CASE 76

Prediction of Urinary Continence Recovery among Patients with Brain Disease Using the Mahalanobis Distance

Abstract: We created an experiment to see how accurately we could predict the rate of independent urination after four weeks using a Mahalanobis distance after one week.

1. Introduction

For our research, we collected data on 232 patients with brain disease, which we classified into four groups, each of which was a possible combination of whether urinary continence was attained by a patient at the end of the first or fourth week after he or she suffered brain trauma (Table 1). Group I consisted of 150 patients who had recovered continence at the end of the first and fourth weeks, respectively. Group II contains only one patient who had recovered continence at the end of the first week but had not at the end of the fourth week. Eighty-one patients who had not recovered continence at the end of the first week belong to groups III and IV. Among them, 30 patients who had recovered at the end of the fourth week were classified as group III, and the remaining 51 patients who could not recover were categorized as group IV.

The most interesting groups in our pattern recognition were groups III and IV, because if we can predict precisely whether a patient who has not been able to recover continence after one week will recover it after four weeks, we can use such information to establish a nursing procedure for each patient.

A Mahalanobis space was constructed using the data of the first week from group III: The patients in this group did not recover continence at the end of the first week but recovered by the end of the fourth week.

As a next step, using after-one-week data for groups III and IV, we computed the Mahalanobis distances. With the central point of a resulting Mahalanobis space defined as a reference point, we predicted the possibility of each patient's continence recovery at the end of the fourth week. Table 2 lists the items.

Items

Twelve factors, including the six diseases shown in Table 2, were used to calculate Mahalanobis distances. Disturbance of consciousness and urinary continence were, according to their degrees, classified into the following categories and quantified as an ordinal number:

1. Disturbance of consciousness 1 (eye-opening level): 4 points
2. Disturbance of consciousness 2 (utterance level): 5 points
3. Disturbance of consciousness 3 (movement level): 6 points
6. Urinary continence level after one week: 8 points

For each of the above, a larger number indicates a better condition. Since 8 points were added to a state of complete continence recovery, all other possible scores from 1 to 7 indicated incomplete recovery.

 Taguchi's Quality Engineering Handbook. Genichi Taguchi, Subir Chowdhury and Yuin Wu

Table 1
Classification of patients by pattern in starting time of urinal continence recovery

Group	Starting Time of Urinal Continence Recovery[a] End of First Week	End of Fourth Week	Number of Patients
I	+	+	150
II[b]	+	−	1
III	−	+	30
IV	−	−	51
		Total	232

[a] +, Continence recovered; −, continence not recovered.
[b] The data in group II can be viewed as exceptional because the patient was operated on again.

Binary Decomposition of Categorical Variables

We defined a male as 0 and a female as 1. This conversion was regarded as a binary expression for categorical variables. Six types of diseases can be expressed by five binary variables (Table 3). Although we can drop any single item, item 10 was removed. For example, when a patient with disease 1 (head injury, operation group) is considered, item 7 is set to 1, whereas items 8 to 12 are set to 0. On the other hand, for a patient with disease 4 (cerebral aneurysm, operation group), 0 was assigned to all items 7 to 12 because item 10 was excluded. For patients with other diseases, we allocated 0 or 1 in a similar manner to that for disease 1.

2. Prediction of Continence Recovery after Four Weeks

To see how accurately we could predict the rate of independent urination after four weeks by using a Mahalanobis distance after one week, we created Figures 1 to 6. The *X*-axis denotes a Mahalanobis distance after one week, and its calculation is expressed as

$$y = 10 \log_{10} \frac{D^2}{f}$$

Table 2
Items used for calculation of Mahalanobis distance

Factor No.	Factor
1	Disturbance of consciousness 1 (eye-opening level)
2	Disturbance of consciousness 2 (utterance level)
3	Disturbance of consciousness 3 (movement level)
4[a]	Gender (male = 0, female = 1)
5	Age
6	Urinary continence level after one week
7[a]	Disease 1 (head injury, operation group)
8[a]	Disease 2 (head injury, nonoperation group)
9[a]	Disease 3 (brain tumor, operation group)
10[a]	Disease 4 (cerebral aneurysm, operation group)
11[a]	Disease 5 (brain infarction, nonoperation group)
12[a]	Disease 6 (brain hemorrhage, operation and nonoperation group)

[a] Categorical variable.

Table 3
Binary expression for six types of diseases

	(7)	(8)	(9)	(11)	(12)
Patient with disease 1	1	0	0	0	0
Patient with disease 2	0	1	0	0	0
Patient with disease 3	0	0	1	0	0
Patient with disease 4	0	0	0	0	0
Patient with disease 5	0	0	0	1	0
Patient with disease 6	0	0	0	0	0

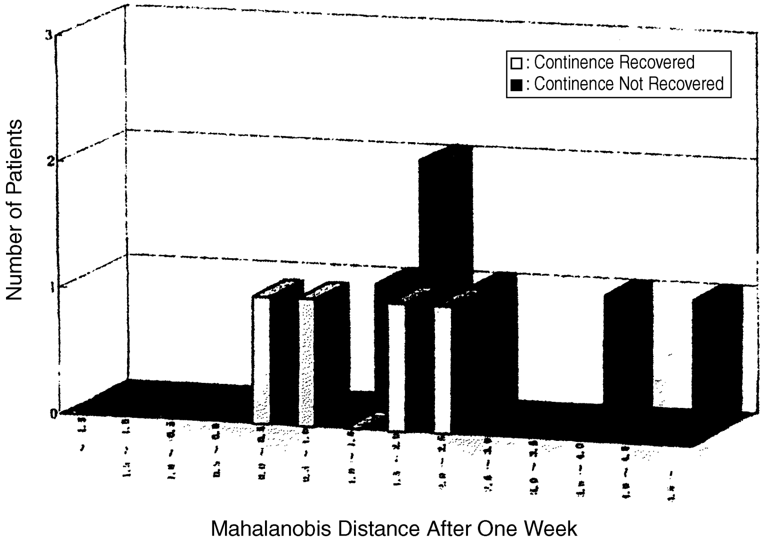

Figure 1
Mahalanobis distances after the end of the first and fourth weeks and the progress of urinary continence recovery for disease 1 (head injury, operation group)

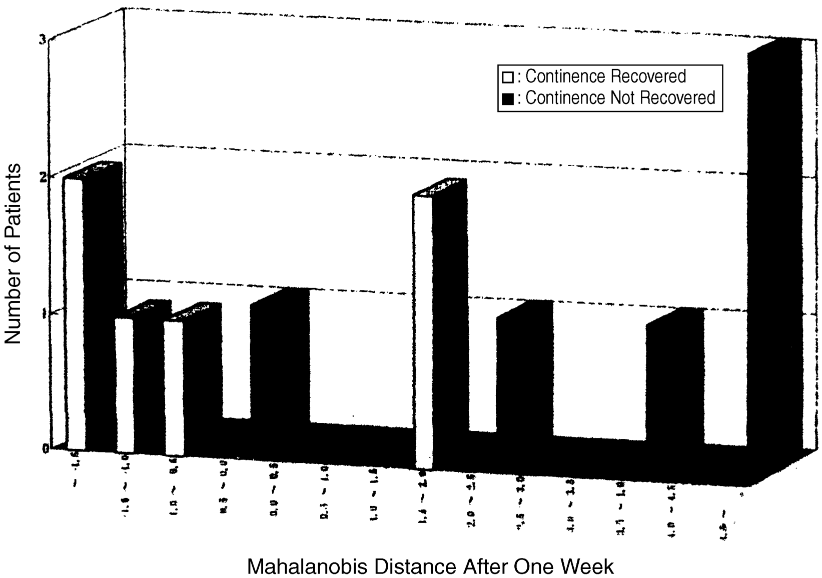

Figure 2
Mahalanobis distances after the end of the first and fourth weeks and the progress of urinary continence recovery for disease 2 (head injury, nonoperation group)

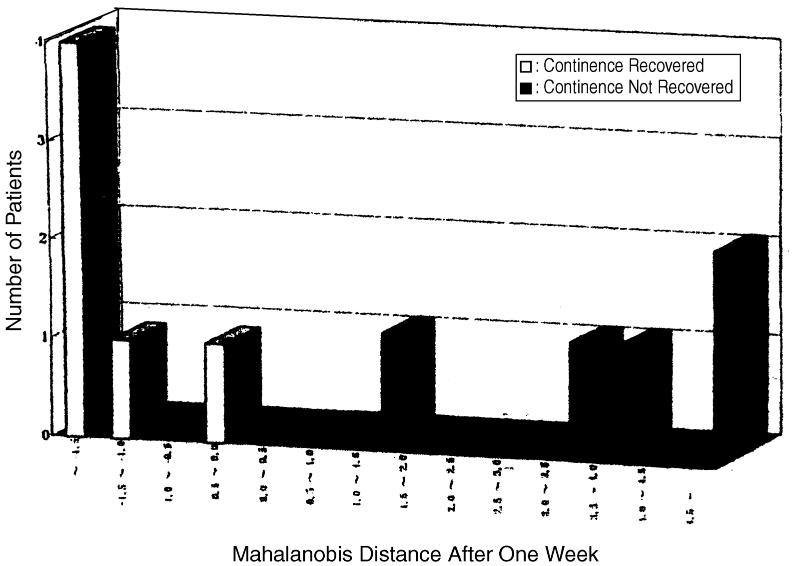

Figure 3
Mahalanobis distances after the end of the first and fourth weeks and the progress of urinary continence recovery for disease 3 (brain tumor, operation group)

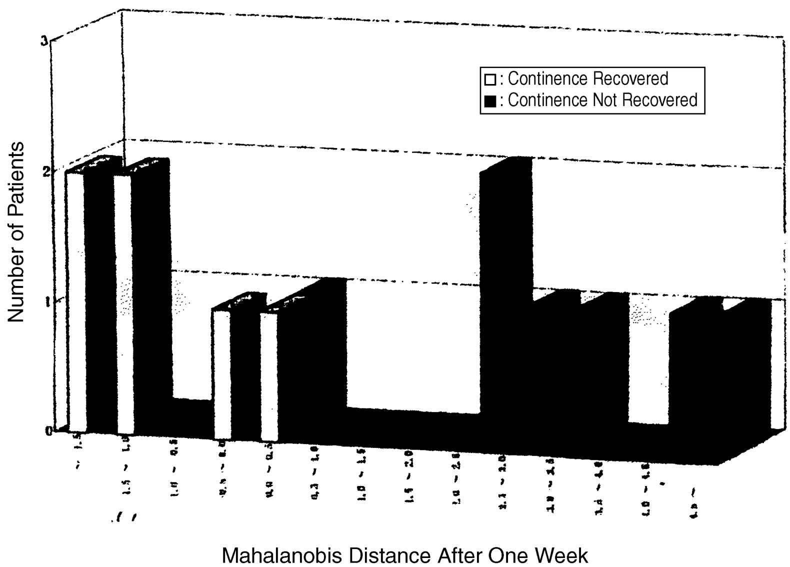

Figure 4
Mahalanobis distances after the end of the first and fourth weeks and the progress of urinary continence recovery for disease 4 (cerebral aneurysm, operation group)

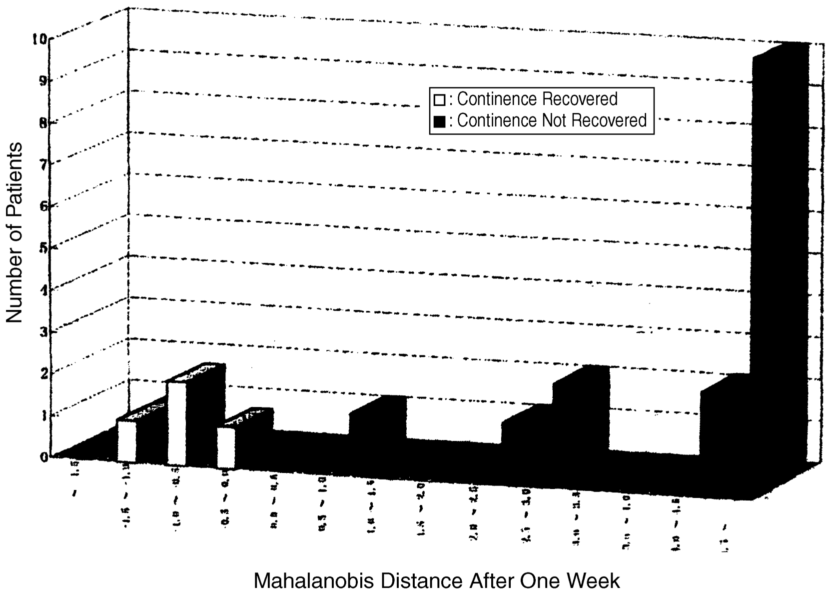

Figure 5
Mahalanobis distances after the end of the first and fourth weeks and the progress of urinary continence recovery for disease 5 (brain infection, nonoperation group)

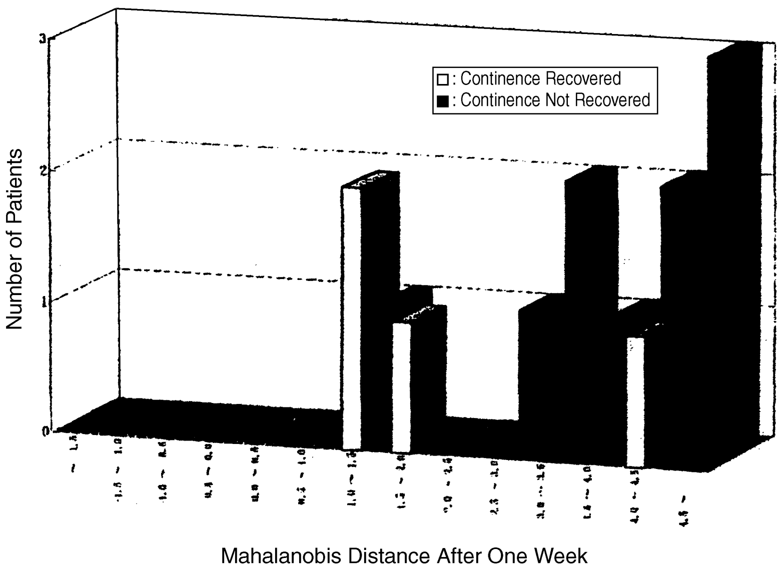

Figure 6
Mahalanobis distances after the end of the first and fourth weeks and the progress of urinary continence recovery for disease 6 (brain hemorrhage, operation and nonoperation groups)

where the Y-axis indicates the number of patients. "Continence recovered" means that a patient can urinate independently at the end of the fourth week, whereas "continence not recovered" signifies that he or she cannot do so.

Our research revealed that for disease 3 (brain tumor, operation group) and disease 5 (brain infarction, nonoperation group), we can predict perfectly from the Mahalanobis distance at the end of the first week the recovery after the fourth week. In addition, for disease 4 (cerebral aneurysm, operation group), we can foresee the progress relatively accurately.

3. Selection of Variables by Orthogonal Array

The number of items used to create a Mahalanobis space was 11, from items 1 to 12 except item 11. However, when handling a larger number of items, the data collection cost could become enormous. How to reduce the cost and make an accurate prediction was the next issue. Using an L_{12} orthogonal array, we assigned items as shown in Table 4.

The levels used are (1) select an item, and (2) do not select the item. Numbers arranged vertically are experiment numbers. For example, in experiment 1 we calculated a Mahalanobis distance with all 11 factors, and in experiment 2 computed MD with five variables, such as factors 3, 4, 6, 8, and 12.

The right side of Table 4 shows the SN ratio and sensitivity. Let the Mahalanobis distances of 30 patients who belong to group III, the patients who recovered continence at the end of the fourth week, be $y_{i,1}, y_{i,2}, \ldots, y_{i,30}$. Also let the Mahalanobis distances of 50 patients who belong to group IV, the patients who could not recover continence, be $y_{i,31}, y_{i,32}, \ldots, y_{i,81}$. The SN ratio and sensitivity were defined as follows. We evaluated the certainty of judgment by the SN ratio and the distance between two groups by sensitivity S:

$$\eta = 10 \log_{10} \frac{(1/r)(S_G - V_e)}{V_e} \qquad (1)$$

$$S = \overline{G}_4 - \overline{G}_3 \qquad (2)$$

where G_3 and G_4 are the averages of Mahalanobis distances belonging to groups III and IV, respectively.

The harmonic mean is expressed as

$$\frac{1}{r} = \frac{1}{2}\left(\frac{1}{30} + \frac{1}{51}\right) \qquad (3)$$

Table 4
SN ratio sensitivity

No.	1	2	3	4	5	6	7	8	9	10	11	η (dB)	S
1	1	1	1	1	1	1	1	1	1	1	1	1.47	4.92
2	1	1	1	1	1	2	2	2	2	2	2	−2.70	4.78
3	1	1	2	2	2	1	1	1	2	2	2	−6.61	2.66
4	1	2	1	2	2	1	2	2	1	1	2	−5.07	3.30
5	1	2	2	1	2	2	1	2	1	2	1	−6.91	2.07
6	1	2	2	2	1	2	2	1	2	1	1	−4.54	2.97
7	2	1	2	2	1	1	2	2	1	2	1	−4.54	3.17
8	2	1	2	1	2	2	2	1	1	1	2	−5.88	2.35
9	2	1	1	2	2	2	1	2	2	1	1	−3.58	4.82
10	2	2	2	1	1	1	1	2	2	1	2	−7.82	2.02
11	2	2	1	2	1	2	1	1	1	2	2	−10.35	2.20
12	2	2	1	1	2	1	2	1	2	2	1	−3.87	4.42

Total variation, S_T, general mean, S_m, variation between groups, S_G, error variation, S_e, and error variance, V_e, were computed as follows:

$$S_T = y_1^2 + y_2^2 + \cdots + y_{81}^2 \qquad (f = 81) \qquad (4)$$

$$S_m = \frac{(y_1 + y_2 + \cdots + y_{81})^2}{81} \qquad (f = 1) \qquad (5)$$

$$S_G = \frac{(y_1^2 + y_2^2 + \cdots + y_{30}^2)}{30} + \frac{(y_{31}^2 + y_{32}^2 + \cdots + y_{81}^2)}{51} \qquad (f = 1) \qquad (6)$$

$$S_e = S_T - S_m - S_G \qquad (f = 79) \qquad (7)$$

$$V_e = \frac{S_e}{79} \qquad (8)$$

Using the equations above, we calculated SN ratios, η, and sensitivities, S, and summarized them in Table 4. Based on the SN ratios and sensitivities shown in the table, we created the response graphs shown in Figure 7, where 1 through 12 indicated above the X-axis represent item numbers and 2 and 1 below the axis indicate the orthogonal array level. Level 1 denotes use of an item, whereas level 2 indicates no use of the item.

The upper part of the plot shows the response graphs for the sensitivity, S. As for the sensitivity S of items 7, 8, and 9, level 1, representing use of a variable, is smaller than level 2, showing negative effects. For other items, use of the items is more effective. The lower part of the plot shows the response graphs for the SN ratio, η, which reveals that use of a variable is better that no use for all items except items 7 and 9.

4. Optimal Selection and Results

Figure 8 shows the relationship between Mahalanobis distances after one week and continence recovery after four weeks when using all items for all diseases. In this case, the SN ratio, η, and sensitivity, S, result in 1.47 dB and 4.92. In contrast, according to the result in the preceding section, Figure 9 illustrates the relationship between the Mahalanobis distances after one week and the recovery after four weeks, when using all items except items 7, 8, and 9. The resulting SN ratio, η, and sensitivity, S, are computed as 0.40 dB and 5.30.

Excluding the three items yields a better sensitivity, S, but a 1.07 dB lower SN ratio than when using all factors. A comparison of Figures 8 and 9 demonstrates that taking all items can lead to better discriminability.

The items excluded, 7, 8, and 9, are some of the items of diseases expressed as binary variables. This example highlights that when we select some cate-

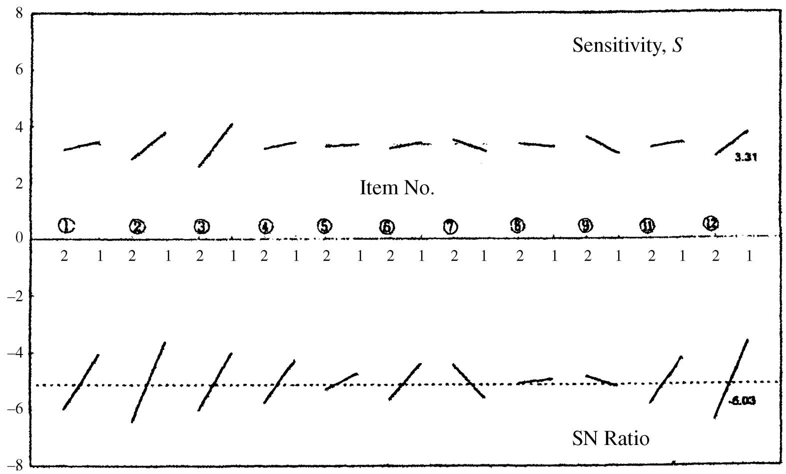

Figure 7
Response graphs for item selection

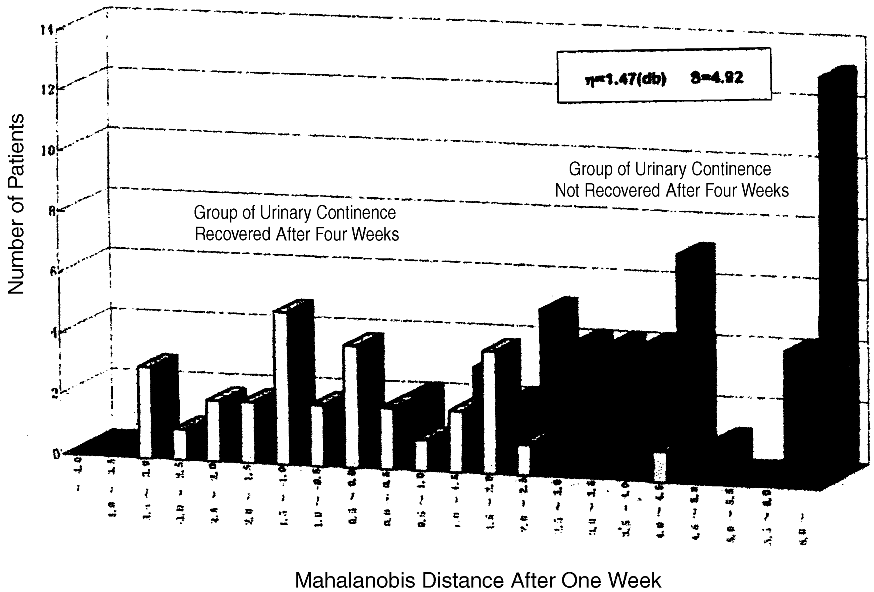

Figure 8
Mahalanobis distances after the end of the first and fourth weeks and the progress in urinary continence recovery for all diseases, including all items

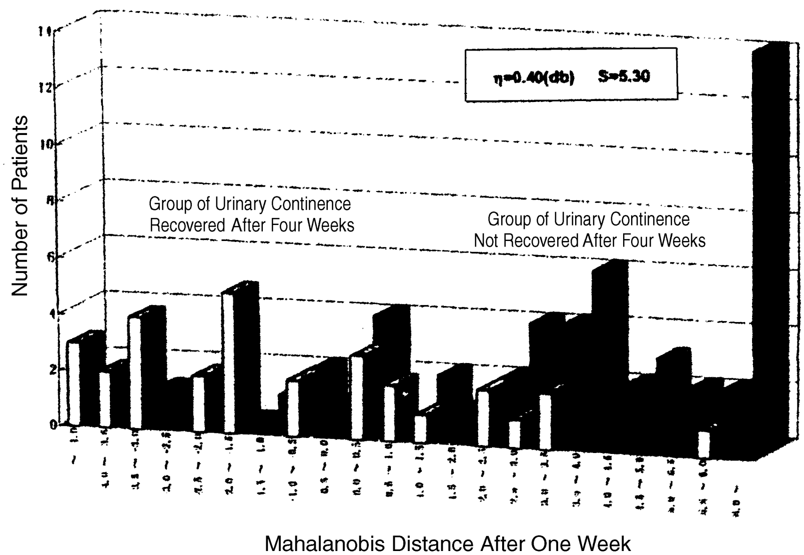

Figure 9
Mahalanobis distances after the end of the first and fourth weeks and the progress in urinary continence recovery for all diseases when items were selected

gorical variables, it is regarded as inappropriate to exclude some of the variables from all the variables. If a selection cannot be averted, we should carefully exclude each variable in turn. This is because each binary variable contains other factors' effects.

In our research, using all the factors was chosen as the optimal selection for creating a Mahalanobis space.

References

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This case study is contributed by Yoshiko Hasegawa and Michiyo Kojima.