35 Introduction to Orthogonal Arrays

35.1. Introduction 584
35.2. Quality Characteristics and Additivity 584
Data Showing Physical Condition 584
Noise Level and Additivity 586
Data Showing Particle Size Distribution 587
35.3. Orthogonal Array L_{18} 589
35.4. Role of Orthogonal Arrays 594
35.5. Types of Orthogonal Arrays 596
References 596

35.1. Introduction

Earlier, reproducibility of conclusions was discussed, and it was recommended that one use orthogonal arrays to check the existence of interactions, the cause of nonreproducibility. Followings are examples of poor-quality characteristics that cause interactions.

35.2. Quality Characteristics and Additivity [1]

Data Showing Physical Condition

At the stage of selecting a characteristic, it must be considered whether that characteristic is the most suitable one to use to achieve the purpose of the experiment.

A characteristic may not always exactly express the purpose itself. As an example, let's discuss an experiment for curing the physical condition of a patient. Since the data showing physical condition is directly related to the purpose of the experiment, a characteristic "condition" is selected. Another example of a characteristic condition would be data recorded under categories such as "delicious" or "unsavory."

Assume that there are three types of medicines: *A*, *B*, and *C*. First, medicine *A* was taken by the patient, then his or her physical condition became better. Such data can be expressed as:

 Taguchi's Quality Engineering Handbook. Genichi Taguchi, Subir Chowdhury and Yuin Wu

A_1:	Did not take A	Bad condition
A_2:	Took A	Slightly better condition

Suppose that the results after taking medicines B and C were as follows:

B_1:	Did not take B	Bad condition
B_2:	Took B	Better condition
C_1:	Did not take C	Bad condition
C_2:	Took C	Slightly better condition

If, for the reason that the three medicines (*A*, *B*, and *C)* did the patient good, and if the patient became much better after taking the three medicines together, we say that there exists additivity in regard to the effects of the three medicines, *A*, *B*, and *C*. However, if the patient became worse or died after taking the three medicines together, we say that there is interaction (a large effect that reverses the results) between *A*, *B*, and *C*.

When there is interaction, it is generally useless to carry out the investigations in such a way that one factor (in this case, medicine) is introduced into the experiment followed by another factor.

For example, suppose that a patient is suffering from diabetes, which is caused by lack of insulin being produced by the pancreas. Also suppose that medicine *A* contains 80%, *B* contains 100%, and *C* contains 150% of the amount of insulin the patient requires. In this case, interaction does occur. Taking *A*, *B*, and C together would result in a 330% insulin content, which would definitely be fatal.

As far as physical condition is concerned, there is no additivity. But if the amount of insulin is measured, it becomes possible to estimate the result of taking *A*, *B*, and *C* together, but the combination can never be tested. If effect *A* is reversed by the other conditions, solely researching effect *A* would turn out to be meaningless. Accordingly, we have to conduct the experiment in regard to all the combinations of *A*, *B*, and *C*.

When there are three two-level factors, there are only eight combinations in total; however, when there are 10 factors to be investigated, 1024 combinations should be included in the experiment, which would be impossible.

If the amount of insulin is measured, there then exists additivity. That is, we can predict that physical condition becomes worse when *A*, *B*, and *C* are taken together. In other words, we do not need the data from all combinations. In this way, even when there are 10 two-level factors, we can estimate the results for all combinations by researching each individual factor, or by using orthogonal arrays (described in other chapters) and then finishing the research work by conducting only about 10 experiments.

"Physical condition" is not really an efficient and scientific characteristic, but "quantity of insulin" is. Research efficiency tends to drop if we cannot find a characteristic by which the results of the individual effect reappears or has consistent effects, no matter how the other conditions, variables, or causes vary. It is therefore most important from the viewpoint of research efficiency to find a characteristic with the additivity of effects in its widest sense.

Noise Level and Additivity

To measure noise level, it is usual to measure the magnitude of noise for an audible frequency range using "phon" or "decibel" as a unit. Generally, there are three types of factors affecting noise. Of these types, additivity exists for the factors of the following two types:

1. Factors that extinguish vibrating energy, the origin of noise.
2. Factors that do not affect vibrating energy itself, but convert the energy caused by vibration into a certain type of energy, such as heat energy. That is, the less vibrating source, the lower the noise level. Also, the more we convert the energy that causes noise into other types of energy, the lower the noise level.

The third type of factors imply:

3. Factors that change the frequency characteristics into frequency levels outside the audible range: for example, changing the shape or dimensions of component parts, changing the gap between component parts, changing surface conditions, or switching the relative positions of the various component parts.

Suppose that the frequency characteristic of a certain noise is as shown in Figure 35.1. There is a large resonance frequency at 500 cycles. It forms half of the total noise. There are two countermeasures, A and B, as follows:

1. *Countermeasure A:* converts resonance frequency from 500 cycles to 50,000 cycles. As a result, the noise level decreases by 50%.
2. *Countermeasure B:* converts resonance frequency from 500 cycles to 5 cycles. As a result, again the noise level decreases by 50%.

If the two countermeasures were instituted at the same time, the noise level might come back to the original magnitude.

Generally, most factors cited in noise research belong to the third type. If so, it is useless to use noise level as a characteristic. Unless frequency characteristics are researched with the conditions outside the audible range, it would turn out to be inefficient; therefore, researching all combinations is required to find the optimum condition.

In many cases, taking data such as the frequency of misoperation of a machine, the rate of error in a communications system or in a calculator, or the magnitude

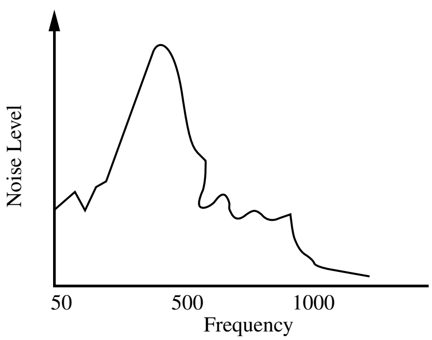

Figure 35.1
Frequency and noise level

of electromagnetic waves when there are frequency changes does not make sense. These errors are not only the function of the signal-to-noise (SN) ratio, but may also be caused by the unbalance of a setting on the machine. In the majority of these cases, it is advisable to calculate the SN ratio, or it is necessary to find another method to analyze these data, which are called *classified attributes*.

Data Showing Particle Size Distribution

Next we discuss an experiment concerned with a crushing process.

❐ Example

The product becomes off grade when it is either too coarse or too fine; the fineness within the range of 15 through 50 mesh is desirable, it being most desirable to increase the yield. But additivity does not exist for yield or percentage in this case. One would not use such a characteristic for the purpose of finding the optimum condition efficiently.

Suppose that the present crushing condition is $A_1B_1C_1$. The resulting yields after changing A_1 to A_2, B_1 to B_2, and C_1 to C_2 are as follows:

$$A_1:\ 40\% \qquad B_1:\ 40\% \qquad C_1:\ 40\%$$

$$A_2:\ 80\% \qquad B_2:\ 90\% \qquad C_2:\ 40\%$$

The data signify that yield increases by 40% when crushing condition A changes from A_1 to A_2; the yield also increases by 50% when B is changed from B_1 to B_2, but yield does not change when C is changed from C_1 to C_2. Judging from these data, the optimum condition is either $A_2B_2C_1$ or $A_2B_2C_2$.

The yield of any one of the optimum conditions is expected to the nearly 100%. But if the crushing process is actually operated under condition $A_2B_2C_2$, and if a yield of less than 10% is obtained, the researcher would be puzzled. However, it

Table 35.1
Particle size distribution

	≤15	15–50	≥50
A_1	60	40	0
A_2	0	80	20
B_1	60	40	0
B_2	0	90	10
C_1	60	40	0
C_2	0	40	60

is very possible to obtain such a result. This is easy to understand from the particle distribution shown in Table 35.1.

At present operating conditions, a 40% low yield is obtained because the particles are too coarse. As to conditions C_1 and C_2, the percentage of particles in the range 15 to 50 mesh is equal, but factor C exerts the largest influence on particle distribution. Only by changing condition C from C_1 to C_2 do the particles become much finer. Furthermore, adding the effects of A and B, the yield of condition $A_2B_2C_2$ tends to be very small. Yield is therefore not a good characteristic for finding an optimum condition.

Even if the yield characteristic were used, the optimum condition could be obtained if all of the combinations were included in the experiment. In this case, the optimum condition is obtainable because there are only eight experiments ($2^3 = 8$). When there are only three two-level factors, the performance of eight experiments is not impractical. But when these factors are three and four levels, the total combinations increase to 27 and 64, respectively.

If there are 10 three-level factors, experiments with 59,049 combinations are required. Assuming that the experiments performed are 10 combinations a day, about 20 years would be required to test for all the combinations. Clearly, in order to find an optimum condition by researching one factor after another, or researching a few factors at one time, a characteristic must have additivity.

The example above is a case where yield becomes 100% when particle size is in a certain range. In such a case, it is nonsense to take yield as data. In addition, we need percentages of the coarser portions and the finer portions. If the three portions are shown, it is clear that condition $A_2B_2C_2$ is the worse condition without conducting any experiment.

There is no additivity with the yield data within the range 15 to 50 mesh, but there is additivity if the total (100%) is divided into three classes, called *classified variables.*

When a physical condition is expressed as good, average, and bad, this type of data would not be additive. The same holds true when yield is expressed as a percentage.

The yield in the case of a reversible chemical reaction, or the yield of a reaction that may be overreacted, would not be considered additive data. What type of scientific field would exist where "condition" or "balance" is seriously discussed, such as the color matching of dyes or paints, the condition of a machine or a furnace, taste, flavor, or physical condition? In such fields, no one could be as competent as an expert worker who has the experience of nearly all the combinations. So far a characteristic with additivity has not been discovered; we can never be out of the working level.

It is difficult in most cases to judge in a specialized technical field whether the characteristic is additive. Accordingly, we need a way to judge whether or not we are lost in a maze. Evaluation of experimental reliability is the evaluation of a characteristic in regard to its additivity. The importance of experiments using orthogonal arrays lies in the feasibility of such evaluations.

35.3. Orthogonal Array L_{18} [2]

In quality engineering, orthogonal arrays L_{12}, L_{18}, and L_{36} are generally used. L_{12} is used for two-level factors, L_{18} for three-level factors, and L_{36} for simulation. In this discussion, L_{18} is used for simplifying the explanation.

❐ Example [1]

An early application of robust design involved the optimization of a tile manufacturing process in Japan in the 1950s. In 1953, a tile manufacturing experiment was conducted by the Ina Seito Company. The flow of the manufacturing process is as follows:

raw material preparation: crushing and mixing → molding → calcining → glazing → calcining

The molded tiles are stacked in the carts and move slowly in a tunnel kiln as burners fire the tiles. The newly constructed kiln did not produce tiles with uniform dimensions. More than 40% of the outside tiles were out of specification. The inside tiles barely met the specification. It was obvious to the engineers at the plant that one of the causes of dimensional variation was the uneven temperature distribution.

The traditional way of improving quality is to remove the cause of variation. In this case it would mean redesigning the tunnel kiln to make temperature distribution more uniform, which was impossible because it was too costly. Instead of removing the cause of variation, the engineers decided to perform an experiment to find the formula for the tile materials that would produce consistently uniform tiles, regardless of their positions within the kiln.

In the study, control factors were assigned to orthogonal arrays, and seven positions in the kiln were assigned to the outer array as noise factors. The interactions between each control factor and the noise factors were studied to find the optimum condition. Today, the SN ratio is used to substitute for the tedious calculation to study the interactions between control and noise factors.

Following are the control factors and their levels:

A: amount of a certain material
- A_1 = 5.0%
- A_2 = 1.0% (current)

B: firmness of material
- B_1 = fine
- B_2 = (current)
- B_3 = coarse

C: amount of agalmatolite
- C_1 = less
- C_2 = (current)
- C_3 = new mix without additive

D: type of agalmatolite
$D_1 = -0.0\%$
$D_2 = 1.0\%$ (current)
$D_3 =$

E: amount charged
E_1 = smaller
E_2 = current
E_3 = larger

F: amount of returned material
F_1 = less
F_2 = medium (current)
F_3 = more

G: amount of feldspar
$G_1 = 7\%$
$G_2 = 4\%$ (current)
$G_3 = 0\%$

H: clay type
H_1 = only K-type
H_2 = half-and-half (current)
H_3 = only G-type

These were assigned to an orthogonal array L_{18} as the inner array, and seven positions in the kiln were assigned to the outer array. Dimensions of tiles for each combination between a control factor and the noise factor are shown in Table 35.2.

Data Analysis

As a quality characteristic, a nominal-the-best SN ratio is used for analysis. (For the SN ratio, see later chapters.) For experiment 1:

$$S_m = \frac{(10.18 + 10.18 + \cdots + 10.20)^2}{7} = 714.8782 \tag{35.1}$$

$$S_T = 10.18^2 + 10.18^2 + \cdots + 10.20^2 = 714.9236 \tag{35.2}$$

$$S_e = S_T - S_m = 714.9236 - 714.8782 = 0.0454 \tag{35.3}$$

$$V_e = \frac{S_e}{7 - 1} = \frac{0.0454}{6} = 0.00757 \tag{35.4}$$

$$\begin{aligned}\eta &= 10 \log \frac{(1/n)(S_m - V_e)}{V_e} \\ &= 10 \log \frac{7(714.8782 - 0.00757)}{0.00757} \\ &= 41.31 \text{ dB}\end{aligned} \tag{35.5}$$

The SN ratios of the other 17 runs are calculated similarly, as shown in Table 35.2.

Table 35.2
Layout and results

L_{18}	*A* 1	*B* 2	*C* 3	*D* 4	*E* 5	*F* 6	*G* 7	*H* 8	P_1	P_2	P_3	P_4	P_5	P_6	P_7	Mean	SN
1	1	1	1	1	1	1	1	1	10.18	10.18	10.12	10.06	10.02	9.98	10.20	10.11	41.31
2	1	1	2	2	2	2	2	2	10.03	10.01	9.98	9.96	9.91	9.89	10.12	9.99	42.19
3	1	1	3	3	3	3	3	3	9.81	9.78	9.74	9.74	9.71	9.68	9.87	9.76	43.65
4	1	2	1	1	2	2	3	3	10.09	10.08	10.07	9.99	9.92	9.88	10.14	10.02	40.36
5	1	2	2	2	3	3	1	1	10.06	10.05	10.05	9.89	9.85	9.78	10.12	9.97	37.74
6	1	2	3	3	1	1	2	2	10.20	10.19	10.18	10.17	10.14	10.13	10.22	10.18	50.03
7	1	3	1	2	1	3	2	3	9.91	9.88	9.88	9.84	9.82	9.80	9.93	9.87	46.34
8	1	3	2	3	2	1	3	1	10.32	10.28	10.25	10.20	10.18	10.18	10.36	10.25	43.21
9	1	3	3	1	3	2	1	2	10.04	10.02	10.01	9.98	9.95	9.89	10.11	10.00	43.13
10	2	1	1	3	3	2	2	1	10.00	9.98	9.93	9.80	9.77	9.70	10.15	9.90	35.99
11	2	1	2	1	1	3	3	2	9.97	9.97	9.91	9.88	9.87	9.85	10.05	9.93	42.88
12	2	1	3	2	2	1	1	3	10.06	9.94	9.90	9.88	9.80	9.72	10.12	9.92	37.05
13	2	2	1	2	3	1	3	2	10.15	10.08	10.04	9.98	9.91	9.90	10.22	10.04	38.46
14	2	2	2	3	1	2	1	3	9.91	9.87	9.86	9.87	9.85	9.80	10.02	9.88	43.15
15	2	2	3	1	2	3	2	1	10.02	10.00	9.95	9.92	9.78	9.71	10.06	9.92	37.70
16	2	3	1	3	2	3	1	2	10.08	10.00	9.99	9.95	9.92	9.85	10.14	9.99	40.23
17	2	3	2	1	3	1	2	3	10.07	10.02	9.89	9.89	9.85	9.76	10.19	9.95	36.60
18	2	3	3	2	1	2	3	1	10.10	10.08	10.05	9.99	9.97	9.95	10.12	10.04	43.48

Tables 35.3 and 35.4 show the response tables for the SN ratio and average. Figure 35.2 shows the response graphs of SN the ratio and average. For example:

$$\text{average SN ratio for } A_1 = \frac{41.31 + 42.19 + \cdots + 43.13}{9} = 43.10 \tag{35.6}$$

$$\text{average SN ratio for } A_2 = \frac{35.99 + 42.88 + \cdots + 43.48}{9} = 39.50 \tag{35.7}$$

$$\text{average SN ratio for } B_1 = \frac{41.31 + 42.19 + \cdots + 37.05}{6} = 40.51 \tag{35.8}$$

Table 35.3
Response table: SN ratio

Level	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*
1	43.10	0.51	40.45	40.33	44.53	41.11	40.44	39.90
2	39.50	41.24	40.96	40.88	40.12	41.38	41.47	42.82
3		42.16	42.51	42.71	39.26	41.42	42.00	41.19
Δ	3.60	1.65	2.06	2.38	5.27	0.31	1.57	2.92
Ranking	2	6	5	4	1	8	7	3

Optimization

The first thing in parameter design is to reduce variability or maximize the SN ratio. Select the optimum combination from the SN ratio response table as $A_1B_3C_3D_3E_1F_3G_3H_2$. Factors *A*, *C*, *D*, *E*, and *H* had a relatively strong impact on variability, whereas *B*, *F*, and *G* had a relatively weak impact. Therefore, A_1, C_3, E_1, and H_2 are definitely selected.

The second step is to adjust the mean. Adjusting the mean is easy by adjusting the dimension of the mold. In general, look for a control factor with a large impact on the mean and a minimum impact on variability.

Estimation and Confirmation

It is important to check the validity of experiments or the reproducibility of results. To do so, the SN ratios of the optimum conditions and initial condition are estimated using the additivity of factorial effects. Their difference, called *gain,* is then compared with the one calculated from the confirmatory experiments under optimum and initial conditions.

Additivity of factorial effects is simply the sum of the gains from the control factors. Every factorial effect may be used (added) to estimate the SN ratios of the optimum and initial conditions. But it is recommended that one exclude weak factorial effects from the prediction to avoid overestimates. In this example, factors *B*,

Table 35.4
Response table: Mean

Level	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*
1	10.02	9.93	9.99	9.99	10.00	10.07	9.98	10.03
2	9.95	10.00	10.00	9.97	10.02	9.97	9.97	10.02
3		10.02	9.97	9.99	9.94	9.91	10.01	9.90
Δ	0.06	0.08	0.03	0.02	0.08	0.17	0.04	0.13
Ranking	5	3	7	8	3	1	6	2

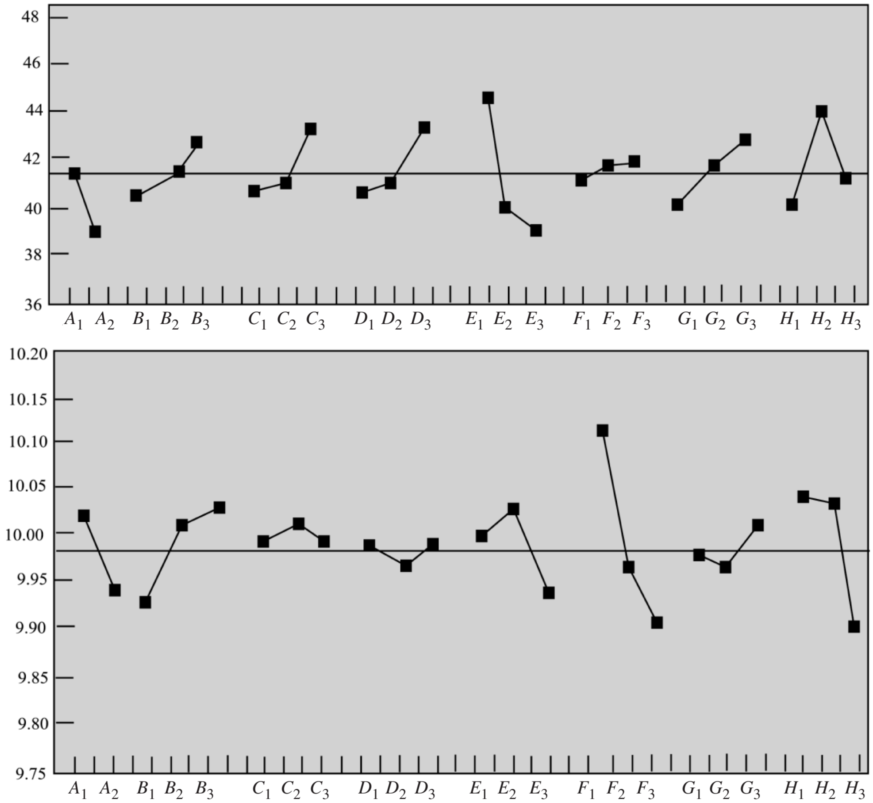

Figure 35.2
Response graphs

F, and G are excluded from the predictions. The SN ratios under the optimum and initial conditions, denoted by η_{opt} and η_{initial}, respectively, are predicted by

$$\begin{aligned}
\eta_{\text{opt}} &= \overline{T} + (\overline{A}_1 - \overline{T}) + (\overline{C}_3 - \overline{T}) + (\overline{D}_3 - \overline{T}) + (\overline{E}_1 - \overline{T}) + (\overline{H}_2 - \overline{T}) \\
&= \overline{A}_1 + \overline{C}_3 + \overline{D}_3 + \overline{E}_1 + \overline{H}_2 - 4\overline{T} \\
&= 43.10 + 42.51 + 42.71 + 44.53 + 42.82 - (4)(41.30) \\
&= 50.47 \text{ dB} \qquad (35.9)
\end{aligned}$$

$$\begin{aligned}
\eta_{\text{initial}} &= \overline{T} + (\overline{A}_2 - \overline{T}) + (\overline{C}_2 - \overline{T}) + (\overline{D}_2 - \overline{T}) + (\overline{E}_2 - \overline{T}) + (\overline{H}_2 - \overline{T}) \\
&= \overline{A}_2 + \overline{C}_2 + \overline{D}_2 + \overline{E}_2 + \overline{H}_2 - 4\overline{T} \\
&= 39.50 + 40.96 + 40.88 + 40.12 + 42.82 - (4)(41.30) \\
&= 39.08 \text{ dB} \qquad (35.10)
\end{aligned}$$

$$\text{gain predicted} = 50.47 - 39.08 = 11.39 \text{ dB} \qquad (35.11)$$

Confirmatory experiments are conducted under the optimum and initial conditions. From these results, two SN ratios are calculated. Their difference is the gain confirmed. When the gain from prediction is close enough to the gain from the confirmatory experiments, it indicates there is additivity and that the conclusions are probably reproducible.

35.4. Role of Orthogonal Arrays

To estimate the eight factorial effects A, B, ... , H, it is not necessary to carry out the experiment by orthogonal arrays as shown in Section 35.3. Traditionally, one-factor-at-a-time experiments have been conducted. In this method, the condition of one factor is varied each time by fixing all conditions of other factors. Such an experiment is much easier to do than the experiments arranged by orthogonal arrays.

In the one-factor-by-one method, levels A_1 and A_2 are compared while other factors, B, C, ... , H, are fixed (to the first level, $B_1C_1D_1E_1F_1G_1H_1$). Generally speaking, the difference between A_1 and A_2 can be obtained very precisely by using this method. In orthogonal arrays, on the contrary, the average effect of A_1 and A_2 (or main effect A) is obtained by varying the levels of other factors. That means to calculate the average of A_1 from the data of the four experiments (No. 1, 2, to 9) and the average of A_2 from No. 10, 11, to 18. But using such a layout is very time consuming, since the conditions of many factors must be varied for each experiment; also, the data variation between experiments generally becomes larger compared with experiments by the one-factor-by-one method.

One way to look at it is that the comparison between single experimental figures in the one-factor-by-one method is less precise than the comparison between the average of four figures from the experiments that used orthogonal arrays. However, this advantage in orthogonal array design might be offset by the disadvantage caused by the tendency for increased variation when the levels of many factors vary from experiment to experiment. As a result, there is no guarantee of obtaining better precision. What, then, is the merit of recommending such time-consuming experiments as orthogonal arrays?

They are stressed because of the high reproducibility of factorial effects. In experiments with orthogonal arrays, the difference of the two levels, A_1 and A_2, is determined as the average effect, while the conditions of other factors vary.

If the influence of A_1 and A_2 on the experimental results is consistent while the conditions of other factors vary, the effect obtained from the experiments on orthogonal arrays tends to be significant. On the other hand, if the difference between A_1 and A_2 either reverses or varies greatly once the levels of other factors change, effect A tends to be insignificant.

If orthogonal arrays are used, a factor having a consistent effect, with other factors varying under different conditions, can be estimated. This means that a large factorial effect (or the order of the preferable levels) obtained from orthogonal arrays does not vary if there are some variations in the levels of other factors. The reliability of such factorial effects is therefore very good.

In experiments using the one-factor-by-one method, on the other hand, the difference between A_1 and A_2 is estimated under a certain constant condition of the other factors. No matter how precisely such an effect is estimated and how neatly the curve is plotted, the effect is correct only for the case where the levels of other factors are exactly identical to the condition that was fixed at the time of the experiment; there is no guarantee at all of obtaining a consistent factorial effect if other factorial conditions change. Accordingly, it is doubtful whether the results obtained from the experimental data of the one-factor-by-one method will be consistent if the researcher changes or if the raw material changes.

It is said that since orthogonal arrays have been used in experiments, the results of small-scale laboratory experiments have become adopted satisfactorily to actual manufacturing. That is, a factor with a consistent effect under the various conditions of other factors has a good possibility of reproducing its effect at a manufacturing scale.

The reason for using orthogonal arrays is not to reduce cost by improving the efficiency of experimentation. When orthogonal arrays such as L_{12}, L_{18} and L_{36} are used, the interactions between control factors are almost evenly distributed to other columns of the orthogonal arrays and confounded to various main effects. If the interactions between control factors are significant, the gain predicted under the optimum condition from the initial condition will become significantly different from the gain from confirmatory experiments.

Using the one-factor-at-a-time method, where all other factor levels are fixed, it is not certain that an effect will be consistent. In other words, the main effect may be different if the conditions of other factors change. So this method is good only when there are no interactions between control factors.

The method of assigning main factors only to an orthogonal array will be successful when there are no interactions. The chance of success is the same as the case of using the one-factor-at-a-time method. Then why is are orthogonal arrays used? It is to check the reproducibility of conclusions by conducting confirmatory experiments. Thus, the success or failure will be clear. If the gain predicted did not agree with the confirmed gain, it tells us that the experiment has failed and the optimum condition was not found.

It is the same as inspection. When a product passes an inspection, the inspection was wasted. Inspection has a value only when a defective is found. The defective product is either scrapped or repaired to prevent problems in the marketplace.

Similarly, an experiment using an orthogonal array is used to inspect a bad experiment. It is to prevent a wrongly designed product from being shipped to the market and causing problems. Therefore, when the gain predicted is reproduced in confirmatory experiments, the use of orthogonal arrays is wasted. The use of using orthogonal arrays is advantageous only when a gain was not reproduced.

To be successful in a study, it is necessary to find a quality characteristic with small interactions between control factors. It is believed in quality engineering that the dynamic SN ratio is based on the functionality of a product or a system. It is also believed that reproducibility using a dynamic SN ratio based on the generic function is superior to use of a nondynamic SN ratio.

35.5. Types of Orthogonal Arrays

There are many orthogonal arrays. $L_4(2^3)$, $L_8(2^7)$, $L_{12}(2^{11})$, $L_{16}(2^{15})$, or $L_{32}(2^{31})$ belong to two-level series; $L_9(3^4)$, $L_{27}(3^{13})$, or $L_{81}(3^{40})$ are three-level series; and $L_{18}(2 \times 3^7)$ or $L_{36}(2^3 \times 3^{13})$ are mixed-level arrays. In quality engineering, it is recommended that one use arrays such as L_{12}, $L_{18,}$ and L_{36} because the interactions are almost evenly distributed to other columns, and there is no worry that an interaction confounds to a specific column or columns, thus leading to confusion.

References

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