كل ما يحتاجه الطالب في جميع الصفوف من أوراق عمل واختبارات ومذكرات، يجده هنا في الروابط التالية لأفضل مواقع تعليمي إماراتي 100 %

تطبيق المناهج الإماراتية	الاجتماعيات	الرياضيات
الصفحة الرسمية على التلغرام	الاسلامية	العلوم
الصفحة الرسمية على الفيسبوك	الانجليزية	
التربية الاخلاقية لجميع الصفوف	اللغة العربية	
التربية الرياضية		

مجموعات التلغرام.	مجموعات الفيسبوك	قنوات تلغرام	قنوات الفيسبوك
الصف الأول	الصف الأول	الصف الأول	الصف الأول
الصف الثاني	الصف الثاني	الصف الثاني	الصف الثاني
الصف الثالث	الصف الثالث	الصف الثالث	الصف الثالث
الصف الرابع	الصف الرابع	الصف الرابع	الصف الرابع
الصف الخامس	الصف الخامس	الصف الخامس	الصف الخامس
الصف السادس	الصف السادس	الصف السادس	الصف السادس
الصف السابع	الصف السابع	الصف السابع	الصف السابع
الصف الثامن	الصف الثامن	الصف الثامن	الصف الثامن
الصف التاسع عام	الصف التاسع عام	الصف التاسع عام	الصف التاسع عام
الصف التاسع متقدم	الصف التاسع متقدم	الصف التاسع متقدم	تاسع متقدم
الصف العاشر عام	الصف العاشر عام	الصف العاشر عام	عاشر عام
الصف العاشر متقدم	الصف العاشر متقدم	الصف العاشر متقدم	عاشر متقدم
الحادي عشر عام	الحادي عشر عام	الحادي عشر عام	حادي عشر عام
الحادي عشر متقدم	الحادي عشر متقدم	الحادي عشر متقدم	حادي عشر متقدم
ثاني عشر عام	الثاني عشر عام	الثاني عشر عام	الثاني عشر عام
ثاني عشر متقدم	الثاني عشر متقدم	ثاني عشر متقدم	ثاني عشر متقدم

Chapter 10: Rotation

Concept Checks

10.1. c **10.2.** c **10.3.** a **10.4.** f **10.5.** b **10.6.** c **10.7.** c **10.8.** b **10.9.** b **10.10** b

Multiple-Choice Questions

10.1. b **10.2.** c **10.3.** b **10.4.** d **10.5.** c **10.6.** c **10.7.** c **10.8.** d **10.9.** b **10.10.** e **10.11.** b **10.12.** a **10.13.** c **10.14.** b **10.15.** c **10.16.** b **10.17.** a **10.18.** b **10.19.** c **10.20.** b

Conceptual Questions

10.21. Rotational kinetic energy is given by $K_{\text{rot}} = \frac{1}{2}cMv^2$

The total kinetic energy for an object rolling without slipping is given by:

$$K_{\text{total}} = K_{\text{trans}} + K_{\text{rot}} = \tfrac{1}{2}Mv^2(1+c) \text{ with } c = 2/5 \text{ for a sphere} \Rightarrow$$

$$\frac{K_{\text{rot}}}{K_{\text{total}}} = \frac{c}{1+c} = \frac{2/5}{1+2/5} = \frac{2}{7}$$

10.22. Assume negligible drag and no slipping. The object that reaches the bottom of the incline first will be the one with the lowest moment of inertia (that is, with the least resistance to rotation). The moments of inertia for the given objects are as follows: Thin ring: $I_{\text{r}} = MR^2$; Solid sphere: $I_{\text{ss}} = \frac{2}{5}MR^2$; Hollow sphere: $I_{\text{hs}} = \frac{2}{3}MR^2$; Homogeneous disk: $I_{\text{d}} = \frac{1}{2}MR^2$. Therefore, the order of the moments of inertia from smallest to greatest (assuming equal mass and radius) is I_{ss}, I_{d}, I_{hs} and I_{r}. Therefore, the order of finish of the objects in the race is: First: solid sphere; Second: homogeneous disk; Third: hollow sphere; Last: thin ring.

10.23. The net translational and rotational forces on both the solid sphere and the thin ring are, respectively, $F_{\text{net}} = ma = mg\sin\theta - f_{\text{static}}$ and $\tau = f_{\text{static}}r = I\alpha$, where the angular acceleration is $\alpha = a/r$. Since the moment of inertia for the solid sphere is $I = \left(2mr^2\right)/5$, the force of static friction is given by $f_{\text{static}} = 2ma/5$. Substitute this expression into the net force equation to solve for the acceleration of the solid sphere: $a_{\text{ss}} = 5g\left(\sin\theta\right)/7$. The moment of inertia for the thin ring is $I = mr^2$. Therefore, the force of static friction in this case is given by $f_{\text{static}} = ma$. Substitute this expression into the net force equation to solve for the acceleration of the thin ring: $a_{\text{r}} = g\left(\sin\theta\right)/2$. Therefore, the ratio of the acceleration is:

$$\frac{a_{\text{r}}}{a_{\text{ss}}} = \frac{\left(g\sin\theta\right)/2}{\left(5g\sin\theta\right)/7} = \frac{7}{10}.$$

10.24. The net translational and rotational forces on the solid sphere on the incline are, respectively, $F_{\text{net}} = ma = mg\sin\theta - f_{\text{static}}$ and $\tau = f_{\text{static}}r = I\alpha$, where the angular acceleration is $\alpha = a/r$. Since the moment of inertia for the solid sphere is $I = \left(2mr^2\right)/5$, the force of static friction is given by $f_{\text{static}} = 2ma/5$. Substituting this expression into the net force equation to solve for the acceleration gives $a = 5g\left(\sin\theta\right)/7$. Thus, $f_{\text{static}} = 2ma/5 = 2mg\left(\sin\theta\right)/7$. The limiting friction corresponding to a coefficient of static friction, μ_{s}, is $\max\left\{f_{\text{static}}\right\} = \mu_{\text{s}}F_{\text{normal}} = \mu_{\text{s}}mg\cos\theta$. For rolling without slipping to take place, it is required that

$$f_{\text{static}} \mu_s mg \max\{f_{\text{static}}\} \Rightarrow \frac{2mg\sin\theta}{7} \le \; _{\text{s}} \quad \cos\theta. \text{ Therefore, } \tan\theta \le \frac{7\mu_{\text{s}}}{2} \Rightarrow \theta \le \tan^{-1}\left(\frac{7\mu_{\text{s}}}{2}\right).$$

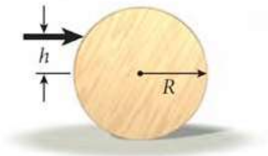
Thus, the maximum angle for which the sphere will roll without slipping is $\theta = \tan^{-1}\left(\frac{7\mu_{\text{s}}}{2}\right)$.

10.25. The "sharp horizontal blow" means that a force of magnitude F acts horizontally on the object along the arrow in the figure. With this force, we have to apply Newton's Second Law for linear motion ($F = Ma$) and Newton's Second Law for rotation ($\tau = I\alpha$). According to the problem text, the round object rolls without slipping. In Section 10.3 we have learned that this condition implies $v = R\omega$ and $a = R\alpha$. The force exerts a torque of magnitude $\tau = Fh$ (= magnitude of force times perpendicular distance) around the center of mass of the round object. Using all these relationships we can write

$$\tau = Fh = (Ma)h = M(R\alpha)h = MRh\alpha = I\alpha.$$

Simplifying and rearranging, we get

$$I = MRh \quad \Rightarrow \quad \frac{I}{MR^2} = \frac{h}{R}.$$



As a double-check, let's calculate h assuming a uniform solid sphere

$$h = \frac{RI}{MR^2} = \frac{R\left(\frac{2}{5}MR^2\right)}{MR^2} = \frac{2}{5}R.$$

So, for example, you might strike a cue ball with a horizontal cue a distance of $2R/5$ above the center of the cue ball to start it rolling without sliding.

10.26. (a) Since the path of the projectile is not a straight line about the origin (which would give an angular momentum of zero), the angular momentum can be determined by considering that the velocity of the projectile changes continuously along its path because of the change in the vertical component of velocity under the gravitational pull. If θ is the angle of projection, the horizontal component of velocity, $v_0 \cos\theta_0$, remains unchanged throughout the path and at the maximum height, the vertical component of velocity is zero and it has only the horizontal $v_0 \cos\theta_0$. The 'lever arm' for angular momentum at the maximum height is the maximum height itself, $\left(v_0^2 \sin^2\theta_0\right)/2g$ so that the angular momentum is

$$L = \frac{\left(mv_0 \cos\theta_0\right)\left(v_0^2 \sin^2\theta_0\right)}{2g}.$$

Since the angular momentum is conserved in this case, the angular momentum above is the same throughout the path.

(b) Since the angular momentum does not change throughout the path, the rate of change is zero.

(c) The rate of change of this angular momentum is the net torque about the origin, which also equals zero, that is:

$$\tau = \frac{dL}{dt} = \frac{d(0)}{dt} = 0.$$

10.27. For each object we convert the initial potential energy into kinetic energy at the bottom of the ramp.

Sphere: $Mgh_0 = \frac{1}{2}Mv^2(1+c_{\text{sphere}}) \Rightarrow v^2 = 2gh_0 / (1+c_{\text{sphere}})$

Cylinder: $Mgh = \frac{1}{2}Mv^2(1+c_{\text{cylinder}}) \Rightarrow v^2 = 2gh / (1+c_{\text{cylinder}})$

If the speed is to be the same in both cases, this means:

$$2gh / (1+c_{\text{cylinder}}) = 2gh_0 / (1+c_{\text{sphere}}) \Rightarrow$$

$$h = h_0 \frac{1+c_{\text{cylinder}}}{1+c_{\text{sphere}}} = h_0 \frac{1+1/2}{1+2/5} = h_0 \frac{3/2}{7/5} = h_0 \frac{15}{14}$$

10.28. To open a door (that is, to rotate a door about the hinges), a force must be applied so as to produce a torque about the hinges. Recall that torque is defined as $\vec{\tau} = \vec{r} \times \vec{F}$. The magnitude of this torque is then given by $\tau = rF\sin\theta$, where θ is the angle between the force applied at a point *p*, and the vector connecting the point *p* to the axis of rotation (to the axis of the hinges in this case). Therefore, torque is maximal when the applied force is perpendicular to the vector $\vec{r}$. That is, when the force is perpendicular to the plane of the door. Similarly, torque is minimal (i.e. zero) when the applied force is parallel to $\vec{r}$ (i.e. along the plane of the door).

10.29. Angular momentum is conserved, however, energy is not conserved; her muscles must provide an additional centripetal acceleration to her hands to pull them inwards. That force times the displacement is equal to the work that she does in pulling them in. Since she is doing work on the system, energy is not conserved.

10.30. Consider a particle of constant mass, *m*, which starts at position, r_0, moving with velocity, *v*, and having no forces acting on it. By Newton's first law of motion, the absence of forces acting on it means that it must continue to move in a straight line at the same speed, so its equation of motion is given by $r = r_0 + vt$. Its linear momentum is *mv*, so its angular momentum relative to the origin is given by $\vec{L} = \vec{r} \times m\vec{v} = (\vec{r}_0 + \vec{v}t) \times m\vec{v}$. The cross product is distributive over addition, so this can be rewritten as $\vec{L} = (\vec{r}_0 \times m\vec{v}) + (\vec{v}t \times m\vec{v})$. Clearly the vectors $\vec{v}t$ and $m\vec{v}$ are parallel, since they are both in the direction of $\vec{v}$, and the cross product of two parallel vectors is zero. So, the last term in the sum above comes to zero, and the expression can be rewritten as $\vec{L} = \vec{r}_0 \times m\vec{v}$. Now $\vec{r}_0$, *m* and $\vec{v}$ are all constants in this system, so it follows that $\vec{L}$ is also constant, as required by the law of conservation of angular momentum. Therefore, whether or not the particle has angular momentum is dependent on the r_0 vector, given non-zero velocity. If the path of the particle crosses the origin, $r_0 = 0$ and the particle has no angular momentum relative to the origin. In every other case, the particle will have constant, non-zero angular momentum relative to the origin.

10.31. Work is given by $W = Fd\cos\theta$ for linear motion, and by $W = \tau\theta$ for angular motion, where the torque, τ, is applied through a revolution of θ.

(a) Gravity points downward, therefore, $W_{\text{gravity}} = mg(s)\sin\theta$.

(b) The normal force acts perpendicular to the displacement. Therefore, $\cos 90 = 0 \Rightarrow W_{\text{normal}} = 0$.

(c) The frictional force considered in this problem is that of static friction since the cylinder is rolling without slipping. The direction of the static friction is opposite to that of the motion. Work done by the frictional force consists of two parts; one is the contribution by translational motion and the other is the contribution by rotational motion.

(i) Translational motion: $W_{\text{translation}} = (f_s)(s)\cos(180°) = -f_s s$

(ii) Rotational motion: $W_{\text{rotational}} = \tau\theta = Rf_s\theta = (f_s)\theta = f_s s$

Therefore, the total contribution to work done by the friction is zero.

10.32. Mechanical energy is conserved for the rolling motion without slipping. Setting the top as the vertical origin, the mechanical energy at the top is $E_t = K + U = 0 + 0$. The mechanical energy at the bottom is $E_b = K(\text{translational} + \text{rotational}) + U = \left[(1+c)mv^2/2\right] + (-mgh)$, where h is the vertical height. By conservation of energy, $E_t = E_b$ implies $v = \sqrt{2gh/(1+c)} = \sqrt{\frac{4gh}{3}}$, $\sin\theta = \frac{h}{s} \Rightarrow v = \sqrt{\frac{4}{3}sg\sin\theta}$, where c is 1/2 for the cylindrical object. Using $v_f^2 - v_i^2 = 2as$, the acceleration is

$$a = \frac{2gh/(1+c)}{2s} = \frac{2gs\sin\theta/(1+c)}{2S} = g\frac{\sin\theta}{(1+c)}.$$

For a cylinder, $c = 1/2$. Therefore, $a = \frac{2}{3}g\sin\theta$.

10.33. Prove that the pivot point about which the torque is calculated is arbitrary. First, consider the definition of torque, $\vec{\tau} = \vec{r} \times \vec{F}$. Therefore, for each of the applied forces, $\vec{F}_1$ and $\vec{F}_2 = -\vec{F}_1$, their contributions to the torque are given by $\vec{\tau}_1 = \vec{r}_1 \times \vec{F}_1$ and $\vec{\tau}_2 = \vec{r}_2 \times \vec{F}_2$, where $\vec{r}_1$ and $\vec{r}_2$ are the respective distances to the pivot point. The net torque is calculated from the algebraic sum of the torque contributions, that is, $\vec{\tau}_{\text{net}} = \vec{\tau}_1 + \vec{\tau}_2 = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 = \vec{r}_1 \times \vec{F}_1 - \vec{r}_2 \times \vec{F}_1$. Since the cross product is distributive, $\vec{\tau}_{\text{net}} = (\vec{r}_1 - \vec{r}_2) \times \vec{F}_1 = \vec{d} \times \vec{F}_1$. Therefore, the net torque produced by a couple depends only on the distance between the forces, and is independent of the actual pivot point about which the contributing torques are calculated or the actual points where the two forces are applied.

10.34. It is actually the act of pulling in her arms that makes the figure skater increase her angular velocity. Since angular momentum is conserved $\left(I\omega_1 = I\omega_2 = \text{constant}\right)$, by reducing her rotational inertia (by means of reducing the distance between her arms and hands to the axis of rotation), the figure skater increases her angular velocity.

10.35. By momentarily turning the handlebars to the left, the contact point of the motorcycle with the ground moves to the left of the center of gravity of the motorcycle so that the motorcycle leans to the right. Now the motorcycle can be turned to the right and the rider can lean to the right. The initial left turn creates a torque that is directed upwards, which deflects the angular momentum of the front tire upward and causes the motorcycle to lean to the right. In addition, as the motorcycle leans to the right, a forward pointing torque is induced that tends to straighten out the front wheel, preventing over-steering and oscillations. At low speeds, these effects are not noticeable, but at high speeds they must be considered.

10.36. The Earth-Moon system, to a good approximation, conserves its angular momentum (though the Sun also causes tides on the Earth). Thus, if the Earth loses angular momentum, the moon must gain it. If there were 400 days in a year in the Devonian period, the day was about 10% shorter, meaning the angular velocity of the Earth was about 10% greater. Since the rotational inertia of the Earth is virtually unchanged, this means that the rotational angular momentum of the Earth was then about 10% greater, and correspondingly the orbital angular momentum of the Moon was about 10% less.

10.37. In this problem, the key is that the monkey is trying to reach the bananas by climbing the rope. Since the monkey has the same mass as the bananas, if he didn't try to climb the rope, both the net torque and total angular momentum on the pulley would be zero. Take counterclockwise to be positive just for aesthetics.

(a) Consider the extra tension provided by the monkey on the rope by climbing (i.e. by pulling on the rope). Average out the force caused by the monkeys pulling with a constant force downwards, $\vec{T}$, on the monkey side. Therefore, the net torque on the pulley axis is provided by this extra force, $\vec{T}$, as

$$\vec{\tau}_{\text{net}} = \left[\left(\vec{F}_{\text{monkey}} + \vec{T}\right) - \vec{F}_{\text{banana}}\right]R = \vec{T}R.$$

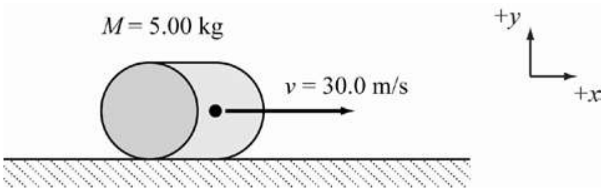
(b) Since there is now a non-zero net torque on the pulley, there is a non-zero total angular momentum given by

$$\vec{\tau}_{\text{net}} = \frac{d\vec{L}}{dt} \Rightarrow \vec{L} = \int \tau\, dt.$$

Using the results of part (a), the above expression can be rewritten as $\vec{L} = \int \vec{T}R dt = \vec{T}Rt.$ Recall that the extra "climbing" force was taken to be constant. In reality, the monkey's pulling will be time dependent and this will affect the final form of the time dependent angular momentum.

Exercises

10.38. **THINK:** Determine the energy of a solid cylinder as it rolls on a horizontal surface. The mass of the cylinder is $M = 5.00$ kg and the translational velocity of the cylinder's center of mass is $v = 30.0$ m/s.
SKETCH:



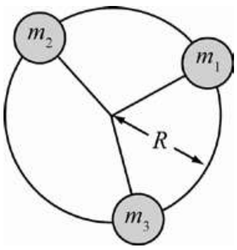
RESEARCH: Since the motion occurs on a horizontal surface, consider only the total kinetic energy of the cylinder, $K_{\text{T}} = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$. $I = cMR^2$ and, for a solid cylinder, $c = 1/2$ as in Table 10.1. For rolling without slipping, $v = \omega R$.

SIMPLIFY: $K_{\text{T}} = \frac{1}{2}Mv^2 + \frac{1}{2}\left(cMR^2\right)\left(\frac{v^2}{R^2}\right) = \frac{1}{2}Mv^2\left(1+c\right)$

CALCULATE: $K_{\text{T}} = \frac{1}{2}\left(5.00 \text{ kg}\right)\left(30.0 \text{ m/s}\right)^2\left(1+1/2\right) = 3375 \text{ J}$

ROUND: Both given values have three significant figures, so the result is rounded to $K_{\text{T}} = 3.38 \cdot 10^3$ J.
DOUBLE-CHECK: The calculated result has Joules as units, which are units of energy. This means that the calculated result is plausible.

10.39. **THINK:** The children can be treated as point particles on the edge of a circle and placed so they are all the same distance, R, from the center. Using the conversion, 1 kg = 2.205 lbs, the three masses are $m_1 = 27.2$ kg, $m_2 = 20.4$ kg and $m_3 = 36.3$ kg. Using the conversion 1 m = 3.281 ft, the distance is $R = 3.657$ m.
SKETCH:



RESEARCH: The moment of inertia for point particles is given by $I = \sum_i m_i r_i^2$.

SIMPLIFY: $I = \left(m_1 + m_2 + m_3\right)R^2$

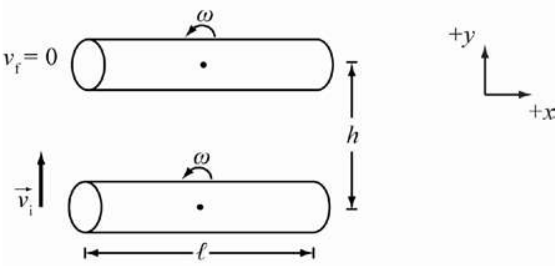
CALCULATE: $I = \left(27.2 \text{ kg} + 20.4 \text{ kg} + 36.3 \text{ kg}\right)\left(3.66 \text{ m}\right)^2 = 1123.9 \text{ kg m}^2$

ROUND: $I = 1.12 \cdot 10^3$ kg m^2

DOUBLE-CHECK: Since the children are located on the edge of the merry-go-round, a large value for I is expected.

10.40. **THINK:** Since the pen with a length of l = 24 cm rotates at a constant rate, rotational kinetic energy remains constant so only the translational energy is converted to potential energy at a height of h = 1.2 m from release. Use kinematics to determine the time it takes the pen to reach the top and make 1.8 revolutions, in order to determine ω.

SKETCH:



RESEARCH: The pen has a translational kinetic energy of $K_T = mv^2/2$, where v_i is the velocity at release. The potential energy at the top is given by $U = mgh$ and $mgh = mv_i^2/2$. The rotational kinetic energy is given by $K_R = I\omega^2/2$, where $I = ml^2/12$. The initial velocity of the pen is determined from $v_f^2 = v_i^2 - 2gh$ and the time of flight is given by $t = -(v_f - v_i)/g$. Angular velocity is given by $\omega = 2\pi(1.8 \text{ rev})/t$.

SIMPLIFY: The final velocity is zero, so the expression reduces to $0 = v_i^2 - 2gh \Rightarrow v_i = \sqrt{2gh}$. The expression for the time of flight also reduces to

$$t = -\frac{(0 - v_i)}{g} = \sqrt{\frac{2h}{g}}.$$

The angular velocity is given by $\omega = \frac{3.6\pi}{\sqrt{2h/g}} = 3.6\pi\sqrt{\frac{g}{2h}}$ rad/s. The ratio is then:

$$\frac{K_R}{K_T} = \frac{\frac{1}{2}I\omega^2}{\frac{1}{2}mv^2} = \frac{\frac{1}{2}\left(\frac{1}{12}ml^2\right)(3.6\pi)^2\frac{g}{2h}}{mgh} = \frac{12.96}{48}\pi^2\frac{l^2}{h^2}.$$

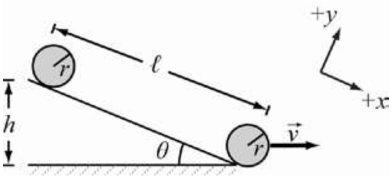
CALCULATE: $\frac{K_R}{K_T} = \frac{12.96}{48}\pi^2\left(\frac{0.24 \text{ m}}{1.2 \text{ m}}\right)^2 = 0.1066$

ROUND: To three significant figures: $\frac{K_R}{K_T} = 0.107.$

DOUBLE-CHECK: Remember that this is not a case of rolling without slipping. Here the translational motion is independent of the rotational motion, and so the ratio between translational and rotational kinetic energies could have almost any value. A simple double-check is thus not easily possible.

10.41. **THINK:** With no friction and no slipping, each object with mass, m = 1.00 kg, conserves energy. Since each object starts at the same height, they all have the same potential energy and hence kinetic energy after they travel a distance l = 3.00 m at an incline of $\theta = 35.0°$. Each ball has a radius of r = 0.100 m. Whichever object has the highest velocity at the bottom should reach the bottom first, and vice versa.

SKETCH:



RESEARCH: The constant *c* is related to the geometry of a figure. The values of *c* for different objects can be found in Table 10.1. The solid sphere has $c_1 = 2/5$, the hollow sphere has $c_2 = 2/3$ and the ice cube has $c_3 = 0$. Since energy is conserved, the velocity of each object at the bottom is $v = \sqrt{2gh/(1+c)}$. The incline shows that $h = l\sin\theta$.

SIMPLIFY: $v_1 = \sqrt{\dfrac{2gl\sin\theta}{1+c_1}}$, $v_2 = \sqrt{\dfrac{2g\sin\theta}{1+c_2}}$, $v_3 = \sqrt{\dfrac{2g\sin\theta}{1+c_3}}$

CALCULATE: $v_1 = \sqrt{\dfrac{10}{7}}\sqrt{gl\sin\theta}$, $v_2 = \sqrt{\dfrac{6}{5}}\sqrt{gl\sin\theta}$, $v_3 = \sqrt{2}\sqrt{gl\sin\theta}$

(a) Since the velocity is inversely proportional to *c*, the object with a smaller *c* will have a larger velocity than that of one with a greater c, and will reach the end first. Since $c_1 < c_2$, the solid sphere reaches the bottom first.

(b) Since $c_3 < c_1$, and the velocity is inversely proportional to *c*, the ice cube travels faster than the solid ball at the base of the incline.

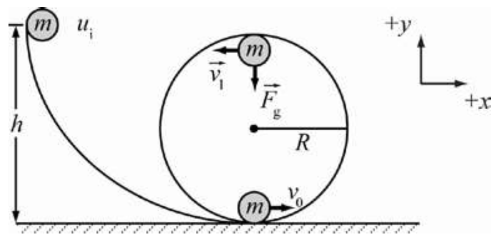
(c) $v_1 = \sqrt{\dfrac{10}{7}}\sqrt{(9.81\text{ m/s}^2)(3.00\text{ m})\sin(35.0°)} = 4.911\text{ m/s}$

ROUND: Parts (a) and (b) do not need to be rounded. (c) $v_1 = 4.91$ m/s

DOUBLE-CHECK: It is reasonable that the ice cube reaches the bottom first since it does not have to contribute any energy to rotational motion. As expected, the velocity of the sphere is less than it would be if it were in freefall $\left(v = \sqrt{2gl} = 8\text{ m/s}\right)$.

10.42. **THINK:** With no friction and no slipping, the object of mass, *m*, and radius, *r*, will conserve energy. Therefore, the potential energy of the ball at height, *h*, should equal the potential energy at the top of the loop of radius, *R*, plus translational and rotational kinetic energy. For the ball to complete the loop, the minimum velocity required is the one where the normal force of the loop on the ball is 0 N, so that the centripetal force is solely the force of gravity on the ball.

SKETCH:



RESEARCH: The only force on the ball at the top of the loop is $F_g = mg = mv_1^2/R$. The initial potential energy is given by $U_i = mgh$ and the final potential energy is given by $U_f = mg(2R)$. The kinetic energy at the top of the loop is $K = (1+c)mv_1^2/2$, where the *c* value for a solid sphere is 2/5.

SIMPLIFY: The conservation of energy is given by

$$U_i = U_f + K \Rightarrow mgh = 2mgR + \frac{1}{2}(1+c)mv_1^2.$$

From the forces, $mg = \frac{mv_1^2}{R} \Rightarrow v_1^2 = gR$. Therefore, $mgh = 2mgR + \frac{1}{2}\left(\frac{7}{5}\right)mgR \Rightarrow h = R\left(2+\frac{7}{10}\right) = \frac{27}{10}R.$

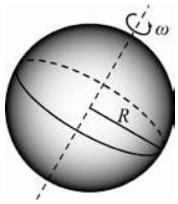
CALCULATE: Not applicable.

ROUND: Not applicable.

DOUBLE-CHECK: The height is greater than $2R$, which neglecting rotational energy would be the minimum energy needed, so the result is reasonable.

10.43. **THINK:** The change in energy should be solely that of the change in rotational kinetic energy. Assume the pulsar is a uniform solid sphere with $m \approx 2\cdot 10^{30}$ kg and $R = 12$ km. Initially, the pulsar rotates at $\omega = 60\pi$ rad/s and has a period, T which is increased by 10^{-5} s after 1 y. We calculate the power emitted by the pulsar by taking the time derivative of the rotational kinetic energy. The power output of the Sun is $P_{\text{Sun}} = 4\cdot 10^{26}$ W.

SKETCH:



RESEARCH: The kinetic energy is given by $K = I\omega^2/2$, so

$$P_{\text{Crab}} = -\frac{dK}{dt} = -I\omega\frac{d\omega}{dt}.$$

The angular velocity is given by $\omega = 2\pi/T$, so

$$\frac{d\omega}{dt} = -\frac{2\pi}{T^2}\frac{dT}{dt} = -\frac{2\pi}{\left(\frac{2\pi}{\omega}\right)^2}\frac{dT}{dt} = -\frac{\omega^2}{2\pi}\frac{dT}{dt}.$$

The moment of inertia of a sphere is

$$I = \frac{2}{5}mR^2.$$

SIMPLIFY: Combining our equations gives us

$$P_{\text{Crab}} = \frac{2}{5}mR^2\omega\frac{\omega^2}{2\pi}\frac{dT}{dt} = \frac{mR^2\omega^3}{5\pi}\frac{dT}{dt}.$$

CALCULATE: First we calculate the change in the period over one year,

$$\frac{dT}{dt} = \frac{10^{-5}\text{ s}}{(365\text{ days})(24\text{ hour/day})(3600\text{ s/hour})} = 3.17\cdot 10^{-13}.$$

The power emitted by the pulsar is

$$P_{\text{Crab}} = \frac{dK}{dt} = \frac{(2\cdot 10^{30}\text{ kg})(12\cdot 10^3\text{ m})^2(60\pi\text{ rad/s})^3}{5\pi}(3.17\cdot 10^{-13}) = 3.89\cdot 10^{31}\text{ W}.$$

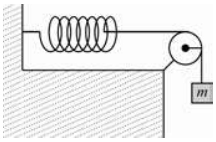
So the ratio of the power emitted by the pulsar to the power emitted by the Sun is

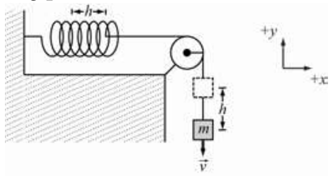
$$\frac{P_{\text{Crab}}}{P_{\text{Sun}}} = \frac{3.89\cdot 10^{31}\text{ W}}{4\cdot 10^{26}\text{ W}} = 9.73\cdot 10^4.$$

ROUND: $\frac{P_{\text{Crab}}}{P_{\text{Sun}}} = 1\cdot 10^5.$

DOUBLE-CHECK: Our result for the ratio of the loss in rotational energy of the Crab Pulsar is close to the expected value of 100,000.

10.44. **THINK:** With no friction and no slipping, mechanical energy is conserved. This means that the potential energy of the block of mass m = 4.00 kg will be converted into the potential energy of the spring with a constant of k = 32.0 N/m, kinetic energy of the block and rotational energy of the pulley with a radius of R = 5.00 cm and mass M = 8.00 kg. If the block falls a distance h, then the spring is extended by a distance h as well. Consider the lower position of the block in parts (a) and (b) to be at zero potential. In part (a), the block falls a distance h = 1.00 m. In part (b), when the block comes to rest, the kinetic energy of the system is zero so that the block's potential energy is converted entirely into spring potential.

SKETCH: (a)


(b)


RESEARCH: The initial energy of the system is $E_i = U_i = mgh$.

(a) The final energy is $E_f = \frac{1}{2}k(x_0 - h)^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$.

(b) The final energy is $E_f = k(x-h)^2/2$. The moment of inertia of the wheel is $MR^2/2$. With no slipping, $R\omega = v$. Let $x_0 = 0$ for the spring equilibrium.

SIMPLIFY:

(a) $E_i = E_f \Rightarrow mgh = \frac{1}{2}kh^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v^2}{R^2}\right) + \frac{1}{2}mv^2$. Therefore,

$$mgh - \frac{1}{2}kh^2 = \left(\frac{1}{4}M + \frac{1}{2}m\right)v^2 \Rightarrow v = \sqrt{\frac{mgh - \frac{1}{2}kh^2}{\frac{1}{4}M + \frac{1}{2}m}}.$$

(b) $E_i = E_f \Rightarrow mgh = \frac{1}{2}kh^2 \Rightarrow h = \frac{2mg}{k}$

CALCULATE:

(a) $$v = \sqrt{\frac{(4.00 \text{ kg})(9.81 \text{ m/s}^2)(1.00 \text{ m}) - \frac{1}{2}(32.0 \text{ N/m})(1.00 \text{ m})^2}{\frac{1}{4}(8.00 \text{ kg}) + \frac{1}{2}(4.00 \text{ kg})}} = 2.410 \text{ m/s}$$

(b) $$h = \frac{2(4.00 \text{ kg})(9.81 \text{ m/s}^2)}{32.0 \text{ N/m}} = 2.45 \text{ m}$$

ROUND: Three significant figures:

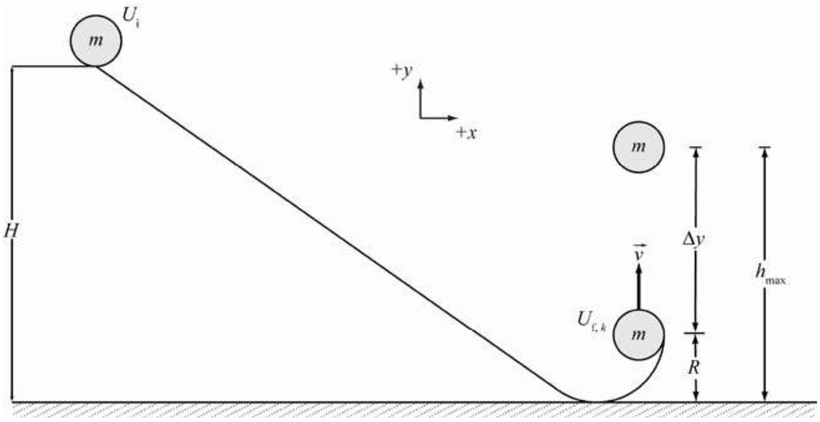
(a) The block has a speed of v = 2.41 m/s after it has fallen 1.00 m.

(b) The maximum extension of the spring is h = 2.45 m.

DOUBLE-CHECK: For part (a), the speed should be less than it is for free fall $\left(v = \sqrt{2gh} = 4.4 \text{ m/s}\right)$, which it is. For part (b), the distance is reasonable.

10.45. **THINK:** With no friction and no slipping, the energy of the object of mass, m, and radius, r, is conserved. This means that the initial potential energy at height H = 6.00 m is equal to the potential energy at height R = 2.50 m, plus its rotational and translational energy. The object has a c value of 0.400. Using conservation of energy, the velocity of the object can be determined. Then, using kinematics, the maximum height the object achieves can be determined. Let the subscript i indicate the ball is at the top of the ramp, and the subscript f indicate the ball is at the end of the ramp, at the launch point.

SKETCH:



RESEARCH: The initial energy of the ball is $E_i = U_i = mgH$. The final energy of the ball is $E_f = U_f + K \Rightarrow E_f = mgR + \left[(1+c)mv^2/2\right]$, where c = 0.400. The kinematics equation for the velocity is $v_f^2 = v_i^2 + 2g\Delta y$. Since the ball is at rest at the top, the equation becomes $v^2 = 2g\Delta y \Rightarrow \Delta y = v^2/2g$. The maximum height achieved is $\Delta y + R = h_{max}$.

SIMPLIFY: $E_i = E_f \Rightarrow mgH = mgR + \frac{1}{2}(1+c)mv^2 \Rightarrow mg(H-R) = \frac{1}{2}(1+c)mv^2 \Rightarrow v^2 = \frac{2g(H-R)}{1+c}$

$h_{max} = \Delta y + R = \frac{v^2}{2g} + R = \frac{(H-R)}{1+c} + R$

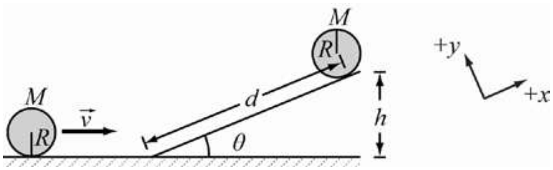
CALCULATE: $h_{max} = \frac{(6.00\text{ m} - 2.50\text{ m})}{1+0.400} + 2.50\text{ m} = 5.000\text{ m}$

ROUND: $h_{max} = 5.00\text{ m}$

DOUBLE-CHECK: If the object did not rotate, the mass is expected to reach its original height of 6 m. Since the object does rotate, the height it reaches should be less than the original height.

10.46. **THINK:** In both cases, energy should be conserved. In part (a), if the ball of mass, *M*, and radius, *R*, continues to spin at the same rate, then there is no change in rotational kinetic energy and only the translational energy is converted to potential energy. In part (b), there is slipping so both rotational and translational kinetic energy are converted to potential energy. The ball has an initial velocity of 3.00 m/s and travels a distance, *d*, up an incline with an angle of $\theta = 23.0°$.

SKETCH:



RESEARCH:

(a) The initial translational kinetic energy is given by $K_T = mv^2/2$, the initial rotational energy is given by $K_R = \frac{1}{2}I\omega^2$, and the final potential energy is given by $U_f = mgh$.

(b) The initial kinetic energy is $K = (1+c)mv^2/2$ and the final potential energy is $U_f = mgh$. The height of the ball is given by the expression $h = d\sin\theta$, and c = 2/5 for a solid sphere.

SIMPLIFY:

(a) $E_{\text{i}} = E_{\text{f}} \;\Rightarrow\; \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgd\sin\theta + \frac{1}{2}I\omega^2 \;\Rightarrow\; d = \frac{1}{2}\frac{v^2}{g\sin\theta}$

(b) $E_{\text{i}} = E_{\text{f}} \;\Rightarrow\; \frac{1}{2}(1+c)mv^2 = mgd\sin\theta \;\Rightarrow\; d = \frac{v^2(1+c)}{2g\sin\theta} = \frac{7}{10}\frac{v^2}{g\sin\theta}$

CALCULATE:

(a) $d = \frac{(3.00\text{ m/s})^2}{2(9.81\text{ m/s}^2)\sin(23.0°)} = 1.174\text{ m}$

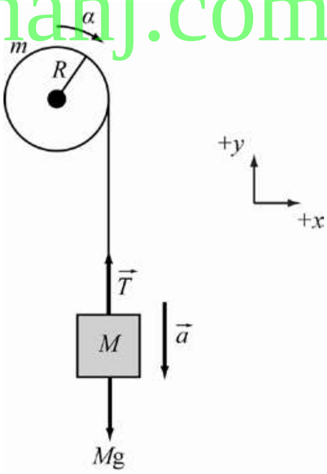
(b) $d = \frac{7(3.00\text{ m/s})^2}{10(9.81\text{ m/s}^2)\sin(23.0°)} = 1.644\text{ m}$

ROUND:
Rounding to three significant figures:
(a) $d = 1.17$ m
(b) $d = 1.65$ m

DOUBLE-CHECK: Since the ball in part (b) does contribute rotational energy to the potential, it is expected to go higher up the ramp and hence have a larger value for d.

10.47. **THINK:** The hanging block with a mass of M = 70.0 kg, will cause a tension, T, in the string that will in turn produce a torque, τ, in the wheel with a mass, m = 30.0 kg, and a radius, R = 40.0 cm. This torque will give the wheel an angular acceleration, α. If there is no slipping, then the angular acceleration of the wheel is directly related to the acceleration of the block.

SKETCH:



RESEARCH: The balance of forces is given by $T - Mg = -Ma$. The torque produced by the tension, T, is given by $\tau = TR = I\alpha$, where I of the wheel is $mR^2/2$. With no slipping, $R\alpha = a$.

SIMPLIFY: First, determine the tension, $T \;\Rightarrow\; T = M(g-a)$. This expression can be substituted into the torque equation to solve for a:

$$M(g-a)R = \frac{1}{2}mR^2\left(\frac{a}{R}\right) \;\Rightarrow\; MgR - MaR = \frac{1}{2}mRa \;\Rightarrow\; Mg = \left(\frac{1}{2}m + M\right)a \;\Rightarrow\; a = \frac{Mg}{\frac{1}{2}m + M}.$$

CALCULATE: $a = \frac{70.0\text{ kg}(9.81\text{ m/s}^2)}{\frac{1}{2}(30.0\text{ kg}) + 70.0\text{ kg}} = 8.079\text{ m/s}^2$

ROUND: $a = 8.08 \text{ m/s}^2$
DOUBLE-CHECK: Since there is tension acting opposite gravity, the overall acceleration of the hanging mass should be less than g.

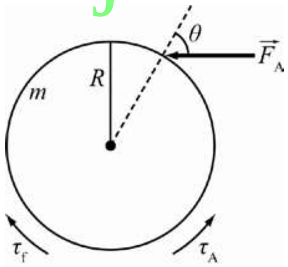
10.48. **THINK:** The torque is simply the cross product of the vectors, $\vec{r} = \left(4\hat{x}+4\hat{y}+4\hat{z}\right)$ m and $\vec{F} = \left(2\hat{x}+3\hat{y}\right)$ N.
SKETCH: Not applicable.
RESEARCH: $\vec{\tau} = \vec{r} \times \vec{F}$
SIMPLIFY: $\vec{\tau} = \left(4\hat{x}+4\hat{y}+4\hat{z}\right)\times\left(2\hat{x}+3\hat{y}\right)$ N m

$$= \left[8\left(\hat{x}\times\hat{x}\right)+12\left(\hat{x}\times\hat{y}\right)+8\left(\hat{y}\times\hat{x}\right)+12\left(\hat{y}\times\hat{y}\right)+8\left(\hat{z}\times\hat{x}\right)+12\left(\hat{z}\times\hat{y}\right)\right] \text{ N m}$$

CALCULATE: $\vec{\tau} = \left[8(0)+12(\hat{z})+8(-\hat{z})+12(0)+8(\hat{y})+12(-\hat{x})\right] \text{ N m} = \left(-12\hat{x}+8\hat{y}+4\hat{z}\right) \text{ N m}$
ROUND: Not applicable.
DOUBLE-CHECK: The magnitude of the calculated torque is about 15. As required, this number is smaller than (or at most equal to) the product of the magnitudes of the force and the position vectors, which is about 25 in this case.

10.49. **THINK:** There are two forces, $F_1 = 70.0$ N (applied from 0 to 2.00 seconds) and $F_2 = 24.0$ N (applied after 2.00 seconds). These forces are applied at an angle of $\theta = 37.0°$ on the surface of a disk of mass, m = 14.0 kg, and diameter of d = 30.0 cm (radius, R = 15.0 cm). After 2.00 seconds, the disk moves at a constant angular speed, ω. This means that the sum of the torques is zero, so the torque produced by friction is equal and opposite the torque produced by the applied force. Assuming the frictional torque is constant, the angular acceleration, α, of the disk from 0 to 2.00 seconds can be calculated and ω can be determined.

SKETCH:



RESEARCH: The torque that F_A produces is $\tau_A = RF_A \sin\theta$. After t = 2.00 s, when ω is constant, $\sum\tau = 0 = \tau_A - \tau_f$, where τ_f is the frictional torque. For t = 0 to t = 2 s, $\sum\tau = \tau_A - \tau_f = \tau_{net}$ and $\tau_{net} = I\alpha$. Starting from rest, $\omega = \alpha t$. The rotational kinetic energy of the wheel after t = 2.00 s is then $K = I\omega^2/2$, where $I = mR^2/2$.
SIMPLIFY:
(a) $\sum\tau = \tau_A - \tau_f = 0 \Rightarrow \tau_A = \tau_f = RF_2\sin\theta$

(b) $\tau_{net} = \tau_A - \tau_f = RF_1\sin\theta - RF_2\sin\theta = I\alpha \Rightarrow \alpha = \dfrac{R\sin\theta\left(F_1-F_2\right)}{\frac{1}{2}mR^2} = \dfrac{2\sin\theta\left(F_1-F_2\right)}{mR}$

$$\omega = \alpha t = \frac{2\sin\theta\left(F_1-F_2\right)}{mR}t$$

(c) $K = \dfrac{1}{2}I\omega^2 = \dfrac{1}{4}mR^2\omega^2$

CALCULATE:

(a) $\tau_{\text{f}} = (0.150\text{ m})(24.0\text{ N})\sin(37.0°) = 2.167\text{ Nm}$

(b) $\omega = \dfrac{2\sin(37.0°)(70.0\text{ N} - 24.0\text{ N})(2.00\text{ s})}{(14.0\text{ kg})(0.150\text{ m})} = 52.73\text{ rad/s}$

(c) $K = \dfrac{1}{4}(14.0\text{ kg})(0.150\text{ m})^2(52.73\text{ rad/s})^2 = 218.96\text{ J}$

ROUND:

(a) $\tau_{\text{f}} = 2.17\text{ Nm}$

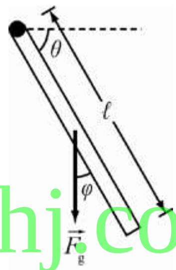
(b) $\omega = 52.7\text{ rad/s}$

(c) $K = 219\text{ J}$

DOUBLE-CHECK: Given the initial variables, these results are reasonable.

10.50. **THINK:** When the rod is at an angle of $\theta = 60.0°$ below the horizontal, the force of gravity acting at the center of mass of the rod, with mass, m = 2.00 kg, and length, l = 1.00 m, will produce a torque, τ, and hence an angular acceleration, α. If the rod has a uniform density, then the center of mass is at the geometric center of the rod.

SKETCH:



RESEARCH: From geometry it can be shown that $\theta + \varphi = 90°$. Therefore, $\varphi = 90° - \theta = 90° - 60° = 30°$. The torque that the force of gravity produces is $\tau = mg(l/2)\sin\varphi = I\alpha$, where $I = ml^2/3$.

SIMPLIFY: $mg\left(\dfrac{l}{2}\right)\sin\varphi = \dfrac{1}{3}ml^2\alpha \Rightarrow \alpha = \dfrac{3g\sin\varphi}{2l}$

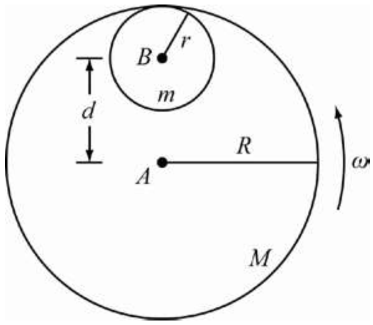
CALCULATE: $\alpha = \dfrac{3(9.81\text{ m/s}^2)\sin(30.0°)}{2(1.00\text{ m})} = 7.35\text{ rad/s}^2$

ROUND: To three significant figures: $\alpha = 7.35\text{ rad/s}^2$

DOUBLE-CHECK: The vertical component of the tangential acceleration at the end is given by $a_{\text{v}} = a_{\text{T}}\sin\theta = l\alpha\sin\theta \approx 6\text{ m/s}^2$, which is less than g. This is expected since the pivot is causing the rod to swing and the vertical displacement of the end is slowing down and a smaller acceleration is expected.

10.51. **THINK:** Each object has its own moment of inertia, I_{A} and I_{B}. Disk *A* with a mass, M = 2.00 kg, and a radius, R = 25.0 cm, rotates about its center of mass while disk *B* with a mass, m = 0.200 kg and a radius, r = 2.50 cm, rotates a distance, $d = R - r$, away from the axis. This means the parallel axis theorem must be used to determine the overall moment of inertia of disk *B*, I'_{B}. The total moment of inertia is the sum of the two. If a torque, $\tau = 0.200\text{ Nm}$, is applied then it will cause an angular acceleration, α. If the disk initially rotates at $\omega = -2\pi\text{ rad/s}$, then kinematics can be used to determine how long it takes to slow down.

SKETCH:



RESEARCH: The moment of inertia of disk A is $I_{\text{A}} = MR^2/2$. The moment of inertia of disk B is $I_{\text{B}} = mr^2/2$. Since disk B is displaced by $d = R - r$ from the axis of rotation, $I'_{\text{B}} = I_{\text{B}} + md^2$, by the parallel axis theorem. Therefore, the total moment of inertia is $I_{\text{tot}} = I_{\text{A}} + I'_{\text{B}}$. The torque that is applied produces $\tau = I_{\text{tot}}\alpha$, where $\alpha = (\omega_{\text{f}} - \omega_{\text{i}})/\Delta t$.

SIMPLIFY:

(a) $I_{\text{tot}} = I_{\text{A}} + I'_{\text{B}} = I_{\text{A}} + I_{\text{B}} + m(R-r)^2$

$$= \frac{1}{2}MR^2 + \frac{1}{2}mr^2 + mR^2 - 2mRr + mr^2 = \left(\frac{1}{2}M + m\right)R^2 + \frac{3}{2}mr^2 - 2mRr$$

(b) $\tau = I_{\text{tot}}\alpha = \frac{I_{\text{tot}}(0 - \omega_{\text{i}})}{t} \Rightarrow t = -\frac{I_{\text{tot}}\omega_{\text{i}}}{\tau}$

CALCULATE:

(a) $I_{\text{tot}} = \left(\frac{1}{2}(2.00) + 0.200\right)(0.250)^2 + \frac{3}{2}(0.200)(0.0250)^2 - 2(0.200)(0.0250)(0.250) = 0.0726875 \text{ kg m}^2$

(b) $t = -\frac{(0.0726875 \text{ kg m}^2)(-2\pi \text{ rad/s})}{0.200 \text{ N m}} = 2.284 \text{ s}$

ROUND:

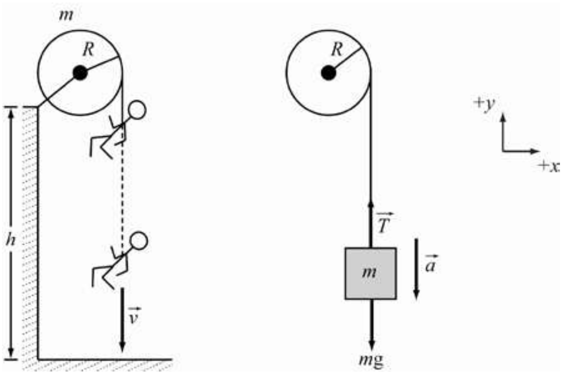
(a) $I_{\text{tot}} = 7.27 \cdot 10^{-2} \text{ kg m}^2$

(b) $t = 2.28$ s

DOUBLE-CHECK: Given the small masses and disk sizes, the moment of inertia should be small. Also, given the small torque and angular velocity, two seconds to come to a stop is reasonable.

10.52. **THINK:** The stuntman with a mass, m = 50.0 kg, will cause a tension, T, in the rope which produces a torque, τ, on the drum of mass, M = 100. kg and radius, R = 0.500 m. This torque will cause the drum to have an angular acceleration, α, and if the rope does not slip, then it will be directly related to the stuntman's translational acceleration, a. If the stuntman starts from rest and needs to accelerate to v = 4.00 m/s after dropping a height, $h = 20.0$ m, then kinematics can be used to determine the acceleration.

SKETCH:



RESEARCH: The sum of the forces yields $T - mg = -ma$. The torque produced by the tension is given by $\tau = TR = I\alpha$. With no slipping, $R\alpha = a$. The velocity of the stuntman after falling a height, h, at an acceleration of a is given by $v_f^2 = v_i^2 + 2ah$, where $v_i = 0$. Also, if there is no slipping, $v = \omega R$. The angle the barrel makes is given by $\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$.

SIMPLIFY:

(a) The tension is given by $T = m(g-a)$. Therefore, the torque is given by $\tau = TR = m(g-a)R = I_0\alpha$. This implies:

$$mgR - maR = I_0\frac{a}{R} \Rightarrow mgR^2 = maR^2 + I_0 a \Rightarrow a = \frac{mgR^2}{mR^2 + I_0}.$$

(b) $v^2 = 0 + 2ah \Rightarrow a = \dfrac{v^2}{2h}$; From part (a), $I_0 = \dfrac{mgR^2 - maR^2}{a} = mR^2\left(\dfrac{g}{a} - 1\right)$.

(c) $\alpha = \dfrac{a}{R}$

(d) $\Delta\theta = \dfrac{\omega_f^2}{2\alpha} = \dfrac{v^2}{2\alpha R^2}$, $\#\text{revolutions} = \dfrac{\Delta\theta}{2\pi} = \dfrac{v^2}{4\pi\alpha R^2}$

CALCULATE:

(a) No calculation is necessary.

(b) $a = \dfrac{(4.00\text{ m/s})^2}{2(20.0\text{ m})} = 0.400\text{ m/s}^2$, $I = (50.0\text{ kg})(0.500\text{ m})^2\left(\dfrac{(9.81\text{ m/s}^2)}{0.400\text{ m/s}^2} - 1\right) = 294.0625\text{ kg m}^2$

(c) $\alpha = \dfrac{0.400\text{ m/s}^2}{0.500\text{ m}} = 0.800\text{ rad/s}^2$

(d) $\#\text{revolutions} = \dfrac{(4.00\text{ m/s})^2}{4\pi(0.800\text{ rad/s}^2)(0.500\text{ m})^2} = 6.366$

ROUND:

Rounding to thre significant figures:

(a) Not applicable.

(b) $a = 0.400\text{ m/s}^2$ and $I = 294\text{ kg m}^2$.

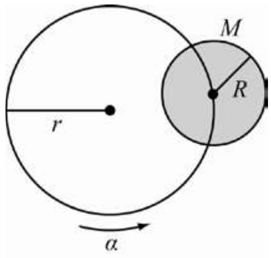
(c) $\alpha = 0.800\text{ rad/s}^2$

(d) $\#\text{revolutions} = 6.37$

DOUBLE-CHECK: Given the large height and the small final velocity, the small accelerations and the few rotations of the drum are reasonable.

10.53. **THINK:** Since the center of mass of the tire with mass, $M = 23.5$ kg, is at a distance, $r = 1.10$ m, from the axis of rotation, the parallel axis theorem is used to determine the overall moment of inertia of the tire. Consider both cases where the tire is a thin hollow cylinder of radius, $R = 0.350$ m, and a thick hollow cylinder with radii, $R_1 = 0.300$ m and $R_2 = 0.400$ m. The torque, $\tau = 20.0$ N m, the athlete applies will cause an angular acceleration, α. Kinematics can be then be used to determine the linear speed after three rotations.

SKETCH:



RESEARCH: From the parallel axis theorem, the moment of inertia of a tire is $I = I_{cm} + Mr^2$. For a thin hollow cylinder, $I_{cm} = MR^2$ and for a thick hollow cylinder, $I_{cm} = M\left(R_1^2 + R_2^2\right)/2$. The torque is given by $\tau = I\alpha$. Then time can be determined from $\Delta\theta = \alpha t^2/2$, where $\Delta\theta$ is three rotations or 6π radians. Since the tire starts from rest, its final angular velocity is $\omega = \alpha t$ and its tangential velocity is $v = \omega r$.

SIMPLIFY:

(a) $\tau = I\alpha = \left(MR^2 + Mr^2\right)\alpha \Rightarrow \alpha = \dfrac{\tau}{M\left(R^2+r^2\right)}, \Delta\theta = \dfrac{1}{2}\alpha t_{throw}^{\ 2} \Rightarrow t_{throw} = \sqrt{\dfrac{2\Delta\theta}{\alpha}} = \sqrt{\dfrac{12\pi\left(M\left(R^2+r^2\right)\right)}{\tau}}$

(b) $v = \omega r = r\alpha t = r\sqrt{2\alpha\Delta\theta} = r\sqrt{\dfrac{12\pi\tau}{M\left(R^2+r^2\right)}}$

(c) $t_{throw} = \sqrt{\dfrac{12\pi\left(M\left(r^2 + \dfrac{R_1^{\ 2}+R_2^{\ 2}}{2}\right)\right)}{\tau}}, \quad v = r\sqrt{\dfrac{12\pi\tau}{M\left(r^2 + \dfrac{R_1^{\ 2}+R_2^{\ 2}}{2}\right)}}$

CALCULATE:

(a) $t_{throw} = \sqrt{\dfrac{12\pi\left((23.5 \text{ kg})\left((0.350 \text{ m})^2 + (1.10 \text{ m})^2\right)\right)}{20.0 \text{ Nm}}} = 7.683 \text{ s}$

(b) $v = (1.10 \text{ m})\sqrt{\dfrac{12\pi(20.0 \text{ Nm})}{(23.5 \text{ kg})\left((0.350 \text{ m})^2 + (1.10 \text{ m})^2\right)}} = 5.39766 \text{ m/s}$

(c) $t_{throw} = \sqrt{\dfrac{12\pi\left((23.5 \text{ kg})\left((1.10 \text{ m})^2 + \dfrac{(0.300 \text{ m})^2 + (0.400 \text{ m})^2}{2}\right)\right)}{20.0 \text{ Nm}}} = 7.690 \text{ s}$

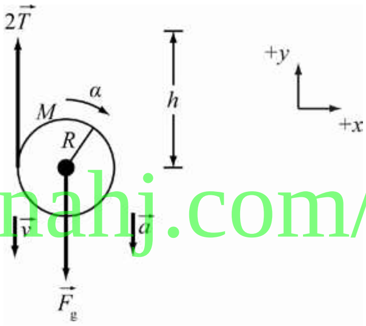
$$v=(1.1\text{ m})\sqrt{\frac{12\pi(20.0\text{ Nm})}{(23.5\text{ kg})\left((1.10\text{ m})^2+\frac{(0.300\text{ m})^2+(0.400\text{ m})^2}{2}\right)}}=5.393\text{ m/s}$$

ROUND:
(a) $t_{\text{throw}}=7.68$ s
(b) $v=5.40$ m/s
(c) $t_{\text{throw}}=7.69$ s and $v=5.39$ m/s

DOUBLE-CHECK: Given the small change in the two moments of inertia between the thin and thick cylinder, virtually identical values in time and velocity are reasonable.

10.54. THINK: Due to the symmetry of the barrel, assume the tension, T, in each rope is equal. The barrel with mass, M = 100. kg, and radius, R = 50.0 cm, will cause a tension, T, in the ropes that in turn produces a torque, τ, on the barrel and hence an angular acceleration, α. If the ropes do not slip, the angular acceleration will be directly related to the linear acceleration of the barrel. Once the linear acceleration is determined, kinematics can be used to determine the velocity of the barrel after it has fallen a distance, h = 10.0 m, assuming it starts from rest.

SKETCH:



RESEARCH: From the kinematic equations, $v_f^2=v_i^2+2ah$, where the initial velocity is zero. The sum of the forces acting on the barrel is given by $2T-Mg=-Ma$. The tension in the ropes also cause a torque, $\tau=2TR=I\alpha$, where $I=MR^2/2$.

SIMPLIFY: Summing the tensions in the ropes gives $2T=M(g-a)$. The torque this tension produces is

$$\tau=2TR=MR(g-a)=\frac{1}{2}MR^2\left(\frac{a}{R}\right)\Rightarrow MgR=\frac{3}{2}MRa\Rightarrow a=\frac{2}{3}g.$$

The velocity is given by $v^2=2ah\Rightarrow v=\sqrt{\frac{4}{3}gh}$. The tension in one rope is $T=M(g-a)/2=Mg/6$.

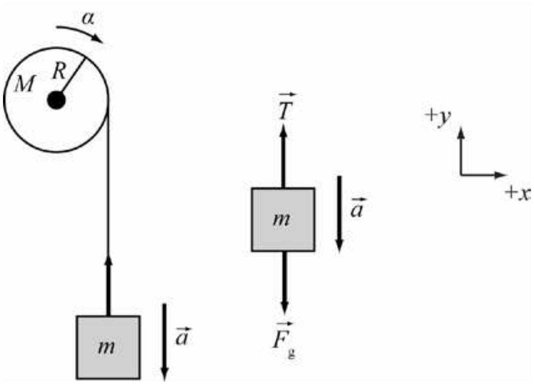
CALCULATE: $v=\sqrt{\frac{4}{3}(9.81\text{ m/s}^2)(10.0\text{ m})}=11.437$ m/s, $T=\frac{1}{6}(100.\text{ kg})(9.81\text{ m/s}^2)=163.5$ N

ROUND: Rounding to three significant figures, $v=11.4$ m/s and T = 164 N.

DOUBLE-CHECK: If the barrel is in free fall, it would have a velocity of 14 m/s after falling 10 m, so a smaller velocity for this result is reasonable.

10.55. THINK: The hanging mass, m = 2.00 kg, will cause a tension, T, in the rope. This tension will then produce a torque, τ, on the wheel with a mass, M = 40.0 kg, a radius, R = 30.0 cm and a c value of 4/9. This torque will then give the wheel an angular acceleration, α. Assuming the rope does not slip, the angular acceleration of the wheel will be directly related to the linear acceleration of the hanging mass.

SKETCH:



RESEARCH: With no slipping, the linear acceleration is given by $a = \alpha R$. The tension can be determined by $T = m(g - a)$, which in turn produces a torque $\tau = TR = I\alpha$, where the moment of inertia of the wheel is $4MR^2/9$.

SIMPLIFY: To determine the angular acceleration:

$$TR = m(g - \alpha R)R = \frac{4}{9}MR^2\alpha \Rightarrow mgR = m\alpha R^2 + \frac{4}{9}MR^2\alpha \Rightarrow \alpha = \frac{mg}{\left(m + \frac{4}{9}M\right)R}.$$

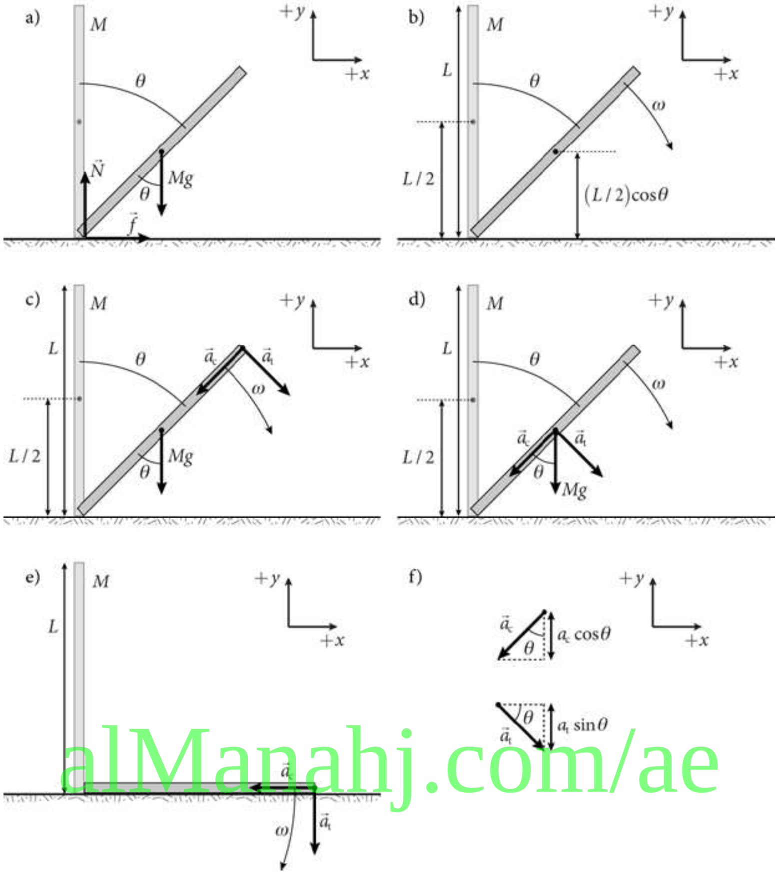
CALCULATE: $\alpha = \dfrac{2.00\text{ kg}\left(9.81\text{ m/s}^2\right)}{\left(2.00\text{ kg} + \frac{4}{9}(40.0\text{ kg})\right)(0.300\text{ m})} = 3.3067\text{ rad/s}^2$

ROUND: $\alpha = 3.31\text{ rad/s}^2$

DOUBLE-CHECK: Given the small hanging mass and the large mass of the wheel, this acceleration is reasonable.

10.56. **THINK:** As the rod with mass, M = 250.0 g and length, L = 50.0 cm, tips over, the torque, τ, caused by the force of gravity on the center of mass will change, which means the angular acceleration, α, of the rod will change with the angle it makes with the vertical. We can use energy conservation to calculate the angular velocity, ω, of the rod for any angle. The linear acceleration of any point on the rod is equal to the sum of the tangential acceleration plus the centripetal acceleration.

SKETCH:



RESEARCH: (a) In part a) of the sketch, we can see that the three forces acting on the rod are the normal force exerted by the table, the force of friction between the rod and the surface of the table, and the force of gravity.

(b1) To calculate the speed of the rod at $\theta = 45.0°$, we can use energy conservation. Conservation of mechanical energy gives us $K + U = K_0 + U_0$. The kinetic energy before the rod begins to fall is zero and at angle θ the kinetic energy is given by the kinetic energy of rotation $K = (1/2)I\omega^2$ where ω is the angular velocity and $I = (1/3)ML^2$. The potential energy before is $U_0 = mg(L/2)$ and the potential energy at angle θ is $U = mg(L/2)\cos\theta$ as illustrated in part b) of the sketch.

(b2) To calculate the vertical acceleration of the moving end of the rod, we need to calculate the tangential acceleration and the centripetal acceleration. The tangential acceleration can be calculated by realizing that that the force of gravity exerts a torque on the rod given by $\tau = mg(L/2)\sin\theta$ assuming the pivot on the table at the end of the rod. The angular acceleration is given by $\tau = I\alpha$ where $I = (1/3)ML^2$. The tangential acceleration can then be calculated from $a_t = L\alpha$. The centripetal acceleration is given by $a_c = L\omega^2$ where ω was obtained in part b1). As shown in part f) of the sketch, the vertical component of the tangential acceleration is $a_t \sin\theta$ and the vertical component of the centripetal acceleration is $a_c \cos\theta$.

(b3) To calculate the normal force exerted by the table on the rod, we need to calculate the vertical component of the tangential and centripetal acceleration of the center of mass of the rod. The angular acceleration is the same as calculated in part b2) so the tangential acceleration is $a_t = (L/2)\alpha$. The centripetal acceleration is $a_c = (L/2)\omega^2$. The vertical components of the tangential and centripetal acceleration are then $a_t \sin\theta$ and $a_c \cos\theta$ respectively. The normal force is then given by $N - Mg = Ma_v$, where a_v is the vertical acceleration of the center of mass of the rod.

(c) When the rod falls on the table, $\theta = 90.0°$ and we have

$$a_t = \frac{3}{2} g \sin 90.0° = \frac{3}{2} g.$$

The centripetal acceleration is given by

$a_c = L\omega^2$ where

$$\omega = \sqrt{\frac{3g}{L}\left(1 - \cos 90.0°\right)} = \sqrt{\frac{3g}{L}} \quad \Rightarrow \quad a_c = L\frac{3g}{L} = 3g.$$

SIMPLIFY:

(b1) We can combine the equations in b1) to obtain

$$\frac{1}{2} I\omega^2 + mg\frac{L}{2}\cos\theta = mg\frac{L}{2}.$$

We can rewrite the previous equation as

$$\frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega^2 = mg\frac{L}{2}\left(1 - \cos\theta\right) \Rightarrow \omega = \sqrt{\frac{3g}{L}\left(1 - \cos\theta\right)}.$$

(b2) We can combine the equations in (b2) to get the tangential acceleration a_t

$$mg\frac{L}{2}\sin\theta = \frac{1}{3}mL^2\frac{a_t}{L} \rightarrow g\frac{1}{2}\sin\theta = \frac{1}{3}a_t \Rightarrow a_t = \frac{3}{2} g \sin\theta.$$

We can then write the vertical component of the acceleration as

$$a_v = a_t \sin\theta + a_c \cos\theta = -\frac{3}{2} g \sin^2\theta - L\omega^2 \cos\theta.$$

(b3) We can combine the equations in (b3) to get

$$mg\frac{L}{2}\sin\theta = \frac{1}{3}mL^2\frac{a_t}{(L/2)} \rightarrow g\frac{1}{2}\sin\theta = \frac{2a_t}{3} \Rightarrow a_t = \frac{3}{4} g \sin\theta.$$

We can now write the vertical component of the acceleration as

$$a_v = -\frac{3}{4} g \sin^2\theta - \frac{L}{2}\omega^2 \cos\theta.$$

The normal force is

$$N = m\left(g + a_v\right) = m\left(g - \frac{3}{4} g \sin^2\theta - \frac{L}{2}\omega^2 \cos\theta\right).$$

(c) The linear acceleration at $\theta = 90.0°$ is

$$a = \sqrt{a_c^2 + a_c^2} = \sqrt{(3g)^2 + \left(\frac{3}{2}g\right)^2} = 3g\sqrt{1 - \frac{1}{4}} = 3g\sqrt{\frac{3}{4}} = \frac{3\sqrt{3}}{2} g.$$

CALCULATE:

(a) Not necessary.

(b1) $\omega = \sqrt{\dfrac{3\left(9.81 \text{ m/s}^2\right)}{0.500 \text{ m}}\left(1 - \cos 45.0°\right)} = 4.1521 \text{ rad/s}.$

(b2) $a_v = -\dfrac{3}{2}\left(9.81 \text{ m/s}^2\right)\sin^2 45.0° - \left(0.500 \text{ m}\right)\left(4.1521 \text{ rad/s}\right)^2 \cos 45.0° = -13.453 \text{ m/s}^2.$

(b3) $N = \left(0.2500 \text{ kg}\right)\left(9.81 \text{ m/s}^2 - \dfrac{3}{4}\left(9.81 \text{ m/s}^2\right)\sin^2 45.0° - \dfrac{0.500 \text{ m}}{2}\left(4.1521 \text{ s}^{-1}\right)^2 \cos 45.0°\right) = 0.77091 \text{ N}.$

(c) $a = \dfrac{3\sqrt{3}}{2}\left(9.81 \text{ m/s}^2\right) = 25.487 \text{ m/s}^2.$

ROUND: Three significant figures:

(a) Not necessary

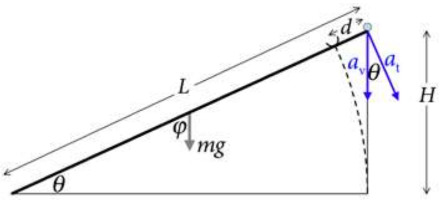
(b) $\omega = 4.15$ rad/s, $a_v = -13.5$ m/s^2 and $N = 0.771$ N.

(c) $a = 25.5$ m/s^2.

DOUBLE-CHECK: When θ 0, $\omega = 0$, i.e. the rod is perfectly upright and not rotating, looking at the equation for the normal force it can be seen that the normal force is equal to the force of gravity. The values for the accelerations may seem surprising because they are larger than g. However, we have to remember that the force of friction and the normal force must provide the centripetal force necessary to keep the rod rotating around one end. Note that the assumption that the friction force can provide the required centripetal force all the way to $\theta = 90.0°$ is unrealistic.

10.57. **THINK:** If we place the ball (blue dot) at the end of the board, it can only be caught by the cup (half circle), if the end of the board falls with a vertical component of the acceleration a_v, which is greater (or at least equal) to g. The cup is to be placed at a distance d away from the end so that it can be vertically under the ball and catch it when the board lands on the ground.

SKETCH:



RESEARCH: The board rotates about its lower end and has the same moment of inertia as a rod, $I = \frac{1}{3}mL^2$. The torque equation is $\tau = I\alpha$, where the torque is given by $\tau = Fr\sin\varphi = mg \cdot \frac{1}{2}L \cdot \sin\varphi$. The angular and tangential acceleration are related to each other via $a_t = \alpha L$. The vertical component of the tangential acceleration is then (see sketch) $a_v = a_t \cos\theta$.

Geometrical relations: Since $\theta = 90° - \varphi$, (see sketch), we find that $\sin\varphi = \cos\theta$. Also from the sketch, we see that the height of the vertical support stick is $H = L\sin\theta$. In addition (dashed circular segment in the sketch), we see that $d = L - L\cos\theta = L(1-\cos\theta)$.

SIMPLIFY:

$\tau = \frac{1}{2}mgL\sin\varphi = \frac{1}{2}mgL\cos\theta = \frac{1}{3}mL^2\alpha = \frac{1}{3}mLa_t \Rightarrow a_t = \frac{3}{2}g\cos\theta$

Inserting this result into $a_v = a_t\cos\theta$ from above, we find $a_v = \frac{3}{2}g\cos^2\theta$. If, as required, $a_v \geq g$, this means $\frac{3}{2}g\cos^2\theta \geq g$ or $\cos\theta \geq \sqrt{\frac{2}{3}}$. Since $\sin^2\theta + \cos^2\theta = 1$, this implies $\sin\theta \leq \sqrt{\frac{1}{3}}$. So, finally, from $H = L\sin\theta$ we see $H \leq \sqrt{\frac{1}{3}}L$.

CALCULATE:

(a) $H_{max} = \sqrt{\frac{1}{3}}L = \sqrt{\frac{1}{3}}(1.00\text{ m}) = 0.57735$ m.

(b) $d = L(1-\cos\theta) = (1.00\text{ m})(1-\sqrt{\frac{2}{3}}) = 0.1835$ m.

ROUND: Rounding to 3 digits leaves us with

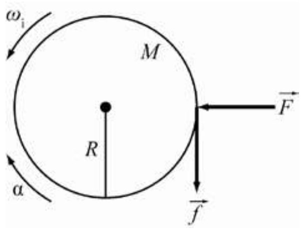
(a) $H_{max} = 0.577$ m.

(b) $d = 0.184$ m.

DOUBLE-CHECK: Clearly, this is a somewhat surprising result. However, it is also a standard lecture demonstration. When you see it you can convince yourself that these calculations for $\sin\theta$ are correct.

10.58. **THINK:** If the brakes applies an inward radial force, F = 100. N, and the contact has a coefficient of friction, $\mu_k = 0.200$, then this frictional force, f, will be perpendicular to F and cause a torque, τ, on the flywheel of mass, M = 120. kg, and radius, R = 80.0 cm. The torque can be used to determine the angular acceleration, α, of the wheel. Kinematics can then be used to determine the number of revolutions, n, the wheel will make and the time it will take for it to come to rest. The work done by torque should be the change in rotational energy, by conservation of energy. The flywheel has an initial angular speed of 500. rpm or $50\pi/3$ rad/s.

SKETCH:



RESEARCH: Similarly to the relation between the normal force and friction, $f = \mu_k F$, the friction causes a torque, $\tau = fR = I\alpha$, where the moment of inertia of the wheel is $I = MR^2/2$. Kinematics is used to determine the number of revolutions and the time it takes to come to an end, $\omega_f^2 = \omega_i^2 - 2\alpha\Delta\theta$ and $\omega_f - \omega_i = -\alpha t$. The work done by the friction is $W = \Delta K = -I\omega_i^2/2$.

SIMPLIFY: The angular acceleration is given by $\alpha = \dfrac{fR}{I} = \dfrac{\mu_k FR}{\frac{1}{2}MR^2} = \dfrac{2\mu_k F}{MR}$. Therefore, the total angular displacement is given by $\omega_f^2 = \omega_0^2 + 2\alpha\Delta\theta \Rightarrow |\Delta\theta| = \dfrac{\omega_0^2}{2\alpha}$. The number of revolutions, n, is given by $\dfrac{\Delta\theta}{2\pi} = \dfrac{\omega_0^2}{4\pi\alpha}$. The time to come to rest is $t = \omega_i/\alpha$. The work done is then $-MR^2\omega_i^2/4$.

CALCULATE:

$$\left(\frac{500 \text{ revolutions}}{1 \text{ min}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ revolution}}\right)\left(\frac{1 \text{ min}}{60 \text{ sec}}\right) = \frac{50\pi}{3} \text{ rad/sec}$$

$$\alpha = \frac{2(0.200)(100.\text{ N})}{(120.\text{ kg})(0.800\text{ m})} = 0.4167 \text{ rad/s}^2,\quad n = \frac{(50\pi/3\text{ rad/s})^2}{4\pi 0.4167\text{ rad/s}^2} = 523.60 \text{ revolutions}$$

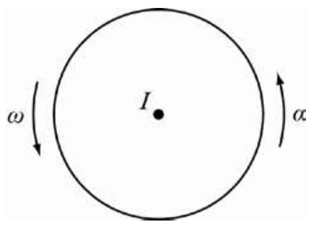
$$t = \frac{50\pi/3\text{ rad/s}}{0.4167\text{ rad/s}^2} = 125.66\text{ s},\quad W = -\frac{1}{4}(120.\text{ kg})(0.800\text{ m})^2\left(\frac{50\pi}{3}\text{ rad/s}\right)^2 = -52638\text{ J}$$

ROUND: To three significant figures: $n = 524$ revolutions, $t = 126$ s and $W = -5.26 \cdot 10^4$ J.

DOUBLE-CHECK: Given the small friction, and hence the small torque, and the fast speed of the wheel, it would take a long time to stop, so the results are reasonable.

10.59. **THINK:** Assuming a constant angular acceleration, α, and $\Delta t = 25$ s, regular kinematics can be used to determine α and $\Delta\theta$. The total work done by torque, τ, should be converted entirely into rotational energy. $I = 25.0\text{ kg m}^2$ and $\omega_f = 150.$ rad/s.

SKETCH:



RESEARCH: From kinematics, $\omega_f - \omega_i = \alpha \Delta t$ and $\Delta\theta = \alpha(\Delta t)^2/2$. The torque on the wheel is $\tau = I\alpha$. Since the torque is constant, the work done by it is $W = \tau\Delta\theta$. The kinetic energy of the turbine is $K = I\omega^2/2$.

SIMPLIFY:

(a) $\omega_f = \alpha\Delta t \Rightarrow \alpha = \omega_f / \Delta t$

(b) $\tau = I\alpha$

(c) $\Delta\theta = \frac{1}{2}\alpha(\Delta t)^2$

(d) $W = \tau\Delta\theta$

(e) $K = \frac{1}{2}I\omega_f^2$

CALCULATE:

(a) $\alpha = \frac{150.\text{ rad/s}}{25.0\text{ s}} = 6.00\text{ rad/s}^2$

(b) $\tau = (25.0\text{ kg m}^2)(6.00\text{ rad/s}^2) = 150.\text{ N m}$

(c) $\Delta\theta = \frac{1}{2}(6.00\text{ rad/s}^2)(25.0\text{ s})^2 = 1875\text{ rad}$

(d) $W = (150.\text{ N m})(1875\text{ rad}) = 281{,}250\text{ J}$

(e) $K = \frac{1}{2}(25.0\text{ kg m}^2)(150.\text{ rad/s})^2 = 281{,}250\text{ J}$

ROUND:

(a) $\alpha = 6.00\text{ rad/s}^2$

(b) $\tau = 150.\text{ N m}$

(c) $\Delta\theta = 1880\text{ rad}$

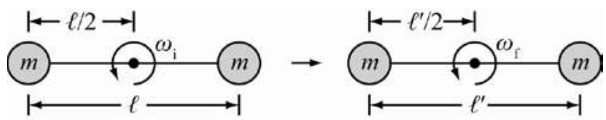
(d) $W = 281\text{ kJ}$

(e) $K = 281\text{ kJ}$

DOUBLE-CHECK: It is expected that the work and kinetic energy are equal. Since they were each determined independently and they are the same value, the procedure must have been correct.

10.60. **THINK:** Since the two masses have an equal mass of m = 6.00 kg, their center of mass will be at the geometric center, $l/2$, which is also the location of the axis of rotation. Initially, l = 1.00 m and then it extends to 1.40 m. When the length increases, the moment of inertia also increases. Since there are no external torques, conservation of angular momentum can be applied. The masses initially rotate at $\omega_i = 5.00$ rad/s.

SKETCH:



RESEARCH: The angular momentum before and after are $L_i = I_i\omega_i$ and $L_f = I_f\omega_f$. The moments of inertia for before and after are $I_i = 2m(l/2)^2$ and $I_f = 2m(l'/2)^2$. The conservation of angular momentum is represented by $L_i = L_f$.

SIMPLIFY: $L_i = L_f \Rightarrow 2m\left(\frac{l^2}{4}\right)\omega_i = 2m\left(\frac{l'^2}{4}\right)\omega_f \Rightarrow l^2\omega_i = l'^2\omega_f \Rightarrow \omega_f = \left(\frac{l}{l'}\right)^2\omega_i$

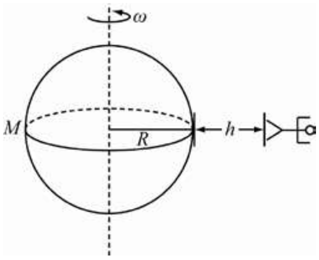
CALCULATE: $\omega_f = \left(\frac{1.00 \text{ m}}{1.40 \text{ m}}\right)^2 5.00 \text{ rad/s} = 2.551 \text{ rad/s}$

ROUND: To three significant figures: $\omega_f = 2.55 \text{ rad/s}$

DOUBLE-CHECK: Since the string length and hence the moment of inertia increases, a smaller rotational speed is expected, since angular momentum is conserved.

10.61. **THINK:** For the moment of inertia of the Earth, treat it as a solid sphere with mass, $M = 5.977 \cdot 10^{24}$ kg, and radius, R = 6371 km. The Chinese ($n = 1.30 \cdot 10^9$ people and m = 70.0 kg each) can be treated as a point mass of total mass, nm, standing on the surface of the Earth and then also at h = 1.00 m above the surface. Conservation of angular momentum relates the change in moment of inertia to the change in angular frequency, and hence period, of the Earth.

SKETCH:



RESEARCH: The Earth's (solid sphere) moment of inertia is $I_E = 2MR^2/5$, while the Chinese have a moment of inertia of $I_C = nmR^2$ on the surface of the Earth and $I'_C = nm(R+h)^2$ when standing on the chair. The angular momentum is $L = I\omega$. The period of the Earth's rotation is 1 day or 86,400 s, and is related to the angular velocity by $\omega = 2\pi / T$.

SIMPLIFY:

(a) The moment of inertia of Earth is $I_E = 2MR^2/5$.

(b) The moment of inertia of the Chinese people on Earth is $I_C = nmR^2$.

(c) The moment of inertia of the Chinese people on chairs is $I'_C = nm(R+1)^2$. The change in the moment of inertia for the Chinese people is $\Delta I_C = I'_C - I_C = nm(R^2 + 2R + 1 - R^2) = nm(2R+1)$.

(d) The conservation of angular momentum states, $(I_E + I_C)\omega_i = (\Delta I_C)\Delta\omega \Rightarrow (I_E + I_C)\frac{2\pi}{T_i} = (\Delta I_C)\frac{2\pi}{\Delta T}$.

Therefore, $\frac{\Delta T}{T} = \frac{\Delta I_C}{I_E + I_C}$ (fractional change) and $\Delta T = \frac{\Delta I_C}{I_E + I_C} T$ (total change).

CALCULATE:

(a) $I_E = \frac{2}{5}(5.977 \cdot 10^{24} \text{ kg})(6{,}371{,}000 \text{ m})^2 = 9.704 \cdot 10^{37} \text{ kg m}^2$

(b) $I_C = (1.30 \cdot 10^9)(70.0 \text{ kg})(6{,}371{,}000 \text{ m})^2 = 3.694 \cdot 10^{24} \text{ kg m}^2$

(c) $\Delta I_C = (1.30 \cdot 10^9)(70.0 \text{ kg})(2(6{,}371{,}000 \text{ m}) + 1.00 \text{ m}) = 1.1595 \cdot 10^{18} \text{ kg m}^2$

(d) $\frac{\Delta T}{T} = \frac{1.1595 \cdot 10^{18} \text{ kg m}^2}{9.704 \cdot 10^{37} \text{ kg m}^2 + 3.694 \cdot 10^{24} \text{ kg m}^2} = 1.1949 \cdot 10^{-20}$

ROUND:

(a) $I_E = 9.704 \cdot 10^{37}\text{ kg m}^2$

(b) $I_C = 3.69 \cdot 10^{24}\text{ kg m}^2$

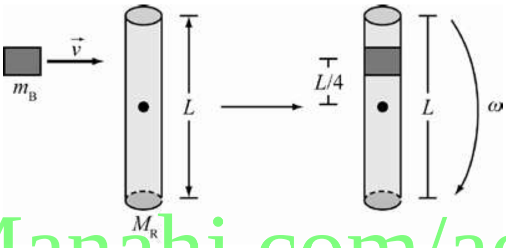
(c) $\Delta I_C = 1.16 \cdot 10^{18}\text{ kg m}^2$

(d) $\dfrac{\Delta T}{T} = 1.19 \cdot 10^{-20}$

DOUBLE-CHECK: Despite the large number of Chinese people, the Earth is so massive that the rotation is hardly affected by their jump onto the surface.

10.62. **THINK:** The bullet with a mass, $m_B = 1.00 \cdot 10^{-2}$ kg, has a linear momentum. When it strikes the rod with length, L = 1.00 m, and mass, $m_R = 5.00$ kg, the rod begins to rotate about its center and thus has an angular momentum. Conservation of momentum means the bullet's linear momentum is equal to the rod and bullet's angular momentum. Likewise, the bullet has a linear kinetic energy and the rod and bullet have a rotational kinetic energy so the change in kinetic energy is the difference between these two. The bullet can be treated as a point particle that is a distance $L/4$ from the axis of rotation.

SKETCH:



RESEARCH: The momentum of the bullet is given by $p = m_B v$. When it hits the rod at $L/4$ from the center, its linear momentum can be converted to angular momentum by $pL/4$. The moment of inertia of the rod is $m_R L^2/12$ and that of the bullet when in the rod is $m(L/4)^2$. The kinetic energy of the bullet is all translational, $K_T = m_B v^2/2$, while the kinetic energy of the bullet and rod together is all rotational, $K_R = I\omega^2/2$.

SIMPLIFY:

(a) The initial angular momentum is $L_i = m_B vL/4$. The final angular momentum is $L_f = I\omega = (m_R/12 + m_B/16)L^2\omega$. The angular velocity is then

$$L_i = L_f \Rightarrow \frac{m_B vL}{4} = \left(\frac{1}{12}m_R + \frac{1}{16}m_B\right)L^2\omega \Rightarrow \omega = \frac{m_B v}{\left(\frac{1}{3}m_R + \frac{1}{4}m_B\right)L}.$$

(b) $K_T = \dfrac{1}{2}m_B v^2$ and $K_R = \dfrac{1}{2}I\omega^2 = \left(\dfrac{1}{24}m_R + \dfrac{1}{32}m_B\right)L^2\omega^2$. Therefore, $\Delta K = K_R - K_T$.

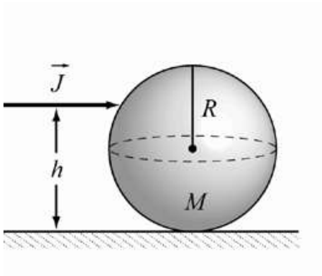
CALCULATE:

(a) $$\omega = \frac{(1.00 \cdot 10^{-2}\text{ kg})(100.\text{ m/s})}{\left(\frac{5.00\text{ kg}}{3} + \frac{0.0100\text{ kg}}{4}\right)(1.00\text{ m})} = 0.5991\text{ rad/s}$$

(b) $$\Delta K = \left(\frac{5.00\text{ kg}}{24} + \frac{1.00 \cdot 10^{-2}\text{ kg}}{32}\right)(1.00\text{ m})^2(0.599\text{ rad/s})^2 - \frac{1}{2}(1.00 \cdot 10^{-2}\text{ kg})(100.\text{ m/s})^2 = -49.925\text{ J}$$

ROUND:
To three significant figures,
(a) $\omega = 0.599$ rad/s
(b) $\Delta K = -49.9$ J
DOUBLE-CHECK: Given that the rod is five hundred times heavier than the bullet and that the bullet will lose energy from imbedding itself in the rod, a small ω and a negative value for ΔK is reasonable.

10.63. **THINK:** The sphere of mass, M, spins clockwise when a horizontal impulse J is exerted at a height h above the tabletop when $R < h < 2R$.
SKETCH:



RESEARCH: To calculate the linear speed after the impulse is applied, we use the fact that the impulse J can be written as $J = \Delta p = M\Delta v$. To get the angular velocity, we write the change in the angular momentum of the sphere as $\Delta L = \Delta p(h-R)$. To calculate the height where the impulse must be applied, we have to apply Newton's Second Law for linear motion, $F = Ma$, and Newton's Second Law for rotation, $\tau = I\alpha$. The torque is given by $\tau = F(h-R)$. The object rolls without slipping so from Section 10.3 we know that $v = R\omega$ and $a = R\alpha$. In addition, we can write the impulse as $J = F\Delta t$.

SIMPLIFY: a) Combining these relationships to get the linear velocity gives us

$$J = \Delta p = M\Delta v = Mv \Rightarrow v = \frac{J}{M}.$$

Combining these relationships to get the angular velocity gives us

$$\Delta L = \Delta p(h-R) = J(h-R)$$

$$\Delta L = I\Delta\omega = I\omega = \frac{2}{5}MR^2\omega$$

$$J(h-R) = \frac{2}{5}MR^2\omega$$

$$\omega = \frac{5J(h-R)}{2MR^2}.$$

b) Combining these relationships to get the height h_0 at which the impulse must be applied for the sphere to roll without slipping we get

$$F = Ma \Rightarrow \frac{J}{\Delta t} = MR\alpha$$

$$F(h_0 - R) = I\alpha \Rightarrow \frac{J}{\Delta t}(h_0 - R) = \frac{2}{5}MR^2\alpha.$$

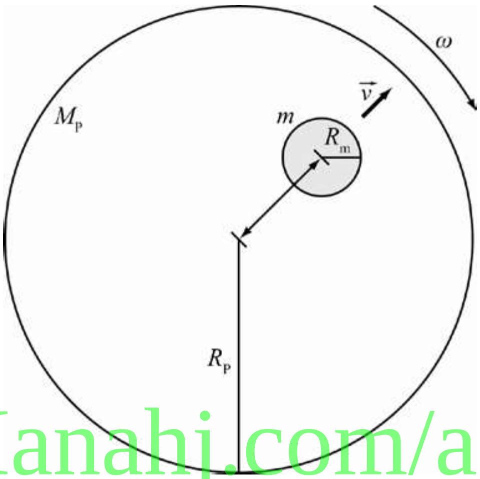
Dividing these two equations gives us

$$h_0 - R = \frac{2}{5}R \Rightarrow h_0 = \frac{7}{5}R.$$

CALCULATE: Not applicable.
ROUND: Not applicable.

DOUBLE-CHECK: The linear velocity will always be positive. However, the angular velocity can be positive or negative, depending on whether $h > R$ or $h < R$. The fact that $h_0 > R$ is consistent with the ball rolling to the right after the impulse is applied.

10.64. **THINK:** If the man, approximated by a cylinder of mass, $m = 80.0$ kg, and radius, $R_m = 0.200$ m, walks at a constant velocity, $v = 0.500$ m/s, then his distance, d, from the center of the platform of mass, $M_p = 400.$ kg, and radius, $R_p = 4.00$ m, will change linearly with time. The platform initially rotates at 6.00 rpm or 0.200π rad/s. Initially, the man and the platform have their center of mass on the axis of rotation, so their moments of inertia are summed. When the man is a distance, d, from the center, the parallel axis theorem is needed to determine his overall moment of inertia.
SKETCH:



RESEARCH: The distance, d, the man is from the center is $d = vt$. The moment of inertia of the platform is $I_p = M_p R_p^2 / 2$. The man has a moment of inertia of $I_m = mR_m^2 / 2$ and by the parallel axis theorem has a final moment of inertia of $I'_m = \left(mR_m^2 / 2\right) + md^2$. Conservation of angular momentum states $L_i = L_f$, where $L = I\omega$.

SIMPLIFY: The man's moment of inertia as a function of time is $I'_m = \left(mR_m^2 / 2\right) + mv^2t^2 = I_m + mv^2t^2$. The initial angular momentum of the system is $L_i = \left(I_p + I_m\right)\omega_i$. The angular momentum at time t is $L_f = \left(I_p + I'_m\right)\omega_f$. Therefore, $\left(I_p + I_m\right)\omega_i = \left(I_p + I_m + mv^2t^2\right)\omega_f$

$$\Rightarrow \omega_f = \frac{\left(I_p + I_m\right)\omega_i}{\left(I_p + I_m + mv^2t^2\right)} \Rightarrow \omega_f(t) = \omega_i\left(1 + \frac{2mv^2t^2}{M_p R_p^2 + mR_m^2}\right)^{-1}.$$

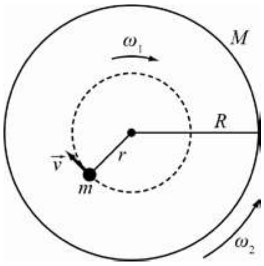
When the man reaches the end, $d = R_p \Rightarrow t = R_p / v$. Therefore, $\omega_f = \omega_i\left(1 + \dfrac{2mR_p^2}{M_p R_p^2 + mR_m^2}\right)^{-1}$.

CALCULATE: $\omega_f = \left(0.200\pi \text{ rad/s}\right)\left(1 + \dfrac{2\left(80.0 \text{ kg}\right)\left(4.00 \text{ m}\right)^2}{\left(400. \text{ kg}\right)\left(4.00 \text{ m}\right)^2 + \left(80.0 \text{ kg}\right)\left(0.200 \text{ m}\right)^2}\right)^{-1} = 0.4489$ rad/s

ROUND: To three significant figures: $\omega_f = 0.449$ rad/s.
DOUBLE-CHECK: As time increases (i.e. the man walks from the center), the overall moment of inertia increases, so a smaller angular velocity is expected.

10.65. **THINK:** Initially, the system has zero angular momentum. The boy with mass, m = 25.0 kg, can be treated as a point particle a distance, r = 2.00 m, from the center of the merry-go-round, which has a moment of inertia, $I_0 = 200.\text{ kg m}^2$. When the boy starts running with a velocity, v = 0.600 m/s, the merry-go-round will begin to rotate in the opposite direction in order to conserve angular momentum.

SKETCH:



RESEARCH: The initial angular momentum is $L_i = 0$. The angular velocity of the boy is $\omega_1 = v/r$. The moment of inertia of the boy is mr^2. The angular momentum is given by $L = I\omega$. The tangential velocity of the merry-go-round at r is $v_2 = \omega_2 r$. The boy's velocity relative to the merry-go-round (which is rotating in the opposite direction) is $v' = v + v_2$.

SIMPLIFY:

(a) $L_f = I_0\omega_2 - mr^2\omega_1 = 0 \Rightarrow \omega_2 = \dfrac{mr^2\omega_1}{I_0} \Rightarrow \omega_2 = \dfrac{mrv}{I_0}$

(b) $v' = v + r\omega_2$

CALCULATE:

(a) $\omega_2 = \dfrac{(25.0\text{ kg})(2.00\text{ m})(0.600\text{ m/s})}{200.\text{ kg m}^2} = 0.150\text{ rad/s}$

(b) $v' = 0.600\text{ m/s} + (2.00\text{ m})(0.150\text{ rad/s}) = 0.900\text{ m/s}$

ROUND:

(a) $\omega_2 = 0.150\text{ rad/s}$

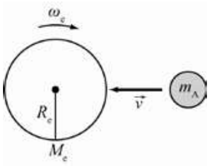
(b) $v' = 0.900\text{ m/s}$

DOUBLE-CHECK: Since the merry-go-round must move opposite to the boy, a relative velocity greater than the velocity compared to the ground makes sense. Also, since the boy and the merry-go-round have comparable moments of inertia, the comparable velocities are reasonable.

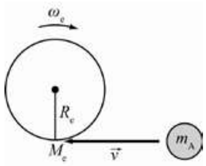
10.66. **THINK:** In every case, the momentum (angular and linear) must be conserved. If the asteroid with mass, $m_A = 1.00 \cdot 10^{22}$ kg, and velocity, $v = 1.40 \cdot 10^3$ m/s, hits the Earth, which has an angular speed of $\omega_E = 7.272 \cdot 10^{-5}$ rad/s, dead on (radially inward), then it will not contribute any of its linear momentum to the angular momentum of the planet, meaning the change in the Earth's rotation is solely a result of it gaining mass. If the asteroid hits the planet tangentially, then the full amount of the asteroid's linear momentum is contributed to the angular momentum. If the asteroid hits in the direction of Earth's rotation, it will add its momentum and the Earth will spin faster and vice versa for the opposite direction. The mass of the Earth is $m_E = 5.977 \cdot 10^{24}$ kg and the radius is $R_E = 6371$ km. The Earth can be treated as a solid sphere.

SKETCH:

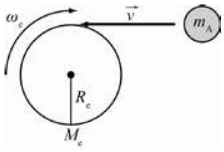
(a)



(b)



(c)



RESEARCH: The moment of inertia of Earth is $I_E = 2M_E R_E^2/5$. After the asteroid has collided, the moment of inertia of the system is then given by $I_T = I_E + m_A R_E^2$. The angular momentum is $L = I\omega$. Conservation of angular momentum applies in each case. The momentum the asteroid contributes is $p = m_A v$ and its linear momentum will be $\pm pR_E$, depending on which way it hits.

SIMPLIFY:

(a) $$I_E\omega_E = I_F\omega_F \Rightarrow \omega_F = \frac{\frac{2}{5}M_E R_E^2}{\frac{2}{5}M_E R_E^2 + M_A R_E^2}\omega_E \Rightarrow \omega_F = \frac{2M_E}{2M_E + 5M_A}\omega_E$$

(b) $$I_E\omega_E + M_A v R_E = I_F\omega_F \Rightarrow \omega_F = \frac{\frac{2}{5}M_E R_E^2\omega_E + M_A v R_E}{\frac{2}{5}M_E R_E^2 + M_A R_E^2} \Rightarrow \omega_F = \frac{\frac{2}{5}M_E R_E\omega_E + M_A v_A}{\frac{2}{5}M_E R_E + M_A R_E}$$

(c) $$I_E\omega_E - M_A v R_E = I_F\omega_F \Rightarrow \omega_F = \frac{\frac{2}{5}M_E R_E\omega_E - M_A v_A}{\frac{2}{5}M_E R_E + M_A R_E}$$

CALCULATE:

(a) $$\omega_F = \frac{2\left(5.977\cdot 10^{24}\text{ kg}\right)\left(7.272\cdot 10^{-5}\text{ rad/s}\right)}{2\left(5.977\cdot 10^{24}\text{ kg}\right) + 5\left(1.00\cdot 10^{22}\text{ kg}\right)} = 7.2417\cdot 10^{-5}\text{ rad/s}$$

(b)

$$\omega_F = \frac{\frac{2}{5}\left(5.977\cdot 10^{24}\text{ kg}\right)\left(6371000\text{ m}\right)\left(7.272\cdot 10^{-5}\text{ rad/s}\right) + \left(1.00\cdot 10^{22}\text{ kg}\right)\left(1.40\cdot 10^{3}\text{ m/s}\right)}{\frac{2}{5}\left(5.977\cdot 10^{24}\text{ kg}\right)\left(6371000\text{ m}\right) + \left(1.00\cdot 10^{22}\text{ kg}\right)\left(6371000\text{ m}\right)} = 7.333\cdot 10^{-5}\text{ rad/s}$$

(c)

$$\omega_F = \frac{\frac{2}{5}\left(5.977\cdot 10^{24}\text{ kg}\right)\left(6371000\text{ m}\right)\left(7.272\cdot 10^{-5}\text{ rad/s}\right) - \left(1.00\cdot 10^{22}\text{ kg}\right)\left(1.40\cdot 10^{3}\text{ m/s}\right)}{\left(\frac{2}{5}\left(5.977\cdot 10^{24}\text{ kg}\right) + \left(1.00\cdot 10^{22}\text{ kg}\right)\right)\left(6371000\text{ m}\right)} = 7.1502\cdot 10^{-5}\text{ rad/s}$$

ROUND:

To three significant figures:

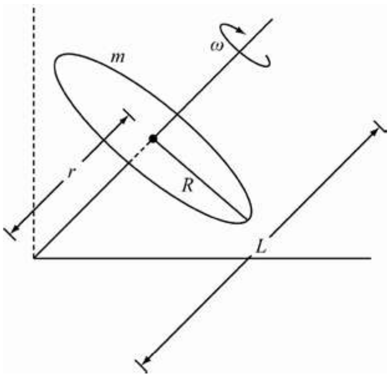
(a) $\omega_F = 7.24\cdot 10^{-5}\text{ rad/s}$

(b) $\omega_F = 7.33 \cdot 10^{-5}$ rad/s

(c) $\omega_F = 7.15 \cdot 10^{-5}$ rad/s

DOUBLE-CHECK: In part (a), it is expected that ω would be reduced very little, since the Earth gains a 0.4% mass on the surface and the moment of inertia is changed only slightly. In part (b), the asteroid would make the Earth spin faster, provided the velocity was great enough. In part (c), the asteroid would definitely make the Earth slow down its rotation.

10.67. **THINK:** If the disk with radius, R = 40.0 cm, is rotating at 30.0 rev/s, then the angular speed, ω, is 60.0π rad/s. The length of the gyroscope is L = 60.0 cm, so that the disk is located at $r = L/2$ from the pivot.
SKETCH:



RESEARCH: The precessional angular speed is given by $\omega_p = rmg / I\omega$. The moment of inertia of the disk is $I = mR^2/2$.

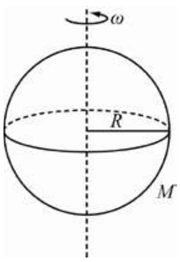
SIMPLIFY: $\omega_p = \dfrac{\frac{L}{2}mg}{\frac{1}{2}mR^2\omega} = \dfrac{Lg}{R^2\omega}$

CALCULATE: $\omega_p = \dfrac{0.600 \text{ m}\left(9.81 \text{ m/s}^2\right)}{\left(0.400 \text{ m}\right)^2 60.0\pi \text{ rad/s}} = 0.19516$ rad/s

ROUND: $\omega_p = 0.195$ rad/s

DOUBLE-CHECK: The precession frequency is supposed to be much less than the frequency of the rotating disk. In this example, the disk frequency is about one thousand times the precession frequency, so it makes sense.

10.68. **THINK:** Assume the star with a mass, $M = 5.00 \cdot 10^{30}$ kg, is a solid sphere. After the star collapses, the total mass remains the same, only the radius of the star has changed. Initially, the star has radius, $R_i = 9.50 \cdot 10^8$ m, and period, $T_i = 30.0 \text{ days} = 2592000$ s, while after the collapse it has a radius, $R_f = 10.0$ km, and a period, T_f. To determine the final period, consider the conservation of angular momentum.
SKETCH:



RESEARCH: The moment of inertia of the star is $I = 2MR^2/5,$ so initially it is $I_i = 2MR_i^2/5$ and afterwards it is $I_f = 2MR_f^2/5.$ Angular momentum is conserved, so $L_i = L_f,$ where $L = I\omega$. The period is related to the angular frequency by $T = 2\pi/\omega$ or $\omega = 2\pi/T.$

SIMPLIFY: $I_i\omega_i = I_f\omega_f \Rightarrow \dfrac{\frac{2}{5}MR_i^2(2\pi)}{T_i} = \dfrac{\frac{2}{5}MR_f^2(2\pi)}{T_f} \Rightarrow T_f = \dfrac{R_f^2}{R_i^2}T_i$

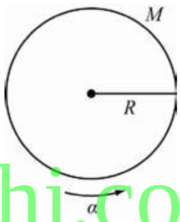
CALCULATE: $T_f = \dfrac{\left(10.0\cdot 10^3\text{ m}\right)^2\left(2{,}592{,}000\text{ s}\right)}{\left(9.50\cdot 10^8\text{ m}\right)^2} = 2.872\cdot 10^{-4}\text{ s}$

ROUND: $T_f = 2.87\cdot 10^{-4}\text{ s}$

DOUBLE-CHECK: Given the huge reduction in size, a large reduction in period, or increase in angular velocity is expected.

10.69. **THINK:** The flywheel with radius, R = 3.00 m, and $M = 1.18\cdot 10^6$ kg, rotates from rest to $\omega_f = 1.95$ rad/s in $\Delta t = 10.0\text{ min} = 600.\text{ s}.$ The wheel can be treated as a solid cylinder. The angular acceleration α, can be determined using kinematics. The angular acceleration is then used to determine the average torque, τ.

SKETCH:



RESEARCH: The energy is all rotational kinetic energy, so $E = I\omega^2/2.$ The moment of inertia of the wheel is $I = MR^2/2.$ From kinematics, $\omega_f - \omega_i = \alpha\Delta t.$ The torque is then given by $\tau = I\alpha.$

SIMPLIFY: The total energy is given by $E = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2 = \frac{1}{4}MR^2\omega^2.$ The angular acceleration is given by $\omega_f - 0 = \alpha\Delta t \Rightarrow \alpha = \omega_f/\Delta t.$ The torque needed is

$$\tau = I\alpha = \left(\frac{1}{2}MR^2\right)\left(\frac{\omega_f}{\Delta t}\right) = \frac{MR^2\omega_f}{2\Delta t}.$$

CALCULATE: $E = \frac{1}{4}\left(1.18\cdot 10^6\text{ kg}\right)\left(3.00\text{ m}\right)^2\left(1.95\text{ rad/s}\right)^2 = 1.0096\cdot 10^7\text{ J}$

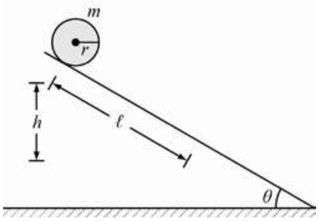
$$\tau = \frac{\left(1.18\cdot 10^6\text{ kg}\right)\left(3.00\text{ m}\right)^2\left(1.95\text{ rad/s}\right)}{2\left(600.\text{ s}\right)} = 17257.5\text{ N m}$$

ROUND: $E = 1.01\cdot 10^7$ J, $\tau = 17{,}300$ N m

DOUBLE-CHECK: The problem mentions that a huge amount of energy is needed for the experiment and the resulting energy is huge. It is reasonable that a huge torque would also be required.

10.70. **THINK:** With no friction and no slipping, energy is conserved. The potential energy of the hoop of mass, m = 2.00 kg, and radius, r = 50.0 cm, will be converted entirely into translational and rotational kinetic energy at l = 10.0 m down the incline with an angle of $\theta = 30.0°.$ For a hoop, c = 1.

SKETCH:



RESEARCH: The change in height of the hoop is $h = l\sin\theta$. The initial potential energy of the hoop is mgh. The kinetic energy of the hoop is $K_f = (1+c)mv^2/2$. The c value for the hoop is 1.

SIMPLIFY: $U_i = K_f \Rightarrow mgl\sin\theta = (1+c)mv^2/2 \Rightarrow v = \sqrt{\dfrac{2gl\sin\theta}{1+c}}$

CALCULATE: $v = \sqrt{\dfrac{2(9.81 \text{ m/s}^2)(10.0 \text{ m})\sin(30.0°)}{1+1}} = 7.004 \text{ m/s}$

ROUND: To three significant figures: $v = 7.00$ m/s

DOUBLE-CHECK: This value is less than the velocity the hoop would have going a distance, h, in free fall ($v = 9.9$ m/s), so it seems reasonable.

10.71. **THINK:** The oxygen atoms, $m = 2.66 \cdot 10^{-26}$ kg, can be treated as point particles a distance, $d/2$ (where $d = 1.21 \cdot 10^{-10}$ m) from the axis of rotation. The angular speed of the atoms is $\omega = 4.60 \cdot 10^{12}$ rad/s.

SKETCH:

RESEARCH: Since the masses are equal point particles, the moment of inertia of the two is $I = 2m(d/2)^2$. The rotational kinetic energy is $K = I\omega^2/2$.

SIMPLIFY:

(a) $I = 2m\left(\dfrac{d^2}{4}\right) = \dfrac{1}{2}md^2$

(b) $K = \dfrac{1}{2}I\omega^2 = \dfrac{1}{4}md^2\omega^2$

CALCULATE:

(a) $I = \dfrac{1}{2}(2.66 \cdot 10^{-26} \text{ kg})(1.21 \cdot 10^{-10} \text{ m})^2 = 1.9473 \cdot 10^{-46} \text{ kg m}^2$

(b) $K = \dfrac{1}{4}(2.66 \cdot 10^{-26} \text{ kg})(1.21 \cdot 10^{-10} \text{ m})^2(4.60 \cdot 10^{12} \text{ rad/s})^2 = 2.06 \cdot 10^{-21} \text{ J}$

ROUND:

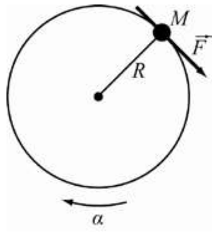
(a) $I = 1.95 \cdot 10^{-46} \text{ kg m}^2$

(b) $K = 2.06 \cdot 10^{-21}$ J

DOUBLE-CHECK: Since an oxygen molecule is so small, a very small moment of inertia and energy are expected.

10.72. **THINK:** If the force, F, is tangent to the circle's radius, then the angle between it and the radius, $R = 0.40$ m, is 90°. The bead with mass, $M = 0.050$ kg, can be treated as a point particle. The required angular acceleration, $\alpha = 6.0 \text{ rad/s}^2$, is then found using the torque, τ.

SKETCH:



RESEARCH: The force produces a torque, $\tau = FR = I\alpha$. The moment of inertia of the bead is $I = MR^2$.

SIMPLIFY: $FR = I\alpha = MR^2\alpha \;\Rightarrow\; F = MR\alpha$.

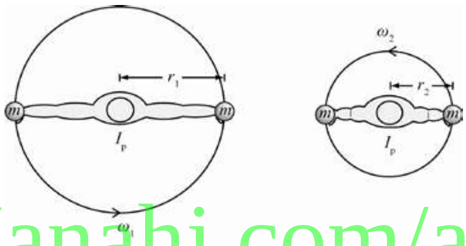
CALCULATE: $F = (0.0500 \text{ kg})(0.400 \text{ m})\left(6.00 \text{ rad/s}^2\right) = 0.120 \text{ N}$

ROUND: To three significant figures: $F = 0.120 \text{ N}$

DOUBLE-CHECK: For a small mass, a small force is reasonable.

10.73. **THINK:** Angular momentum will be conserved when the professor brings his arms and the two masses, because there is no external torque.

SKETCH:



RESEARCH: Conservation of angular momentum states $I_i\omega_i = I_f\omega_f$, and the moment of inertia at any point is $I = I_{\text{body}} + 2r^2m$. We assume that $I_{\text{body,i}} = I_{\text{body,f}}$.

SIMPLIFY: Substituting the moments of inertia into the conservation equation gives

$$\omega_f = \frac{I_i}{I_f}\omega_i = \frac{I_{\text{body,i}} + 2r_i^2 m}{I_{\text{body,f}} + 2r_f^2 m}\omega_i .$$

CALCULATE:

The initial angular speed is $\omega_i = 2\pi f = 2\pi\left(1.00 \text{ rev/min}\right) = 0.1047 \text{ rad/s}$.

So the final angular speed is

$$\omega_f = \frac{(2.80 \text{ kg m}^2) + 2(1.20 \text{ m})^2(5.00 \text{ kg})}{(2.80 \text{ kg m}^2) + 2(0.300 \text{ m})^2(5.00 \text{ kg})}(0.1047 \text{ rad/s}) = 0.4867135 \text{ rad/s}^{-1}.$$

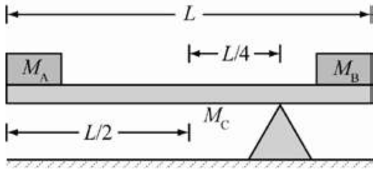
ROUND:

$\omega_f = 0.487 \text{ rad/s}$.

DOUBLE-CHECK: We find that the angular velocity increases from 0.105 rad/s to 0.487 rad/s. Does it make sense that the professor speeds up by pulling in the arms? If you have ever watched a figure skating competition, you know that the answer is yes, and that speeding up the rate of rotation by a factor of ~3 is very reasonable.

10.74. **THINK:** Determine the angular acceleration, which can be obtained by first determining the total torque. Make sure that the moments of inertia are calculated with respect to the pivot point. $M_A = 1.00$ kg, $M_B = 10.0$ kg, $M_C = 20.0$ kg and $L = 5.00$ m.

SKETCH:



RESEARCH: $\sum \tau = I\alpha$

For M_A: $I_A = M_A(3L/4)^2$. For M_B: $I_B = M_B(L/4)^2$. For the rod: $I_C = (1/12)M_C L^2 + M_C(L/4)^2$.
$I = I_A + I_B + I_C$, $\sum \tau = \tau_A + \tau_C - \tau_B$, $\tau_A = M_A g(3L/4)$, $\tau_B = M_B g(L/4)$ and $\tau_C = M_C g(L/4)$.

SIMPLIFY: $\sum \tau = gL\left(\frac{3}{4}M_A - \frac{1}{4}M_B + \frac{1}{4}M_C\right) = \frac{gL}{4}(3M_A - M_B + M_C)$

$$I = L^2\left(\frac{9}{16}M_A + \frac{1}{16}M_B + \left(\frac{1}{12}+\frac{1}{16}\right)M_C\right) = \frac{L^2}{16}\left(9M_A + M_B + \frac{7}{3}M_C\right),\quad \sum \tau = I\alpha \Rightarrow \alpha = \frac{\sum \tau}{I}$$

$$\alpha = \frac{gL}{4}\left(\frac{16}{L^2}\right)\left(\frac{3M_A - M_B + M_C}{9M_A + M_B + \frac{7}{3}M_C}\right) = \frac{4g}{L}\left(\frac{3M_A - M_B + M_C}{9M_A + M_B + \frac{7}{3}M_C}\right)$$

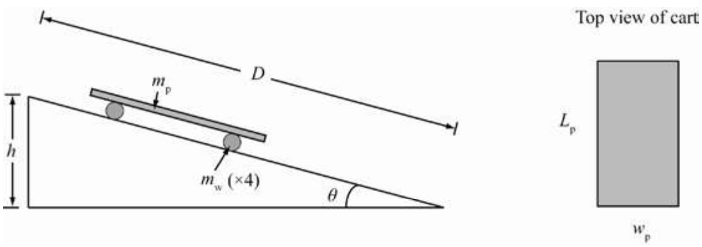
CALCULATE: $\alpha = \frac{4(9.81 \text{ m/s}^2)}{5.00 \text{ m}}\left(\frac{3(1.00 \text{ kg}) - 10.0 \text{ kg} + 20.0 \text{ kg}}{9(1.00 \text{ kg}) + 10.0 \text{ kg} + \frac{7}{3}20.0 \text{ kg}}\right) = 1.55 \text{ rad/s}^2$

ROUND: Using three significant figures, $\alpha = 1.55$ rad/s^2. The positive sign indicates that the angular acceleration is counter clockwise.

DOUBLE-CHECK: Note that α decreases as L increases. This makes sense because I increases faster with L (L is squared) than does τ.

10.75. **THINK:** To determine the cart's final speed, use the conservation of energy. The initial gravitational potential energy is converted to kinetic energy. The total kinetic energy at the bottom is the sum of the translational and rotational kinetic energies. Use $m_p = 8.00$ kg, $m_w = 2.00$ kg, $L_p = 1.20$ m, $w_p = 60.0$ cm, $r = 10.0$ cm, $D = 30.0$ m and $\theta = 15.0°$.

SKETCH:



RESEARCH: The initial energy is $E_{tot} = U$ (potential energy). The final energy is $E_{tot} = K$ (kinetic energy). $U = M_{tot}gh$, $h = D\sin\theta$, $M_{tot} = m_p + 4m_w$, $K = M_{tot}v^2/2 + I\omega^2/2$, $\omega = v/r$ and $I = 4(m_w r^2/2)$.

SIMPLIFY:

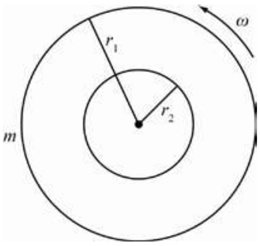
$$U = K \Rightarrow M_{tot}gh = \frac{1}{2}M_{tot}v^2 + \frac{1}{2}I\omega^2 \Rightarrow (m_p + 4m_w)gD\sin\theta = \frac{1}{2}(m_p + 4m_w)v^2 + \frac{1}{2}(4)\left(\frac{1}{2}m_w r^2\right)\left(\frac{v^2}{r^2}\right)$$

$$\Rightarrow \left(m_{\text{p}}+4m_{\text{w}}\right)gD\sin\theta=\left(\frac{1}{2}m_{\text{p}}+2m_{\text{w}}+m_{\text{w}}\right)v^2 \Rightarrow v=\sqrt{\frac{\left(m_{\text{p}}+4m_{\text{w}}\right)gD\sin\theta}{\frac{1}{2}m_{\text{p}}+3m_{\text{w}}}}$$

CALCULATE: $v=\sqrt{\dfrac{\left(8.00\text{ kg}+4\left(2.00\text{ kg}\right)\right)\left(9.81\text{ m/s}^2\right)\left(30.0\text{ m}\right)\sin 15.0^\circ}{\frac{1}{2}\left(8.00\text{ kg}\right)+3\left(2.00\text{ kg}\right)}}=11.04\text{ m/s}$

ROUND: The length of the incline is given to three significant figures, so the result should be rounded to $v = 11.0$ m/s.
DOUBLE-CHECK: This velocity is rather fast. In reality, the friction would slow the cart down. Note also that the radii of the wheels play no role.

10.76. **THINK:** Determining the moment of inertia is straightforward. To determine the torque, first determine the angular acceleration, α, and both $\Delta\omega$ and $\Delta\theta$ are known. Knowing α and I, the torque can be determined. $m = 15.0$ g, $r_1 = 1.5$ cm/2, $r_2 = 11.9$ cm/2, $\omega_{\text{i}} = 0$, $\omega_{\text{f}} = 4.3$ rev/s and $\Delta\theta = 0.25$ revs.
SKETCH:



RESEARCH: $\tau = I\alpha$, $I=\frac{1}{2}m\left(r_1^2+r_2^2\right)$, $\omega_{\text{f}}^2=\omega_{\text{i}}^2+2\alpha\Delta\theta$

SIMPLIFY:

(a) $I=\frac{1}{2}m\left(r_1^2+r_2^2\right)$

(b) $\alpha=\dfrac{\left(\omega_{\text{f}}^2-\omega_{\text{i}}^2\right)}{2\Delta\theta}$, $\tau=I\alpha=\dfrac{m}{4\Delta\theta}\left(r_1^2+r_2^2\right)\omega_{\text{f}}^2\ \left(\omega_{\text{i}}=0\right)$

CALCULATE:

(a) $I=\frac{1}{2}\left(15.0\cdot 10^{-3}\text{ kg}\right)\left(\left(\frac{1.50\cdot 10^{-2}}{2}\text{ m}\right)^2+\left(\frac{11.9\cdot 10^{-2}\text{ m}}{2}\right)^2\right)=2.697\cdot 10^{-5}\text{ kg m}^2$

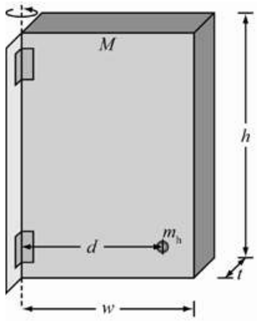
(b) $\Delta\theta=0.250\text{ revs}\left(\frac{2\pi\text{ rad}}{\text{rev}}\right)=1.571\text{ rad}$, $\omega_{\text{f}}=4.30\text{ rev/s}\left(\frac{2\pi\text{ rad}}{\text{rev}}\right)=27.02\text{ rad/s}$

$\alpha=\dfrac{\left(27.02\text{ rad/s}\right)^2}{2\left(1.571\text{ rad}\right)}=232.4\text{ rad/s}^2$, $\tau=2.697\cdot 10^{-5}\text{ kg m}^2\left(232.4\text{ rad/s}^2\right)=6.267\cdot 10^{-3}\text{ N m}$

ROUND: Rounding to three significant figures, (a) $I=2.70\cdot 10^{-5}\text{ kg m}^2$ and (b) $\tau=6.27\cdot 10^{-3}\text{ N m}$.
DOUBLE-CHECK: These results are reasonable for the given values.

10.77. **THINK:** Begin with the moment of inertia of the door about an axis passing through its center of mass, then use the parallel axis theorem to shift the axis to the edge of the door, and then add the contribution of the handle, which can be treated as a point particle. $\rho = 550.\text{ kg/m}^3$, $w = 0.550$ m, $h = 0.790$ m, $t = 0.0130$ m, $d = 0.450$ m and $m_{\text{h}} = 0.150$ kg.

SKETCH:



RESEARCH: $M = \rho V = \rho wht$, $I_{\text{center}} = \frac{1}{12}M\left(w^2 + t^2\right)$, $I_{\text{edge}} = I_{\text{center}} + M\left(\frac{w}{2}\right)^2$, $I_{\text{handle}} = m_{\text{h}}d^2$

$I = I_{\text{edge}} + I_{\text{handle}}$

SIMPLIFY: $I = \frac{M}{12}\left(w^2 + t^2\right) + \frac{M}{4}w^2 + m_{\text{h}}d^2 = \frac{M}{12}\left(4w^2 + t^2\right) + m_{\text{h}}d^2$

CALCULATE: Substituting $M = \rho wht$ into the above equation yields:

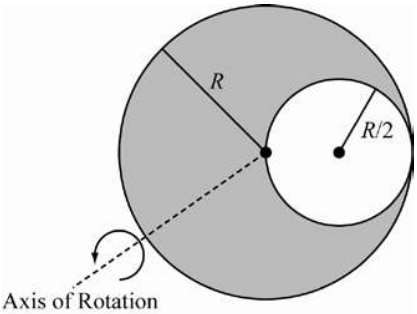
$$I = \frac{1}{12}\left(550.\text{ kg/m}^3\right)\left(0.550\text{ m}\right)\left(0.790\text{ m}\right)\left(0.0130\text{ m}\right)\left(4\left(0.550\text{ m}\right)^2 + \left(0.0130\text{ m}\right)^2\right) + \left(0.150\text{ kg}\right)\left(0.450\text{ m}\right)^2$$
$$= 0.3437\text{ kg m}^2.$$

ROUND: Rounding to three significant figures gives $I = 0.344\text{ kg m}^2$.

DOUBLE-CHECK: This is a reasonable result for a door of this size. Note that the height of the door enters into only the calculation of the door's mass.

10.78. **THINK:** The moment of inertia of the machine part is the moment of inertia of the initial solid disk about its center, minus the moment of inertia of a solid disk of the amount of mass removed about its outside edge (which is at the center of the disk). M = mass of the disk without the hole cut out, and m = mass of the material cut out to make a hole.

SKETCH:



RESEARCH: $I_{\text{center}} = MR^2/2$ (disk spinning about its center). $I_{\text{edge}} = \frac{1}{2}m\left(R/2\right)^2 + m\left(R/2\right)^2$ (disk spinning about its edge). The area of the hole is $\pi R^2/4$. The area of the disk without the hole is πR^2. The area of the disk with the hole is $\pi R^2 - \pi R^2/4 = 3\pi R^2/4$. The area of the hole is 1/4 the area of the disk without the hole; therefore, because the disk has uniform density, $m = M/4$. The moment of inertia is

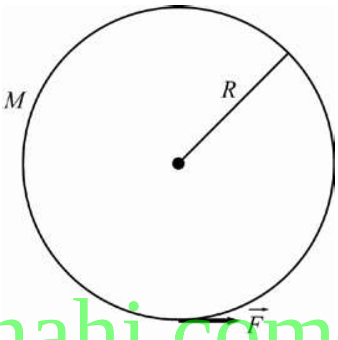
$$I = \frac{1}{2}MR^2 - \left(\frac{1}{2}m\left(\frac{R}{2}\right)^2 + m\left(\frac{R}{2}\right)^2\right).$$

SIMPLIFY: Substitute $m = M/4$ into the above equation to get

$$I = \frac{1}{2}MR^2 - \left[\frac{M}{8}\left(\frac{R^2}{4}\right) + \frac{M}{4}\left(\frac{R^2}{4}\right)\right] = \frac{16}{32}MR^2 - \frac{1}{32}MR^2 - \frac{2}{32}MR^2 = \frac{13}{32}MR^2.$$

CALCULATE: This step is not necessary.
ROUND: This step is not necessary.
DOUBLE-CHECK: As expected, the moment of inertia decreases when the hole is cut out.

10.79. **THINK:** If the angular momentum and the torque are determined, the time can be determined by recalling that torque is the time rate of change of angular momentum. To determine the angular momentum, first determine the angular speed required to produce a centripetal acceleration equal to Earth's gravitational acceleration. From this, the angular momentum, L, of the space station can be determined. Finally, the torque can be determined from the given force and the radius of the space station. $R = 50.0$ m, $M = 2.40 \cdot 10^5$ kg and $F = 1.40 \cdot 10^2$ N.
SKETCH:



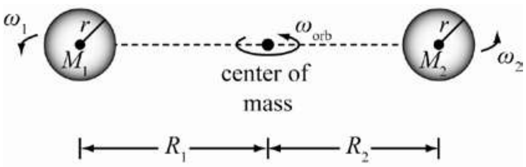
RESEARCH: $I = MR^2$, $L = I\omega$, $v = \omega R$, $\frac{v^2}{R} = g$, $\tau = FR$, $\tau = \frac{\Delta L}{\Delta t}$

SIMPLIFY: $\Delta t = \frac{\Delta L}{\tau} = \frac{I\omega - 0}{FR} = \frac{MR^2\omega}{FR} = \frac{MR\omega}{F}$, $\omega = \frac{v}{R} = \frac{1}{R}\sqrt{Rg} = \sqrt{\frac{g}{R}}$, $\Delta t = \frac{MR}{F}\sqrt{\frac{g}{R}} = \frac{M\sqrt{Rg}}{F}$

CALCULATE: $\Delta t = \frac{2.40 \cdot 10^5 \text{ kg}\sqrt{(50.0 \text{ m})(9.81 \text{ m/s}^2)}}{1.40 \cdot 10^2 \text{ N}} = 3.797 \cdot 10^4 \text{ s}$

ROUND: The radius of the space station is given to three significant figures, so the result should be rounded to $\Delta t = 3.80 \cdot 10^4$ s.
DOUBLE-CHECK: The result is equal to about 10 hours. For such a relatively small thrust, this result is reasonable. As expected, this time interval increases if either the thrust decreases or the mass increases.

10.80. **THINK:** There is enough information given to determine the stars' rotational and translational kinetic energies directly and subsequently determine their ratio. Note that the orbital period is given as 2.4 hours. Use the values: $M_1 = 1.250M_{\text{Sun}}$, $M_2 = 1.337M_{\text{Sun}}$, $\omega_1 = 2\pi$ rad/2.8 s, $\omega_2 = 2\pi$ rad/0.023 s, r = 20.0 km, $R_1 = 4.54 \cdot 10^8$ m, $R_2 = 4.23 \cdot 10^8$ m and $\omega_{\text{orb}} = 2\pi$ rad/2.4 h.
SKETCH:



RESEARCH: $K_{\text{rot}} = \frac{1}{2}I\omega^2$, $I = \frac{2}{5}MR^2$, $K_{\text{orb}} = \frac{1}{2}Mv^2 = \frac{1}{2}MR^2\omega^2$

SIMPLIFY:

(a) $\dfrac{K_{1,\text{rot}}}{K_{2,\text{rot}}}=\dfrac{I_1\omega_1^2}{I_2\omega_2^2}=\dfrac{M_1 r_1^2\omega_1^2}{M_2 r_2^2\omega_2^2}=\dfrac{M_1\omega_1^2}{M_2\omega_2^2}$

(b) $\dfrac{K_{1,\text{rot}}}{K_{1,\text{orb}}}=\dfrac{\frac{1}{5}M_1 r_1^2\omega_1^2}{\frac{1}{2}M_1 R_1^2\omega_{\text{orb}}^2}=\dfrac{2r_1^2\omega_1^2}{5R_1^2\omega_{\text{orb}}^2},\quad \dfrac{K_{2,\text{rot}}}{K_{2,\text{orb}}}=\dfrac{2r_2^2\omega_2^2}{5R_2^2\omega_{\text{orb}}^2}$

CALCULATE:

(a) $\dfrac{K_{1,\text{rot}}}{K_{2,\text{rot}}}=\dfrac{1.25/(2.8)^2}{1.337/(0.023)^2}=6.308\cdot 10^{-5}$

(b) $\dfrac{K_{1,\text{orb}}}{K_{1,\text{rot}}}=\dfrac{2\left(20\cdot 10^3\right)^2/(2.8\text{ s})^2}{5\left(4.54\cdot 10^8\right)^2/(2.4\text{ h}\cdot 3600\text{ s/h})^2}=0.00739128$

$\dfrac{K_{2,\text{rot}}}{K_{2,\text{orb}}}=\dfrac{2\left(20\cdot 10^3\text{ m}\right)^2/(0.023\text{ s})^2}{5\left(4.23\cdot 10^8\right)^2/(2.4\text{ h}\cdot 3600\text{ s/h})^2}=126.186$

ROUND: The rotation periods are given to at least three significant figures, so the results should be rounded to:

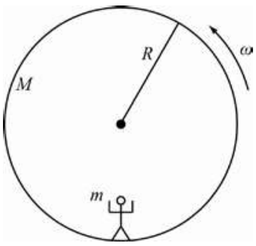
(a) $K_{1,\text{rot}}/K_{2,\text{rot}}=6.31\cdot 10^{-5}$

(b) $K_{1,\text{orb}}/K_{1,\text{rot}}=7.39\cdot 10^{-3},\quad K_{2,\text{orb}}/K_{2,\text{rot}}=126$

DOUBLE-CHECK: M_2 has a much faster rotational speed than M_1. The kinetic energy for M_1 is dominated by the orbit, while for M_2 it is dominated by rotational motion.

10.81. **THINK:** Conservation of angular momentum can be considered to determine the angular momentum of the merry-go-round. From this, the mass, M, of the merry-go-round can be determined. For parts (b) and (c), use the uniform acceleration equations to answer the problem. R = 1.50 m, $\omega = 1.30$ rad/s, m = 52.0 kg and v = 6.80 m/s (speed of the student just prior to jumping on).

SKETCH:



RESEARCH: $L_{\text{student}}=Rmv,\quad L=\omega I$ (merry-go-round), $I=\frac{1}{2}MR^2+mR^2,\quad \Delta\theta=\frac{1}{2}\alpha t^2+\omega_{\text{i}}t,$

$\omega_{\text{f}}=\alpha t+\omega_{\text{i}}$, and $\tau=I\alpha$.

SIMPLIFY:

(a) $L_{\text{student}}=I\omega \Rightarrow Rmv=\left(\frac{1}{2}MR^2+mR^2\right)\omega \Rightarrow \frac{1}{2}MR^2\omega=Rmv-mR^2\omega$

$\Rightarrow M=\dfrac{2}{\omega R^2}\left(Rmv-mR^2\omega\right) \Rightarrow \dfrac{2m}{R\omega}\left(v-R\omega\right)=2m\left(\dfrac{v}{\omega R}-1\right)$

(b) $\tau=I\alpha=\left(\frac{1}{2}MR^2+mR^2\right)\alpha,\quad \alpha=\dfrac{\Delta\omega}{\Delta t}=-\dfrac{\omega}{t}\quad\left(\omega_{\text{i}}=\omega,\ \omega_{\text{f}}=0,\ t_{\text{f}}=t,\ t_{\text{i}}=0\right),\quad \tau=-\dfrac{\omega R^2}{t}\left(\dfrac{M}{2}+m\right)$

(c) $\Delta\theta = \frac{1}{2}\alpha t^2 + \omega_i t = \frac{1}{2}\left(-\frac{\omega}{t}\right)t^2 + \omega t = -\frac{1}{2}\omega t + \omega t = \frac{1}{2}\omega t$

CALCULATE:

(a) $M = 2(52.0 \text{ kg})\left(\frac{6.80 \text{ m/s}}{1.30 \text{ rad s}^{-1}(1.50 \text{ m})} - 1\right) = 258.7 \text{ kg}$

(b) $\tau = -\frac{(1.30 \text{ rad s}^{-1})(1.50 \text{ m})^2}{35.0 \text{ s}}\left(\frac{258.7 \text{ kg}}{2} + 52.0 \text{ kg}\right) = -15.15 \text{ N m}$

(c) $\Delta\theta = \frac{1}{2}(1.30 \text{ rad/s})(35.0 \text{ s}) = 22.75 \text{ rad} = 22.75 \text{ rad}\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = 3.621 \text{ rev}$

ROUND: Round the results to three significant figures.

(a) $M = 259$ kg

(b) $\tau = -15.2$ N m

(c) $\Delta\theta = 3.62$ revolutions

DOUBLE-CHECK: The results are all consistent with the given information.

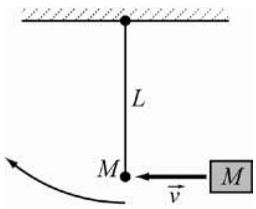
10.82. THINK:

(a) The speed of the pendulum just after the collision can be determined by considering the conservation of linear momentum. From the conservation of energy, the maximum height of the pendulum can be determined, since at this point, all of the initial kinetic energy will be stored as gravitational potential energy.

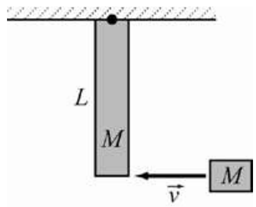
(b) From the conservation of angular momentum, the rotation speed of the pendulum just after collision can be determined. From the conservation of energy, the maximum height of the pendulum can be determined, since at this point, all of the initial rotational kinetic energy will be stored as gravitational potential energy. $L = 0.48$ m and $v = 3.6$ m/s.

SKETCH:

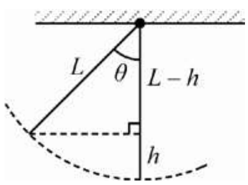
(a)



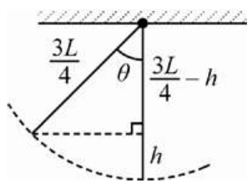
(b)



(c)



(d)



RESEARCH:

(a) $E = \text{constant} = K + U, \quad K = \frac{1}{2}mv^2, \quad U = mgh$

(b) $L = \text{constant} = I\omega, \quad I_{\text{rod}} = \frac{1}{3}ML^2, \quad I_{\text{proj}} = ML^2$

SIMPLIFY: v_0 is the speed of the projectile just prior to collision. v_p is the speed of the pendulum at the lower edge just after collision.

(a) $P_\text{i} = P_\text{f} \;\Rightarrow\; Mv_0 = (M+M)v_\text{p} \;\Rightarrow\; v_\text{p} = \frac{1}{2}v_0$; At the pendulum's maximum height,

$$K_\text{f} = (M+M)gh = \frac{1}{2}(M+M)v_\text{p}^{\;2}.$$

$$h = \frac{v_p^{\;2}}{2g} = \frac{v_0^{\;2}}{8g} \;\Rightarrow\; \cos\theta = 1 - \frac{h}{L} = 1 - \frac{v_0^{\;2}}{8gL} \;\Rightarrow\; \theta = \cos^{-1}\left(1 - \frac{v_0^{\;2}}{8gL}\right)$$

(b) $L = I\omega$, $\omega = v/L$, $L_\text{i} = I_\text{i}\frac{v_0}{L} = MLv_0$

$$L_\text{f} = \left(I_\text{rod} + ML^2\right)\omega_\text{f} = L_\text{i} = MLv_0 \;\Rightarrow\; \frac{4}{3}ML^2\omega_\text{f} = MLv_0 \;\Rightarrow\; \omega_\text{f} = \frac{3}{4L^2}Lv_0 = \frac{3v_0}{4L} \;\Rightarrow\; \frac{3v_0}{4L} = \frac{v_\text{p}}{L} \;\Rightarrow\; v_\text{p} = \frac{3}{4}v_0$$

At maximum height: $\frac{1}{2}I\omega_\text{f}^{\;2} = mgh \;\Rightarrow\; \frac{1}{2}\left(\frac{4}{3}ML^2\right)\left(\frac{3v_0}{4L}\right)^2 = 2Mgh \;\Rightarrow\; \frac{3}{8}v_0^{\;2} = 2gh \;\Rightarrow\; h = \frac{3v_0^{\;2}}{16g}.$

This is the height attained by the center of mass of the pendulum and projectile system. By symmetry, the center of mass of the system is located $3L/4$ from the top. So,

$$\cos\theta = 1 - \frac{h}{3L/4} = 1 - \frac{4h}{3L} = 1 - \frac{4}{3L}\cdot\frac{3v_0^{\;2}}{16g} = 1 - \frac{v_0^{\;2}}{4gL} \;\Rightarrow\; \theta = \cos^{-1}\left(1 - \frac{v_0^{\;2}}{4gL}\right).$$

CALCULATE:

(a) $\theta = \cos^{-1}\left(1 - \frac{(3.6 \text{ m/s})^2}{8(9.81 \text{ m/s}^2)(0.48 \text{ m})}\right) = 49.01°$

(b) $\theta = \cos^{-1}\left(1 - \frac{(3.6 \text{ m/s})^2}{4(9.81 \text{ m/s}^2)(0.48 \text{ m})}\right) = 71.82°$

ROUND: Round the results to three significant figures.

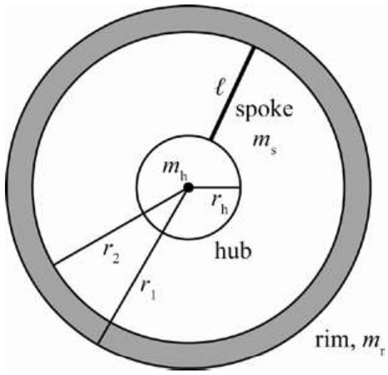
(a) $\theta = 49.0°$

(b) $\theta = 71.8°$

DOUBLE-CHECK: The rod swings higher. This is expected since the center of mass is higher than for the pendulum. The projectile exerts a greater torque on the rod.

10.83. **THINK:** The quantity of interest can be calculated directly from the given information. $m_\text{r} = 5.20$ kg, $m_\text{h} = 3.40$ kg, $m_\text{s} = 1.10$ kg, $r_1 = 0.900$ m, $r_2 = 0.860$ m, $r_\text{h} = 0.120$ m and $l = r_2 - r_h$.

SKETCH:



RESEARCH: $I_{\text{rim}} = \frac{1}{2}m_{\text{r}}\left(r_1^2 + r_2^2\right)$, $I_{\text{spoke}} = \frac{1}{12}m_{\text{s}}l^2 + m_{\text{s}}d^2$, (with $d = \frac{1}{2}l + r_{\text{h}}$), $I_{\text{hub}} = \frac{1}{2}m_{\text{h}}r_{\text{h}}^2$

SIMPLIFY: $I = I_{\text{rim}} + I_{\text{hub}} + 12I_{\text{spoke}}$, $M = m_{\text{r}} + m_{\text{h}} + 12m_{\text{s}}$, $R = r_1$

CALCULATE: $I_{\text{rim}} = \frac{1}{2}5.20 \text{ kg}\left(\left(0.900\right)^2 + \left(0.860\right)^2\right) \text{ m}^2 = 4.029 \text{ kg m}^2$

$I_{\text{hub}} = \frac{1}{2}\left(3.40 \text{ kg}\right)\left(0.120 \text{ m}\right)^2 = 2.448 \cdot 10^{-2} \text{ kg m}^2$

$I_{\text{spoke}} = \frac{1}{12}\left(1.10 \text{ kg}\right)\left(0.860 \text{ m} - 0.120 \text{ m}\right)^2 + \left(1.10 \text{ kg}\right)\left(0.490 \text{ m}\right)^2 = 3.143 \cdot 10^{-1} \text{ kg m}^2$

$I = I_{\text{rim}} + I_{\text{hub}} + 12I_{\text{spoke}} = 7.825 \text{ kg m}^2$, $M = \left[5.20 + 3.40 + 12\left(1.10\right)\right] \text{ kg} = 21.8 \text{ kg}$

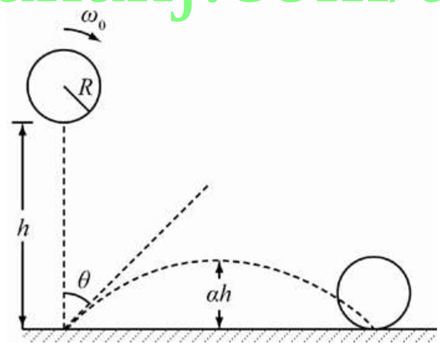
$$c = \frac{I}{MR^2} = \frac{7.825 \text{ kgm}^2}{\left(21.8 \text{ kg}\right)\left(0.900 \text{ m}\right)^2} = 0.4431$$

ROUND: Rounding to three significant figures, $c = 0.443$.

DOUBLE-CHECK: It is reasonable that the moment of inertia is dominated by the rim and the spokes, and the hub is negligible.

10.84. **THINK:** To determine the angles in parts (a), the vertical and horizontal components of the velocity just after impact must be determined. To determine the vertical velocity, consider the conservation of energy. To determine the horizontal velocity, consider the linear and angular impulses experienced in either of the following two situations. Situation I: The ball slips on the floor during the entire impact time. Kinetic friction must be considered the entire time. Situation II: The ball stops slipping on the floor at some point during the impact. From this point for the duration of the impact, rolling motion is attained, and the usual equations relating angular and rotational speeds are applicable.

SKETCH:



RESEARCH: Energy conservation is given by $mgh = mv_0^2/2$. v_0 is the speed of the ball just prior to the impact for the first time. Also, from energy conservation: $mg\left(\alpha h\right) = \frac{1}{2}mv_{2y}^2$, where v_{2y} is the vertical velocity just after the impact of the ball with the ground. Linear impulse is given by

$$\int_{t_1}^{t_2} F\left(t\right)dt = p\left(t_2\right) - p\left(t_1\right).$$

Angular impulse is given by $\int_{t_1}^{t_2} \tau\left(t\right)dt = L\left(t_2\right) - L\left(t_1\right) = I\left(\omega_0 - \omega_2\right)$.

SIMPLIFY: Just prior to impact: $mgh = mv_0^2/2 \Rightarrow v_0 = \sqrt{2gh}$. Just after impact:

$$\alpha mgh = \frac{1}{2}mv_{2y}^2 \Rightarrow v_{2y} = +\sqrt{2\alpha gh}.$$

Situation I:
n is the normal force and μn is the frictional force. The impulses are as follows:

$$I_y = \int_{t_1}^{t_2} n(t)\,dt = m\sqrt{\alpha}v_0 - m(-v_0) = (1+\sqrt{\alpha})mv_0,\quad I_x = \int_{t_1}^{t_2} \mu n\,dt = \mu I_y = \mu m(1+\sqrt{\alpha})v_0 = mv_{2x}$$

$$I_\theta = \int_{t_1}^{t_2} \mu n R\,dt = R\mu I_y = R\mu m(1+\sqrt{\alpha})v_0 = I(\omega_0 - \omega_2) \Rightarrow v_{2x} = \mu(1+\sqrt{\alpha})v_0 = \mu(1+\sqrt{\alpha})\sqrt{2gh}$$

$$\omega_2 = \omega_0 - \frac{1}{I}R\mu m(1+\sqrt{\alpha})v_0 = \omega_0 - \frac{R}{I}\mu m(1+\sqrt{\alpha})\sqrt{2gh} = \omega_0 - \frac{Rmv_{2x}}{I}$$

(a) $\tan\theta = \left(\frac{v_{2x}}{v_{2y}}\right)^{-1} = \left(\frac{\mu(1+\sqrt{\alpha})\sqrt{2gh}}{\sqrt{2\alpha gh}}\right)^{-1} = \left(\frac{\mu(1+\sqrt{\alpha})}{\sqrt{\alpha}}\right)^{-1}$

(b) The time, t, it takes for the ball to fall is $t = \frac{2v_{2y}}{g} = \frac{2}{g}\sqrt{2\alpha gh} = \sqrt{\frac{8\alpha h}{g}}$.

The distance traveled during this time is $d = v_{2x}t = \mu(1+\sqrt{\alpha})\left(\sqrt{2gh}\right)\left(\sqrt{\frac{8\alpha h}{g}}\right) = 4\mu(1+\sqrt{\alpha})\left(\sqrt{\alpha}h\right)$.

(c) $\omega_2 = \omega_0 - \frac{Rmv_{2x}}{I}$. The minimum ω_0 occurs when $R\omega_2 = v_{2x}$, where $R\omega_2$ is the velocity of the contact point. $R\omega_{0,\min} - \frac{R^2mv_{2x}}{I} = v_{2x}$, $\omega_{0,\min} = v_{2x}\left(\frac{1}{R} + \frac{Rm}{I}\right) = \frac{v_{2x}}{R}\left(1 + \frac{mR^2}{I}\right) = \frac{\mu(1+\sqrt{\alpha})\sqrt{2gh}}{R}\left(1+\frac{mR^2}{I}\right)$

Situation II:
After the ball stops slipping, there is a rolling motion and $\omega_2 R = v_{2x}$. The impulses are as follows.

$$I_x = \int_{t_1}^{t_2} \mu n\,dt = mv_{2x},\quad I_\theta = \int_{t_1}^{t_2} \mu n R\,dt = I(\omega_0 - \omega_2) = RI_x$$

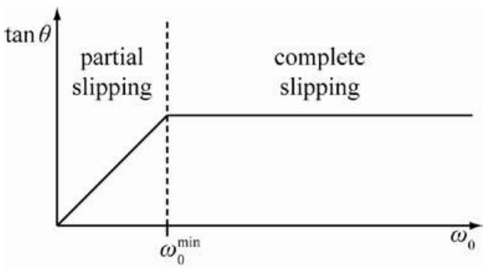
$$\Rightarrow I(\omega_0 - \omega_2) = Rmv_{2x} \Rightarrow I\left(\omega_0 - \frac{v_{2x}}{R}\right) = Rmv_{2x}.$$

Solve for v_{2x} by substituting $I = (2mR^2/5)$ into the above equation to get:

$$\frac{2}{5}mR^2\left(\omega_0 - \frac{v_{2x}}{R}\right) = Rmv_{2x} \Rightarrow \frac{2}{5}(R\omega_0 - v_{2x}) = v_{2x} \Rightarrow v_{2x}\left(1+\frac{2}{5}\right) = \frac{2}{5}\omega_0 R \Rightarrow v_{2x} = \frac{5}{7}\left(\frac{2}{5}\omega_0 R\right) = \frac{2}{7}\omega_0 R.$$

(d) $\tan\theta = \frac{v_{2x}}{v_{2y}} = \frac{\frac{2}{7}\omega_0 R}{\sqrt{2\alpha gh}}$

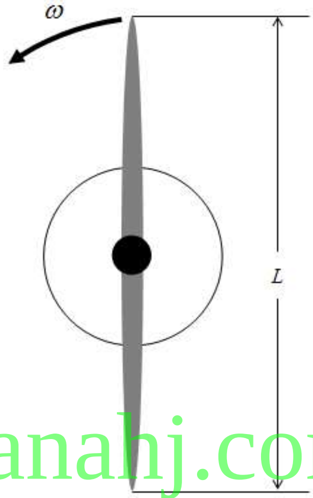
(e) $d = v_{2x}t = \frac{2}{7}\omega_0 R\sqrt{\frac{8\alpha h}{g}}$



CALCULATE: This step is not necessary.
ROUND: This step is not necessary.
DOUBLE-CHECK: When only partial slipping occurs, the horizontal distance traveled should and does depend on ω_0.

Multi-Version Exercises

10.85. **THINK:** The length and mass of the propeller, as well as the frequency with which it is rotating, are given. To find the kinetic energy of rotation, it is necessary to find the moment of inertia, which can be calculated from the mass and radius by approximating the propeller as a rod with constant mass density.
SKETCH: The propeller is shown as it would be seen looking directly at it from in front of the plane.



RESEARCH: The kinetic energy of rotation is related to the moment of inertia and angular speed by the equation $K_{\text{rot}} = \frac{1}{2}I\omega^2$. The angular speed $\omega = 2\pi f$ can be computed from the frequency of the propeller's rotation. Approximating the propeller as a rod with constant mass density means that the formula $I = \frac{1}{12}mL^2$ for a long, thin rod rotating about its center of mass can be used.
SIMPLIFY: Combine the equations for the moment of inertia and angular speed to get a single equation for the kinetic energy $K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{12}mL^2\right)\cdot(2\pi f)^2$. Using algebra, this can be simplified to $K_{\text{rot}} = \frac{m}{6}(\pi L f)^2$. Since the angular speed is given in revolutions per minute, the conversion 1 minute = 60 seconds will also be needed.
CALCULATE: The propeller weighs m = 17.36 kg, it is L = 2.012 m long, and it rotates at a frequency of f = 3280. rpm. The rotational kinetic energy is

$$\begin{aligned} K_{\text{rot}} &= \frac{m}{6}(\pi L f)^2 \\ &= \frac{17.36\text{ kg}}{6}\left(\pi \cdot 2.012\text{ m}\cdot 3280.\text{ rpm}\cdot\frac{1\text{min}}{60\text{sec}}\right)^2 \\ &= 345{,}461.2621\text{ J} \end{aligned}$$

ROUND: The values in the problem are all given to four significant figures, so the final answer should have four figures. The propeller has a rotational kinetic energy of $3.455\cdot 10^5$ J or 345.5 kJ.
DOUBLE-CHECK: Given the large amount of force needed to lift a plane, it seems reasonable that the energy in the propeller would be in the order of hundreds of kilojoules. Working backwards, if a propeller weighing 17.36 kg and having length 2.012 m has rotational kinetic energy 345.5 kJ, then it is turning at

$f = \frac{1}{\pi L}\sqrt{\frac{6K_{\text{rot}}}{m}} = \frac{1}{\pi \cdot 2.012\text{ m}}\sqrt{\frac{6 \cdot 3.455 \cdot 10^5\text{ J}}{17.36\text{ kg}}}$. This is 54.667 revolutions per second, which agrees with the given value of 54.667·60 = 3280 rpm. This confirms that the calculations were correct.

10.86. $K_{\text{rot}} = \frac{m}{6}(\pi L f)^2$

$$f = \frac{1}{\pi L}\sqrt{\frac{6K_{\text{rot}}}{m}}$$

$$= \frac{1}{\pi(2.092\text{ m})}\sqrt{\frac{6(422.8 \cdot 10^3\text{ J})}{17.56\text{ kg}}} = 57.8\text{ rev/s} = 3470.\text{ rpm}$$

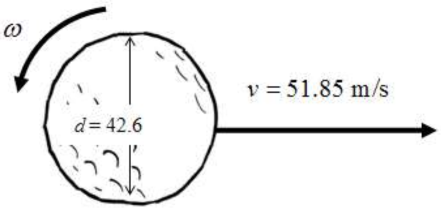
10.87. $K_{\text{rot}} = \frac{m}{6}(\pi L f)^2$

$$m = \frac{6K_{\text{rot}}}{(\pi L f)^2}$$

$$= \frac{6(124.3 \cdot 10^3\text{ J})}{\left[\pi(1.812\text{ m})\left(2160.\text{ rpm} \cdot \frac{1}{60}\text{ s/min}\right)\right]^2} = 17.76\text{ kg}$$

10.88. **THINK:** The total kinetic energy of the golf ball is the sum of the rotational kinetic energy and the translational kinetic energy. The translational kinetic energy can be calculated from the mass of the ball and the speed of the center of mass of the golf ball, both of which are given in the question. To find the rotational kinetic energy, it is necessary to find the moment of inertia of the golf ball. Though the golf ball is not a perfect sphere, it is close enough that the moment of inertia can be computed from the mass and diameter of the golf ball using the approximation for a sphere.

SKETCH: The golf ball has both rotational and translational motion.



RESEARCH: The total kinetic energy is equal to the translational kinetic energy plus the rotational kinetic energy $K = K_{\text{trans}} + K_{\text{rot}}$. The translational kinetic energy is computed from the speed and the mass of the golf ball using the equation $K_{\text{trans}} = \frac{1}{2}mv^2$. The rotational kinetic energy is computed from the moment of inertia and the angular speed by $K_{\text{rot}} = \frac{1}{2}I\omega^2$. It is necessary to compute the moment of inertia and the angular speed. The angular speed $\omega = 2\pi f$ depends only on the frequency. To find the moment of inertia, first note that golf balls are roughly spherical. The moment of inertia of a sphere is given by $I = \frac{2}{5}mr^2$. The question gives the diameter d which is twice the radius ($d / 2 = r$). Since the frequency is given in revolutions per minute and the speed is given in meters per second, the conversion factor $\frac{1\text{ min}}{60\text{ sec}}$ will be necessary.

SIMPLIFY: First, find the moment of inertia of the golf ball in terms of the mass and diameter to get $I = \frac{1}{10}md^2$. Substituted for the angular speed and moment of inertia in the equation for rotational kinetic energy to get $K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{10}md^2\right)(2\pi f)^2$. Finally, use the equations $K_{\text{trans}} = \frac{1}{2}mv^2$ and $K_{\text{rot}} = \frac{1}{2}\left(\frac{1}{10}md^2\right)(2\pi f)^2$ to find the total kinetic energy and simplify using algebra:

$$\begin{aligned} K &= K_{\text{trans}} + K_{\text{rot}} \\ &= \tfrac{1}{2}mv^2 + \tfrac{1}{2}\left(\tfrac{1}{10}md^2\right)(2\pi f)^2 \\ &= \tfrac{1}{2}mv^2 + \tfrac{1}{5}m(\pi df)^2 \end{aligned}$$

CALCULATE: The mass of the golf ball is 45.90 g = 0.04590 kg, its diameter is 42.60 mm = 0.04260 m, and its speed is 51.85 m/s. The golf ball rotates at a frequency of 2857 revolutions per minute. The total kinetic energy is

$$\begin{aligned} K &= \tfrac{1}{2}mv^2 + \tfrac{1}{5}m(\pi df)^2 \\ &= \tfrac{1}{2}(0.04590\text{ kg})(51.85\text{ m/s})^2 + \tfrac{1}{5}(0.04590\text{ kg})\left(\pi \cdot 0.04260\text{ m} \cdot 2857\text{ rpm} \cdot \frac{1\text{ min}}{60\text{ sec}}\right)^2 \\ &= 62.07209955\text{ J} \end{aligned}$$

ROUND: The mass, speed, frequency, and diameter of the golf ball are all given to four significant figures, so the translational and rotational kinetic energies should both have four significant figures, as should their sum. The total energy of the golf ball is 62.07 J.

DOUBLE-CHECK: The golf ball's translational kinetic energy alone is equal to $\frac{1}{2}(0.04590\text{ kg})(51.85\text{ m/s})^2 = 61.7\text{ J}$, and it makes sense that a well-driven golf ball would have much more energy of translation than energy of rotation.

10.89. $K = K_{\text{trans}} + K_{\text{rot}} = \frac{1}{2}mv^2 + \frac{1}{5}m(\pi df)^2$

$$f = \frac{1}{\pi d}\sqrt{5\left(\frac{K}{m} - \frac{1}{2}v^2\right)}$$

$$= \frac{1}{\pi(0.04260\text{ m})}\sqrt{5\left(\frac{67.67\text{ J}}{0.04590\text{ kg}} - \frac{1}{2}(54.15\text{ m/s})^2\right)} = 47.79\text{ rev/s} = 2867\text{ rpm}$$

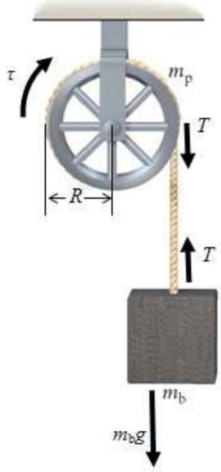
10.90. $K = K_{\text{trans}} + K_{\text{rot}} = \frac{1}{2}mv^2 + \frac{1}{5}m(\pi df)^2$

$$v = \sqrt{2\left(\frac{K}{m} - \frac{1}{5}(\pi df)^2\right)}$$

$$= \sqrt{2\left(\frac{73.51\text{ J}}{0.04590\text{ kg}} - \frac{1}{5}\left(\pi \cdot 0.04260\text{ m} \cdot 2875\text{ rpm} \cdot \frac{1}{60}\text{ min/s}\right)^2\right)} = 56.45\text{ m/s}$$

10.91. **THINK:** The gravitational force on the block is transmitted through the rope, causing a torque on the pulley. The torque causes an angular acceleration, and the linear acceleration is calculated from the angular acceleration.

SKETCH: Use the figure from the text:



RESEARCH: The torque on the pulley is given by the tension on the rope times the radius of the pulley $\tau = TR$. This torque will cause an angular acceleration $\tau = I\alpha$, where the moment of inertia of the pulley is given by $I = m_p R^2$. The tension on the rope is given by $T - m_b g = -m_b a$ (the minus indicates that the block is accelerating downward). The linear acceleration of the block a is related to the angular acceleration of the pulley α by the equation $a = R\alpha$.

SIMPLIFY: First, substitute for the tension on the pulley $T = m_b g - m_b a$ in the equation for the torque τ to get $\tau = (m_b g - m_b a)R$. Then, substitute for the moment of inertia ($I = m_p R^2$) and angular acceleration ($\alpha = a/R$) in the equation $\tau = I\alpha$ to get $\tau = (m_p R^2)\left(\frac{a}{R}\right) = m_p Ra$. Combine these two expressions for the torque to get $(m_b g - m_b a)R = m_p Ra$. Finally, solve this expression for the linear acceleration a of the block:

$$(m_b g - m_b a)R = m_p aR$$
$$m_b gR - m_b aR + m_b aR = m_p aR + m_b aR$$
$$m_b gR = (m_p R + m_b R)a$$
$$\frac{m_b gR}{m_p R + m_b R} = a$$
$$a = \frac{m_b g}{m_p + m_b}$$

CALCULATE: The mass of the block is m_b = 4.243 kg and the mass of the pulley is m_p = 5.907 kg. The acceleration due to gravity is –9.81 m/s^2. So, the total (linear) acceleration of the block is

$$a = \frac{m_b g}{m_p + m_b} = \frac{-9.81 \text{ m/s}^2 \cdot 4.243 \text{ kg}}{5.907 \text{ kg} + 4.243 \text{ kg}} = -4.100869951 \text{ m/s}^2.$$

ROUND: The masses of the pulley and block are given to four significant figures, and the sum of their masses has five figures. On the other hand, the gravitational constant g is given only to three significant figures. So, the final answer should have three significant figures. The block accelerates downward at a rate of 4.10 m/s^2.

DOUBLE-CHECK: A block falling freely would accelerate (due to gravity near the surface of the Earth) at a rate of 9.81 m/s^2 towards the ground. The block attached to the pulley will still accelerate downward, but the rate of acceleration will be less (the potential energy lost when the block falls 1 meter will equal the kinetic energy of a block in free fall, but it will equal the kinetic energy of the block falling plus the rotational kinetic energy of the pulley in the problem). The mass of the pulley is close to, but a bit larger than, the mass of the block, so the acceleration of the block attached to the pulley should be a bit less than half of the acceleration of the block in free fall. This agrees with the final acceleration of 4.10 m/s^2, which is a bit less than half of the acceleration due to gravity.

10.92. $$a = \frac{m_b g}{m_p + m_b}$$

$$m_p = \frac{m_b g}{a} - m_b = m_b\left(\frac{g}{a} - 1\right) = (4.701 \text{ kg})\left(\frac{9.81 \text{ m/s}^2}{4.330 \text{ m/s}^2} - 1\right) = 5.95 \text{ kg}$$

10.93. $$a = \frac{m_b g}{m_p + m_b}$$

$$m_p = \frac{m_b g}{a} - m_b = m_b\left(\frac{g}{a} - 1\right)$$

$$\Rightarrow m_b = \frac{m_p}{\frac{g}{a} - 1} = \frac{5.991 \text{ m}}{\frac{9.81 \text{ m/s}^2}{4.539 \text{ m/s}^2} - 1} = 5.16 \text{ kg}$$