Appendix D

CURRENT DENSITY DISTRIBUTION IN A CONDUCTOR

Let us consider a sinusoidal current flowing in a homogeneous conducting half space with conductivity σ and permeability μ (see Fig. D.1), and let us assume that the current density is parallel to the surface, oriented in the x-direction, and dependent only on the coordinate orthogonal to the surface (z), that is,

$$\vec{J}(x,y,z) = J_x(z)\,\vec{x} \tag{D.1}$$

In order to obtain the distribution of current in the conductor, we use the Maxwell's curl equations, taking into account that the displacement current can be neglected, that is,

$$\nabla \times \vec{E} = -j\omega\mu\,\vec{H} \tag{D.2a}$$

$$\nabla \times \vec{H} = \vec{J} \tag{D.2b}$$

Since $\vec{J} = \sigma\vec{E}$, expression (D.2a) can be written as follows:

$$\nabla \times \vec{J} = -j\omega\mu\sigma\,\vec{H} \tag{D.3}$$

From the Biot–Savart law and symmetry considerations, it follows that there is only a y component of the magnetic field, and Equations (D.2) and (D.3) give

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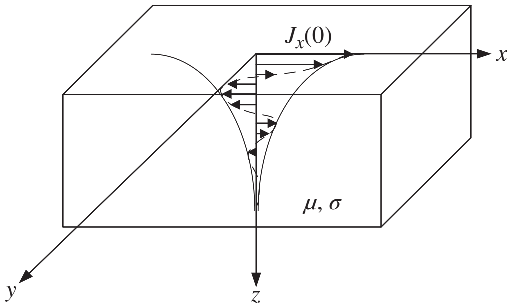

FIGURE D.1 Portion of the homogeneous conducting half space (extending everywhere for $z > 0$), and instantaneous current density distribution.

$$\frac{\partial J_x}{\partial z} = -j\omega\mu\sigma H_y \tag{D.4a}$$

$$-\frac{\partial H_y}{\partial z} = J_x \tag{D.4b}$$

Combining the previous equations, the following second-order differential equation for J_x is obtained:

$$\frac{\partial^2 J_x}{\partial z^2} = j\omega\mu\sigma J_x \tag{D.5}$$

The general solution of this equation is of the following form:

$$J_x(z) = J_1 e^{\gamma z} + J_2 e^{-\gamma z} \tag{D.6}$$

with

$$\gamma = \sqrt{j\omega\mu\sigma} = \frac{1+j}{\delta_{\mathrm{p}}} \tag{D.7}$$

and δ_{p} defined in (1.79). Notice that γ is identical to the propagation constant given in (1.78). Obviously, the particular solution must have $J_1 = 0$ since the current cannot increase indefinitely from the surface. Considering that the current in the surface is given by the surface field as $J_x(z = 0) = \sigma E_{\mathrm{o}}$, the current can be expressed as follows:

$$J_x(z) = \sigma E_{\mathrm{o}} e^{-z/\delta_{\mathrm{p}}} \cdot e^{-jz/\delta_{\mathrm{p}}} \tag{D.8}$$

which is identical to expression (1.84).