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Legal Indeterminacy and Constitutional Interpretation

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sions.[37] '$O(q \vee r)$' is a logical consequence (in the logic of relevance as well) of *(1)* and *(2)* and, therefore, a justified decision.

ii) Second, I will now analyze the criticism Anderson and Belnap direct against Lewis's deduction, commented earlier, of 'q' from '$p \vee \neg p$'. In the authors' words:

„In rejecting the principle of the disjunctive syllogism, we intend to restrict our rejection to the case in which the 'or' is taken truth functionally. In general and with respect to our ordinary reasoning this would not be the case; perhaps always when the principle is used in reasoning one has in mind an intensional meaning of 'or', where there is relevance between the disjuncts. But for the intensional meaning of 'or', it seems clear that the analogues of $A \rightarrow (A \vee B)$ are invalid, since this would hold only if the simple truth of A were sufficient for the relevance of A to B; hence, there is a sense in which the real flaw in Lewis's argument is not a fallacy of relevance but rather a fallacy of ambiguity. The passage from *b* to *d* [i. e., from 'p' to 'p $\vee$ q'] is valid only if the '$\vee$' is read truth functionally, while the passage from *c* and *d* to *e* [i. e., from '$\neg$p' and 'p $\vee$ q' to 'q'] is valid only if the '$\vee$' is taken intensionally." (Anderson/Belnap 1975, 165 f.)

But to reject the disjunctive syllogism also means, as the authors admit (Anderson/Belnap 1975, 165), to reject *modus ponens* for material implication (though, of course, not for their relevant implication), since „'$A \vee B$' and '$\neg A$' imply 'B'" is equivalent to „'$\neg A \rightarrow B$' and '$\neg A$' imply 'B'". And *modus ponens* is a rule which, in the words of the authors (ibid.), „has perhaps never been seriously questioned before".

With the loss of *modus ponens*, inferential capacity is greatly reduced. As Sanford (1989, 131) observes, to reject *modus ponens* (or the disjunctive syllogism) „appears to many a worse cure than the disease it aims to abolish": „Relevance logic rejects too much".[38]

For these two reasons, the logic of relevance is unable to eliminate those irrelevancies that are most important for the deductive conception of the justification of judicial decisions and, instead, deprives us of important inferential mechanisms without which we cannot justify decisions we all regard as justified.

c) Logic Plus Relevance

In this section, I will try to defend the following thesis: The content of a judicial decision is justified if and only if it is a *relevant* logical consequence (in the sense of classical logic, extended to account for deontic logic) of the normative premise(s) and the statements describing the facts of the case.

In order to do this, I need a criterion for distinguishing, among the logical consequences entailed by a set of premises, those that are *relevant* and those that are *irrele-*

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[37] Hernández Marín (1989, 306 f.) has used examples like this one to criticize the so-called logical conception of the application of the law.

[38] One can agree with Hunter (1993, 283) that „one thing a hospital should not do is spread disease; and one thing logic should not do is teach people that invalid arguments are valid"; but it is also true that medicine should not kill the patient, and relevance should not destroy the inferential capacity of logic.

vant. To this effect, I will use the notion of relevant (logical) conclusion as it appears in a recent paper by Schurz (1991).[39] Schurz's idea is simple, yet elegant:

DF 1. Assume $\{P\} \vdash C$. Then: C is a *relevant conclusion* of {P} if and only if no propositional variable in C is replaceable on some of its ocurrences by any other propositional variable, salva validitate of $\{P\} \vdash C$. Otherwise, C is an *irrelevant conclusion* of {P}.[40]

The irrelevance of a conclusion *C* can result from the addition of propositional variables (through disjunction introduction or similar rules). The paradoxical examples of irrelevant judicial decisions presented above (cf. ch. I.3.c) are of this type. It is true that '$p \rightarrow Oq$' and 'p' imply '$O(q \vee r)$' as well as '$r \rightarrow Oq$'. But these are cases of irrelevant conclusions, since in both conclusions 'r' can be replaced by any other propositional variable *salva validitate.*

Irrelevance, however, can also be due to the presence, as conclusions, of logical truths for which the presence of the premises is superfluous. From '$p \rightarrow Oq$' and 'p', '$Oq \vee \neg Oq$' can be deduced. But that conclusion is irrelevant because the propositional variable 'q' can be replaced by any other in both its occurences in the formula.

This is an interesting case, because the logic of relevance has attempted to show the irrelevance of those implications (regarded as paradoxes of material or strict implication). For this notion of relevant conclusion, the two inferences

$p \vdash q \vee \neg q$ (*verum ex quodlibet*) and
$p \wedge \neg p \vdash q$ (*ex falso quodlibet*),

which are valid in sentential logic, are cases of irrelevant logical consequences. In both cases, variable q can be replaced (in both its occurrences in the first formula, and in its single occurrence in the second) by any other formula *salva validitate* of the implication.

This is the kind of irrelevance von Wright intended to avoid with his notion of *entailment*. As he observes,

„p entails q, if and only if, by means of logic, it is possible to come to know the truth of $p \rightarrow q$ without coming to know the falsehood of p or the truth of q" (von Wright 1957, 181).

Von Wright adds that the possibility of coming to know, by means of logic, whether or not a proposition is true means that that proposition is *demonstrable*. Thus, the concept of entailment is intimately linked to the concept of demonstrability. With this idea, the definition can be reformulated in the following way:

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[39] In that paper, Schurz attempts to show how his notion serves to eliminate a great number of paradoxes resulting mainly from the rule for disjunction introduction (also known as addition rule). Ross's Paradox is among the paradoxes analyzed by Schurz.

[40] Cf. Schurz 1991, 409. His definition is somewhat more complex, in order to be able to apply it to predicate logic too. But we can ignore that complication here, since I only use sentential or propositional logic.

„p entails q, if and only if, by means of logic, $p \rightarrow q$ is demonstrable independently of demonstrating the falsehood of p or the truth of q." (von Wright 1957, 181)[41]

Schurz (1991, 411) points out that von Wright's notion can be explicated with his own notion of irrelevant logical consequence or, more precisely, with his notion of *completely* irrelevant logical consequence, which is a consequence where all occurrences of propositional variables can be replaced *salva validitate*.

Now, it may be useful to analyze what is gained and what is lost with that concept of relevant logical consequence in comparison with a logic of relevance like that of Anderson and Belnap.

One thing we gain is consistency with our presystematic intuitions about apparently paradoxical consequences. Disjunction introduction, which is a valid rule in relevance logic, is the paradigm of irrelevance here. And for this gain we do not need to reduce drastically the inferential capacity of our logic: The disjunctive syllogism (and with it, *modus ponens*) is not irrelevant in our logic. 'q' is an irrelevant consequence of '$p \wedge \neg p$' not because the disjunctive syllogism is irrelevant, but because of the irrelevance (in Lewis's proof) introduced by the rule for disjunction introduction (Schurz 1991, 413).

Furthermore, we gain the possibility of continuing to use the classical concept of logical consequence (extended to account for deontic logic). The point here is not to defend the view that the classical notion of logical consequence is the only correct one. It is simply that this notion belongs to the innermost core of our conceptual structure and, following Quine's arguments,[42] should be revised only *in extremis*. As I try to show, the problems we encounter in justifying judicial decisions do not constitute so extreme an emergency as to call for sacrificing the notion of logical consequence.[43]

However, there is also some loss, as compared to the logic of relevance. Above all, we lose certain elegant properties of the notion of relevant consequence (Schurz 1991, 412 f.). Thus, the notion of logical consequence of Anderson and Belnap's logic of relevance (in what follows: *LCAB*) — just like the standard notion of logical consequence — is *closed under substitution*, whereas the notion of relevant logical consequence *(RLC)* is not. Hence, while 'p' is an *LCAB* of '$p \wedge \neg p$', because it is an instance

[41] Von Wright (ibid.) adds that, using the symbols 'M' for 'possible' and 'D' for 'demonstrated', we can express that definition as follows: 'p entails q' =df '$M\,(D(p \rightarrow q) \wedge \neg D\neg p \wedge \neg Dq)$'.

[42] In Quine's words (1970, 100): „Logic is in principle no less open to revision than quantum mechanics or the theory of relativity. The goal is, in each, a world system — in Newton's phrase — that is as smooth and simple as may be and that nicely accommodates observations around the edges. If revisions are seldom proposed that cut so deep as to touch logic, there is a clear enough reason for that: the maxim of minimum mutilation."

[43] Perhaps the philosophical status of my position on relevance is close to that of Orayen (1989, 234-255; this work, besides, contains an excellent exposition of the ideas of Anderson and Belnap, as well as a sharp critique of them): „I am inclined to think that there are no theoretical reasons that justify abandoning the classical analysis of deducibility for one offered by some logic of the relevant type. This involves rejecting relevant logic as a logic diverging from classical logic: it would not be a good substitute. Still, perhaps relevant logic can have some other use, despite of being regarded ill-suited to replace the *official* logic."

of '$A \wedge \neg A \vdash A$', it is not a *RLC*, since 'p' can be replaced by any other propositional variable *salva validitate*. *LCAB* satisfies the properties of *transitivity* and *monotonicity*, whereas *RLC* does not.[44] Still, this is not so very serious if we recall that the notion of relevance is not intended to replace the notion of logical consequence, but only to constitute a criterion for distinguishing relevant from irrelevant logical consequences.

Another problem we must address is particular to deontic logic. Standard deontic logic accepts the inference according to which $Op \vdash Pp$ (as a consequence of an axiom or a theorem, depending on how the calculus is presented). And yet we would be surprised to see a judge arguing conclusion 'Pq' as a result of the premises $\{p \rightarrow Oq, p\}$.[45] If we want to say that 'Pq' is an irrelevant logical conclusion of the premises, we must have a notion of relevance that not only affects propositional variables, but deontic operators as well.

One way of responding can be found in the distinction presented at the beginning of this chapter, between *maximal* and *partial* solutions. It should be recalled that only maximal solutions completely determine actions from the normative point of view.

Thus, we can present the following definition of a relevant logical conclusion with respect to deontic operators:

DF 2. Assume $\{P\} \vdash C$. *A logical conclusion C is relevant with respect to deontic operators* if and only if those deontic operators of C that constitute partial solutions cannot be deduced from other formulae that are consequences of $\{P\}$ as well and that contain deontic operators that constitute maximal solutions.

Thus, a partial solution is a relevant logical conclusion of a set of premises A only when A does not provide sufficient information for deriving a maximal solution. Otherwise, the partial solution is irrelevant. That is what happens with the derivation of 'Pq' from $\{p \rightarrow Oq, p\}$, since from these premises we can obtain the maximal solution 'Oq', from which, in turn, 'Pq' can be derived. On the other hand, *DF 2* implies that if all deontic operators in C are maximal solutions, then C is relevant with respect to its deontic operators.

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[44] Schurz (1991, 414) gives the following examples. The first is to show the non-transitivity of RLC: $(p \vee q) \wedge r \vdash p \vee (q \wedge r)$ and $p \vee (q \wedge r) \vdash (p \vee q) \wedge (p \vee r)$; in contrast, $(p \vee q) \wedge r \vdash (p \vee q) \wedge (p \vee r)$ is not a case of relevant deduction, because the second occurrence of p in $(p \vee q) \wedge (p \vee r)$ can be replaced by any other formula *salva validitate*; that is, it is possible for B to be a relevant consequence of A, and for C to be a relevant consequence of B, while C is an irrelevant consequence of A. Regarding monotonicity (the rule according to which if $A \vdash B$ and $A \subseteq C$, then $C \vdash B$), the following case shows that the notion of RLC is not monotonous: $p \vee q \vdash p \vee q$, but $\{p, p \vee q\} \vdash p \vee q$ is not a case of relevant deduction, since q can be replaced by any other formula *salva validitate*.

[45] It would be surprising if a judge argued as follows:

(1) If x does A, then x must be punished with sanction S.
(2) x does A.

Therefore, *(3)* x *may* be punished with sanction S.

The content of *(3)* is deduced, in deontic logic, from the premises; but (as in the cases in which the rule for disjunction introduction is used) it is weaker than it needs to be.

If we call Schurz's definition of a relevant conclusion *(DF 1)* the definition of a *relevant logical conclusion with respect to propositional variables*, we can say:

DF 3. A logical conclusion is relevant in deontic logic if and only if it is relevant with respect to propositional variables and with respect to deontic operators (or, what is the same, if and only if it is *DF 1-relevant* and *DF 2-relevant*).

d) Relevance and Normative Systems

This notion of relevant conclusion in deontic logic can by useful for a revision of our notion of a normative system. As I will explain in the next chapter, *legal propositions* contained in statements like 'Legally, all F ought to do ϕ' presuppose that certain norms belong to the legal system — in this case, the norm contained in the norm-formulation 'All F ought to do ϕ'. Now, since 'All F ought to do ϕ' implies 'All F ought to do ϕ or ought to do σ', the proposition contained in 'Legally, all F ought to do ϕ or ought to do σ' is true as well. But this conclusion is counterintuitive; and one can hardly attribute to a norm-authority the intention (if only an implicit one) of enacting the irrelevant logical consequences of its explicitly enacted norms.

In this context, Raz (1994a, 211 f.) has distinguished between the *source* thesis according to which all law is based on certain social acts of norm-creation, and the *incorporation* thesis according to which all law is either based on sources or implied by law based on sources. In order to explain that distinction, Raz compares sets of norms with sets of beliefs and suggests that one usually does not attribute to a person *all* the logical consequences of what she explicitly believes. Similarly, Raz thinks that authorities do not prescribe all the things implied by the norms they explicitly enact and that, therefore, the incorporation thesis is incompatible with an approach to law as invested with authority.

There are other reasons, however, for which it is important to preserve the notion of a normative system as a deductive system containing *all* its logical consequences. The most important of those reasons has to do with the questions of *legal dynamics* which will be treated in Chapter III. But it will be important later to keep in mind the distinction between relevant and irrelevant logical consequences in the set of all logical consequences of a normative system.

In what follows, I will show this importance with respect to the topic of *normative contradictions*.

One of the aspects that have sometimes been criticized about the conception of legal systems as normative systems is that in the case where a set of formulated norms contains a contradiction, because of the *ex falso quodlibet* rule, any norm will belong to that normative system. Thus, one can say that a normative system S is inconsistent if and only if any norm belongs to S.

Føllesdal and Hilpinen (1971, 16) define what they call the principle of the consistency of a normative system as follows:

„If a set of sentences A is consistent and $\{Of_1, Of_2, ..., Of_n, Pg\} \subseteq A$, then $\{f_1, f_2, ..., f_n, g\}$ is consistent."[46]

That means that if a set of norms contains a norm ordering not-*p* as well as another norm permitting *p*, then it is inconsistent since $\{p, \neg p\}$ is inconsistent. The corresponding normative system would thus contain any norm, since a contradictory set implies any statement. As I have shown, however, none of these consequences is a *relevant* consequence, since in each of the formulae each of the propositional variables can be replaced by any other *salva validitate* (i. e., they are *completely* irrelevant consequences). Thus, from the normative system $\{O\neg p, Pp\}$ we can deduce *Oq*, *Phq*, *Fq*, etc., but none of these formulae is a relevant consequence, since variable *q* can be replaced by any other *salva validitate* (including, of course, by variable *p*). We can thus define the notion of an inconsistent normative system (containing a normative contradiction) with the help of the notion of relevant logical consequence:

A normative system S is *inconsistent* if and only if it lacks relevant consequences, or all its consequences are *completely* irrelevant.

This definition may help us understand in what sense an inconsistent normative system is defective: It contains any norm, but none of these norms is relevant.[47]

[46] The authors add: „It should be observed that [this principle] does not require that all permitted states of affairs can be realized simultaneously, but only that *each* permission is compatible with all obligatory states of affairs" (emphasis added).

[47] Recently, Atienza (1992, 1017 f.) has criticized Alchourrón and Bulygin's notion of a legal system as a normative system precisely because, in that case, if a system contains a normative contradiction then that would imply that any norm belongs to that system: „According to this notion, the jurist who wants to reconstruct some part of the legal order would have to take into consideration that the existence of a contradiction in that part — however small it may be — leads to all kinds of consequences (since from two contradictory statements any other statement follows). For example, if the question is to reconstruct the constitutional subsystem, and he detects a contradiction in it, in the sense that, say, for the approval of a certain type of law one constitutional norm requires a majority of two thirds, and another only a simple majority, the jurist who strictly wants to apply Alchourrón and Bulygin's notion of a legal system would have to conclude that in that legal order the constitution stipulates that 'anything goes': laws must be approved by parliament, or not; citizens have the right to *habeas corpus*, or not; etc. Obviously, no jurist in his right mind — and not even a somewhat deranged jurist — would accept this. So how is it possible that Alchourrón and Bulygin have — at least implicitly — sustained this, and — for more than twenty years! — have not done anything to correct that notion of a legal system? Can one even correct it without abandoning the classical notion of logical consequence?" In my opinion, the introduction of the notion of relevant logical consequence shows that we need not abandon the notion of logical consequence. We must only remember that jurists are interested in the relevant normative consequences of a legal system, and that an inconsistent system lacks such consequences. Therefore, what must be done is to 'dissolve' the contradiction, i. e. to reformulate the inconsistent system — or subsystem — to make it consistent (which is, besides, what jurists usually do).

II. A Logical Analysis of Legal Propositions

1. Introduction

The purpose of the present chapter is to present a logical analysis of statements like the following:

(1) Legally, all F ought to do ϕ.
(2) Legally, x ought to do ϕ.
(3) Legally, all F are ϕ.
(4) Legally, x is a ϕ.

Such statements are canonical formulations of expressions that do not always come in the same grammatical form. For instance, the word 'legally' is often omitted. Thus, one says 'x is real estate', where the term 'legally' is implied by the context, or 'x has the obligation to pay that tax', where the same is implied. Obviously, 'legally' in those cases means 'according to some specific *legal system*'. (1) and (2) are regarded as the canonical formulations of all deontic expressions, such as

(1a) Legally, all F ought not to do ϕ (i. e., all F are prohibited to do ϕ);
(1b) Legally, all F may do ϕ (i. e., all F are permitted to do ϕ or, what amounts to the same, it is not obligatory for an F to omit ϕ);

and

(2a) Legally, x is prohibited to do ϕ.
(2b) Legally, x is permitted (in some contexts: x has the right) to do ϕ.

I will call such statements *legal statements.* And I will say that legal statements express *legal propositions.*

2. Deontic Legal Statements and Conceptual Legal Statements

I have assumed that norms belonging to a legal system *LS* are of two kinds: *a)* norms in the strict sense or *prescriptive rules*, and *b) conceptual rules.*

A legal statement like formulation (1) implies that a certain prescriptive rule is a normative consequence of *LS*. Now, we can simplify legal statements by realizing that to say that ϕ is legally obligatory ('$O\phi$') is equivalent to saying that '$O\phi$' belongs to the normative consequences of *LS*: '$O\phi$' $\in$ *NC (LS)*; and to assert that ϕ is not legally obligatory is to assert that '$\neg O\phi$' belongs to the normative consequences of *LS*: '$\neg O\phi$' $\in$ *NC (LS)*.

Since '$\neg O\phi$' is equivalent to '$P\neg\phi$', to say that legally ϕ is not obligatory is the same as saying that legally it is permitted that not-ϕ.[1] I will call legal statements that refer to normative consequences *deontic legal statements.*

A legal statement like (3) presupposes that a certain conceptual rule is a normative consequence of *LS*. For reasons of simplicity, I will, for the time being, represent conceptual rules through propositional variables. To say that legally p (where p can be replaced by statements like 'Persons 18 years or older are of age', or 'Killing another person is manslaughter', etc.) is the same as saying that p is a consequence of the respective legal system: 'p' $\in$ *NC (LS).* To say that legally *not-p* is to say that $\neg p$ is a consequence of LS: '$\neg p$' $\in$ *NC (LS).*

Legal statements like (3) and (4) will be called *conceptual legal statements.*

3. Pure Legal Statements and Applicative Legal Statements

The truth of propositions expressed by statements like (1) and (3) depends *exclusively* on the existence of certain legal norms in a given system; if a norm like 'All *F* ought to do ϕ' is among the consequences of legal system *LS*, then (1) is true, and if a conceptual rule like 'All *F* are ϕ' is among the consequences of the legal system, then (3) is true. Such statements will be called *pure legal statements.*

In contrast, the truth of propositions expressed by statements like (2) and (4) can depend on the existence of certain legal norms *and* on the truth of certain statements of fact.[2] Thus, (2) can be analyzed in the following terms:

(2c) The norm 'All *F* ought to do ϕ' belongs to *NC (LS)*, and x is an *F*.

An analysis of (4) would be:

(4a) The conceptual rule 'All *F* are ϕ' belongs to *NC (LS)*, and x is an *F*.

Often, whether x has a particular property (whether it is real estate, an owner, an accomplice, of age, etc.) depends on certain legal qualifications. Thus, if a norm-authority is-

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[1] If we take into consideration the distinction between *strong* and *weak* permission (von Wright 1963a, Alchourrón/Bulygin 1984), 'It is legally permitted that not ϕ' must be understood as 'It is permitted in the strong sense'. A conduct is permitted in the strong sense in a system *S* if, and only if, a normative consequence of *S* explicitly permits it. A conduct is permitted in the weak sense in *S* if, and only if, no normative consequence of *S* prohibits it. Weak permission of a conduct does not guarantee the truth of the corresponding legal statement. I will come to this later.

[2] If, in the system in question, an individual norm like 'x ought to do ϕ' or a conceptual rule like 'x is a ϕ' can be deduced, then statements (2) and (4) would also be pure statements (the truth of the propositions they express would depend exclusively on whether or not those individual norms belong to a certain system). For reasons of simplicity, I will assume that legal systems contain only general norms. Thus, the truth of the propositions expressed in (2) and (4) will always depend on whether or not certain general norms can be shown to belong to the system *and* whether or not the propositions expressed by certain statements of fact are true.

sues a norm according to which persons of age have the obligation to vote and a conceptual rule like 'Persons aged 18 years or older are of age', then whether x is of age depends on that conceptual rule. In those cases, we must assume that (2c) refers to the normative concequence of both the prescriptive and the conceptual rule, i. e., that 'All F ought to do ϕ' stands for 'Persons aged 18 years or older must vote'.

Statements of types (2) and (4) will be called *applicative legal statements*.[3]

4. Truth-Conditions of Legal Statements

The proposition expressed by legal statement (1) is true if, and only if, there is a normative consequence of *LS* that makes it obligatory for all F to do ϕ. But when is that proposition false? I will say that it is false when there is a normative consequence of *LS* that does not make it obligatory for all F to do ϕ, i. e., that permits an F to omit ϕ. What happens if in *LS* there is neither a normative consequence making it obligatory to do ϕ nor one permitting to omit ϕ? In that case, I will say that the proposition in question has no truth-value: it is then neither true nor false that legally all F ought to do ϕ.

But are there propositions without a truth-value? If the answer to this question is affirmative, as I wish to suggest, we must revise our conception of logic: the law of bivalence, which says that all propositions are either true or false, cannot then be retained unconditionally. In contemporary philosophy, this idea is often linked to Dummett's (1978, 1991) view that there is a strong connection between realism and bivalence. According to Dummett, there are three interrelated kinds of realism, such that if you reject one of them, you must also reject the others (cf. Engel 1991, 129):

(I) *Metaphysical* realism: Independently of our knowledge, there is something in the world which can make our propositions true.
(II) *Semantic* realism: The meaning of a statement is determined by its truth conditions, independently of how we may be able to verify them.
(III) Realism in *logic*: The principle of *bivalence*, i. e., the principle that all propositions are either true or false, is accepted as a fundamental principle of logic.

Dummett's position, in short, is that all these forms of realism must be rejected and replaced by:

(I') *Metaphysical* antirealism: Our propositions are not true because of a reality that is independent of our ability to verify it; reality is relative to the knowledge we have of it.
(II') *Semantic* antirealism: The meaning of our statements is not determined by their truth-conditions, but by their conditions of assertability and use.
(III') Antirealism in *logic*, or *revisionism*: The principle of bivalence, as well as the law of excluded middle, is rejected and instead, the revision of the laws of

[3] The distinction between pure and applicative statements can be found in Raz (1970, 45-50; 1979, 62; 1980, 218; 1994b, 181 f.) and, expressed in different terms, in Hernández Marín (1989, 270-284).

classical logic implied by this rejection in favour of an *intuitionist* conception of logic is defended.

Dummett's project is ambitious, and it has been widely reviewed and discussed (see, e. g., Wright 1993, 1-43). My purpose here is more modest. I do not wish to defend a global antirealist position. Realism may be an adequate conception for some sectors of human knowledge. But since the law is a human construction, it seems plausible to hold an antirealist or *constructivist* conception with respect to the class of legal propositions.

A parable often used in the literature (e. g., Blackburn 1984, 203-210; Dworkin 1985, 146-166) can throw some light on what I mean. In that literature, statements of literary criticism, referring to objects of fiction,[4] are compared to legal statements. What are the truth-conditions of statements referring, e. g., to aspects of the fictional character of Madame Bovary? Obviously, the truth of such statements depends on the story constructed by Flaubert. Here, it does not matter what theory of literature we subscribe; what matters is only that it seems plausible to assume that statements like 'In Flaubert's novel, Madame Bovary's blood group was A' have no truth-value. What makes this assumption plausible is precisely metaphysical antirealism with regard to what we can call the *Flaubert-world.* The Flaubert-world does not exist independently of our possible knowledge of Flaubert's novel. There is no Flaubert-world independently of Flaubert's novel and, therefore, of our knowledge of that novel, that could make statements about Madame Bovary true or false. This metaphysical antirealism in literature is accompanied by semantic antirealism: Only when we are able to assert some property of Madame Bovary we can attribute truth or falsity to the statement that, according to Flaubert's novel, attributes that property to Madame Bovary. But then, there are reasons for rejecting bivalence, since if it cannot be shown that Madame Bovary, according to Flaubert's novel, is a ϕ, nor that she is not a ϕ, then the statement 'In Flaubert's novel, Madame Bovary is a ϕ' has no truth-value.[5]

Something similar happens in law (cf. Patterson 1996, 3-21). The truth of legal statements too depends on whether certain consequences can be obtained in some *LS*. The truth of the statement 'Legally Gaius is of age' may depend on whether one can obtain — whether one can prove — a consequence like 'Persons aged 18 or older are of age' in a particular *LS* (as in current Spanish law, in virtue of art. 12 of the Constitution), and on whether Gaius is 18 years or older. But if we ask about the truth-value of the statement 'Legally, Gaius is a fan of FC Barcelona', it seems plausible to say (as in

[4] For a distinction between literature and fiction, cf. Searle 1979, 58-60.

[5] Dummett himself (1978, 230) has held this view for fictional characters: „Thus, to say that the fictional characters are the creations of imagination is to say that a statement about a fictional character can be true only if it is imagined as being true, that a fictional character can have only those properties which it is part of the story that he has; to say that something is an object of sense — that for it *esse est percipi* — is to say that it has only those properties it is perceived as having: in both cases, the ontological thesis is a ground for rejecting the law of excluded middle as applied to statements about objects. Thus we cannot separate the question of the ontological status of a class of objects from the question of the correct notion of truth for statements about thse objects; i. e. of the kind of thing in virtue of which such statements are true, when they are true.“

the case of Madame Bovary's blood group) that, since there is no corresponding norm in *LS*, that statement is neither true nor false.

We thus get the *metaphysical* thesis of *legal antirealism* or *constructivism*: There is no legal world, beyond our capacity of knowing the law as constructed by human beings, that can make legal propositions true or false.[6]

And we also have the *semantic* thesis of *legal constructivism*: The meaning of legal statements is determined by their condition of *assertability*, that is, by the ability to prove that certain consequences obtain in some legal system *LS*.[7]

These theses, in turn, imply the *logical* thesis of *legal constructivism*: Not all legal propositions are true or false.

But then, what is the logic that applies to legal propositions? *Revisionism* in logic which proposes to accept an intuitionist logic raises problems I cannot treat here (cf. Haack 1978, 216-220). For this and other reasons, I choose what can perhaps be called an intermediate way: the *truth-logic* (or logics) constructed by G. H. von Wright (1984c, 1988, 1989).[8] In truth-logic, a minimal deviation from classical logic brings with it the ability to account for the problems encountered in treating propositions without a truth-value.

The only *heterodox* characteristic of truth-logic *(TL)* is that the notion of truth is introduced in the object-language with the symbol *T*. *T* functions as a modal operator; prefixing it to a well-formed expression produces a new well-formed expression. It is read as 'it is true that', and forms what I will call *T*-expressions.

With the help of *T*, we can distinguish two ways of negating a statement: *external* negation $\neg T$, to be read as 'it is not true that', and *internal* negation $T\neg$, meaning 'it is true that not'. An internal negation asserts *falsity* and can therefore also be read as 'it is false that'. In *TL*, falsity and not-truth are not the same: not-truth is weaker than falsity. A false proposition is always not-true, but not all not-true propositions are false (as, for example, propositions that are neither true nor false). This is the novelty in *TL*: it can account for propositions without a truth-value.

But, can there really be propositions without a truth-value? Are not all propositions necessarily true or false? In another paper, von Wright (1984a) *demystifies* the notion of a proposition as follows: The basic notion is that of a *sentence* (a grammatically well-formed expression). A proposition is then defined as 'a move of language': a grammatically well-formed sentence expresses a proposition if, and only if, the sentence obtained by prefixing it with the expression 'it is true that' is well-formed too. If one

[6] This thesis is intimately linked to legal positivism, with its thesis that the existence of law in a society depends only on certain social facts, i. e., on human acts. Cf. Moreso 1994a, 353 f.

[7] Dworkin (1977a, 8) posits a strong relationship between positivism and semantic antirealism, with special reference to Dummett's characterization: „I think that many positivists rely, more or less consciously, on an antirealist theory of meaning. They think that no sense can be assigned to a proposition unless those who use that proposition are all agreed about how the proposition could, at least in theory, be proved conclusively."

[8] Although the most important ideas can already be found in von Wright 1984c, the system I will present is one of those constructed in von Wright 1989, which was already anticipated in von Wright 1988 and which has some important particularities, among them especially the acceptance of mixed formulas.

accepts that notion of a proposition, perhaps it will be easier to accept that some propositions have no truth-value. Thus, 'Prime numbers are blue' seems to express a proposition without a truth-value. That it expresses a proposition can be verified by noting that 'It is true that prime numbers are blue' is a well-formed sentence.[9]

Although von Wright himself has developed some ideas for constructing a quantificational truth-logic (von Wright 1984b), *TL* is an extended propositional logic.

The basic symbols of *TL* are:

1) *Propositional variables*: p, q, r.
2) *Sentential connectives*: $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$.
3) *Brackets:* (,).
4) *Operator:* T.

The notion of a *formula* of *TL* is defined recursively:

1') All propositional variables are formulae of *TL.*
2') If α is a formula of *TL*, then so is $\neg\alpha$.
3') If α and β are formulae of *TL*, then so are $\alpha \wedge \beta$, $\alpha \vee \beta$, $\alpha \rightarrow \beta$ y $\alpha \leftrightarrow$ b.
4') If α is a formula of *TL*, then so is $T\alpha$.[10]

The axioms of *TL* are the following:

A0. All formulae obtained from tautologies of classical, two-valued, propositional logic by putting the letter T immediately in front of every variable occurring in the tautologous formula.
A1. $Tp \leftrightarrow T\neg\neg p$. A proposition is true if, and only if, its negation is false.
A2. $T(p \wedge q) \leftrightarrow Tp \wedge Tq$. A conjunction is true if, and only if, all its conjuncts are true.
A3. $T\neg(p \wedge q) \leftrightarrow T\neg p \vee T\neg q$. A conjunction is false if, and only if, at least one of its conjuncts are false.
A4. $Tp \rightarrow p$. If it is true that p, then p.

The rules of inference or transformation are as follows:

R1. Substitution of formulae for variables (a variable is also a formula).
R2. Detachment (*Modus ponens).*

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[9] Other examples more widely discussed in philosophical scholarship are statements about contingent futures, like 'Tomorrow there will be a naval battle'; statements containing descriptions referring to inexisting entities, like 'The King of France is bald' (cf. Appendix); some counterfactual conditionals, like 'If Kelsen had been born in Barcelona, he would have spoken Catalan'; or statements containing vague terms, when they refer to cases that fall within the penumbra of the referent of those expressions, like 'Maria (referring to a person aged 65) is old'. I will say more about this last type later.

[10] The conventions for brackets are the usual ones.

R3. The Rule of Truth: if α is an axiom or theorem of *TL*, then so is $T\alpha$.[11]

TL can be interpreted as a three-valued logic: true, false, and neither-true-nor-false — indetermined — (represented in what follows by '+', '-', and '?'). A truth-table may help us understand some of the special characteristics of *TL*:

p	$\neg p$	Tp	$T\neg p$	$\neg Tp$	$\neg T\neg p$
+	-	+	-	-	+
-	+	-	+	+	-
?	?	-	-	+	+

In the same way, we can construct the truth-tables for all the other connectives of *TL*.[12]

Now, some comments about *TL* are warranted.:

(i) Although propositional variables can express propositions without a truth-value, *T*-expressions in *TL* always express propositions that are true or false. Therefore, the heterodoxy of *TL* is limited: *T*-expressions follow the rules of classical logic.

(ii) In classical logic, the law of excluded middle is usually represented by $p \vee \neg p$: all propositions are either true or not true. But $T(p \vee \neg p)$ is not a theorem of *TL*. The law of excluded middle is retained in *TL* in a more restricted form: $Tp \vee \neg Tp$.

(iii) The law of bivalence, according to which all propositions are either true or false — $Tp \vee T\neg p$ —, is not retained in *TL* either, since it is possible that $\neg Tp \wedge \neg T\neg p$. That is precisely what happens when p has no truth-value. But we can give a restricted version of the law of bivalence, and that version is retained in *TL*: $T(p \vee \neg p) \leftrightarrow Tp \vee T\neg p$ (it is true that p-or-not-p if and only if it is true that p or it is true that not-p). And it

[11] This is one of the systems of von Wright 1989 (called *TLM*). In von Wright 1989, 25 f., it is shown that the systems of 1984c and of 1988 which in addition to *A0* - *A3* contained the axiom

A5. $Tp \rightarrow \neg T\neg p$

and (only the system of von Wright 1984c) the axiom

A6. $T\neg Tp \leftrightarrow \neg Tp$

are contained in the system of 1989. This means that *A5* and *A6* are demonstrable in that system (as von Wright explicitly acknowledges, the proof is due to Carlos Alchourrón).

[12] Here are the truth-tables for conjunction, disjunction, conditional and biconditional:

p	q	$p \wedge q$	$p \vee q$	$p \rightarrow q$	$p \leftrightarrow q$
+	+	+	+	+	+
+	-	-	+	-	-
+	?	?	+	?	?
-	+	-	+	+	-
-	-	-	-	+	+
-	?	-	?	+	?
?	+	?	+	+	?
?	-	-	?	?	?
?	?	?	?	?	?

is also a logical law of *TL* that it is either true that p is true or it is false that p is true: $TTp \vee T\neg Tp$. That law is perhaps the best-suited candidate for being called the law of bivalence in that logic.

(iv) Since $\neg T\neg p$ is equivalent to $T\neg T\neg p$, from the theorem $Tp \rightarrow \neg T\neg p$ (cf. n. 11) it follows that $Tp \rightarrow T\neg T\neg p$. That means that if a proposition is true, then it is false that that proposition is false. But the converse does not hold. As von Wright remarks (1988, 13), this characteristic, together with the rejection of the classical law of bivalence, is what makes *TL* similar, though not equivalent, to an *intuitionist* logic.

(v) It is precisely this similarity to intuitionist logics which makes *TL* an adequate system for the logical treatment of legal propositions in a constructivist approach. Intuitionists like L. E. J. Brouwer and A. Heyting (cf. Haack 1978, 216-220) claim that classical logic cannot adequately account for mathematical reasoning. Their constructivist approach to mathematics sees it basically as a mental activity, and numbers as mental entities. Thus, to say that there is a certain number with certain properties is to say that such a number can be constructed; if we cannot *prove* that a certain number has property P, then that there is such a number that is P has no truth-value. In the opinion of intuitionists, there is no mathematical reality independently of our capacity to construct mathematical systems. And that metaphysical thesis (rejecting Platonism in mathematics) is accompanied by the semantic thesis that truth in mathematics is provability, and by the logical thesis that rejects bivalence in the classical sense (that every proposition is either true or false). Thus, Dummett's interest (1978, 215-247) in the philosophical consequences of intuitionism in logic is not surprising.

In my view, in that sense, legal propositions are similar to mathematical propositions. Thus, it is legally true that all F ought to do ϕ if, and only if, there is (can be proved to be) a normative consequence of *LS* making it obligatory for all F to do ϕ, and it is legally false that all F ought to do ϕ if, and only if, there is a normative consequence of *LS* permitting an F not to do ϕ. If in *LS* there is no norm making it obligatory to do ϕ or permitting not to do ϕ, then I will say that legally it is not true that all F ought to do ϕ, and that legally it is not false that all F ought to do ϕ.

The distinction introduced in Chapter I between *relevant* and *irrelevant* logical consequences of a set of statements can help sharpen this approach to legal propositions. It would seem absurd to say that since the norm 'All F must vote' implies 'All F must vote or sleep fifteen hours a day' the legal proposition expressed by the statement 'Legally, x (who is an F) must vote or sleep fifteen hours a day' is true. In order to avoid such absurdities, I will say that the truth of legal propositions depends on whether some *relevant* consequence can be proved to exist in a system. Therefore:

A legal proposition is true with respect to *LS* if and only if the norm it refers to is a provable *relevant* consequence in *LS*, and it is false if the negation of the norm it refers to is a provable *relevant* consequence in *LS*. If neither the norm nor its negation are provable relevant consequences in *LS*, then that legal proposition has no truth-value.[13]

[13] For reasons of simplicity, unless otherwise indicated, in what follows the expression 'provable in the system' should be understood to mean 'provable as a *relevant* consequence in the system'.

We can now use *TL* for a logical analysis of legal statements like (1) 'Legally, all *F* ought to do ϕ'. If we insert the expression 'it is (not) true that' or 'it is (not) false that' between 'Legally' and the rest of (1), we get what I will call *legal T-statements.* Legal *T*-statements always have a truth-value. The legal *T*-statement corresponding to (1) has the following possible forms:

(LTS1a) Legally, it is true that all *F* ought to do ϕ.
(LTS1b) Legally, it is false that all *F* ought to do ϕ.
(LTS1c) Legally, it is not true that all *F* ought to do ϕ.
(LTS1d) Legally, it is not false that all *F* ought to do ϕ.

(LTS1a) implies *(LTS1d)*, and *(LTS1b)* implies *(LTS1c)*. But neither does *(LTS1d)* imply *(LTS1a)*, nor does *(LTS1c)* imply *(LTS1b)*. In the case where (1) has no truth-value, although legally it is not true that all *F* ought to do ϕ, neither is it false that all *F* ought to do ϕ, i. e., *(LTS1c)* and *(LTS1d)* can both be true.

The same can be said for statements like (2). The legal *T*-statements corresponding to (2) are:

(LTS2a) Legally, it is true that all *F* are ϕ.
(LTS2b) Legally, it is false that all *F* are ϕ.
(LTS2c) Legally, it is not true that all *F* are ϕ.
(LTS2d) Legally, it is not false that all *F* are ϕ.

Note that 'Legally, it is true that' and 'Legally, it is false that' do not exhaust the logical space, whereas 'Legally, it is true that' and 'Legally, it is not true that' do.

The analysis of legal *T*-statements corresponding to (3) and (4) must be more elaborate. As I have shown, (3) and (4) are conjunctive statements.

Let us begin with (3). The analysis of (3) must be in terms of

(LTS3a) Legally, it is true that *x* ought to do ϕ,

because *(LTS3)* must show the molecular structure of (3) in a form like

(LTS3a') Legally, it is true that all *F* ought to do ϕ, and it is true that *x* is an *F*.

Thus, for

(LTS3b) Legally, it is false that *x* ought to do ϕ

we have correspondingly:

(LTS3b') Legally, it is false that all *F* ought to do ϕ, or it is false that *x* is an *F*.

This is merely an application of axioms *A2* and *A3* of truth-logic, according to which $T(p \wedge q) \leftrightarrow Tp \wedge Tq$ and $T\neg(p \wedge q) \leftrightarrow T\neg p \vee T\neg q$, and the assumption that the expression 'legally' only is prefixed to 'it is (not) true' or 'it is (not) false' in the case of statements referring to normative consequences of *LS*.

> *(LTS3c)* Legally, it is not true that x ought to do ϕ

will be analyzed with the help of

> *(LTS3c')* Legally, it is not true that all F ought to do ϕ or it is not true that x is F.

And finally,

> *(LTS3d)* Legally, it is not false that x ought to do ϕ

is to be analyzed with

> *(LTS3d')* Legally, it is not false that all F ought to do ϕ or it is not false that x is an F.

With the necessary changes, the same applies to the legal T-statements corresponding to (4).

Now, is it possible that a statement like 'x is an F' has no truth-value? In other words: Is the expression 'It is true that x is an F' equivalent to 'It is not false that x is an F'? Factual propositions attributing properties to individuals (e. g., the property of being 18 years or older to x) or events (e. g., the property of being a murder to an action of x) may behave according to the laws of classical logic, and their truth-value may depend on their correspondence with empirical reality, since truth in that case is not equivalent to demonstrability in a legal system. But as we will see later, it is important to preserve the possibility that such propositions have no truth-value, because it allows us to approach the treatment of factual propositions containing vague concepts.

5. *Legal Statements and Contradictions: The Law Speaks With Many Voices*

If it could be shown in *LS* — if it were a consequence of *LS* — that all F are ϕ and that all F are not ϕ, or that all F ought to do not-ϕ and that all F may do ϕ, then the propositions expressed by the legal T-statements 'Legally, it is true that all F are ϕ' and 'Legally, it is true that all F ought to do not-ϕ' would be at the same time true and false. This idea can be expressed by saying that the legal world corresponding to that *LS* is an impossible legal world. There is no logical reason that could prevent an authority to issue commands or qualifications (through conceptual rules) that are contradictory (in the sense, already explained, that there is no possible world that could make such a normative system effective). Although this leads to unfortunate consequences for the addressees of such norms, it is a rather common situation in complex legal systems.

But, as I have shown in Chapter I, those normative consequences are irrelevant, since an *LS* that contains a contradiction has no *relevant consequences*. If the truth-value of a legal proposition depends on the demonstrability in *LS* of a relevant consequence, then the legal statements 'Legally, all *F* ought to do not-ϕ' and 'Legally, all *F* may do ϕ' express propositions without a truth-value, since all the logical consequences of *LS* are *irrelevant*.

The conclusion of all this is that all legal propositions, expressed by legal statements, that refer to a legal system *LS* containing contradictions, i. e., an *LS* with antinomies, have no truth-value; and all propositions expressed by legal *T*-statements like 'Legally, it is not true that ...' and 'Legally, it is not false that ...' referring to that *LS* are true.

To say that a normative system that contains contradictions has no relevant consequences can perhaps explain why such a system is not an adequate system for the regulation of human behaviour: it cannot serve as a guideline for action.

6. Legal Statements and Normative Gaps: The Law Is Silent

The question of whether legal systems are, by definition, complete or whether they can have gaps has been — and still is — widely discussed in legal philosophy.[14]

Since the word 'gap' has a wide variety of uses in legal theory, an attempt to reconstruct and elucidate those different uses seems to be needed. In my view, the best investigation of gaps in the law is that of Alchourrón and Bulygin (1971, 17-21, 31-34, 94-115, 116-143).

What the different uses of the word 'gap' in legal contexts seem to have in common is that whenever gaps are mentioned, there is some degree of *indeterminacy* in the application of the law. Alchourrón and Bulygin (1971, 31) observe that this indeterminacy can derive from „problems of a *conceptual* type arising at the level of generic cases and general norms" or from „*empirical* and *semantic* problems arising from the *application* of general norms to individual cases". These two cases must be carefully distinguished — which has not always been done in legal theory — in order to clarify the different questions treated under the label of 'gap'.

One of the most important problems arising in the application of general norms to individual cases is the classification of the individual case, i. e., what jurists call 'subsumption' or 'legal qualification'.

The problems arising in the context of subsumption can have one of two possible sources. The first is lack of information about the facts of the case. Alchourrón and Bulygin call such cases *gaps of knowledge*.[15] If a general norm prescribes that all persons who in a given year earned more than $ 10.000 have the obligation of presenting a declaration of income, our ignorance of whether or not Ticius had a higher income may

[14] For the thesis that legal systems can have no gaps, cf. Kelsen 1960, 251-255; Dworkin 1989, 128-134.

[15] Alchourrón/Bulygin 1971, 33: „Where, through ignorance of some of the properties of the fact, we do not know whether a certain individual case belongs to a certain generic case, we shall speak of a *gap of knowledge*."

prevent us from knowing whether his individual case can be subsumed under the generic case regulated by that general norm. We then do not know the truth-value of the legal statement 'Legally, Ticius has the obligation of presenting a declaration of income' because we do not know whether the statement 'Ticius earned more than $ 10.000' is true. That statement, however, is true or false with respect to the real world; therefore (assuming a *realist* conception of the real world) it does have a truth-value, although we may not know which.

But the difficulty may also arise, not out of our ignorance of the facts of the case, which perhaps we have perfect knowledge of, but because we cannot determine, e. g., whether certain gifts Ticius received from some of his clients should count as *income*. The problem then is one of *semantic indeterminacy* or *vagueness*, not of ignorance, because the problem subsists even if we know perfectly well how much the gifts received by Ticius are worth. Alchourrón and Bulygin call such cases *gaps of recognition*.[16]

In the next section, I will argue that in that case the statement 'Ticius had an income of more than $ 10.000' has no truth-value and, more generally, that problems of vagueness stemming from the use of certain general expressions referring to cases of *penumbra* give rise to statements which express propositions that have no truth-value. Here, I only wish to stress that problems from a lack of knowledge, or ignorance, are different from problems from vagueness, or semantic indeterminacy.[17] We will very likely never find out the truth-value of the statement 'On September 25, 1995, there was an uneven number of blades of grass on the Bellaterra Campus of Barcelona's Autonomous University';[18] yet, that statement is either true or false (to that class of statements, bivalence applies). This question is different from that of whether or not a particular blade of grass on the Bellaterra Campus is green. If it is greenish, with a yellowish overtone, it may be impossible to decide whether or not it is green, even though we possess all the information one can have. Our doubts in that case do not arise from a lack of information.

The problem I wish to treat in this section, however, concerns another use of the expression 'gap', namely, what Alchourrón and Bulygin call *normative gaps*, i. e. situations where a *generic case* of a universe of cases is not correlated with any maximal normative solution.[19] This, again, is not a problem of the application of general norms to individual cases, but a conceptual question: the absence of a normative solution for a generic case.

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[16] Alchourrón/Bulygin 1971, 33: „Where, through semantic indeterminacy of the concepts which characterize a generic case, we do not know whether a certain individual case belongs to it, we shall speak of a gap of recognition."

[17] Recently the thesis has been defended, however, that problems of vagueness are merely a special kind of problems of ignorance. Cf. Williamson 1994. I will come back to that question in the next section.

[18] Cf. Quine 1981, 31-37.

[19] Strictly speaking, I should add: 'or where a generic case is correlated with a disjunction of two or more maximal solutions' (this would be what Alchourrón/Bulygin [1971, 20 f.] call a *partial gap*). For reasons of simplicity, in this chapter I will omit this complication. — Alchourrón and Bulygin analyze still another use of the word 'gap', i. e., what they call 'axiological gaps'. I will come to them in Chapter IV.

The Spanish Civil Code, for instance, regulates the reimbursement of expenses and improvements in case of a transference of possession. Now, does a former possessor have the right to be reimbursed for expenses and improvements (or to withdraw improvements consisting in separable objects)? The Spanish Civil Code regulates such cases (arts. 453-455), taking into account the following criteria: whether the expenses were necessary, useful or superfluous, and whether the claimant possessed the object in good or in bad faith. I am not interested here in the complex solution the Code gives to those cases; I only wish to point out that the case of useful expenses made by a possessor in bad faith has no normative solution (Alonso 1995).[20] Thus, while the legal statement 'Legally, Gaius (a former possessor in good faith who carried out necessary improvements of the object in his possession) has the right to be reimbursed for what he spent on the improvement of the object' (which means that the new possessor has the obligation of paying Gaius back what he spent) is a statement expressing a true proposition, the legal statement 'Legally, Ticius (a former possessor in bad faith who carried out useful improvements of the object) has the right to be reimbursed for what he spent on the improvement of the object' expresses a proposition without a truth-value, since there is no normative consequence in the normative system in question that correlates the fact of useful expenses made by a possessor in bad faith with a normative solution.

Therefore, when there are legal gaps, legal statements referring to cases falling within those gaps express legal propositions without a truth-value.

But this is a very controversial conclusion. In what follows, I will briefly consider the positions of Kelsen, Dworkin and Raz who, for reasons that do not fully coincide, reject that conclusion and assert the contrary: that the law is always complete, and that in cases falling within an area of normative gaps, legal statements always express propositions that are either true or false.

a) Kelsen: gaps as fiction

Kelsen's thesis that legal systems never have (i. e., cannot have) gaps is related to the question of a judge's obligations in the application of the law. However, in my presentation I will try to keep the two questions apart, because in my view the answers are not necessarily connected.

Kelsen's conception can be summarized in the following two theses (cf., e. g., Kelsen 1945, 146-148; 1960, 251-255):

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[20] Simplifying somewhat, one can say that the Spanish Civil Code regulates those cases by conceding the more rights the more the claimant possessed the object in good faith and the greater the *need* of the expenses and improvements. Thus, if the expenses were necessary and the claimant possessed the object in good faith, then he has the right to be reimbursed, and to withhold the property until the new possessor has paid him for the expenses. In the case of a former possessor in bad faith who carried out unnecessary improvements, he only has the right to keep those improvements that consist in separable objects, provided the new possessor prefers not to buy them from him at the price of the value of the objects at the time of transference. The problem is that in arts. 453-455, which regulate that situation, no mention is made of useful expenses or improvements made by a possessor in bad faith.

(I) Legal system cannot have gaps, because a behaviour that is not prohibited by the law is permitted.[21]

(II) In cases where there is no norm obligating a particular person to a certain behaviour, if that person is sued by another, then the competent judge *applies the law* by dismissing the case.[22]

The link between (I) and (II) is expressed by Kelsen in two different ways: Until 1960, he maintained that a judge acquits the defendant (or dismisses the case) by applying a negative rule according to which no-one can be obligated to behave in ways that are not prescribed by law (cf. Kelsen 1945, 147); but in 1960, Kelsen asserts that „by dismissing the case or acquitting the defendant, the court applies the legal order which permits the defendant the behaviour against which the action or charge, without foundation in the legal order, was directed".[23] That means that in 1960 Kelsen holds that judges apply the legal order as a whole, and no longer uses the argument of the negative rule for acquitting the defendant or dismissing the suit in the case of a legal gap.

Thesis (I) has been convincingly criticized by Alchourrón and Bulygin (1971, 119-124). The Argentine authors distinguish two versions of thesis (I) which they call the *Principle of Prohibition* ('Everything which is not prohibited is permitted'). The two versions correspond to two possible meanings of the term 'permitted' in the Principle of Prohibition: 'permitted' in the *strong* sense, and 'permitted' in the *weak* sense (von Wright 1963a, 86-90; Alchourrón 1969, Alchourrón/Bulygin 1971, 121 f.; Hernández Marín 1989, 337 f.; Moreso/Navarro 1992):

A behaviour *b* is *strongly* permitted in the (generic) case *c* in a legal system *LS* if, and only if, a normative consequence can be inferred from *LS* which permits *b* in case *c*.

A behaviour *b* is *weakly* permitted in the (generic) case *c* in a legal system *LS* if and only if no normative consequence can be inferred from *LS* which prohibits *b* in case *c*.

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[21] Cf. Kelsen 1945, 147: „Just because no norm exists which obligates the defendant to the behavior claimed by the plaintiff, the defendant is free according to positive law, and has not committed any delict by his behavior"; and Kelsen 1960, 251, where he asserts that „wenn die Rechtsordnung keine Pflicht eines Individuums zu einem bestimmten Verhalten statuiert, sie dieses Verhalten erlaubt".

[22] Kelsen 1945, 147: „If the judge dismisses the suit, he applies, as it were, the negative rule that nobody must be forced to observe conduct to which he is not obliged by law"; Kelsen 1960, 251: „Die Anwendung der geltenden Rechtsordnung ist in dem Fall, in dem die traditionelle Theorie eine Lücke annimmt, nicht logisch unmöglich. Zwar ist in diesem Falle die Anwendung einer einzelnen Rechtsnorm nicht möglich, aber die Anwendung der Rechtsordnung, und auch das ist Rechtsanwendung, ist möglich. Rechtsanwendung ist nicht logisch ausgeschlossen."

[23] Kelsen 1960, 248: „Indem das Gericht die Klage abweist oder den Angeklagten freispricht, wendet es die Rechtsordnung an, die dem Beklagten oder Angeklagten das Verhalten erlaubt, gegen das sich die in der Rechtsordnung nicht begründete Klage oder Anklage gerichtet hat."

The two meanings of 'permitted' give rise to two versions of the Principle of Prohibition: the *weak* version of that Principle says that 'Everything which is not prohibited is weakly permitted', and the *strong* version says that 'Everything which is not prohibited is strongly permitted'.

The weak version of the Principle of Prohibition is an analytic statement that is necessarily true. It says that all behaviour that is not prohibited in a legal system is not prohibited in it. But the truth of the weak version of the principle does not guarantee that there are no gaps. Rather, it is compatible with the existence of gaps: Whenever there is a normative gap (a generic case without a normative solution), the corresponding behaviour is weakly permitted.

If the strong version of the Principle of Prohibition, according to which for every behaviour that is not prohibited in a legal system *LS* there is a normative consequence in *LS* expressly permitting that behaviour, were true, it would guarantee that all normative systems are closed, i. e., have no gaps. But the truth of the strong version of the principle is contingent. In the words of Alchourrón and Bulygin (1971, 127):

„For from the mere fact that a certain norm (the norm to the effect that *p* is prohibited in *q*) does not belong to a certain system, it does not follow that another, different norm (namely, the norm to the effect that *p* is permitted in *q*) belongs to the system. Therefore the Principle of Prohibition, in its strong version, is not necessarily true."

To sum up: the Principle of Prohibition — Kelsen's thesis (I) — does not preclude the presence of gaps in legal systems. In its *weak version*, though expressing a proposition that is necessarily true, it is compatible with the existence of unregulated cases, of gaps. In its *strong version*, it expresses a proposition that is only contingently true and therefore only guarantees the absence of gaps for those systems where it expresses a true proposition.

For the analysis of Kelsen's thesis (II), according to which in cases where there is no legal norm that makes a particular behaviour obligatory the competent judge applies the law in dismissing the case, we will need to introduce the notion of *applicability* and the distinction between norms that belong to a legal system and norms that are applicable in a legal system (which will be developed in the next chapter). For now, it is sufficient to recall a few obvious points:

Not all branches of the law behave in the same way (that is, have the same criteria of applicability) in unregulated cases: While in criminal law, under the rule of law judges are prohibited (by the so-called *principle of legality*) to sanction behaviour that is not subject to any norm, in civil law they may (or must) make use of the so-called *analogical* application of norms, with the consequence that suits based on unregulated facts can (or must) be accepted (as in the civil-law case I referred to earlier). To say that a judge *applies* the law in those cases by dismissing the suit would be regarded as a highly counterintuitive consequence. Kelsen himself seems to be aware of this:

„But it is also possible that the legal order empowers the court not to dismiss the case or not to acquit the defendant, in case it cannot find a general legal norm imposing on the defendant the duty violation of which is alleged by the private plaintiff or the public prosecutor, and instead to admit the suit or convict the accused, if it regards the lack of such a general legal norm as unjust, or unfair, i. e. as not satisfactory."[24]

As I have shown, Kelsen's transition from thesis (I) to thesis (II) is supported either by the doctrine of the application of a negative rule (until 1960) or by the doctrine of the application of the legal order as a whole (after 1960; cf. also 1979, xxxi). Alchourrón and Bulygin maintain that this shift of argument is due to an evolution in Kelsen's conception of the meaning of what I have called thesis (I). According to Alchourrón and Bulygin (1971, 130-134), until 1960 Kelsen interprets the Principle of Prohibition in its *strong version* (i. e., 'Everything which is not prohibited by the law is explicitly permitted by the law'), whereas beginning in 1960 Kelsen interprets it in its *weak version* (i. e., 'Everything which is not prohibited by the law is weakly permitted'). The reason they give for this change in Kelsen's thinking is the following: Kelsen (1960, 249) now recognizes that there may be a behaviour that is not prohibited, and in that sense is permitted, to some individual and which prevents another individual to perform another behaviour which also is not prohibited and, therefore, is permitted. In that case, Kelsen adds, „there is ... a conflict of interest not prevented by the legal order; and no legal order can prevent all possible conflicts of interest".[25] That is why Alchourrón and Bulygin (1971, 132 f.) hold that Kelsen changed his position and came to adopt the weak version of the Principle of Prohibition, *implicitly* admitting that there may be gaps in the form of conflicts of interest not prevented by the legal order.

More recently, Ruiz Manero (1990, 41-45) has rejected the interpretation of Alchourrón and Bulygin, arguing that the version of the Principle of Prohibition of the Kelsen of 1960 also is the strong version. According to Ruiz Manero, in the cases Kelsen calls „conflicts of interest not prevented by the legal order", all behaviour in question is permitted in the strong sense, and therefore judges in such cases have the duty to dismiss the case (if it is a civil suit) or to acquit the defendant (in case of criminal proceedings). In that sense, Kelsen still would not distinguish cases of strong and of weak permission. For Kelsen, the two following situations would be equivalent: A legal system LS_1 containing a norm permitting behaviour p by person x, and another norm permitting behaviour q by person y which prevents the performance of p; and a legal system LS_2 containing no norm prohibiting behaviour p by person x and no norm prohibiting behaviour q by person y, where the latter prevents the performance of p. For Kel-

[24] Kelsen 1960, 249: „Es ist aber auch möglich, dass die Rechtsordnung das Gericht ermächtigt, falls es keine generelle Rechtsnorm feststellen kann, die dem Beklagten oder Angeklagten die Pflicht auferlegt, deren Verletzung der private Kläger oder öffentliche Ankläger behauptet, die Klage nicht abzuweisen oder den Angeklagten nicht freizusprechen, sondern, wenn es das Fehlen einer solchen generellen Rechtsnorm für ungerecht, unbillig, das heisst für nicht befriedigend hält, der Klage stattzugeben oder den Angeklagten zu verurteilen." Kelsen's repeated quotation (1945, 147; 1960, 252) in this context of the first paragraph of the Swiss civil code („A défaut d'une disposition légale applicable, le juge prononce selon le droit coutumier, et au défaut d'une coutume, selon les règles qu'il établirait s'il avait à faire acte de législateur") also seems to point in that direction.

[25] „... liegt ... ein Interessenkonflikt vor, dem die Rechtsordnung nicht vorbeugt; und keine Rechtsordnung kann allen möglichen Interessenkonflikten vorbeugen".

sen, the two systems would be equivalent, and insofar, Ruiz Manero is right in saying that there is an interpretation of the expression 'conflicts of interest not prevented by the legal order' which does not imply the acceptance of gaps. System LS_1 has no normative gap, though there is a conflict of interest left without a solution, whereas system LS_2 does have gaps and, therefore, unresolved conflicts of interest. The fact that Kelsen regards the two systems as equivalent shows that his interpretation of the Principle of Prohibition is, as Ruiz Manero claims, the interpretation of the strong version.

Still, Alchourrón's and Bulygin's doubts seem justified since, as we have seen, Kelsen himself recognizes that a legal order may contain a norm authorizing judges to accept a suit or convict a defendant in the case of a system like LS_2. This assertion by Kelsen suggests that he does see a difference between a system like LS_1 and a system like LS_2. In LS_1 judges must dismiss a suit of y against x (or of x against y) because of the norm that permits y to do q (or x to do p). In LS_2 the duties of a judge depend on what other norms possibly say. If some person z is sued for having done r, and r is permitted by the legal system in the strong sense, then the judge must dismiss the case (otherwise we would not say that r is permitted); in contrast, if z is sued for having done r, and r is permitted in the weak sense — there is no norm prohibiting r — then the duties of the judge will depend on other norms of the system which — as a contingent matter — not always require him to dismiss the case against z.[26]

We can thus conclude that Kelsen did grasp, albeit not very clearly, the two senses of the Principle of Prohibition, as Alchourrón and Bulygin say; but, as Ruiz Manero holds, he still did not change his interpretation of the principle in his work of 1960.

Finally, Kelsen (1945, 147-149; 1960, 253-255) points out — and his commentators (Ruiz Manero 1990, 45-48) emphasize — that the traditional doctrine of gaps is a fiction designed to authorize judges to issue individual norms whose content is not determined by general norms, in order to avoid results that would be unsatisfactory and unjust in the light of their moral and political beliefs. Without questioning the correctness of Kelsen's opinion on this matter, I only wish to point out that his idea (e. g., 1960, 251) that unregulated cases and (according to some criterion of justice) unjustly regulated cases are equivalent is totally unfounded. We can, and I think we should, distinguish the two cases: We do have a clear criterion for determining cases of *normative gaps* (and we do not need any criterion of value for this); and they are different from cases that are regulated *unjustly*, which Alchourrón and Bulygin call 'axiological gaps' (here, we do need a criterion of value). I will treat that matter in Chapter IV.

b) Dworkin and the completeness of the law

At least on one occasion, Dworkin (1989, 127 f.) has raised the question of gaps in a way very similar to the one used here:

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[26] In the case mentioned earlier, concerning the Spanish Civil Code, it would be considered contrary to law if a judge were to dismiss a suit filed by a former possessor in bad faith against the new possessor for the reimbursement of his expenses.

„I would like to explain logically what is understood by a 'gap' in the law. In order to do this, I think we must start from a specific proposition. Thus, let us take Neil MacCormick's example: 'The law authorizes me to ride my bicycle through the Bois de Boulogne today.' According to the thesis admitting the existence of gaps in the law, there are certain — maybe many — equally specific propositions that are neither true nor false. That is what the theory of gaps in the law says, and it is essential to distinguish this thesis — which I think is wrong — from a radically different one according to which it may turn out to be uncertain whether or not a legal proposition is true or false, there may be room for controversy about that question, and it may even be doubtful whether it can be solved. I wish to point out that this last kind of propositions, which speak of uncertainty and controversy, are compatible with realist theses about the ontology of law; they represent a realist approach, whereas the theory of gaps — a proposition is neither true nor false — represent an anti-realist approach."[27]

As can be seen, Dworkin uses the term 'gap' in a wide sense, including all cases in which legal statements express propositions without a truth-value (and which, therefore, lead to indeterminacy in the application of the law). Dworkin's arguments against an *antirealist* (or *constructivist*) conception of legal propositions, like the one advocated here, are of a mixed kind, but they are based on an interpretive conception of the law according to which a legal proposition is true if and only if it forms part, or is a consequence, of the best possible interpretation of a community's legal-political history (Dworkin 1989, 129, and esp. 1986, chs. 2 and 3). In any case, the analysis and critique of that conception and of how it guarantees the completeness of the law will be offered in the last chapter.

For the time being, I only wish to analyze one of Dworkin's arguments against the idea that the law is incomplete (and that, therefore, there are legal propositions which are neither true nor false), and that is an arguement which is independent of his interpretive conception.[28]

I mean the following argument: Dworkin (1985, 129 f.) holds that even if there are legal statements of the form 'x is ϕ' which are true, others which are false, and still others which are neither true nor false, „indeterminacy will not result if a principle of legislation is adopted which requires that if 'x is ϕ' is not true, it be treated as false". This is how bivalence is restored; 'Legally, it is not true that x ought to do ϕ' becomes

[27] „Je voudrais expliquer de manière logique ce qu'on entend par 'lacune' du droit. Pour cela, je crois qu'il faut partir d'une proposition concrète. Ainsi, prenons l'exemple de Neil MacCormick: 'Le droit m'autorise à circuler en bicyclette dans le Bois de Boulougne aujourd'hui.' D'après la thèse qui admet des lacunes dans le droit, il existe certains propositions, peut-être nombreuses, d'un type aussi concret qui ne sont ni vraies ni fausses. Voilà la théorie des lacunes du droit, et il est essentiel de distinguer cette thèse — pour moi erronée — d'une thèse radicalement différente selon laquelle il peut se révéler incertain de dire si une proposition juridique est vraie ou fausse, il peut y avoir place à controverse sur cette question, et même il peut se révéler douteux de pouvoir le trancher. Je tiens à signaler que ces derniéres propositions, qui parlent d'incertitude et de controverse, sont compatibles avec les thèses réalistes en matière d'ontologie du droit; elles représentent une approche réaliste, tandis que la théorie des lacunes — une proposition n'est ni vraie ni fausse — représente une approche antiréaliste."

[28] Dworkin actually presents his argument in the context of a discussion of the vagueness of certain concepts used in the law (of gaps of recognition). I will say more about his point of view in the next section. I do think, however, that his argument also applies to so-called *normative gaps*.

equivalent to 'Legally, it is false that x ought to do ϕ' (which implies the truth of 'Legally, it is permitted for x not to do ϕ').[29]

Two brief comments on this argument of Dworkin:

(i) Dworkin presents the argument against *V*, a fictitious author who defends the trivalence of legal propositions. *V* could reject Dworkin's argument by grounding his position on the truth-logic *TL* and holding that the restauration of bivalence is what his *constructivism* tries to avoid. He could add that if the principle of legislation proposed by Dworkin is understood to say that if a proposition is not true, then it is false, it leads to a contradiction in *TL*. In the notation of *TL*, the principle of legislation *(PL)* can be represented as follows:

$$(PL)\ \neg Tp \rightarrow T\neg p.$$

Now, *V* assumes that there are legal propositions without a truth-value, that is,

$$(1)\ \neg Tp \wedge \neg T\neg p$$

(1) and *(PL)* are contradictory, as can easily be seen:

(2) $\neg Tp$	(elimination, (1))
(3) $T\neg p$	(modus ponens, (2) and *PL*)
(4) $\neg T\neg p$	(elimination, (1))
(5) $T\neg p \wedge \neg T\neg p$	((3) and (4))

Hence, we must reject either (1) or *PL*. *V* would conclude that *PL*, as proposed by Dworkin, does not restore bivalence in a trivalent system, but rather *presupposes* it, which means that it is inconsistent with a trivalent system.

(ii) Perhaps *PL* should not be interpreted in this way. Dworkin could be suggesting that *PL* is a norm for judges which prohibits them to convict people who have performed actions that are not explicitly prohibited (cf. Blackburn 1984, 206-210).[30] Now, Dworkin's argument resembles Kelsen's Principle of Prohibition in its strong version. As I have argued before, whether or not that principle (sometimes called a *closure rule*) belongs to a legal system is a contingent matter. What's more, in many spheres of the law, there is no such principle; thus, in civil law, that there is no norm explicitly prohibiting some conduct does not mean that suits filed against persons for performing that conduct always ought to be dismissed. The application of norms by analogy is grounded precisely on the opposite assumption.

[29] For a more detailed discussion of Dworkin's argument, cf. Endicott 1997.

[30] From this, Blackburn (1984, 209 f.) infers that it is wrong to identify a bivalent logical practice (like that of the judge in Dworkin's case) with the acceptance of realism.