Appendix G

PARAMETER EXTRACTION

Parameter extraction is a technique for the determination of the element values of the circuit model describing certain electromagnetic (EM) structure. The number of required conditions to find these values is equal to the number of elements of the circuit model, and these conditions are inferred either from the EM simulation or from the measured response of the structure. In this appendix, the specific methods for parameter extraction in SRR-, CSRR-, OSRR-, and OCSRR-loaded lines are reported. The lossless EM simulation is the considered response in all the cases; and coherently, the circuit models do not include the effect of losses.

G.1 PARAMETER EXTRACTION IN CSRR-LOADED LINES

The parameters of the circuit of Figure 3.38a can be extracted from the EM simulation of the CSRR-loaded (unit cell) line according to the following procedure. First, the reflection coefficient, S_{11}, is represented in the Smith chart. At the intercept of S_{11} with the unit resistance circle, the shunt branch opens and hence the resonance frequency of the CSRR can be determined:

$$f_o = \frac{1}{2\pi\sqrt{L_c C_c}} \tag{G.1}$$

The series reactance at this frequency, χ, directly provides the value of the inductance L, namely

Artificial Transmission Lines for RF and Microwave Applications, First Edition. Ferran Martín.

$$L = \frac{\chi}{2\pi f_o} \tag{G.2}$$

To univocally determine the three-circuit elements of the shunt branch, two additional conditions, apart from expression (G.1), are needed. One of them is the transmission zero frequency, f_z,

$$f_z = \frac{1}{2\pi\sqrt{L_c(C + C_c)}} \tag{G.3}$$

that can be easily determined from the representation of the magnitude of S_{21} with frequency. The third required condition can be derived from (2.33), which can be rewritten as follows:

$$\cos(\phi) = 1 + \frac{Z_s(\omega)}{Z_p(\omega)} \tag{G.4}$$

where $Z_s(\omega)$ and $Z_p(\omega)$ are the series and shunt impedance of the T-circuit model of the unit cell. Forcing $\phi = \beta l = \pi/2$, it follows that

$$Z_s(\omega_{\pi/2}) = -Z_p(\omega_{\pi/2}) \tag{G.5}$$

where $\omega_{\pi/2}$ is the angular frequency where $\phi = \pi/2$, which means that the phase of the transmission coefficient is $\phi_{21} = -\pi/2$ and can be easily computed. Thus, from (G.1), (G.3), and (G.5) the three reactive element values that contribute to the shunt impedance can be determined.

The circuit of Figure 3.38b has an additional parameter (C_g). Therefore, an additional condition to fully determine the circuit parameters of CSRR/gap-loaded lines is needed. This condition can be the resonance frequency of the series branch

$$f_s = \frac{1}{2\pi\sqrt{LC_g}} \tag{G.6}$$

which is given by the intercept of S_{11} with the unit conductance circle, where the impedance of the series branch nulls. Notice that for CSRR/gap-loaded lines, (G.5) rewrites as follows:

$$Z_s(\omega_{-\pi/2}) = -Z_p(\omega_{-\pi/2}) \tag{G.7}$$

$\omega_{-\pi/2}$ being the angular frequency of the left-handed (LH) band, where $\phi = -\pi/2$, that is, $\phi_{21} = \pi/2$.[1] Moreover, notice that for CSRR/gap-loaded lines, the series reactance at

[1] Although condition (G.4), as it is, can in principle be used to determine the element values of CSRR/gap-loaded lines, its use implies that a condition in the second (right-handed) transmission band is imposed. Since the model of CSRR-based lines is not very accurate in the second (right-handed) band, it is more convenient to use (G.6), which is a condition for an angular frequency ($\omega_{-\pi/2}$) within the first (LH) band.

f_o, inferred from the Smith chart, gives a condition involving L and C_g (not only involving L as given by expression G.2). Combining this condition with (G.6) allows us to determine L and C_g.

The parameter extraction method for CSRR- and CSRR/gap-loaded lines was first reported in Ref. [1], where the effects of losses were also considered (by adding a parallel resistance to the L_c-C_c tank), and parameter extraction was inferred from the measured responses of the considered CSRR-based lines.

G.2 PARAMETER EXTRACTION IN SRR-LOADED LINES

Parameter extraction in SRR-loaded lines, described by the circuit of Figure 3.34d (if the magnetic wall concept is applied) or by the circuit of Figure 3.35b by excluding L'_p (in this case, the magnetic wall concept is not applied), follows a similar procedure (the method was first reported in Ref. [2]). Since parameter extraction is obtained from the EM simulation of real structures (i.e., including the two halves), the considered model of SRR-loaded lines for parameter extraction is the one depicted in Figure 3.35b with the exclusion of L'_p. From the representation of the reflection coefficient, S_{11}, in the Smith chart, two conditions are obtained. On the one hand, the frequency that nulls the series reactance, f_s, is inferred from the intercept of S_{11} with the unit conductance circle. This frequency is given by the following expression:

$$f_s = \frac{1}{2\pi}\sqrt{\frac{1}{L'_s C'_s} + \frac{1}{L' C'_s}} \tag{G.8}$$

On the other hand, the susceptance, B, of the unit cell seen from the ports at f_s, which can be inferred from the Smith chart, directly gives C:

$$C = \frac{B}{2\pi f_s} \tag{G.9}$$

Another condition concerns the parallel resonator of the series branch. At the resonance frequency of this resonator, given by

$$f_z = \frac{1}{2\pi}\sqrt{\frac{1}{L'_s C'_s}} \tag{G.10}$$

the series branch opens, and the transmission coefficient exhibits a transmission zero at this frequency. The fourth condition is simply expression (G.5), applied to the circuit model of Figure 3.35b without L'_p.

For SRR/strip-loaded lines, an additional (fifth) condition is required since there are five elements in the circuit model of Figure 3.35b, namely, the shunt inductance L'_p is now included. The fifth condition is given by the frequency that opens the shunt branch, which can be inferred from the intersection of the S_{11} trace in the Smith chart with the unit resistance circle. This frequency is given by

$$f_p = \frac{1}{2\pi}\sqrt{\frac{2}{L'_p C}} \tag{G.11}$$

Moreover, for SRR/strip-loaded lines, condition (G.7), rather than (G.5), must be used (similar to CSRR/gap-loaded lines).

G.3 PARAMETER EXTRACTION IN OSRR-LOADED LINES

The parameters of the circuit model of a CPW loaded with an OSRR (Fig. 3.53e) can be extracted from the EM simulation of the structure following a straightforward procedure (notice that only three parameters are involved in the circuit model) [3]. From the intercept of S_{11} with the unit conductance circle in the Smith chart, we can directly infer the value of the shunt capacitance according to

$$C = \frac{B}{4\pi f_s} \tag{G.12}$$

where B is the susceptance at the intercept point. The frequency at this intercept point is the resonance frequency of the series branch:

$$f_s = \frac{1}{2\pi}\sqrt{\frac{1}{C_s L'_s}} \tag{G.13}$$

To determine the two element values of this branch, another condition is needed. This condition comes from the fact that at the reflection zero frequency, f_z, (maximum transmission) the characteristic impedance of the structure is $Z_o =$ 50 Ω (the typical value of the reference impedance of the ports). In this π-circuit, the characteristic impedance is given by (3.84). Thus, by forcing this impedance to 50 Ω, the second condition results. By inverting equations (G.13) and (3.84), the element values of the series branch can be determined. The following results are obtained:

$$C_s = \left[\frac{f_z^2}{f_s^2} - 1\right] \cdot \left\{\frac{1}{8\pi^2 Z_o^2 f_z^2 C} + \frac{C}{2}\right\} \tag{G.14}$$

$$L'_s = \frac{1}{4\pi^2 f_s^2 C_s} \tag{G.15}$$

G.4 PARAMETER EXTRACTION IN OCSRR-LOADED LINES

The parameters of the circuit model of a CPW loaded with an OCSRR (Fig. 3.54e) can be extracted following a similar procedure. In this case, the intercept of S_{11} with the unit resistance circle in the Smith chart gives the value of the series inductance:

$$L = \frac{\chi}{4\pi f_p} \tag{G.16}$$

where χ is the reactance at the intercept point. The shunt branch resonates at this frequency; that is,

$$f_p = \frac{1}{2\pi}\sqrt{\frac{1}{L'_p C'_p}} \tag{G.17}$$

Finally, at the reflection zero frequency (f_z), the characteristic impedance, given by (2.30) must be forced to be 50 Ω. From these two latter conditions, we finally obtain the following:

$$L'_p = \left[\frac{f_z^2}{f_p^2} - 1\right] \cdot \left\{\frac{Z_o^2}{8\pi^2 f_z^2 L} + \frac{L}{2}\right\} \tag{G.18}$$

$$C'_p = \frac{1}{4\pi^2 f_p^2 L'_p} \tag{G.19}$$

and the element values are determined.

If the wideband model of Figure 3.55a, including the inductance L_{sh}, is considered, an additional condition for the determination of the elements of the shunt branch, such as (G.5), is required. Alternatively, the transmission zero frequency, given by the frequency that nulls the shunt impedance, can also be used.

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