**Factor**	**1**	**2**	**3**
A	6.0	6.5	7.0	*H*	9.6	10.6	11.6
B	31.5	33.5	35.5	*I*	80.0	82.0	84.0
C	31.24	33.24	35.24	*J*	80.0	82.0	84.0
D	9.45	10.45	11.45	*K*	23.5	25.5	27.5
E	2.2	2.5	2.8	*L*	61.0	63.0	65.0
F	45.0	47.0	49.0	*M*	16.0	16.5	17.0
G	7.03	7.83	8.63				

[a]The unit for all levels is millimeters, except for *F* (angle), which is radians.

Table 3.4
Qualitative effect of noise factor on objective function

Factor	Effect	Factor	Effect
A'	+	H'	−
B'	−	I'	−
C'	−	J'	+
D'	+	K'	+
E'	+	L'	−
F'	+	M'	+
G'	−		

Calculation of Standard SN Ratio and Optimal Condition

For experiment 1 in the orthogonal array (L_{27} in this case), each sliding displacement for each rotational angle under the standard conditions, N_1 and N_2 is shown in Table 3.6. Decomposition of the total variation is shown in Table 3.7.

Now we obtain the SN ratio:

$$\eta = 10 \log \frac{(1/2r)(S_\beta - V_e)}{V_N} = 3.51 \text{ dB} \tag{3.4}$$

For other experimental runs, by resetting sliding displacement under standard conditions as a signal, we should compute SN ratios. Each signal level value is the average of output values at both N_1 and N_2 for each experiment. Therefore, even if we use this approximation method or the normal procedure based on standard output values, the signal value for each of experiments 1 to 18 is different.

Tables 3.8 and 3.9 and Figure 3.1 show factor assignment and SN ratios, level totals for SN ratios, and response graphs, respectively. The optimal condition is $A_1B_3C_3D_1E_1F_1G_3H_1I_1J_1K_1L_3M_3$, which is the same as that of Oki Electric Industry. Table 3.10 summarizes estimation and confirmation. This simulation calculation strongly influenced U.S. engineers because it involved strategic two-stage optimi-

Table 3.5
Two levels of compounded noise factor

Factor	N_1	N_2	Factor	N_1	N_2
A'	−0.1	+0.1	H'	+0.1	−0.1
B'	+0.1	−0.1	I'	+0.1	−0.1
C'	+0.1	−0.1	J'	−0.15	+0.15
D'	−0.1	+0.1	K'	+0.15	−0.15
E'	−0.05	+0.5	L'	+0.15	−0.15
F'	−0.5	+0.5	M'	−0.15	+0.15
G′	+0.1	−0.1			

Table 3.6
Data of experiment 1

Rotational Angle:	1.3°	2.6°	3.9°	5.2°	6.5°	Loss Function
Standard Condition:	3.255	6.245	9.089	11.926	14.825	
N_1	2.914	5.664	8.386	11.180	14.117	463.694309
N_2	3.596	6.826	9.792	12.672	15.533	524.735835

zation and noise selection in simulation. However, quite often in the United States, instead of compounding noise factors, they assign them to an outer orthogonal array. That is, after assigning three levels of each noise factor to an L_{27} orthogonal array, they run a simulation for each combination (direct product design) formed by them and control factor levels.

Analysis for Tuning

At the optimal SN ratio condition in simulation, shown in the preceding section, data at N_0 and the average values at both N_1 and N_2 are calculated as follows:

Signal factor M (deg):	1.3	2.6	3.9	5.2	6.5
Target value m (mm):	2.685	5.385	8.097	10.821	13.555
Output value y (mm):	2.890	5.722	8.600	11.633	14.948

These data can be converted to Figure 3.2 by means of a graphical plot. Looking at Figure 3.2, we notice that each (standard) output value at the optimal condition digresses with a constant ratio from a corresponding target value, and this tendency makes the linear coefficient of output differ from 1. Now, since the coefficient β_1 is more than 1, instead of adjusting β_1 to 1, we calibrate M, that is, alter a signal M to M^*:

Table 3.7
Decomposition of total variation

Source	*f*	*S*	*V*
β	1	988.430144	988.430144
$N\beta$	1	3.769682	3.769682
e	8	0.241979	0.030247
N	9	4.011662	0.445740
Total	10	992.441806	

Table 3.8
Factor assignment and SN ratio based on average value

No.	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*K*	*L*	*M*	η (dB)
1	1	1	1	1	1	1	1	1	1	1	1	1	1	3.51
2	1	1	1	1	2	2	2	2	2	2	2	2	2	2.18
3	1	1	1	1	3	3	3	3	3	3	3	3	3	0.91
4	1	2	2	2	1	1	1	2	2	2	3	3	3	6.70
5	1	2	2	2	2	2	2	3	3	3	1	1	1	1.43
6	1	2	2	2	3	3	3	1	1	1	2	2	2	3.67
7	1	3	3	3	1	1	1	3	3	3	2	2	2	5.15
8	1	3	3	3	2	2	2	1	1	1	3	3	3	8.61
9	1	3	3	3	3	3	3	2	2	2	1	1	1	2.76
10	2	1	2	3	1	2	3	1	2	3	1	2	3	3.42
11	2	1	2	3	2	3	1	2	3	1	2	3	1	2.57
12	2	1	2	3	3	1	2	3	1	2	3	1	2	0.39
13	2	2	3	1	1	2	3	2	3	1	3	1	2	5.82
14	2	2	3	1	2	3	1	3	1	2	1	2	3	6.06
15	2	2	3	1	3	1	2	1	2	3	2	3	1	5.29
16	2	3	1	2	1	2	3	3	1	2	2	3	1	3.06
17	2	3	1	2	2	3	1	1	2	3	3	1	2	−2.48
18	2	3	1	2	3	1	2	2	3	1	1	2	3	3.86
19	3	1	3	2	1	3	2	1	3	2	1	3	2	5.62
20	3	1	3	2	2	1	3	2	1	3	2	1	3	3.26
21	3	1	3	2	3	2	1	3	2	1	3	2	1	2.40
22	3	2	1	3	1	3	2	2	1	3	3	2	1	−2.43
23	3	2	1	3	2	1	3	3	2	1	1	3	2	3.77
24	3	2	1	3	3	2	1	1	3	2	2	1	3	−1.76
25	3	3	2	1	1	3	2	3	2	1	2	1	3	3.17
26	3	3	2	1	2	1	3	1	3	2	3	2	1	3.47
27	3	3	2	1	3	2	1	2	1	3	1	3	2	2.51

Table 3.9
Level totals of SN ratio

	1	2	3		1	2	3
A	34.92	27.99	20.01	*H*	29.35	27.23	26.34
B	24.26	28.55	30.11	*I*	28.67	27.21	27.07
C	10.62	27.33	44.97	*J*	37.38	28.48	17.06
D	32.92	27.52	22.48	*K*	32.94	26.59	23.39
E	34.02	28.87	20.03	*L*	16.10	27.78	39.04
F	35.40	27.67	19.85	*M*	22.06	26.63	34.23
G	24.66	28.12	30.14				

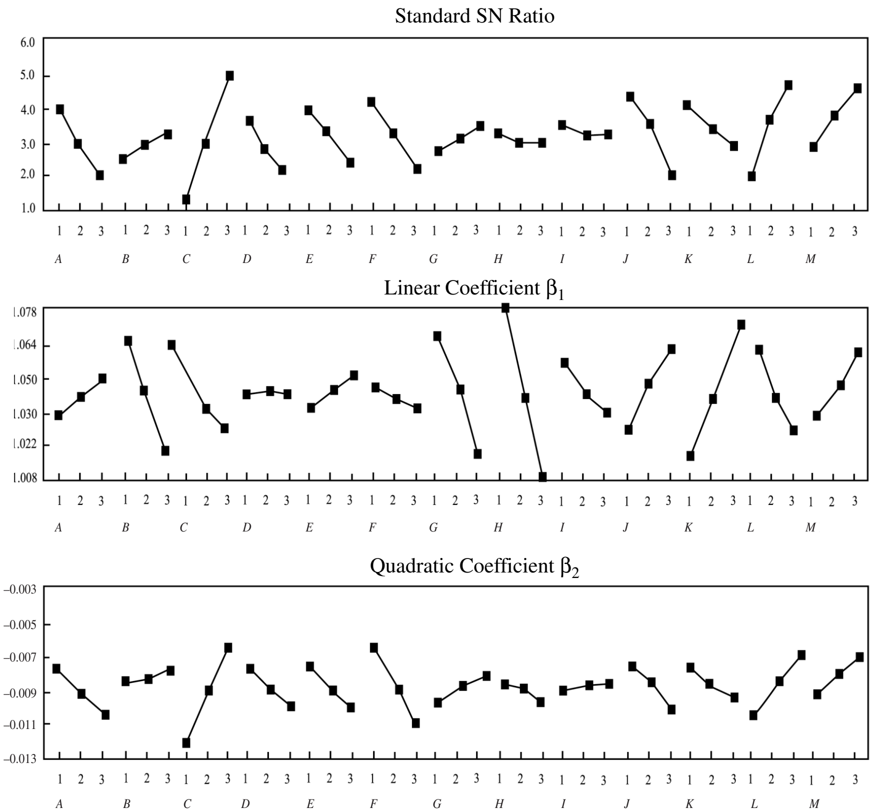

Figure 3.1
Factor effect plots of standard SN ratio, linear coefficient, and quadratic coefficient

Table 3.10
Estimation and confirmation of optimal SN ratio (dB)

	Estimation by All Factors	Confirmatory Calculation
Optimal condition	12.50	14.58
Initial condition	3.15	3.54
Gain	9.35	11.04

$$M^* = \frac{1}{\beta_1} M \quad (3.5)$$

However, in some cases we cannot correct M to M^* in accordance with (3.5). Although, normally in the case of $\beta_1 \neq 1$, we calibrate using a signal factor, we change β_1 to 1 using control factors on some occasions. Quality engineering has not prescribed which ways should be selected by designers because tuning is regarded as straightforward. We explain a procedure by analyzing data using control factors, whereas we should keep in mind that a calibration method using a signal is more common.

Looking at the data above, there is a gap between a target value m and an output value y under the standard conditions of N_0. To minimize this gap is called *adjustment*, where accuracy is our primary concern. To achieve this, using an output value y, we decompose the first and second terms in the orthogonal expansion equation (3.5).

Primarily, we calculate the total output variation for S_T with 5 degrees of freedom as follows:

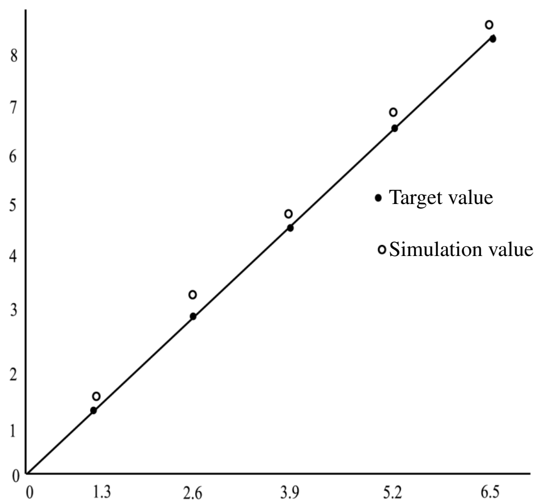

Figure 3.2
Standard output value at optimal S/N ratio condition

$$S_T = 2.890^2 + \cdots + 14.948^2 = 473.812784 \qquad (f = 5) \tag{3.6}$$

Secondarily, the variation of a linear term $S_{\beta 1}$ is computed as follows:

$$\begin{aligned} S_{\beta 1} &= \frac{(m_1 y_1 + m_2 y_2 + \cdots + m_5 y_5)^2}{m_1^2 + m_2^2 + \cdots + m_5^2} \\ &= \frac{(2.685)(2.890) + \cdots + (13.555)(14.948)^2}{2.685^2 + \cdots + 13.555^2} \\ &= 473.695042 \end{aligned} \tag{3.7}$$

On the other hand, a linear term's coefficient β_1 is estimated by the following equation:

$$\begin{aligned} \beta_1 &= \frac{m_1 y_1 + m_2 y_2 + \cdots + m_5 y_5}{m_1^2 + m_2^2 + \cdots + m_5^2} \\ &= \frac{436.707653}{402.600925} \\ &= 1.0847 \end{aligned} \tag{3.8}$$

In this case, a calibration method using a signal is to multiply a signal of angle M by 1/1.0847. In contrast, in a calibration procedure by control factors, we select one control factor that significantly affects β_1.

As a next step for analyzing the quadratic term, we derive the second-order equation after calculating constants K_1 and K_2:

$$\begin{aligned} K_2 &= \tfrac{1}{5}(m_1^2 + m_2^2 + \cdots + m_5^2) \\ &= \tfrac{1}{5}(2.685^2 + \cdots + 13.555^2) \\ &= 80.520185 \end{aligned} \tag{3.9}$$

$$\begin{aligned} K_3 &= \tfrac{1}{5}(m_1^3 + m_2^3 + \cdots + m_5^3) \\ &= \tfrac{1}{5}(2.685^3 + \cdots + 13.555^3) \\ &= 892.801297 \end{aligned} \tag{3.10}$$

Thus, the second-order term can be calculated according to the expansion equation (3.2) as follows:

$$\begin{aligned} \beta_2\left(m^2 - \frac{K_3}{K_2} m\right) &= \beta_2\left(m^2 - \frac{892.801297}{80.520185} m\right) \\ &= \beta_2(m^2 - 11.0897m) \end{aligned} \tag{3.11}$$

Next, to compute β_2 and variation in the second-order term, we need to calculate the second-order term's coefficients $w_1, w_2, \ldots, w_5$ for $m_1, m_2, \ldots, m_5$ using the following formula:

$$w_i = m_i^2 - 11.0897 m_i \qquad (i = 1,2, \ldots, 5) \tag{3.12}$$

Now we obtain

$$w_1 = 2.685^2 - (11.0897)(2.685) = -22.567 \quad (3.13)$$

$$w_2 = 5.385^2 - (11.0897)(5.385) = -\ 30.720 \quad (3.14)$$

$$w_3 = 8.097^2 - (11.0897)(8.097) = -24.232 \quad (3.15)$$

$$w_4 = 10.821^2 - (11.0897)(10.821) = -2.9076 \quad (3.16)$$

$$w_5 = 13.555^2 - (11.0897)(13.555) = 33.417 \quad (3.17)$$

Using these coefficients $w_1, w_2, \ldots, w_5$, we compute the linear equation of the second-order term L_2:

$$\begin{aligned} L_2 &= w_1y_1 + w_2y_2 + \cdots + w_5y_5 \\ &= -(22.567)(2.890) + \cdots + (33.477)(14.948) \\ &= 16.2995 \end{aligned} \quad (3.18)$$

Then, to calculate variation in the second-order term $S_{\beta 2}$, we compute r_2, which denotes a sum of squared values of coefficients $w_1, w_2, \ldots, w_5$ in the linear equation L_2:

$$\begin{aligned} r_2 &= w_1^2 + w_2^2 + \cdots + w_5^2 \\ &= (-22.567)^2 + \cdots + 33.477^2 \\ &= 3165.33 \end{aligned} \quad (3.19)$$

By dividing the square of L_2 by this result, we can arrive at the variation in the second-order term $S_{\beta 2}$. That is, we can compute the variation of a linear equation by dividing the square of L_2 by the number of units (sum of squared coefficients). Then we have

$$S_{\beta 2} = \frac{16.2955^2}{3165.33} = 0.083932 \quad (3.20)$$

Finally, we can estimate the second-order term's coefficient β_2 as

$$\beta_2 = \frac{L_2}{r_2} = \frac{16.2995}{3165.33} = 0.005149 \quad (3.21)$$

Therefore, after optimizing the SN ratio and computing output data under the standard conditions N_0, we can arrive at Table 3.11 for the analysis of variance to compare a target value and an output value. This step of decomposing total variation to compare target function and output is quite significant before adjustment (tuning) because we can predict adjustment accuracy prior to tuning.

Economic Evaluation of Tuning

Unlike evaluation of an SN ratio, economic evaluation of tuning, is absolute because we can do an economic assessment prior to tuning. In this case, considering that the functional limit is 1.45 mm, we can express loss function L as follows:

Table 3.11
ANOVA for tuning (to quadratic term)

Source	*f*	*S*	*V*
β_1	1	473.695042	
β_2	1	0.083932	
e	3	0.033900	0.011300
Total	5	473.812874	
$\beta_2 + e$	(4)	(0.117832)	(0.029458)

$$L = \frac{A_0}{\Delta_0^2}\sigma^2 = \frac{A_0}{1.45^2}\sigma^2 \tag{3.22}$$

A_0 represents loss when a product surpasses its function limit of 1.45 mm in the market and is regarded as cost needed to deal with a user's complaint (e.g., repair cost). We assume that A_0 = \$300. σ^2 includes tuning and SN ratio errors, each of which is mutually independent.

It is quite difficult to evaluate an SN ratio on an absolute scale because absolute values of errors due to environment, deterioration, or variability among products are unknown. In short, SN ratio gain is based on a relative comparison of each loss. In contrast, for adjustment errors we can do an absolute assessment by computing the error variance according to the following procedure. When adjusting only the linear term, the error variance σ_1^2 is:

$$\sigma_1^2 = \frac{S_{\beta 2} + S_e}{4} = \frac{0.083932 + 0.033930}{4} = 0.029458 \tag{3.23}$$

When adjusting up to the quadratic term, the error variance σ_2^2 is

$$\sigma_2^2 = \frac{S_e}{3} = \frac{0.033930}{3} = 0.001130 \tag{3.24}$$

Plugging these into (3.22), we can obtain the absolute loss. For the case of tuning only the linear term, the loss is

$$L_1 = \frac{300}{1.45^2}(0.029458) = \$4.20 \tag{3.25}$$

For the case of adjusting the linear and quadratic terms,

$$L_2 = \frac{300}{1.45^2}(0.001130) = 16 \text{ cents} \tag{3.26}$$

Thus, if we tune to the quadratic term, as compared to tuning only the linear term, we can save

$$420.3 - 16.1 = \$4.04 \tag{3.27}$$

Suppose a monthly production volume of only 20,000 units, approximately $80,000 can be saved.

Tuning Procedure

To tune in a practical situation, we generally use control factors that affect β_1 and β_2 significantly. Since only β_1 can be adjusted by a signal *M*, we sometimes use *only control factors that affect* β_2. Traditionally, we have used two tuning methods, one to change individual control factors and the other to use various factors at a time based on the least-squares method. In both cases, tuning has often failed because the selection of control factors for adjustment has depended on each designer's ability.

To study adjustment procedures, we calculate coefficients of the first- and second-order terms at each level of the L_{27} orthogonal array shown in Table 3.6. Table 3.12 summarizes the level average of coefficients, and Figure 3.1 plots the factorial effects. Even if we do not adjust the linear term's coefficient, to 1, quite often we can do tuning by changing a range of rotational angles. Since in this case each linear term's coefficient is 8% larger, we can adjust by decreasing the rotational angle by 8%. This implies that elimination of quadratic terms is a key point in adjustment. Since the quadratic term's coefficient $\beta_2 = 0.005$, by referring to the response graphs of the quadratic term, we wish to make $\beta_2 = 0$ without worsening the SN ratio. While *B*, *G*, *H*, and *I* have a small effect on SN ratios, none of

Table 3.12
Level average of linear and quadratic coefficients

	Linear Coefficient			Quadratic Coefficient		
Factor	**1**	**2**	**3**	**1**	**2**	**3**
A	1.0368	1.0427	1.501	−0.0079	−0.0094	−0.0107
B	1.0651	1.0472	1.0173	−0.0095	−0.0094	−0.0091
C	1.0653	1.0387	1.0255	−0.0124	−0.0091	−0.0064
D	1.0435	1.0434	1.0427	−0.0083	−0.0093	−0.0104
E	1.0376	1.0441	1.0478	−0.0081	−0.0094	−0.0105
F	1.0449	1.0434	1.0412	−0.0077	−0.0093	−0.0109
G	1.0654	1.0456	1.0185	−0.0100	−0.0092	−0.0088
H	1.0779	1.0427	1.0089	−0.0090	−0.0092	−0.0098
I	1.0534	1.0430	1.0331	−0.0094	−0.0093	−0.0093
J	1.0271	1.0431	1.0593	−0.0076	−0.0093	−0.0111
K	1.0157	1.0404	1.0734	−0.0083	−0.0094	−0.0103
L	1.0635	1.0405	1.0256	−0.0113	−0.0095	−0.0073
M	1.0295	1.0430	1.0570	−0.010	−0.0095	−0.0084

them has a great effect on the quadratic term. Therefore, even if we use them, we cannot lower the quadratic term's coefficient. As the next candidates, we can choose *K* or *D*. If the *K*′ level is changed from 1 to 2, the coefficient is expected to decrease to zero. Indeed, this change leads to increasing the linear term's coefficient from the original value; however, it is not vital because we can adjust using a signal. On the other hand, if we do not wish to eliminate the quadratic term's effect with a minimal number of factors, we can select *K* or *D*. In actuality, extrapolation can also be utilized. The procedure discussed here is a generic strategy applicable to all types of design by simulation, called the *Taguchi method* or *Taguchi quality engineering* in the United States and other countries.