1-

(b) $\frac{1}{3} \ln 2$ — 1

2-

(d) 2 — 1

3-

$\because$ The area of the circular sector $= 4$

$\therefore \frac{1}{2} r L = 4 \Rightarrow L = \frac{8}{r}$ — $\frac{1}{2}$

let the perimeter of the sector $= y$

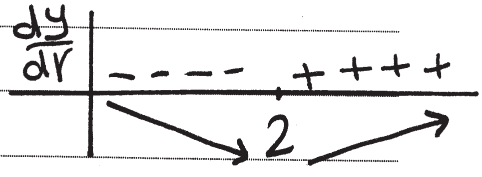
$\therefore y = 2r + L \Rightarrow y = 2r + \frac{8}{r}$ — $\frac{1}{2}$

$\therefore \frac{dy}{dr} = 2 - \frac{8}{r^2}$ — $\frac{1}{2}$

at $\frac{dy}{dr} = 0 \Rightarrow \frac{8}{r^2} = 2 \Rightarrow r^2 = 4$

$\Rightarrow r = 2$ cm — $\frac{1}{2}$

Discuss the sign of the function:



at $r = 2$ cm — $\frac{1}{2}$

The perimeter is minimum as possible

$\therefore L = \frac{8}{2} = 4 \quad , \theta^{rad} = \frac{L}{r} = \frac{4}{2} = 2^{rad.}$ — $\frac{1}{2}$

4-

To determine the intersection point:

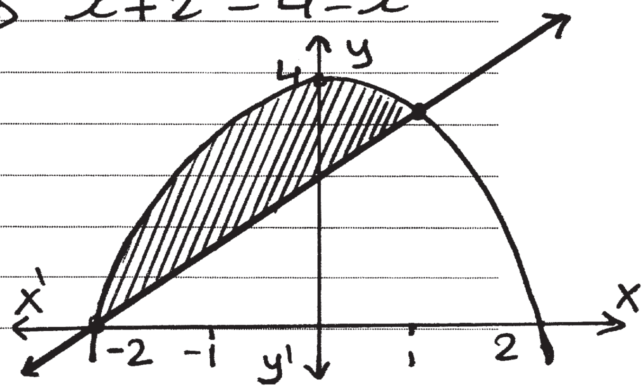
let $y_2 = y_1 \Rightarrow x+2 = 4-x^2$

$\therefore x^2+x-2=0$

$(x+2)(x-1)=0$

$x=-2$ $\frac{1}{2}$

or $x=1$



$$\text{Area} = \int_{-2}^{1} \left[(4-x^2)-(x+2)\right] dx \qquad \frac{1}{2}$$

$$= \int_{-2}^{1} [4-x^2-x-2]\, dx$$

$$= \int_{-2}^{1} [-x^2-x+2]\, dx$$

$$= \left[-\frac{1}{3}x^3 - \frac{1}{2}x^2 + 2x\right]_{-2}^{1} \qquad \frac{1}{2}$$

$$= \left(-\frac{1}{3}-\frac{1}{2}+2\right) - \left(\frac{8}{3}-2-4\right)$$

$$= \frac{9}{2} \text{ area unit} \qquad \frac{1}{2}$$

(تراعى الحلول الأخرى)

5-

(b) $\log_a b$ 1

6-

(c) 4 1

7-

a) $\int x^3 (x^2+1)^6 \, dx$

Let $y = x^2+1 \Rightarrow x^2 = y-1$

$\therefore 2x \, dx = dy \Rightarrow dx = \frac{dy}{2x}$ $\frac{1}{2}$

$\therefore \int x^2 . x (x^2+1)^6 dx$ $\frac{1}{2}$

$= \int (y-1) . x . y^6 . \frac{dy}{2x}$ $\frac{1}{2}$

$= \int \left(\frac{1}{2} y^7 - \frac{1}{2} y^6\right) dy$ $\frac{1}{2}$

$= \frac{1}{16} y^8 - \frac{1}{14} y^7 + C$ $\frac{1}{2}$

$= \frac{1}{16} (x^2+1)^8 - \frac{1}{14} (x^2+1)^7 + C$ $\frac{1}{2}$

b) $\int (x-3) e^{2x} dx$

Let $u = x-3$, $dv = e^{2x} dx$

$du = dx$, $v = \frac{1}{2} e^{2x}$ 1

$= (x-3)\left(\frac{1}{2} e^{2x}\right) - \int \frac{1}{2} e^{2x} dx$ 1

$= \frac{(x-3)}{2} e^{2x} - \left[\frac{1}{2} \times \frac{1}{2} e^{2x}\right] + C$ $\frac{1}{2}$

$= \frac{(x-3)}{2} e^{2x} - \frac{1}{4} e^{2x} + C$ $\frac{1}{2}$

(تراعى الحلول الأخرى)

8-

(a) $-\ln|\cos\theta| + c$ 1

9-

(b) Zero 1

10-

a $f(x) = x^3 - 3x - 2$

$f'(x) = 3x^2 - 3$, $f''(x) = 6x$ $\frac{1}{2}$

at $f'(x) = 0 \Rightarrow 3x^2 - 3 = 0$

$3(x-1)(x+1) = 0$

$\therefore x = 1$ or $x = -1$ $\frac{1}{2}$

From the Discuss of the sign of the function:

$f'(x)$	+ + +	-1	- - - -	1	+ + +
$f(x)$	↗		↘		↗

• f(x) has a local max. value at $x = -1$ which is $f(-1) = \text{zero}$ $\frac{1}{2}$

• f(x) has a local min. value at $x = 1$ which is $f(1) = -4$ $\frac{1}{2}$

at $f''(x) = 0 \Rightarrow 6x = 0 \Rightarrow x = 0$

From the discuss of the sign of the function: $\frac{1}{2}$

$f''(x)$	- - - - -	0	+ + + + +
$f(x)$	∩		∪

• f(x) has an infliction point at $x = 0$ which is $(0, f(0))$ $\frac{1}{2}$

$(0, -2)$ $\frac{1}{2}$

b $f(x) = x(x^2 - 12) = x^3 - 12x$

$\therefore f'(x) = 3x^2 - 12$ $\frac{1}{2}$

at $f'(x) = 0$ $\frac{1}{2}$ $\Rightarrow 3(x^2 - 4) = 0 \Rightarrow x = \pm 2$

$2 \in [-1, 4]$ & $-2 \notin [-1, 4]$ $\frac{1}{2}$

$f(-1) = 11$, $f(2) = -16$, $f(4) = 16$

$\frac{1}{2}$ absolute min. value $\frac{1}{2}$ absolute max. value $\frac{1}{2}$

(تراعى الحلول الأخرى)

11-

(a) -50 $\triangle 1$

12-

(b) $]-\infty, 0[$ $\triangle 1$

13-

$x = \sec\theta \quad , \quad y = \tan\theta$

at $\theta = \frac{\pi}{6} \Rightarrow x = \sec\frac{\pi}{6} \Rightarrow x = \frac{2\sqrt{3}}{3}$

$\Rightarrow y = \tan\frac{\pi}{6} \Rightarrow y = \frac{\sqrt{3}}{3}$

$\left(\frac{2\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right) \in$ Curve $\triangle \frac{1}{2}$

$\because \frac{dx}{d\theta} = \sec\theta . \tan\theta \quad , \quad \frac{dy}{d\theta} = \sec^2\theta$ $\triangle \frac{1}{2}$

$\therefore \frac{dy}{dx} = \frac{\sec^2\theta}{\sec\theta . \tan\theta} = \frac{\sec\theta}{\tan\theta} = \csc\theta$ $\triangle \frac{1}{2}$

at $\theta = \frac{\pi}{6}$ The slope $= 2$ $\triangle \frac{1}{2}$

The equation of the tangent:

$y - \frac{\sqrt{3}}{3} = 2\left(x - \frac{2\sqrt{3}}{3}\right)$ $\triangle \frac{1}{2}$

The equation of the normal:

$y - \frac{\sqrt{3}}{3} = -\frac{1}{2}\left(x - \frac{2\sqrt{3}}{3}\right)$ $\triangle \frac{1}{2}$

14-

$$\sin y + \cos 2x = 0$$

$$\cos y \frac{dy}{dx} - 2\sin 2x = 0 \quad \frac{1}{2}$$

$$\cos y \frac{d^2y}{dx^2} + \left(-\sin y \frac{dy}{dx}\right)\left(\frac{dy}{dx}\right) - 4\cos 2x = 0 \quad \frac{1}{2}$$

$$\cos y \frac{d^2y}{dx^2} - \sin y \left(\frac{dy}{dx}\right)^2 = 4\cos 2x \quad \div(\cos y) \quad \frac{1}{2}$$

$$\frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^2 \tan y = 4\cos 2x \sec y \, . \quad \frac{1}{2}$$

(تراعى الحلول الأخرى)

15-

(b) 3 or 2 $\quad$ 1

16-

(b) The Curve of the function f has a local minimum value at $x=3$ $\quad$ 1

17-

$$\because y = a x^{b}$$

$$\therefore \frac{dy}{dt} = ab\, x^{b-1} \frac{dx}{dt} \quad \frac{1}{2}$$

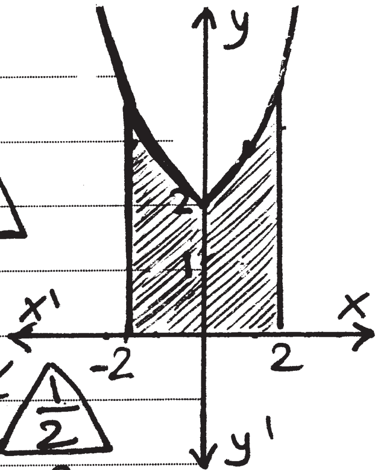
$$\frac{dy}{dt} = \frac{ab\, x^{b}}{x} \cdot \frac{dx}{dt} \quad \frac{1}{2}$$

Multiply by $\frac{1}{y}$

$$\therefore \frac{1}{y} \cdot \frac{dy}{dt} = \frac{1}{\cancel{a}\cancel{x^{b}}} \cdot \frac{\cancel{a}b\,\cancel{x^{b}}}{x} \cdot \frac{dx}{dt} \quad \frac{1}{2}$$

$$\frac{1}{y} \cdot \frac{dy}{dX} = \frac{b}{x} \cdot \frac{dx}{dt} \quad \frac{1}{2}$$

18-



$$V = \pi \int_{-2}^{2} y^2 \, dx$$ $\frac{1}{2}$

$$= 2\pi \int_{0}^{2} (x^2+2)^2 \, dx$$

$$= 2\pi \int_{0}^{2} (x^4 + 4x^2 + 4) \, dx$$ $\frac{1}{2}$

$$= 2\pi \left[\frac{1}{5}x^5 + \frac{4}{3}x^3 + 4x \right]_{0}^{2}$$ $\frac{1}{2}$

$$= 2\pi \left[\frac{32}{5} + \frac{32}{3} + 8 \right]$$

$$= \frac{752}{15}\pi$$ Volume unit. $\frac{1}{2}$

(تراعى الحلول الأخرى)

(انتهت الإجابة وتراعى الحلول الأخرى)