Appendix F

AVERAGING THE EFFECTIVE DIELECTRIC CONSTANT IN EBG-BASED TRANSMISSION LINES[1]

The approximate analytical solutions of the coupled mode equations given in Section 2.4.3 have been obtained by neglecting the dependence on z of the phase constant, β, or the effective dielectric constant, ε_{re} (these parameters are related by 2.49). However, it is possible to improve the accuracy of the solutions by considering the z-dependence of β. To this end, let us introduce a new spatial variable, ζ, defined as follows:

$$\zeta(z) = \int_0^z \sqrt{\varepsilon_{re}(z')} \cdot dz' \tag{F.1}$$

in the first of the coupled-mode equations (2.47a). With this new variable, (2.47a) can be written as follows:

$$\frac{da^+}{d\zeta}\frac{d\zeta}{dz} + j\beta_o \frac{d\zeta}{dz} \cdot a^+ = K(\zeta)\frac{d\zeta}{dz} \cdot a^- \tag{F.2}$$

where $\beta_o = \omega/c$ and

$$\frac{d\zeta(z)}{dz} = \sqrt{\varepsilon_{re}} \tag{F.3}$$

[1] This appendix has been co-authored with Txema Lopetegi (Public University of Navarre, Spain).

Artificial Transmission Lines for RF and Microwave Applications, First Edition. Ferran Martín.

as results by differentiating (F.1). The coupling coefficient in terms of this new variable is simply

$$K(z) = -\frac{1}{2Z_o}\frac{dZ_o}{d\zeta}\frac{d\zeta}{dz} = K(\zeta)\frac{d\zeta}{dz} \tag{F.4}$$

The derivative of ζ with respect to z can be eliminated in (F.2), and the resulting equation is

$$\frac{da^+}{d\zeta} + j\beta_o a^+ = K(\zeta)a^- \tag{F.5a}$$

Similarly, the second coupled mode equation (2.47b) can be expressed in terms of ζ as follows:

$$\frac{da^-}{d\zeta} - j\beta_o a^- = K(\zeta)a^+ \tag{F.5b}$$

Notice that Equations F.5 are formally identical to Equations 2.47 if β is considered to be constant. Since this approximation (i.e., $\beta \neq \beta(z)$) has been used in order to obtain analytical solutions of the coupled-mode equations, it follows that the solutions of (F.5) must be the same (provided β_o is constant), simply replacing z with ζ. In particular, the frequency of maximum reflectivity, corresponding to $\Delta\beta = 0$, can be inferred from

$$\beta_{o,\max} = \frac{n\pi}{\zeta(z=l)} \tag{F.6}$$

or

$$\frac{2\pi f_{\max}}{c} = \frac{n\pi}{\int_0^l \sqrt{\varepsilon_{re}(z')}.dz'} \tag{F.7}$$

Comparing (F.7) and (2.84), the averaging value of (2.57) is justified.