١

1-

(c) $\sqrt{2}\, e^{\frac{-3}{4}\pi i}$ 1

2-

(d) $\|\overrightarrow{A}\|^2 \|\overrightarrow{B}\|^2$ 1

3-

$T_4 = 7 \Rightarrow {}^8C_3\, x^3 = 7$ $\frac{1}{2}$

$\therefore x^3 = \frac{7}{56} = \frac{1}{8}$ $\frac{1}{2}$ $\Rightarrow x = \frac{1}{2}$ $\frac{1}{2}$

The order of the middle term $= \frac{8}{2} + 1 = 5$

$\therefore$ The middle term is T_5 $\frac{1}{2}$

$\frac{T_6}{T_5} = \frac{8-5+1}{5} \times \frac{x}{1}$ $\frac{1}{2}$ $= \frac{4}{5} \times \frac{1}{2} = \frac{2}{5}$ $\frac{1}{2}$

4-

$\because$ The two straight lines are parallel

$\therefore$ They have the same direction

$\therefore$ The direction vector $= (2, 4, 3)$ $\quad \frac{1}{2}$

$\therefore$ The vector form:

$$\vec{r} = (-2, 3, 5) + t\,(2, 4, 3) \quad \frac{1}{2}$$

• The Parametric form:

$$x = -2 + 2t$$
$$y = 3 + 4t \quad \frac{1}{2}$$
$$z = 5 + 3t$$

• The Cartesin form:

$$\frac{x+2}{2} = \frac{y-3}{4} = \frac{z-5}{3} \quad \frac{1}{2}$$

(تراعى الحلول الأخرى)

5-

(a) 1 $\triangle 1$

6-

$\vec{n} \cdot \vec{r} = \vec{n} \cdot \vec{A}$ $\frac{1}{2}$

$(2,-3,4) \cdot \vec{r} = (2,-3,4) \cdot (1,-1,4)$ $\frac{1}{2}$

$(2,-3,4) \cdot \vec{r} = 21$ The vector form. $\frac{1}{2}$

$2(x-1) - 3(y+1) + 4(z-4) = 0$ $\frac{1}{2}$

The standard form

$2x - 3y + 4z - 21 = 0$ $\frac{1}{2}$

The general equation.

7-

$$\because \begin{pmatrix} 1 & -2 & 0 \\ -1 & 1 & 2 \\ 1 & 0 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ -1 \end{pmatrix} \qquad AX = B$$

$$|A| = \begin{vmatrix} 1 & -2 & 0 \\ -1 & 1 & 2 \\ 1 & 0 & -2 \end{vmatrix} = 1(-2) + 2(0) + 0 = -2 \neq 0 \qquad \frac{1}{2}$$

$$Adj(A) = \begin{pmatrix} -2 & 0 & -1 \\ -4 & -2 & 2 \\ -4 & -2 & -1 \end{pmatrix}^t \; \frac{1}{2} = \begin{pmatrix} -2 & -4 & -4 \\ 0 & -2 & -2 \\ -1 & 2 & -1 \end{pmatrix} \qquad \frac{1}{2}$$

$$A^{-1} = \frac{1}{|A|} \; Adj(A) = \frac{1}{-2} \begin{pmatrix} -2 & -4 & -4 \\ 0 & -2 & -2 \\ -1 & 2 & -1 \end{pmatrix} \qquad \frac{1}{2}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 2 & 4 & 4 \\ 0 & 2 & 2 \\ 1 & -2 & 1 \end{pmatrix} \begin{pmatrix} 5 \\ 0 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 6 \\ -2 \\ 4 \end{pmatrix} \qquad \frac{1}{2}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \qquad \frac{1}{2}$$

$$\therefore x = 3 \; , \; y = -1 \; , \; z = 2$$

(تراعى الحلول الأخرى)

8-

(b) ${}^5C_4 \times {}^8C_6 + {}^5C_5 \times {}^8C_5$ 1

9-

(b) 1 1

10-

(d) (3,1,2) 1

11-

(a) $Z = \frac{16}{1-\sqrt{3}i} \times \frac{1+\sqrt{3}i}{1+\sqrt{3}i} = \frac{16(1+\sqrt{3}i)}{1+3}$

$\therefore Z = 4+4\sqrt{3}i$, $|Z| = \sqrt{16+48} = 8$ $\frac{1}{2}$

$\tan\theta = \sqrt{3}$ $\therefore \theta = \frac{\pi}{3}$

$\therefore Z = 8\left[\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right]$ $\frac{1}{2}$

$\therefore \sqrt[3]{Z} = 2\left[\cos\frac{\frac{\pi}{3}+2\pi n}{3} + i\sin\frac{\frac{\pi}{3}+2\pi n}{3}\right]$, $n = 0, 1, -1$ $\frac{1}{2}$

at $n=0$ $\therefore \sqrt[3]{Z} = 2\left(\cos\frac{\pi}{9} + i\sin\frac{\pi}{9}\right) = 2e^{\frac{\pi}{9}i}$ $\frac{1}{2}$

at $n=1$ $\therefore \sqrt[3]{Z} = 2\left(\cos\frac{7\pi}{9} + i\sin\frac{7\pi}{9}\right) = 2e^{\frac{7\pi}{9}i}$ $\frac{1}{2}$

at $n=-1$ $\therefore \sqrt[3]{Z} = 2\left(\cos\frac{-5\pi}{9} + i\sin\frac{-5\pi}{9}\right) = 2e^{-\frac{5\pi}{9}i}$ $\frac{1}{2}$

[b] $\dfrac{1+10\omega+10\omega^2}{1-3\omega-3\omega^2} = \dfrac{1+10(\omega+\omega^2)}{1-3(\omega+\omega^2)}$ $\frac{1}{2}$

$= \dfrac{1-10}{1+3} = \dfrac{-9}{4}$ $\frac{1}{2}$

$\therefore \; K^2 i^2 = \dfrac{-9}{4}$ $\frac{1}{2}$

$-K^2 = -\dfrac{9}{4} \Rightarrow K^2 = \dfrac{9}{4}$ $\frac{1}{2}$

$\therefore \; K = -\dfrac{3}{2}$ $\frac{1}{2}$

or $K = \dfrac{3}{2}$ $\frac{1}{2}$

(تراعى الحلول الأخرى)

12-

(c) $r<5$ 1

13-

(b) 10 1

14-

(d) 41 1

15-

ⓐ $\overrightarrow{AC} = \vec{C} - \vec{A} = (m-1, -1, 3-10m)$

$\overrightarrow{AB} = \vec{B} - \vec{A} = (2, 3, 1)$

(i) $\because$ A, B and C are collinear

$\therefore \overrightarrow{AC} = K\,\overrightarrow{AB}$

$(m-1, -1, 3-10m) = K(2, 3, 1)$

$m-1 = 2K \quad , \quad -1 = 3K \quad , \quad 3-10m = K$

$\therefore K = -\frac{1}{3}$ $\frac{1}{2}$ $\frac{1}{2}$

$m-1 = \frac{-2}{3} \Rightarrow m = \frac{1}{3}$

(ii) $\because \overrightarrow{AB} \perp \overrightarrow{AC} \quad \therefore \overrightarrow{AB} \cdot \overrightarrow{AC} = 0$

$(2,3,1) \cdot (m-1, -1, 3-10m) = 0$ $\frac{1}{2}$

$2m - 2 - 3 + 3 - 10m = 0$

$-8m = 2 \quad \therefore m = \frac{-1}{4}$ $\frac{1}{2}$

$\boxed{b}$ (i) $\overrightarrow{AB} = \overrightarrow{B} - \overrightarrow{A} = (3, 2, 3)$

$\overrightarrow{DC} = \overrightarrow{C} - \overrightarrow{D} = (3, 2, 3)$

$\overrightarrow{BC} = \overrightarrow{C} - \overrightarrow{B} = (-2, 2, 0)$

$\overrightarrow{AD} = \overrightarrow{D} - \overrightarrow{A} = (-2, 2, 0)$

$\because \overrightarrow{AB} = \overrightarrow{DC}$ & $\overrightarrow{BC} = \overrightarrow{AD}$

$\therefore$ ABCD is a parallelogram $\quad \frac{1}{2}$

$$\because \overrightarrow{AB} \times \overrightarrow{AD} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 2 & 3 \\ -2 & 2 & 0 \end{vmatrix}$$

$$= -6\vec{i} - 6\vec{j} + 10\vec{k}$$

$$\therefore \|\overrightarrow{AB} \times \overrightarrow{AD}\| = \sqrt{36 + 36 + 100}$$

$$= 2\sqrt{43} \quad \frac{1}{2}$$

$\therefore$ Area of $\square = \|\overrightarrow{AB} \times \overrightarrow{AD}\| = 2\sqrt{43}$ area units

(ii) The perpendicular unit vector $= \dfrac{\overrightarrow{AB} \times \overrightarrow{AD}}{\|\overrightarrow{AB} \times \overrightarrow{AD}\|} \quad \frac{1}{2}$

$$= \frac{(3, 2, 3) \times (-2, 2, 0)}{2\sqrt{43}}$$

$$= \frac{-6\vec{i} - 6\vec{j} + 10\vec{k}}{2\sqrt{43}}$$

$$= \left(\frac{-6}{2\sqrt{43}}, \frac{-6}{2\sqrt{43}}, \frac{10}{2\sqrt{43}}\right) \quad \frac{1}{2}$$

(تراعى الحلول الأخرى)

16-

(c) $-x^{10}$ 1

17-

(a) 13 1

18-

(d) 2 1

19-

$$\begin{vmatrix} a & b & c \\ b & a & c \\ b & c & a \end{vmatrix} \qquad C_1 \longrightarrow C_1 + C_2 + C_3$$

$$= \begin{vmatrix} a+b+c & b & c \\ a+b+c & a & c \\ a+b+c & c & a \end{vmatrix} \;\frac{1}{2}\; = (a+b+c) \begin{vmatrix} 1 & b & c \\ 1 & a & c \\ 1 & c & a \end{vmatrix} \;\frac{1}{2}$$

$$= (a+b+c) \begin{vmatrix} 1 & b & c \\ 0 & a-b & 0 \\ 0 & c-b & a-c \end{vmatrix} \;\frac{1}{2}\; \xRightarrow{R_3 - R_1,\ R_2 - R_1}$$

$$= (a+b+c)(a-b) \begin{vmatrix} 1 & b & c \\ 0 & 1 & 0 \\ 0 & c-b & a-c \end{vmatrix} \;\frac{1}{2}\; \xRightarrow{-(c-b)R_2 + R_3}$$

$$= (a+b+c)(a-b) \begin{vmatrix} 1 & b & c \\ 0 & 1 & 0 \\ 0 & 0 & a-c \end{vmatrix} \;\frac{1}{2}$$

$$= (a+b+c)(a-b)(a-c) \;\frac{1}{2}$$

(تراعى الحلول الأخرى)

(انتهت الإجابة وتراعى الحلول الأخرى)