1-

(a) $\sqrt{2}$ 1

2-

(d) 10 1

3-

$$Z(n) = 20\left(\frac{n}{12} - \ln\left(\frac{n}{12}\right)\right) + 30$$

$$Z'(n) = 20\left(\frac{1}{12} - \frac{12}{n} \times \frac{1}{12}\right)$$ 1

$$Z'(n) = 20\left(\frac{n-12}{12n}\right)$$

at $Z'(n) = 0 \Rightarrow n - 12 = 0 \Rightarrow n = 12$ $\frac{1}{2}$

(i) from the discussion of the sign of the function $Z'(n)$

Z': $- - -$ | $+ + +$ (at 12)

Z: $\searrow$ 12 $\nearrow$

$\frac{1}{2}$

the number of bacteria be minimum when $n = 12$ days. $\frac{1}{2}$

(ii) The least number of bacteria $= Z(12)$

$$= 20\left(\frac{12}{12} - \ln\left(\frac{12}{12}\right)\right) + 30 = 50 \text{ cm}^3$$ $\frac{1}{2}$

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4-

To determine the intersection point:

let $y_2 = y_1 \Rightarrow x^2 = 3x - 2$

$\therefore x^2 - 3x + 2 = 0 \Rightarrow (x-2)(x-1) = 0$

$\therefore x = 1$ or $x = 2$ $\quad \frac{1}{2}$

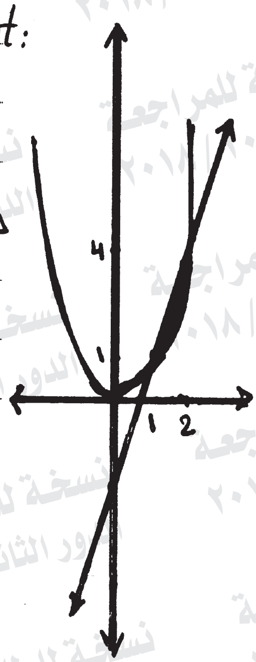
$$\text{Volume} = \pi \int_1^2 \left[(3x-2)^2 - (x^2)^2\right] dx \quad \frac{1}{2}$$

$$= \pi \int_1^2 \left((3x-2)^2 - x^4\right) dx \quad \frac{1}{2}$$

$$= \pi \left[\frac{(3x-2)^3}{3 \times 3} - \frac{1}{5}x^5\right]_1^2 \quad \frac{1}{2}$$

$$= \pi \left[\left(\frac{32}{45}\right) - \left(\frac{-4}{45}\right)\right]$$

$$= \frac{4}{5}\pi \text{ cubic unit} \quad \frac{1}{2}$$



(تراعى الحلول الأخرى)

5-

(c) e 1

6-

(b) Zero 1

7-

a) $\int x(x+2)^6\,dx$

let $y = x+2 \Rightarrow x = y-2$ & $dx = dy$ $\quad \frac{1}{2}$

$\therefore \int x(x+2)^6\,dx = \int (y-2)\,y^6\,dy$ $\quad \frac{1}{2}$

$= \int (y^7 - 2y^6)\,dy$

$= \frac{1}{8}y^8 - \frac{2}{7}y^7 + C$ $\quad \frac{1}{2}$

$= \frac{1}{8}(x+2)^8 - \frac{2}{7}(x+2)^7 + C$ $\quad \frac{1}{2}$

b) $\int (x+5)e^x\,dx$

$= (x+5)e^x - \int e^x\,dx$ $\quad \frac{1}{2}$

$= (x+5)\,e^x - e^x + C$ $\quad \frac{1}{2}$

$= e^x\,(x+4) + C$

where

$\frac{1}{2}$ $\quad u = x+5 \quad \xrightarrow{(\times)} \quad dv = e^x\,dx$

$\therefore du = dx \quad \xleftarrow{(-)} \quad v = e^x$ $\quad \frac{1}{2}$

(تراعى الحلول الأخرى)

8-

(d) $x + \ln|x+1| + c$ $\triangle 1$

9-

(b) $\frac{1}{2}$ $\triangle 1$

10-

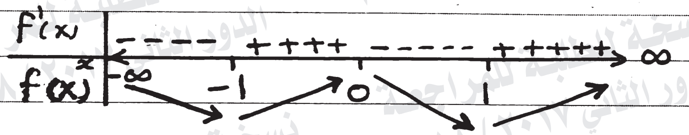
ⓐ $f(x) = x^4 - 2x^2$

$\therefore f'(x) = 4x^3 - 4x = 4x(x^2 - 1)$ $\frac{1}{2}$

at $f'(x) = 0$ $\quad 4x(x-1)(x+1) = 0$ $\frac{1}{2}$

$\therefore x = 0$ or $x = 1$ or $x = -1$

by discussing the sign of the function:



$\frac{1}{2}$

at $x = -1$, the function has a local min. value $\frac{1}{2}$

$f(-1) = -1$

at $x = 0$, the function has a local max. value $\frac{1}{2}$

$f(0) = 0$

at $x = 1$, the function has a local min. value $\frac{1}{2}$

$f(1) = -1$

$\boxed{b}$ $f(x) = \dfrac{4x}{x^2+1}$

$\therefore f'(x) = \dfrac{(x^2+1)(4) - (4x)(2x)}{(x^2+1)^2}$ $\frac{1}{2}$

$f'(x) = \dfrac{-4x^2+4}{(x^2+1)^2}$ $\frac{1}{2}$

at $f'(x) = 0 \Rightarrow -4x^2 + 4 = 0$ $\frac{1}{2}$

$-4(x-1)(x+1) = 0$ $\quad \therefore x = 1$ or $x = -1$ $\frac{1}{2}$

$f(-1) = -2$, $f(1) = 2$, $f(3) = \frac{6}{5}$

$\therefore$ The absolute max. value $= 2$ $\frac{1}{2}$

The absolute min. value $= -2$ $\frac{1}{2}$

(تراعى الحلول الأخرى)

11-

(d) e^3 1

12-

(b) -3 1

13-

$y = 3 + \sec x$

at $x = \frac{2\pi}{3} \Rightarrow y = 3 + \sec\frac{2\pi}{3} = 1$

$\therefore \left(\frac{2\pi}{3}, 1\right) \in$ the Curve $\frac{1}{2}$

$\because \frac{dy}{dx} = \sec x \tan x$ $\frac{1}{2}$

$\therefore$ the slope at $x = \frac{2\pi}{3}$ $= \sec\frac{2\pi}{3} \tan\frac{2\pi}{3}$ $\frac{1}{2}$

$= (-2)(-\sqrt{3}) = 2\sqrt{3}$ $\frac{1}{2}$

The equation of the tangent:

$y - 1 = 2\sqrt{3}\left(x - \frac{2\pi}{3}\right)$ $\frac{1}{2}$

The equation of the normal:

$y - 1 = \frac{-1}{2\sqrt{3}}\left(x - \frac{2\pi}{3}\right)$ $\frac{1}{2}$

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14-

To determine the intersection point,

let $y_1 = y_2 \Rightarrow x = \sqrt{2x}$

$\therefore x^2 = 2x \quad \therefore x^2 - 2x = 0$

$x(x-2) = 0 \quad \therefore x = 0$ or $x = 2$ $\quad \frac{1}{2}$

$\therefore$ The area $= \int_0^2 (\sqrt{2x} - x)\, dx$ $\quad \frac{1}{2}$

$= \left[\frac{2\sqrt{2}}{3} x^{\frac{3}{2}} - \frac{1}{2} x^2\right]_0^2$ $\quad \frac{1}{2}$

$= \left[\left(\frac{2\sqrt{2}}{3}(2)^{\frac{3}{2}} - \frac{1}{2}(2)^2\right) - 0\right] = \frac{2}{3}$ area unit $\quad \frac{1}{2}$

(تراعى الحلول الأخرى)

15-

(b) $3t$ $\triangle 1$

16-

(d) $]0,\infty[$ $\triangle 1$

17-

$\sin x = xy$ (derivative with respect to x)

$\cos x = xy' + y$ $\triangle \frac{1}{2}$ $\Rightarrow y' = \dfrac{\cos x - y}{x}$ $\triangle \frac{1}{2}$

$-\sin x = xy'' + y' + y'$ $\triangle \frac{1}{2}$

$-\sin x = xy'' + 2y'$

$-xy = xy'' + 2 \times \dfrac{\cos x - y}{x}$ $\triangle \frac{1}{2}$

$-x^2 y = x^2 y'' + 2\cos x - 2y$ $\triangle \frac{1}{2}$

$x^2 y + x^2 y'' + 2\cos x = 2y$ $\triangle \frac{1}{2}$

$x^2 (y + y'') + 2\cos x = 2y$

18-

$$x e^{y} = 2 - \ln 2 + \ln x$$

$$x e^{y} \frac{dy}{dt} + e^{y} \frac{dx}{dt} = \frac{1}{x} \frac{dx}{dt}$$ (1)

$$\because \frac{dx}{dt} = 6 , \quad x = 2 , \quad y = 0$$

$$\therefore 2 e^{0} \frac{dy}{dt} + e^{0} \times 6 = \frac{1}{2} \times 6$$ ($\frac{1}{2}$)

$$\therefore 2 \frac{dy}{dt} = 3 - 6$$

$$2 \frac{dy}{dt} = -3$$

$$\frac{dy}{dt} = \frac{-3}{2}$$ ($\frac{1}{2}$)

(تراعى الحلول الأخرى)

(انتهت الإجابة وتراعى الحلول الأخرى)