Part III

Quality Loss Function

Taguchi's Quality Engineering Handbook. Genichi Taguchi, Subir Chowdhury and Yuin Wu

7 Introduction to the Quality Loss Function

7.1. Introduction

Definition of Quality

A product is sold by virtue of its product species and its price. *Product species* relate to product function and market size. *Product quality* relates to loss and market size. Quality is often referred to as *conformance to specifications*. However, Taguchi proposes a different view of quality, one that relates it to cost and loss in dollars, not just to the manufacturer at the time of production but to the consumer and to society as a whole.

Loss is usually thought of as additional manufacturing costs incurred up to the point that a product is shipped. After that, it is society, the customer, who bears the cost for loss of quality. Initially, the manufacturer pays in warranty costs. After the warranty period expires, the customer may pay for repair on a product. But indirectly, the manufacturer will ultimately "foot the bill" as a result of negative customer reaction and costs that are difficult to capture and account for, such as customer inconvenience and dissatisfaction, and time and money spent by customers. As a result, the company's reputation will be damaged, and eventually the market share will be lost.

Real growth comes from the market, cost, and customer satisfaction. The money the customer spends for a product and the perceived loss due to poor quality ultimately come back as long-term loss to the manufacturer. Taguchi defines *quality* as "the loss imparted by the product to the society from the time the product is shipped." The objective of the quality loss function is quantitative evaluation of loss caused by functional variation of a product.

❒ Example

Once a serious problem occurred concerning the specification of the thickness of vinyl sheets used to build vinyl houses for agricultural production. Figure 7.1 shows the relationship between thickness, cost, and quality versus loss. Curve L shows the sum of cost and quality. The midvalue of specification, or target value, is the lowest point on curve L. Assume that the specification given by the Vinyl Sheet Manufacturing Association is

$$1.0 \pm 0.2 \text{ mm} \tag{7.1}$$

A vinyl sheet manufacturer succeeded in narrowing the variation of thickness down to ±0.02 mm after a series of quality improvement efforts. To reduce material cost, the manufacturer changed its production specification to

$$0.82 \pm 0.02 \text{ mm} \tag{7.2}$$

Although the production specification above still meets the one given by the association, many complaints were lodged after the products within the specification of equation (7.2) were sold.

When the thickness of vinyl sheets is increased, the production cost (including material cost, processing cost, and material-handling cost) increases as shown by curve C. On the other hand, when the thickness is decreased, sheets become easy to break and farmers must replace or repair them; thus, the cost caused by inferior quality increases following curve Q.

Initially, the midvalue of specification (denoted by *m*) is the lowest point on curve L, which shows the sum of the two curves. When the average thickness is lowered from $m = 1.0$ mm to 0.82 mm, the manufacturing cost is cut by *B* dollars,

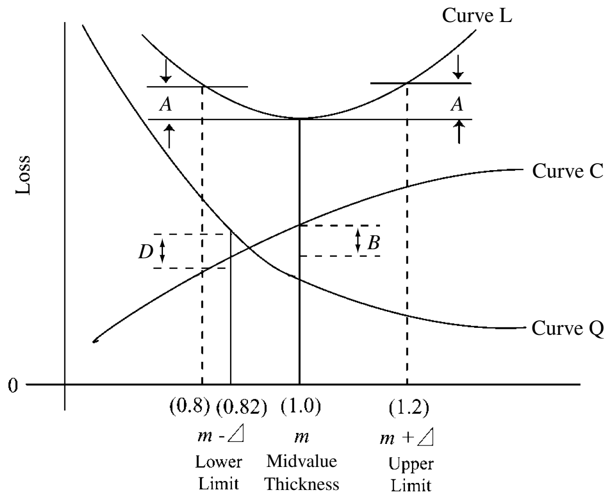

Figure 7.1
Objective value and its limits

but consumer loss is increased by *D* dollars, which is much larger than *B* dollars. Also, if the thickness varies by 0.2 mm on either side of the target value (*m*), the manufacturer incurs a total loss of *A* dollars.

Because of many complaints, the manufacturer had to adjust the midvalue of thickness back to the original value of 1.0 mm. The association still maintained the original specification after adding a clause that the average thickness must be 1.0 mm.

Quality Loss Function and Process Capability

Traditionally, the core of quality control is the fraction defective and its countermeasures. If defective products are shipped, it creates quality problems. If defectives are *not* shipped, it causes loss to the manufacturer. To avoid damage to the company's reputation, it is important to forecast the quality before products are shipped. Since the products within specifications are shipped, it is necessary to forecast the quality level of nondefective products. For this purpose, the *process capability index* has been used. This index is calculated from the tolerance divided by 6σ, which is quite abstract. (In general, root mean square is used instead of sigma. This is described in a later chapter.) Since there is no economic issue and no standard to follow a particular value, such as 0.8 or 1.3 or 2.0, it is difficult for management to understand the concept.

The loss function is calculated from the square of the reciprocal of the process capability index after multiplying a constant related to economy. It is an economic forecasting value that is imparted to the customer in the market.

The process capability index, C_p, is calculated by the following equation:

$$C_p = \frac{\text{tolerance}}{6(\text{standard deviation})} \tag{7.3}$$

The loss function is given as

$$L = k(y - m)^2 \tag{7.4}$$

where L is the loss in dollars when the quality characteristic is equal to y, y the value of the quality characteristic (i.e., length, width, concentration, surface finish, flatness, etc.), m the target value of y, and k a constant to be defined later. As shown in Figure 7.2, this quadratic representation of the loss function $L(y)$ is

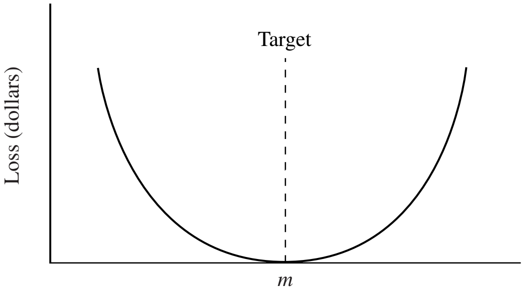

Figure 7.2
Quality loss function

minimum at $y = m$, increases as y deviates from m, and is expressed in monetary units.

However, one might still question whether equation (7.4) is a reasonable representation. The loss function L can be expanded in a Taylor series around the target value m:

$$\begin{aligned} L &= L(m + y - m) \\ &= L(m) + \frac{L'(m)}{1!}(y - m) + \frac{L''}{2!}(y - m)^2 + \cdots \end{aligned} \tag{7.5}$$

Because L is a minimum at $y = m$, $L'(m) = 0$. $L(m)$ is always a constant and is ignored since its effect is to raise or lower the value of $L(y)$ uniformly at all values of y. The $(y - m)^2$ term is the dominant term in equation (7.5) (higher-order terms are neglected). Therefore, we use the following equation as an approximation:

$$\begin{aligned} L &= \frac{L''}{2!}(y - m)^2 \\ &= k(y - m)^2 \end{aligned} \tag{7.6}$$

In reality, for each quality characteristic there exists some function that uniquely defines the relationship between economic loss and the deviation of the quality characteristic from its target value. The time and resources required to obtain such a relationship for each quality characteristic would represent a considerable investment. Taguchi has found the quadratic representation of the quality loss function to be an efficient and effective way to assess the loss due to deviation of a quality characteristic from its target value (i.e., due to poor quality). The concept involved in Taguchi methods is that useful results must be obtained quickly and at low cost. Use of a quadratic, parabolic approximation for the quality loss function is consistent with this philosophy.

For a product with a target value m, from most customers' point of view, $m \pm \Delta_0$ represents the deviation at which functional failure of the product or component occurs. When a product is manufactured with its quality characteristic at the extremes, $m + \Delta_0$ or at $m - \Delta_0$, some countermeasure must be undertaken by the average customer (i.e., the product must be discarded, replaced, repaired, etc.). The cost of the countermeasure is A_0 since the quality loss function is

$$L = k(y - m)^2 \tag{7.4}$$

at

$$y = m + \Delta_0 \tag{7.7}$$

$$A_0 = k(m + \Delta_0 - m)^2 \tag{7.8}$$

$$k = \frac{A_0}{\Delta_0^2} \tag{7.9}$$

This value of k, which is constant for a given quality characteristic, and the target value, m, completely define the quality loss function curve (Figure 7.3).

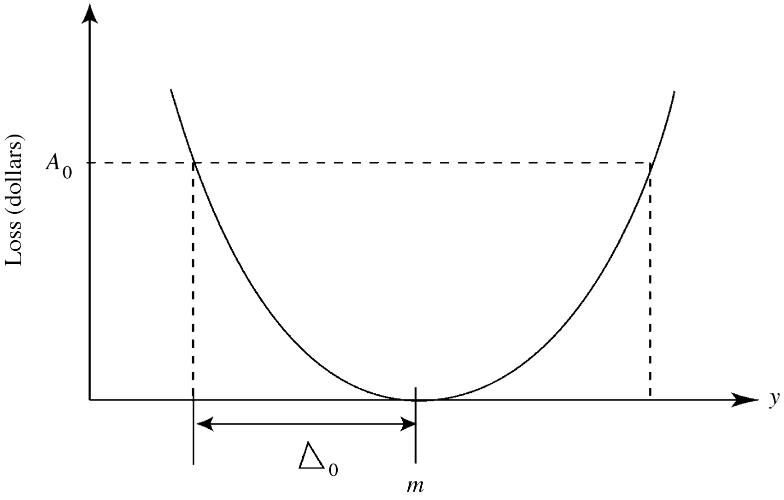

Figure 7.3
Quality loss function

Up to this point, the quality loss function is explained with one piece of product. In general, there are multiple pieces of products. In that case, the loss function is given by

$$L = k\delta^2 \tag{7.10}$$

Equation (7.3) can be written as

$$C_p = \frac{\text{tolerance}}{6(\text{standard deviation})} = \frac{2\Delta_0}{6\sigma} \tag{7.11}$$

Therefore,

$$L = \frac{A}{\Delta_0^2}\sigma^2 = \frac{A}{9}\frac{1}{C_p^2} \tag{7.12}$$

It is seen that the loss function can be expressed mathematically by the process capability index. But the former has dollars as a unit and the latter has no unit. For example, the process capability index of last month was 0.9, and it is 0.92 this month. This fact does not explain the difference from the economic or productivity viewpoint. Using the loss function, monetary expression can be made and productivity can be compared.

There are two aspects to the quality loss function. The first is the *justice-related* aspect. Take, for example, a product specified to have a 100-g content. If the amount is 97 g instead of 100, the customer loses 3 g. If the price is 10 cents a gram, the customer loses 30 cents. In this case, it is easy to understand.

If the content is 105 g, the customer may be happy to have a free 5 g of product. However, it might be a loss to the customer. If the manufacturer aimed for 105 g in a package, there would be a 5% production loss. If a package is supposed to contain 100 g, a planned production of 10,000 packages would result in producing only 9500 packages in the case of 105-g packages. The manufacturer normally would calculate the manufacturing cost based on 9500 units; then overhead and profit would be added. The product selling price is a few times the manufacturing cost. Selling a 100-g package based on a 105-g package cost calculation is simply cheating.

The second aspect is a *quality* issue. Assume that a package of medicine specified to contain 100 g actually contains 105 g. If the customer measured the quantity accurately and used it as prescribed, there would be a 5-g waste. But if the customer used the package of 105 g as 100 g, there might be some problems and the problems could be fatal. It is therefore important that the quality level be at the target from both the legal and quality aspects.

7.2. Steps in Product Design

The objective function of a TV set's power circuit is to convert an ac input of 100 V into a dc output of 115 V. If the power circuit maintains a 115-V output anytime, anywhere, the quality of its power circuit is perfect as far as voltage is concerned. The variables causing the voltage to move away from the target of 115 V, called *noise variables*, are classified as follows:

1. *Outer noise:* variation of environmental conditions (e.g., temperature, humidity, dust, input voltage)
2. *Inner noise:* deterioration of elements or materials comprising the circuit (e.g., the resistance of a resistor increases by 10% in 10 years, or the parameter of a transistor changes with time)
3. *Between-product noise:* manufacturing imperfection resulting in different outputs

To minimize the effects caused by these noise sources, some countermeasures may be considered. Of these countermeasures, the most important is the countermeasure by design. It consists of (1) system design, (2) parameter design, and (3) tolerance design.

System Design

Knowledge from a specialized field is applied to system design: from electronics, research into the type of circuit used to convert alternating current into direct current; and from the chemical industry, studies concerning the specific chemical reaction needed to obtain the best process given the available technology.

Automatic controlling systems are included in the scope of this step. For example, an output voltage is measured from time to time. When there is a deviation from the target voltage, a parameter in the circuit is changed automatically. However, it is difficult to control the deterioration or variation of an automatic controlling system. Not only that but there is an additional cost for the automatic control mechanism. Therefore, it is not the design of a stable circuit at a low cost. Our goal is to design a stable and low-cost circuit by introducing a drastic change to the system. However, we must rely on knowledge from specialized fields. This is the specialist's territory, and neither quality control nor the design of experiments can help.

Many systems are conceivable for a certain function. We select a system that matches our objective from countless possible systems. It is desirable to select a new system that does not exist anywhere else in the world. Patents protect new systems. System selection is often determined at a meeting of the persons in charge, where opinions are exchanged regarding the advantages and disadvantages of each item in the functions, price, and life that are set at the planning stage. Judgment of each item is only quantitative, but this is important in order to convince executives to go along with it. When the development section has the resources, two or three systems may be developed simultaneously.

If social loss is great in case a function fails, considerations of safety are also a part of system selection. Safety design for improvement of productivity is an important factor in system selection. One of the most important duties of a system designer is to include a safety mechanism that does not endanger human life or cause significant financial loss if the function fails, rather than simply continuing to increase reliability.

Parameter Design: Two-Step Optimization

Once the system design is finished, the next step is to determine the optimum level of individual parameters of the system in both quality and cost. *Parameter design* is a method used to reduce the influence of sources of variation. It has been called *utilization of nonlinearity* or *utilization of interaction between control and noise factors.* In the technical world, analytical methods have been used for parameter design.

Research on this has been neglected, especially in developing countries, where researchers look for information from literature, designing power circuits that "seem to be the best." A prototype is made, then charged with 100 V ac current. If the output voltage equals the target value, it is labeled a success; and if the result is unsatisfactory, the value of a parameter that seemed effective in the circuit is changed. This approach is dated, reminiscent of the dyer of old. Dyers had color samples and tried to create dyes to match. If a dyer got a color identical to a sample, he succeeded; if not, he changed either the mixing ratio or the dyeing conditions and tried again.

This is not a design, merely operational work, called a *modification* or *calibration.* "Designs" in developing countries are similar to the dyers' methods; their engineers proceed in a similar fashion.

❐ Example

An element transistor *A* and resistor *B* in a power circuit affect the output voltage as shown in Figure 7.4.

Suppose that a design engineer forecasts the midvalues of the elements in a circuit, assemblies these to get a circuit, and puts 100 V ac into it. If the output voltage so obtained is only 100 V instead of 115 V, he changes resistance *A* from $A_1 = 200\ \Omega$ to $A' = 250\ \Omega$ to adjust the 15-V deviation from the target value. From the standpoint of quality control, this is very poor methodology. Assume that the resistance used in the circuit is the cheapest grade. It either varies or deteriorates to a maximum range of ±10% during its service period as the power source of a TV set. From Figure 7.4, we see that the output voltage varies within a range of ±6 V. In other words, the output voltage varies by the influence of noise, such as the original variation in the resistance itself, or due to deterioration. If resistance

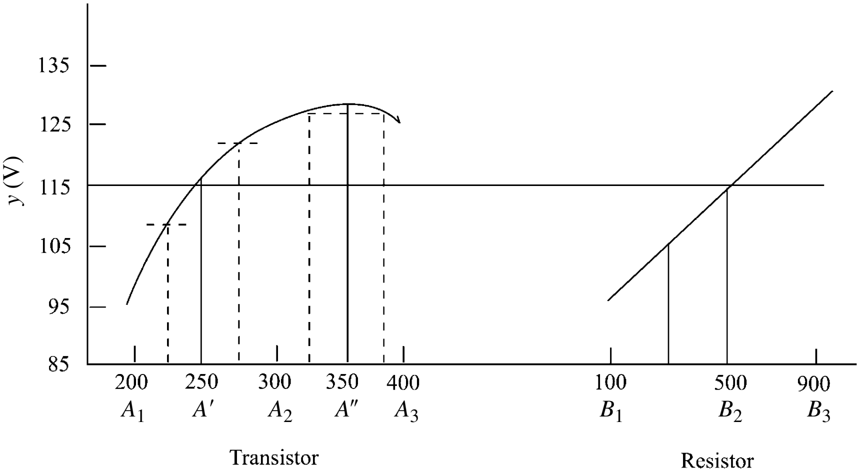

Figure 7.4 Relationship between factors A and B and the output voltage

A'', which has a midvalue of 350 Ω, is used instead of A', its influence on the output voltage is only ±1 V, even if its variation is ±10% (35 Ω). However, the output voltage goes up about 10 V, as shown in the figure. Such a deviation can be adjusted by a factor like B with influence on the output voltage almost linear. In this case, 200 Ω is selected instead of 500 Ω for level B. A factor like B, with a differential coefficient to the output voltage that is nearly constant no matter what level is selected, is not useful for a high-level quality control purpose; it is used merely to adjust a deviation from the target value.

Parameter design is the most important step in developing stable and reliable products or manufacturing processes. With this technique, nonlinearity may be utilized positively. (Factor A has this property.) In this step we find a combination of parameter levels that are capable of damping the influences not only of inner noise but of all noise sources, while keeping the output voltage constant. At the heart of research lives a conflict: to design a product that is reliable within a wide range of performance conditions but at the lowest price. Naturally, elements or component parts with a short life and wide tolerance variation are used. The aim of design of experiments is the utilization of nonlinearity. Most important in applying design of experiments is to cite factors or to select objective characteristics with this intention.

There is a difference between the traditional quality improvement approach and parameter design. In the traditional approach, the first step is to try to hit the target, then reduce variability. The countermeasures of reducing variability depend on the tolerance design approach: improving quality at high cost increase. In parameter design, quality is improved by selecting control factor combinations. After the robustness of a product is improved, the target is adjusted by selecting a control factor whose level change affects the average but affects variability minimally. For this reason, parameter design is also called *two-step optimization.*

Tolerance Design

Once parameter design is completed, we have the midvalues of factors comprising the system elements. The next step is to determine the tolerances of these factors, or to rationally select the grades of parts and materials. Once the system design is complete, we have the midvalues of the factors comprising the system elements. The next step is to determine the tolerances of these factors. To make this determination, we must consider environmental conditions as well as the system elements. The midvalues and varying ranges of these factors and conditions are considered as noise factors and are arranged in orthogonal arrays so that the magnitude of their influence on the final output characteristics may be determined. A narrower allowance will be given to those noise factors, imparting a large influence on the output. Cost considerations determine the allowance. Different from parameter design, this step results in “cost-up” by controlling noise in narrower ranges; therefore, quality-controlling countermeasures should be achieved through parameter design analysis. If this is not an option, tolerance design is the only countermeasure left that will allow for influencing factors with a narrow range.