

النموذج (ب)

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(b) 15



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(c) $(x-2)^2 + (y+3)^2 + (z-4)^2 = 16$



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(b) $Z = 5$



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$$\text{i)} \overrightarrow{BA} = \vec{A} - \vec{B} = (-1, -2, -3)$$

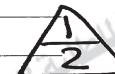
$$\overrightarrow{BC} = \vec{C} - \vec{B} = (-1, 4, 0)$$

$$\text{ii)} \therefore \cos(\angle ABC) = \frac{\overrightarrow{BA} \cdot \overrightarrow{BC}}{\|\overrightarrow{BA}\| \|\overrightarrow{BC}\|}$$

$$= \frac{(-1, -2, -3) \cdot (-1, 4, 0)}{\sqrt{1+4+9} \sqrt{1+16+0}}$$

$$= \frac{1-8+0}{\sqrt{14} \sqrt{17}} = \frac{-7}{\sqrt{14} \sqrt{17}}$$

$$\therefore m(\angle ABC) \approx 117^\circ$$



$$\text{iii)} \overrightarrow{BC} = \vec{C} - \vec{B} \Rightarrow \vec{C} = \overrightarrow{BC} + \vec{B}$$

$$\vec{C} = (-1, 4, 0) + (3, 5, 4) = (2, 9, 4)$$

$$\therefore \overrightarrow{AC} = \vec{C} - \vec{A} = (0, 6, 3)$$

$$\text{The direction Component} = \left(\frac{\overrightarrow{AC} \cdot \overrightarrow{AB}}{\|\overrightarrow{AB}\|^2} \right) \cdot \overrightarrow{AB}$$



$$= \frac{(0, 6, 3) \cdot (1, 2, 3)}{1+4+9} \cdot (1, 2, 3)$$

$$= \frac{0+12+9}{14} \cdot (1, 2, 3) = \left(\frac{3}{2}, 3, \frac{9}{2} \right)$$



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b) The volume of the parallelepiped

$$\begin{aligned}
 &= |\vec{A} \cdot \vec{B} \times \vec{C}| \\
 &= \begin{vmatrix} 1 & 4 & 2 \\ -3 & 2 & 1 \\ -1 & 1 & 4 \end{vmatrix} \\
 &= 1(7) - 4(-11) + 2(-1) = 49
 \end{aligned}$$



(i) The base area = $\|\vec{A} \times \vec{B}\|$

$$\therefore \vec{A} \times \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 4 & 2 \\ -3 & 2 & 1 \end{vmatrix} = -7\vec{j} + 14\vec{k}$$

$$\therefore \|\vec{A} \times \vec{B}\| = \sqrt{(-7)^2 + (14)^2} = 7\sqrt{5}$$



\therefore The base area = $7\sqrt{5}$ area unit

The height of the parallelepiped = $\frac{\text{Volume}}{\text{base area}}$

$$\begin{aligned}
 &= \frac{49}{7\sqrt{5}} = \frac{7\sqrt{5}}{5} \text{ length unit.} \\
 &\approx 3.13 \text{ length unit.}
 \end{aligned}$$



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5-

(b) 15



6-

(a) 10



7-

(b) 2



النموذج (ب)

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$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} \quad \frac{C_2 - C_1}{C_3 - C_1} \Rightarrow$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ a & b-a & c-a \\ a^2 & b^2-a^2 & c^2-a^2 \end{vmatrix} \quad \triangle 1$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ a & (b-a) & (c-a) \\ a^2 & (b-a)(b+a) & (c-a)(c+a) \end{vmatrix} \quad \triangle 2$$

$$= (b-a)(c-a) \begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 1 \\ a^2 & b+a & c+a \end{vmatrix} \quad \frac{1}{2} \triangle$$

$$C_3 - C_2 \Rightarrow$$

$$= (b-a)(c-a) \begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ a^2 & b+a & c-b \end{vmatrix} \quad \triangle 2$$

$$= (b-a)(c-a)(c-b) \quad \frac{1}{2} \triangle$$

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(d) π



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(c) $\tan \theta$



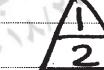
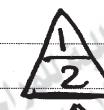
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Let the term Contains x^4 be T_{r+1}

$$T_{r+1} = {}^{12}C_r \left(-\frac{1}{x^2}\right)^r (x^2)^{12-r}$$

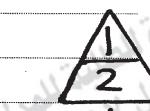
$$= {}^{12}C_r (-1)^r x^{24-4r}$$

$$\text{let } 24-4r = 4 \Rightarrow r = 5$$



\therefore The term Contains x^4 is T_6

$$T_6 = {}^{12}C_5 (-1)^5 (x^4) = -792x^4$$



• The order of the middle terms : $\frac{12}{2} + 1 = 7$

$$T_7 = {}^{12}C_6 \times \left(-\frac{1}{x^2}\right)^6 \times (x^2)^6 = {}^{12}C_6 = 924$$

$$\therefore \frac{\text{The coeff. of } T_6}{\text{The coeff. of } T_7} = \frac{-792}{924} = \frac{-6}{7}$$

$$\text{or } \frac{\text{The coeff. of } T_6}{\text{The coeff. of } T_7} = \frac{r}{n-r+1} \times \frac{\text{coeff of 1st}}{\text{coeff of 2nd}}$$

$$= \frac{6}{12-6+1} \times \frac{1}{-1}$$

$$= -\frac{6}{7}$$



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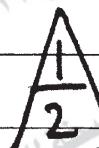
The equation of the plane:

$$\vec{n} \cdot \vec{r} = \vec{n} \cdot \vec{A}$$



$$(4, 10, -7) \cdot \vec{r} = (4, 10, -7) \cdot (2, -1, 0)$$

$$(4, 10, -7) \cdot \vec{r} = -2$$



The vector form

$$4(x-2) + 10(y+1) - 7z = 0$$



The standard form

$$4x + 10y - 7z + 2 = 0$$



The general form

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(b) $(1, 1)$

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\therefore The st. line makes equal angles with the positive directions of the Coordinated axes

$$\therefore \cos \theta_x = \cos \theta_y = \cos \theta_z$$

$$\therefore \cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z = 1$$

$$\therefore \cos^2 \theta_x = \cos^2 \theta_y = \cos^2 \theta_z = \frac{1}{3}$$

$$\therefore \cos \theta_x = \cos \theta_y = \cos \theta_z = \pm \frac{1}{\sqrt{3}}$$

\therefore The directrvector of the st. line = $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right)$
or $= (1, 1, 1)$

$$\therefore \vec{r} = (3, 2, -1) + t \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right)$$

$$\text{or } \therefore \vec{r} = (3, 2, -1) + t (1, 1, 1)$$

$$\bullet x = 3+t, y = 2+t, z = -1+t$$

$$\text{or } x = 3 + \frac{1}{\sqrt{3}}t, y = 2 + \frac{1}{\sqrt{3}}t, z = -1 + \frac{1}{\sqrt{3}}t$$

$$\bullet x-3 = y-2 = z+1$$

$$\text{or } \frac{x-3}{\left(\frac{1}{\sqrt{3}}\right)} = \frac{y-2}{\left(\frac{1}{\sqrt{3}}\right)} = \frac{z+1}{\left(\frac{1}{\sqrt{3}}\right)}$$

النموذج (ب)

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$$\begin{pmatrix} 0 & -3 & 2 \\ 5 & 1 & 0 \\ 1 & -2 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7 \\ 4 \\ 1 \end{pmatrix}$$

$$AX = B$$

$$|A| = \begin{vmatrix} 0 & -3 & 2 \\ 5 & 1 & 0 \\ 1 & -2 & -1 \end{vmatrix} = 3(-5) + 2(-11) = -37 \neq 0$$

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$$\text{Adj}(A) = \begin{pmatrix} -1 & 5 & -11 \\ -7 & -2 & -3 \\ -2 & 10 & 15 \end{pmatrix}^t = \begin{pmatrix} -1 & -7 & -2 \\ 5 & -2 & 10 \\ -11 & -3 & 15 \end{pmatrix}$$

△
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$$A^{-1} = \frac{1}{|A|} \text{Adj}(A)$$

$$\therefore A^{-1} = \frac{1}{-37} \begin{pmatrix} -1 & -7 & -2 \\ 5 & -2 & 10 \\ -11 & -3 & 15 \end{pmatrix}$$

△
2

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{-37} \begin{pmatrix} -1 & -7 & -2 \\ 5 & -2 & 10 \\ -11 & -3 & 15 \end{pmatrix} \begin{pmatrix} 7 \\ 4 \\ 1 \end{pmatrix}$$

△
2

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{-37} \begin{pmatrix} -37 \\ 37 \\ -74 \end{pmatrix}$$

△
2

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

$$\therefore x = 1, y = -1, z = 2$$

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النموذج (ب)

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$$(b) {}^3C_1 \times {}^9C_3$$



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$$(b) 2 \cos \theta$$



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$$(d) \vec{r} = (2, 1, -3) + k(-1, 1, -2) \quad \triangle$$

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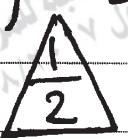
$$\textcircled{a} \quad Z = \frac{8(\sqrt{3}+i)}{(\sqrt{3}-i)} \times \frac{\sqrt{3}+i}{\sqrt{3}+i}$$

$$Z = \frac{8(3+2\sqrt{3}i-1)}{4}$$



$$Z = 8 \left(\frac{1}{2} + \frac{\sqrt{3}}{2} i \right) = 8 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$$

$$Z = 8 e^{\frac{\pi}{3}i}$$



$$\sqrt[3]{Z} = 2 e^{\frac{\frac{\pi}{3}+2\pi n}{3}i}, \quad n=0, 1, -1$$



$$\text{at } n=0 \quad \therefore \sqrt[3]{Z} = 2 e^{\frac{\pi}{9}i}$$



$$\text{at } n=1 \quad \therefore \sqrt[3]{Z} = 2 e^{\frac{4\pi}{9}i}$$



$$\text{at } n=-1 \quad \therefore \sqrt[3]{Z} = 2 e^{-\frac{5\pi}{9}i}$$



النموذج (ب)

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$$\begin{aligned}
 b) (x+yi)(1-3i) &= 37 \left(\frac{7+4\omega^2+3-4\omega^2}{(3-4\omega^2)(7+4\omega^2)} \right) \triangle 1 \\
 &= 37 \left(\frac{10}{(3-4\omega^2)(7+4\omega^2)} \right) \\
 &= 37 \left(\frac{10}{21-16\omega^2-16\omega} \right) \triangle 2 \\
 &= 37 \left(\frac{10}{21-16(\omega^2+\omega)} \right) \triangle 1 = \frac{370}{37} = 10 \triangle 2 \\
 \therefore x+yi &= \frac{10}{1-3i} \times \frac{1+3i}{1+3i} = \frac{10(1+3i)}{10} \triangle 1 \\
 &\triangle 2
 \end{aligned}$$

$$x+yi = 1+3i \Rightarrow x=1 \text{ and } y=3 \triangle 2$$

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(انتهت الإجابة وتراعى الحلول الأخرى)