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Ministry of Education &
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Central Administration of
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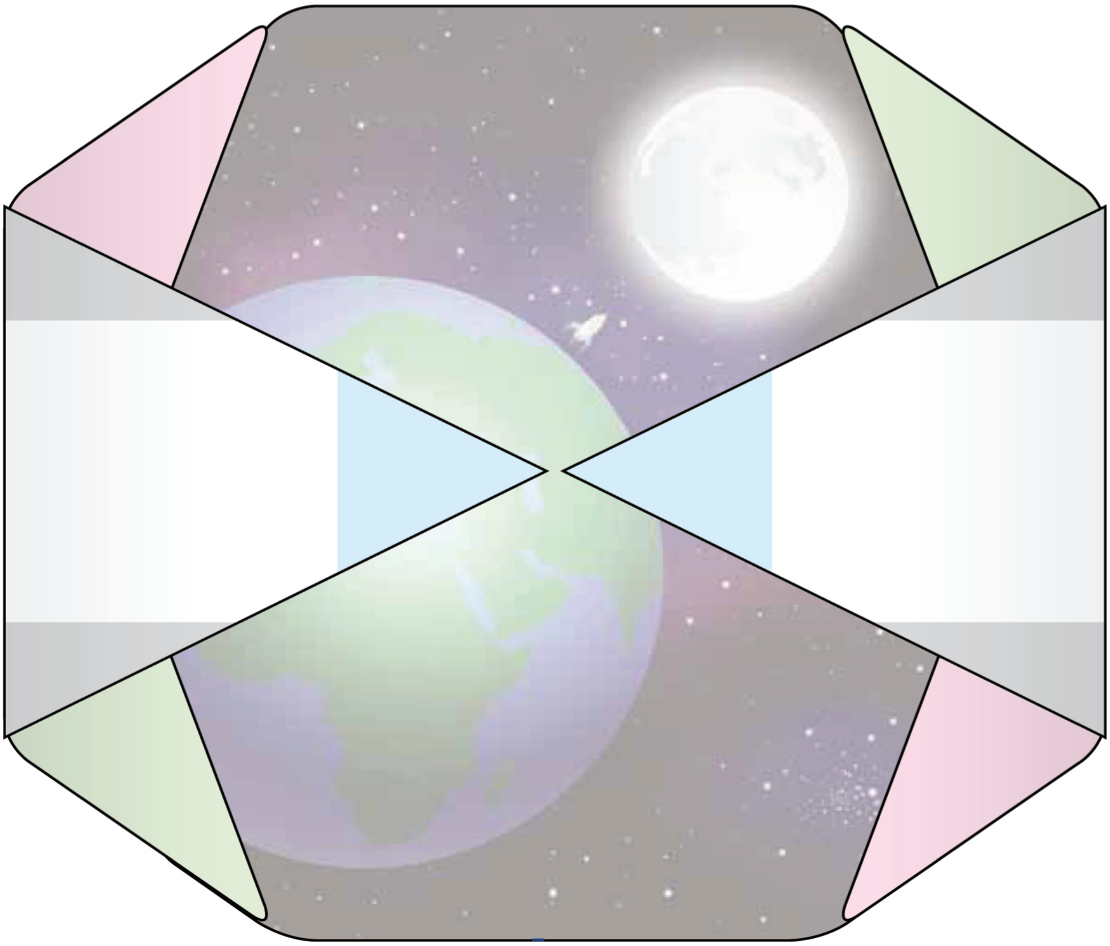
MATHEMATICS

Student's Book

For Preparatory Year three — *First Term*



2019-2020

غير مصرح بتداول هذا الكتاب خارج وزارة التربية والتعليم والتعليم الفنى

بسم اللّٰه الرحمن الرحيم

Authors

Mr. **Omar Fouad Gaballa**

Prof.Dr. **Afaf Abo-ElFoutoh Saleh**

Dr. **Essam Wasfy Rouphaiel**

Mr. **Serafiem Elias Skander**

Mr. **Kamal yones kabsha**

Revised by

Samir Mohamed Sedawy

Issued: 2010
D.N: 8925/2010
ISBN: 978-977-705-000-5

Introduction

Dear students:

It is extremely great pleasure to introduce the mathematics book for third preparatory. We have been specially cautious to make learning mathematics enjoyable and useful since it has many practical applications in real life as well as in other subjects. This gives you a chance to be aware of the importance of learning mathematics, to determine its value and to appreciate mathematicians roles.

This book sheds new lights on the activities as a basic objective. Additionally, we have tried to introduce the subject simply and excitingly to help attaining mathematical knowledge as well as gaining patterns of positive thinking which pave your way to creativity.

This book has been divided into units, each unit contains lessons. Colors and pictures are effectively used to illustrate some mathematical concepts and the properties of figures. Lingual level of previous study has been taken into consideration.

Our great interest here is to help you get the information independently in order to improve your self-study skills.

Calculators and computer sets are used when needed. Exercises, practices, general exams, portfolios, unit test, general tests, and final term tests attached with model answers have been involved to help you review the curriculum completely.

Eventually, we hope getting the right track for the benefits of our students as well as for our dearest Egypt hoping bright future to our dearest students.

Authors

Contents

Trigonometry

Unit (4): Trigonometry

Coordinate geometry

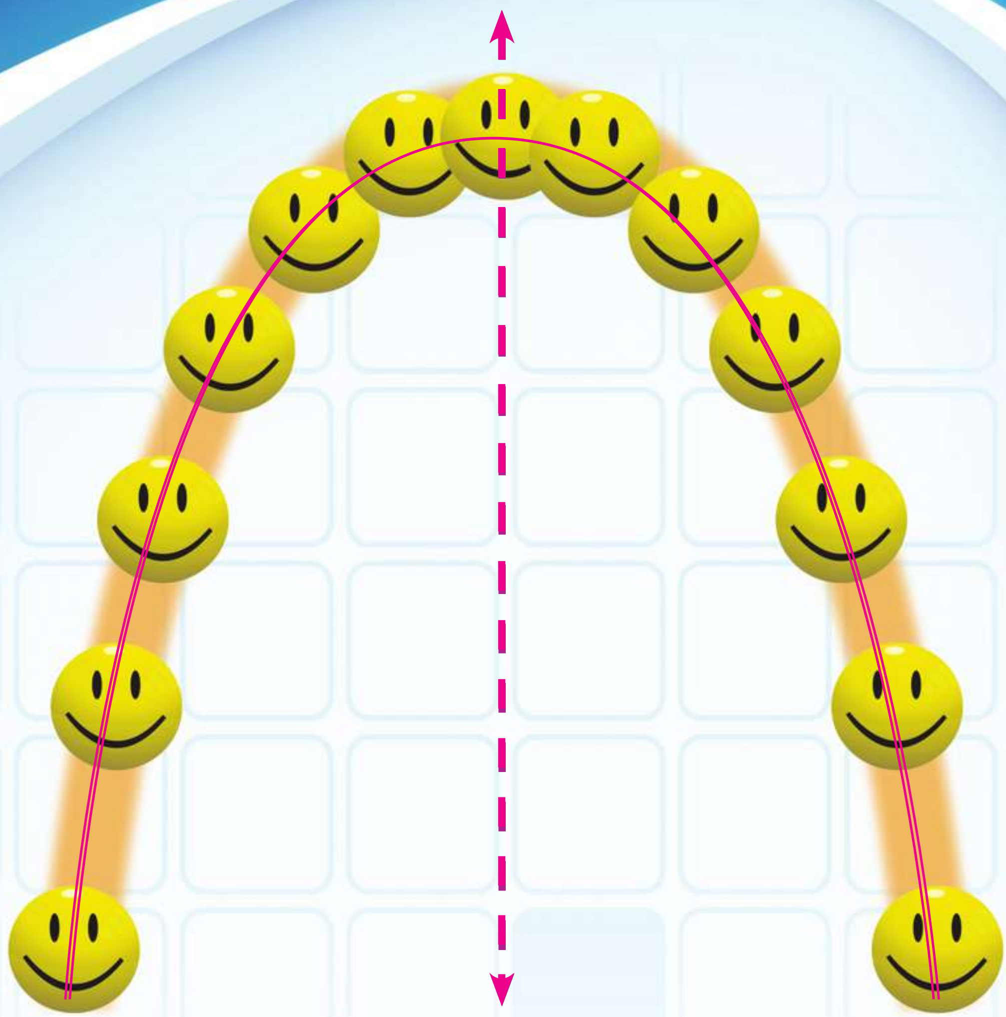
Unit 5: Coordinate geometry

MATHEMATICAL NOTATION

N	The set of natural numbers	$\perp$	Perpendicular to
Z	The set of integers	//	Parallel to
Q	The set of rational numbers	$\overline{AB}$	Straight segment AB
Q`	The set of irrational numbers	$\overrightarrow{AB}$	Ray AB
R	The set of real number	$\overleftrightarrow{AB}$	Straight line AB
$\sqrt{A}$	The Square root of A	m ($\angle$ A)	Measure of angle A
$\sqrt[3]{A}$	The Cube root of A	m ($\overset{\frown}{AB}$)	Measure of arc AB
[a, b]	Closed interval	$\sim$	Similarity
]a, b[	Open interval	$>$	Grater than
[a, b[	Half-open interval	$\geqslant$	Grater than or equal to
]a,b]	Half-open interval	$<$	Less than
[a, ∞ [	Infinite interval	$\leqslant$	Less than or equal to
$\equiv$	Is congruent to	p(e)	Probability of occurring event
n (A)	Number of elements of A	$\overline{x}$	Mean
s	Sample space	σ	Standard deviation
		$\sum$	Sum

Unit 1

Relations and Functions



One of the players threw the ball so, it took the direction shown in the figure.

This figure represents one of the functions which you will study and is called " a quadratic function"

1 - 1

Cartesian product

What you'll learn

- ☆ Cartesian product of two non-empty sets.

Key terms

- ☆ Ordered pair.
- ☆ A cartesian product.
- ☆ An arrow diagram.
- ☆ A cartesian diagram.
- ☆ Relation.

Think and Discuss

You have previously studied relation between two variables x, y

1. Find a set of the ordered pairs which satisfy the relation: $y = 2x - 1$ when $x = 0$ and $x = 1$, $x = 2$
2. Represent these ordered pairs graphically in the coordinate plane.
3. Does the ordered pair (3, 5) equal the ordered pair (5, 3)? **(Use the graph)**.

From the previous, we notice:

1. In each ordered pair (a , b), a is called the first projection, and b is called the second projection.
2. Each pair is represented by one and only one point in the coordinate plane.
3. *If* $a \neq b$ *then* $(a, b) \neq (b, a)$, Why?
4. $(a, b) \neq \{a, b\}$.
5. *If* $(a, b) = (x, y)$ *then* $a = x$, $b = y$

Example 1

Find x, y if: $(x - 2, 3) = (5, y + 1)$

Solution

$x - 2 = 5 \quad \therefore x = 7 \quad , \quad 3 = y + 1 \quad \therefore y = 2$

Drill

Find **a and b in each of the following:**

A. $(a, b) = (-5, 9)$

B. $(a - 2, b + 1) = (2, -3)$

C. $(6, b - 3) = (2 - a, - 1)$

D. $(a - 7, 26) = (-2, b^3 - 1)$

Example 2

If X = {a , b}, Y = { -1 , 0 , 3) then find: X × Y , Y × X, **What do you notice?**

Solution

To find the cartesian product of the set X and Y which is denoted by the symbol X × Y. write the set of all the ordered pairs in which its, first projection is an element of X, and its second projection is an element belongs to Y , and it is written as:

X × Y = {a , b} × { -1 , 0 , 3} = { (a , -1) , (a , 0), (a , 3) , (b , -1), (b , 0), (b, 3)}

Y × X = { -1, 0, 3} × {a, b } = { (-1, a), (-1, b), (0, a), (0, b), (3, 0), (3, b)}

So: $X \times Y \neq Y \times X$

We can get X × Y and Y × X from the following tables

×		Second projection		
		-1	0	3
First Projection	a	(a, -1)	(a, 0)	(a, 3)
	b	(b, -1)	(b, 0)	(b, 3)

×		Second Projection	
		a	b
First projection	-1	(-1, a)	(-1, b)
	0	(0, a)	(0, b)
	3	(3, a)	(3,b)

Think:

1. When X × Y = Y × X?
2. Are the number of elements of X × Y = the number of elements of Y × X?

We notice that :

1. *If* X and Y are two finite and non empty sets then :

   $X \times Y = \{(a , b) : a \in X , b \in Y\}$

2. $X \times Y \neq Y \times X$ *where* : $X \neq Y$

   $n (X \times Y) = n (Y \times X) = n (X) \times n (Y)$

   where n denotes the number of set elements .

3. *If* $(k, m) \in X \times Y$ *then* $k \in X$, $m \in Y$

4. *If* X is a non-empty set,

   then: $X \times X = \{(a , b) : a , b \in X\}$

   and written as X^2 and it is read as (X two).

Example 3

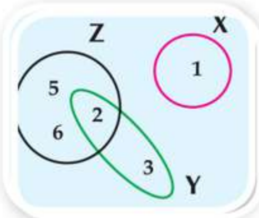
If X = {1}, Y = {2, 3}, Z = {2, 5, 6} represent the sets of X , Y , Z with venn diagram then find:

First : A $X \times Y$ B $Y \times Z$ C $X \times Z$ D Y^2

Second: $(X \times Y) \cup (Y \times Z)$ **Third:** $X \times (Y \cap Z)$

Fourth: $(X \times Y) \cap (X \times Z)$ **Fifth:** $(Z - Y) \times (X \cup Y)$

Solution



First :

A $X \times Y = \{1\} \times \{2, 3\} = \{(1, 2), (1, 3)\}$

B $Y \times Z = \{2, 3\} \times \{2, 5, 6\}$

$= \{(2, 2), (2, 5), (2, 6), (3, 2), (3, 5), (3, 6)\}$.

C $X \times Z = \{1\} \times \{2, 5, 6\} = \{(1, 2), (1, 5), (1, 6)\}$

D $Y^2 = Y \times Y = \{2, 3\} \times \{2, 3\}$

$= \{(2, 2), (2, 3), (3, 2), (3, 3)\}$

Second : $(X \times Y) \cup (Y \times Z) = \{(1, 2), (1, 3), (2, 2), (2, 5), (2, 6), (3, 2), (3, 5), (3, 6)\}$

Third : $X \times (Y \cap Z) = \{1\} \times \{2\} = \{(1, 2)\}$

Fourth : $(X \times Y) \cap (X \times Z) = \{(1, 2), (1, 3)\} \cap \{(1, 2), (1, 5), (1, 6)\} = \{(1, 2)\}$.

Fifth : $Z - Y = \{5, 6\}$ $\therefore (Z - Y) \times (X \cup Y) =$ ....   Complete

Drill

If X = {2 , - 1}, Y = {4 , 0} , Z = {4 , 5 , -2} Find

A $X \times Y$ B $Y \times Z$ C X^2

D $n(X \times Z)$ E $n(Y^2)$ F $n(Z^2)$

The representation of the cartesian product:

Example 4

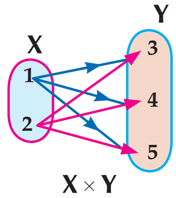
1 If X = {1, 2}, Y {3, 4, 5} ***Find:*** $X \times Y$, and represent it:

First: by the arrow diagram.   **Second: by the cartesian diagram.**

Solution

$X \times Y = \{1, 2\} \times \{3, 4, 5\} = \{ (1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5)\}$

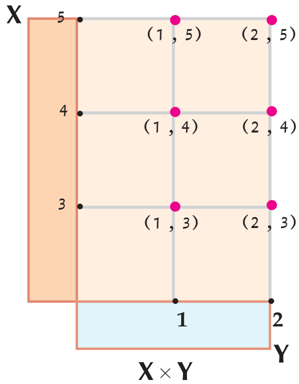
Where the cartesian product of $X \times Y$ is represented by an arrow diagram, or a grahpical net, as follows:



First: An arrow diagram

Draw an arrow from each element that represents the first projection (The elements of set of X) to each element that represents the second projection (The elements of set of Y)

i . e: The arrow diagram of the cartesian product represents each ordered pair by an arrow that starts from its first projection and ends at the second projection.



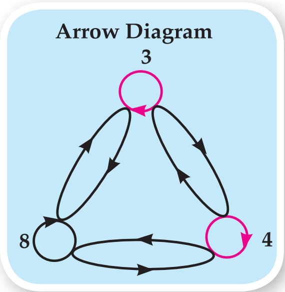
Second: Cartesian diagram (the perpendicular graphical net.

On a perpendicular graph net, the elements of set X is represented horizontally and the elements of set Y vertically. The intersection points of the horizontal and vertical lines represent the ordered pairs of the elements of the cartesian product X x Y.

Example 5

If $X = \{3, 4 , 8\}$ then find, $X \times X$ and represent it with an arrow diagram.

Solution



$X \times X = \{3, 4, 8\} \times \{3, 4, 8\}$

$=\{ (3, 3), (3, 4), (3, 8), (4, 3), (4, 4), (4, 8), (8, 3), (8, 4), (8, 8)\}$.

Notice in the figure: the ordered pairs are represented by arrows, and the ordered pairs in which the first projection is equal to the second projection as: (3 , 3) , (4, 4), (8, 8) are represented by a buttonhole to show that the arrow comes from a point and ends in the same point.

Notice that: $n(X) = 3$, then $n(X \times X) = 3 \times 3 = 9$

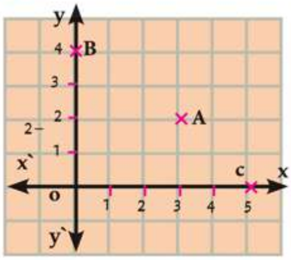
In this case, the cartesian product $X \times X$ can be represented graphically by 9 points where each point represents an ordered pair. But if X is an infinite set، then the number of elements of $X \times X$ is infinite.

Think: How can you represent the cartesian product of each of the following?

$N \times N$, $Z \times Z$, $Q \times Q$ and $R \times R$.

The cartesiem product of the infinite sets and its graphical representation:

First: **To represent the cartesian product of N× N = {(x, y) : x ∈ N, y ∈ N}**



1. Draw two perpendicular straight lines, one of them is $\overleftrightarrow{xx`}$ horizontialy and the other $\overleftrightarrow{yy`}$ vertically and are intersected at point o.
2. Represent the natural numbers N on each of the horizontal and vertical straight lines starting with the origin point 0 which represents the number zero.
3. Draw vertical straight lines and horizontal straight lines from the points which represent the natural numbers, you will get the opposite figure, and thus, the points of intersection of the set of these straight lines are represented by the perpendicular graphical net of the cartesian product of N × N.

Notice that: Each point of this net represents one the ordered pairs in the cartesian product of N × N.

For Example: point A represents the ordered pair (3 , 2) and point B represents (0, 4).

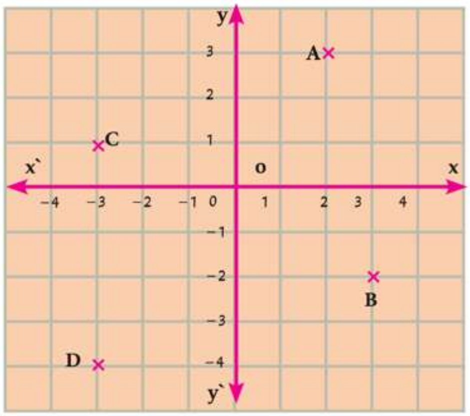
Complete: point C represents the ordered pair (...... , ...), point O represents the ordered pair (...... , ...).

Second: **To represent the cartesien product of Z × Z = {(x, y): x ∈ Z, y ∈ Z}.**

We represent the set of integers on each of the two horizontal and vertical straight lines where the point (O) represents the ordered pair (0, 0).

Thus, each point of the net points represents one of the pairs in the cartesian product Z × Z

This net is known as the coordinat plane of Z × Z.



Drill

Identify the ordered pairs which represented by the points A , B , C and D in the previous graphical net.

Third: **To represent the cartesian product Q × Q = {(x, y) : x ∈ Q, y ∈ Q}**

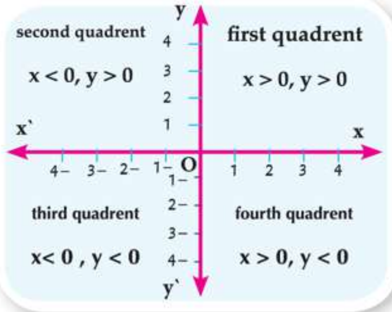
Draw a perpendicular graphical net and represent the set of rational numbers Q on the two horizontal and vertical straight lines, then identify the points: A $(3, \frac{5}{2})$, B $(-\frac{3}{2}, 4)$, C $(-3, -\frac{3}{2})$ and D $(\frac{5}{2}, -\frac{3}{2})$

Fourth: Representing the cartesian product RXR = {(x, y) : x ∈ R, y ∈ R}

the set of real numbers can be represented on each of the two horizontal and vertical straight lines, and point O represents the ordered pair (o , o).

The horizontal straight line $\overleftrightarrow{xx'}$ is called the x - axis, and the vertical straight line $\overleftrightarrow{yy'}$ is called the y - axis.

Then, the net is divided into four parts (quadrants) as in the opposite figure:



Example 6

Draw a perpendicular square net of the cartesian product R × R, then tell the quadrant or the axis where each of the following points is located:

A (3, 3), B (3, -2), C (-4, -2), D (-4, 3), E (0, - 3),F (2, 0)

Solution

A (3, 3) is located in the first quadrant.

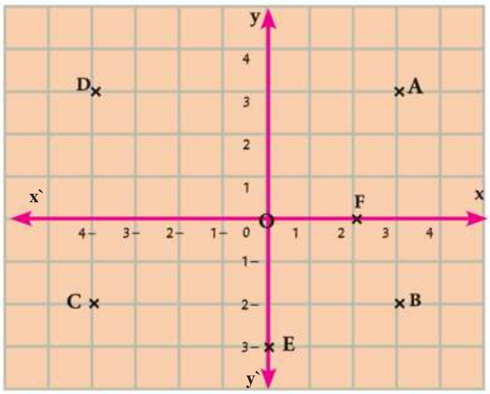
B (3, -2) is located in the fourth quadrant.

C (- 4, - 2) is the located in the third qudrant

D (- 4, 3) is the located in the second qudrant

E (0, - 3) is the locate on y - axis

F (2, 0) is the located on x - axis.



Drill

If X = [- 2 , 3] find the location which represents X × X.

Show which of the following points belongs to the cartesian product of X × X

A (1, 2), B (3, - 1), C (- 1, 4) and D (-2, 0)

Exercises 1-1

First : Complete the following:

1. If (a + 5, 3) = (8, b - 1) then a = ..., b = ...
2. If $(x^5, y+1) = (32, \sqrt[3]{27})$ then x = ..., y = ...

3. $(x - 1, 11) = (8, y + 3)$ then $\sqrt{x + 2y}$ = ...

4. If $n(X^2) = 9$, , then $n(X)$ = ...

5. If $X \times Y = \{(2, 6), (2, 9), (3, 6), (3, 9), (5, 6), (5, 9)\}$, then

   X = ... , Y = ...

Second: Choose the correct answer from the given answers::

1. If $n(X) = 3$, $n(X \times Y) = 12$ then $n(Y)$ equals: ....

   A 4 B 9 C 15 D 36

2. If $(3, 5) \in \{3, 6\} \times \{x, 8\}$ then x = ...

   A 8 B 6 C 5 D 3

3. If the point $(5, b - 7)$ is located on the X - axis then b =

   A 2 B 5 C 7 D 12

4. If the point $(x- 4, 2 - x)$ where $x \in Z$ is located in the third quadrant, then x equals:

   A 2 B 3 C 4 D 6

Third:

1. If $X = \{2, 3\}$, $Y \{3, 4, 5\}$ then find:

   A $X \times Y$ and represent it by an arrow diagram and and a cartesian diagram

   B $n(X \times Y)$ C $n(Y^2)$ D $(X \times Y) \cap Y^2$

2. If $X \times Y = \{(1, 1), (1, 3), (1, 5)\}$ then find:

   A X, Y B $Y \times X$ C Y^2

3. If: $X = \{3, 4\}$, $Y = \{4, 5\}$, $Z = \{6, 5\}$ then find:

   A $X \times (Y \cap Z)$ B $(X - Y) \times Z$ C $(X - Y) \times (Y- Z)$

4. Identify the following points on a perpendicular graphical net of the cartesian product $R \times R$:

   A (4, 5), B (6, -3), C (-2, 7), D (-1, 6), E (-4, -5), M (0, 6), K (9, 0)

   Then mention the quadrant that each point is located on the perpendicular graphichal net.

   Or the axis it belongs to.

1 - 2

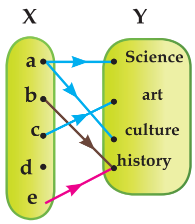
Relations

Think and Discuss

In the festival "Reading for All", five students represent the set of X = {a , b, c , d , e} went to the school library to read some books which are represented by the set Y = [science, literature, culture and history] student A read a book in science and a book in culture, student b read a book in histroy, student c read a literary book, pupil e read a book of the historical books, but student d didn't read any of these books.

1. Write the previous statements in the form of ordered pairs from X to Y.
2. Represent a set of the ordered pairs in the form of an arrow diagram.

Notice that: The expression " read" connects some of the elements of the set X with the elements of set Y, and it determines a relation between X and Y which is denoted by the symbol R. This relation can be represented by an arrow diagram - as shown in the opposite figure , where we draw an arrow beginning from the student and ending at the type of books he reads.



We can also express the relation from X to Y by the net of the following ordered pairs:

{(a, Science), (a, Culture), (b, Histrory), (c, Literature), (e, History)}.

This set of ordered pairs are called the relation R.

Think:Is R a subset from the cartesian product $X \times Z$?

Example 1

If X = {-1, 1, 2}, Y = {2, 4, 6, 8}, and R is a relation from X to Y where a R b means: «b = 2 a + 4», for each $a \in X$, $b \in Y$

Write and represent R once in an arrow diagram and another by a cartesian diagram.

What you'll learn

- A relation of set of X to the set of Y.
- A relation from a set on it self.

Key terms

- Relation.

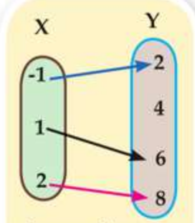
Solution

When: A = -1 $\therefore$ B = 2 × (-1) + 4 = 2

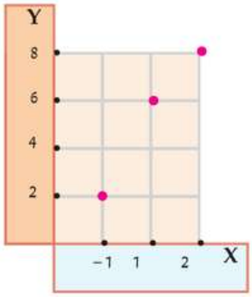
When: A = 1 $\therefore$ B = 2 × 1 + 4 = 6

When: A = 2 $\therefore$ B = 2 × 2 + 4 = 8

$\therefore$ R = {(-1, 2), (1, 6), (2, 8)}



Arrow diagram



Graph digram

From the previous, we deduce that

1. The relation from X to Y where X, Y are two non-empty sets is a relation, connecting some or all the elements of X with some or all the elements of Y.
2. X × Y is the set of ordered pairs where the first projection in each ordered pair belongs to X and the second projection belongs to Y.
3. If R is a relation from X to Y , then R ⊂ X × Y .

The relation from a set to itself

If R is a relation from a set X to X (itself) then R is called a relation on X and R ⊂ X × X.

Example 2

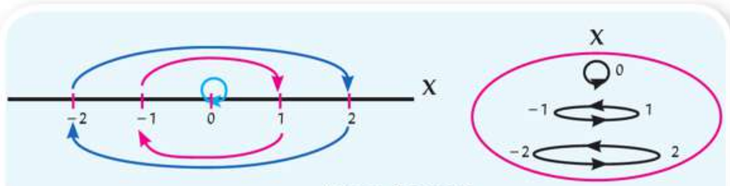
If X = {-2, -1, 0, 1, 2} and R is a given relation on X where a R b means:

«The number a is the additive inverse of the number b» for each of a , b ∈ X

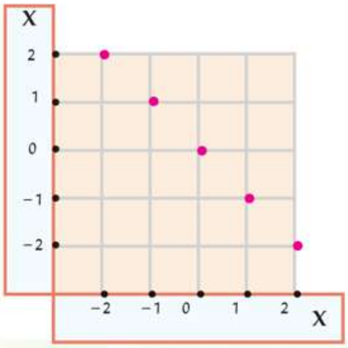
Write the relation R and represent it by an arrow diagram and also by, cartesian diagram.

Solution

R = { (-2, 2), (-1, 1), (0, 0), (1, -1), (2, -2)}



Arrow diagram



Cartesian Diagram

Drill

If X = {1, 2, 3}, Y = {12, 21, 47, 52}, and R is the relation from X to Y where a R b means :
(a is a digit from the digits of b), for each $a \in X$, $b \in Y$

First: Write R and represent it by an arrow diagram and also, by a cartasian diagram.

Second: Show which of the following relations are correct and why?

1 R 52 2 R 21 3 R 47

Exercises 1-2

1. If X = {1, 2, 4, 6, 10}, and R is a relation on X, where a R b means **(a is a multiple of b)**, for each of a , $b \in X$. Write R and represent it by an arrow diagram and also, by a cartesian diagram.

2. If X = {2, 4, 5, 7}, Y = {4, 5, 6, 7, 9} and R is a relation from X to Y where a R b means ($a \leqslant b$) for each of $a \in X$ and $b \in Y$. Write R and represent it by an arrow diagram and also, by a cartasian diaram.

3. If X = {1, 2, 3}, Y = {1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$} and R is a relation from X to Y. where a R b means **«The number a is the multiplicative inverse of the number b»** for each of $a \in X$, $b \in Y$, Write R and represent it by an arrow diagrm and also, by a cartasian diagram.

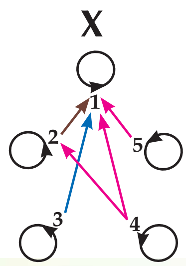
4. If X = {1, 3, 4, 5}, Y = {1, 2, 3, 4, 5, 6} and R is relation from X to Y where a R b means **«a + b = 7»** for each of $a \in X$, $b \in Y$. Write R and represent it by an arrow diagram and also by a cartesian diagram.

5. If X = {-1, 0, 1, 2, 3}, Y = {0, 1, 4, 6, 9} and R is a relation from X to Y where a R b means **«$a^2 = b$»** for each of $a \in X$, $b \in Y$. Write R and represent it by an arrow diagram and also by a cartesian diagram.

6. If X = {- 2, -1, 1, 2}, Y = {$\frac{1}{8}$, $\frac{1}{3}$, 1, 3, 8} and R is the relation from X to Y where a R b means «$a^3 = b$» for each of $a \in X$, $b \in Y$. Write R and represent it by an arrow diagram and also cartesian diagram.

7. If X = {2, 3, 4}, Y = {6, 8, 10, 11, 15} and R is a relation from X to Y. where a R b means **«a divides b»** for eah of $a \in X$, $b \in Y$ write the relation R .

8. **The opposite figure:**
Represents the arrow diagram of the given relation R on the set X = {1, 2 , 3, 4 , 5}. Write the relation R and represent it by a cartesian diagram.



1 - 3 Functions (Mapping)

What you'll learn

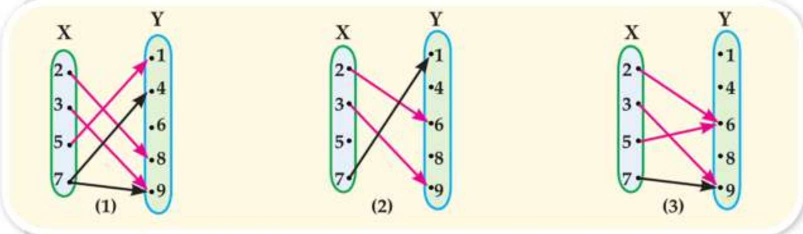
- ☆ Concept of the function.
- ☆ Symbolical expression of the function.

Key terms

- ☆ Functions.
- ☆ Domain
- ☆ Codomain
- ☆ Range

Think and Discuss

The following figures represent three relations from X to Y.



1. Write each relation and represent it by a cartesian diagram.
2. Which of these relations satifies the following condition: each element of X is connected to only one element of Y.

Definition:

A relation from X to Y is said to be a function if:

Each of the elements of X appears only once as a first projection in one of the ordered pairs of the relation.

The Symbolic representation of the function:

1. The function is denoted by one of the following symbols: f or m or Q or... and the function f from the set X to the set Y.

   is written mathimatecally as:

   $f : X \rightarrow Y$ and is read as: «f is a fuction from X to Y».

Notes:

1. If f is a function from X to itself, we say that f is a function on X.
2. If the ordered pair (x, y) belongs to the function, then the element y is called the image of the element x by the function f, and we express it by one of the following two forms:

   $f : x \mapsto y$ is read as : the function: f maps x to y

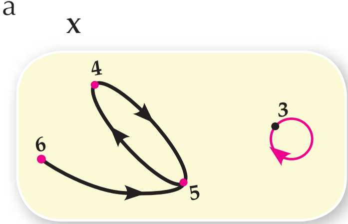
   Or $f(x) = y$ it is read as: f is a function where f (x) = y

Example 1

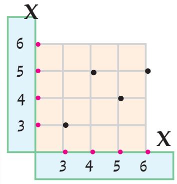
If f is a function on X where: X = {3, 4, 5, 6} and f (3) = 3, f (4) = 5, f (5) = 4, f (6) = 5.
Represent f by an arrow diagram and also, by a cartesian diagram.

Solution

f = { (3, 3), (4, 5), (5, 4), (6, 5)}



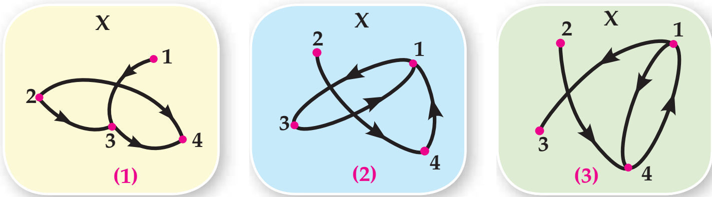
Arrow digaram



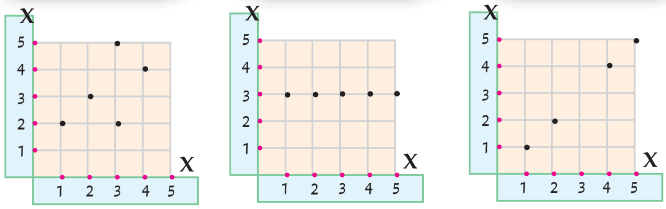
Cartesian diagram

Drill

1. If X = {1, 2, 3, 4} which of the following arrow diagrams represent a function on the set X?



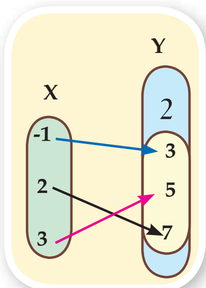
2. Which of the following cartesian diagrams represent a fuction from X to X.



Think: **Is every relation a function? Explain your answer and give examples.**

The Domain, the codomain and the range

If f is a function from X to Y.
i. e: f : X → Y , then
The set X is called the domain of the function f.
The set Y is called the codomain of the function f.
The set of images of the elements of the domain of X by the function f, is called the range of the function.



For example: If f : X → Y.

, X = { - 1, 2, 3}, Y = {2, 3, 5, 7}, f = { (-1, 3), (3, 5), (2, 7)} then:

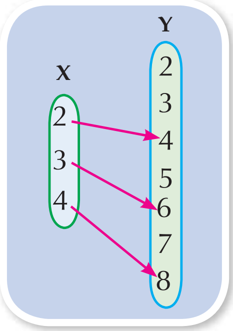
1. The domain of the function f is the set X = { - 1, 2, 3}
2. The codomain of the function f is the set Y = {2, 3, 5, 7}
3. The range of the function f is the set of the images of the elements of X by the function f and equal to { 3, 5, 7}.

Note that: The range is a subset of the codomain of the function.

Example 2

If X = {2, 3, 4}, Y = {y : y ∈ N, 2 ≤ y < 9} where N is the set of natural numbers, and R is a relation from X to Y where a R b means: « $a = \frac{1}{2}b$» for each of a ∈ X, b ∈ Y, write R and represent it by an arrow diagram show that R is a function from X to Y and find its range.

Solution



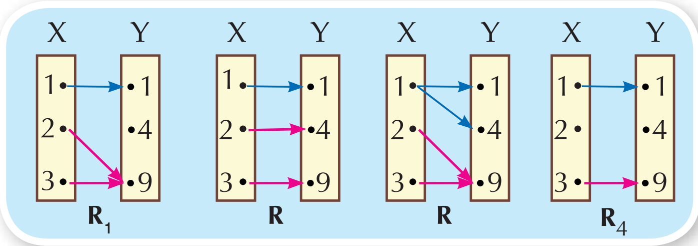
Y = {2, 3, 4, 5, 6, 7, 8} , R = {(2, 4), (3, 6), (4, 8)}

R is a function because every element of the X has only one arrow coming out to one element of Y.

The function range = {4, 6, 8}

Exercises 1-3

1. Which of the following relations represent a function from X to Y? If the relation represents a function, then find the function range?.



2. If X = {2, 5, 8}, Y = {10, 16, 24, 30} and R is a relation from X to Y where a R b means «a is a factor of b» for each a ∈ X, b ∈ Y. Write R and represent it by an arrow diagram and by cartesian diagram **Is R a function? and Why?**

3. If X = {0, 1, 4, 7} Y = {1, 3, 5, 6} and R is a relation from X to Y where a R b means «a + b < 8» for each a ∈ X, b ∈ Y, write R and represent it by an arrow diagram and also, by a cartesian diagram. Is R a function? and why?

4. If X ={1, 2 , 4 , 6 , 10} and R is a relation on X where a R b means: "a is twice b" for each of a , b ∈ X. Write R, and represent it by an arrow diagram and also, by a cartesian diagram. Is R a function? and why?

5. If X = {1 , 2 , 3 , 6, 11} and R is a relation on X where a R b means: "a + 2 b = an odd number "for each of a, b ∈ X, write R and represent it by an arrow diagram. Is R a function? and why?

Polynomial functions

1 - 4

Think and Discuss

In the functions f: R →R , $f_1(x) = 5$

f: R →R , $f_2(x) = 3x - 8$

f: R →R , $f_3(x) = 4x^2 - 5x + 8$

We notice that:

1. The domain and the codomain of the function is the set of the real numbers R.
2. The rule of function (image of x) is a term or an algebraic expression.
3. What the power of the variable x in the previous functions?

Definition

The function f : R → R where:

$f(x) = a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n$ where $a_0, a_1, a_2, \dots a_n \in R$

$n \in N$, $a_n \neq 0$, is called a polynomial of degree n.

And thus: the degree of the polynomial is the highest power of the variable in the function rule.

Drill

1. Which of the following functions represents polynomial:

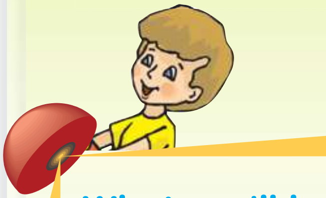
   A $f_1(x) = x^3 + x^2 + 3$ B $F_2(x) = x^3 + \frac{1}{x} + 7$

   C $f_3(x) = x^2 + \sqrt{x} + 8$ D $F_4(x) = x(x + \frac{1}{x} - 2)$

2. If f : R → R then mention the degree of the function in the following:

   A $f(x) = 3 - 2x$ B $f(x) = x^2 - (x^2 - 3)$

   C $f(x) = x(x - 2x^2)$ D $f(x) = x^2(x - 3)^2$



What you'll learn

- ☆ The linear function and its graphical representation.

Key terms

- ☆ Polynomial function.
- ☆ Linear function.
- ☆ quadratic Function
- ☆ The graphical representation of function.

Example 1

If $f(x) = x^2 - x + 3$ then find: $f(-2)$, $f(0)$, $f(\sqrt{3})$

Solution

$\because f(x) = x^2 - x + 3 \quad \therefore f(-2) = (-2)^2 - (-2) + 3 = 4 + 2 + 3 = 9$

$f(0) = 3$, $f(\sqrt{3}) = (\sqrt{3})^2 - \sqrt{3} + 3 = 6 - \sqrt{3}$

Drill

If $f(x) = x^2 - 3x$, $g(x) = x - 3$

A Find $f(\sqrt{2}) + 3g(\sqrt{2})$

B Prove that $f(3) = g(3) = 0$

Linear function

Definition

The function $f: R \to R$ where $f(x) = ax + b$, $a, b \in R$, $a \neq 0$ this function is called a linear function or a function of the first degree.

The graphical representation of the linear fuction:

Example 2

Represent graphically the function $f: R \to R$, $f(x) = 2x - 3$

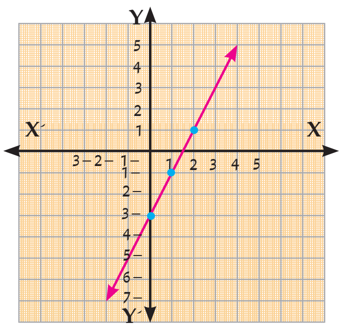
Solution

$\because f(x) = 2x - 3$

$\therefore f(0) = 0 - 3 = -3$, $f(1) = 2 - 3 = -1$, $f(2) = 4 - 3 = 1$

You can put these ordered pairs in a table as the following:

x	0	1	2
y = f(x)	-3	-1	1



The ordered pairs of the cartesian product of $R \times R$ is represented on the square net.

Remarks:

1. It is enough to find two ordered pairs belonging to the function, it is prefered to find third ordered pairs to check the graph.
2. If $f : R \rightarrow R$, $f(x) = a x$, where $a \neq 0$ then it represents graphically by a straight line passing through the origin (0, 0)

Drill

Reprsent graphically each of the following functions:

1. f: $f(x) = x + 2$
2. $g : g(x) = 3x$
3. $\ell : \ell(x) = -2x$

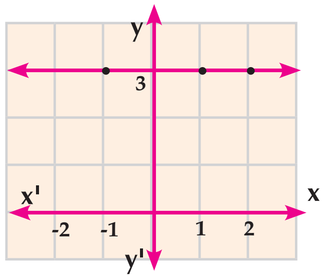
Special case: If $f : R \rightarrow R$, $f(x) = b$ where $b \in R$

then f is called a constant function.

For example: $f(x) = 3$

and it is written as $y = 3$

x	-1	1	2
y = f(x)	3	3	3



It is represented by a straight line paralled to the x-axis.

Drill

Represent the following functions graphically:

1. $f(x) = 5$
2. $f(x) = -4$
3. $f(x) = 0$
4. $f(x) = 2\frac{1}{2}$

The quadratic function

The function $f : R \rightarrow R$ where $f(x) = a x^2 + b x + c$, a, b, c are real numbers, $a \neq 0$ is called a quadratic function and it is a function of second degree.

The graphical representation of the quadratic function.

Example 3

Represent graphically the quadratic function f, where $f(x) = x^2$, $x \in R$ consider $x \in [-3, 3]$

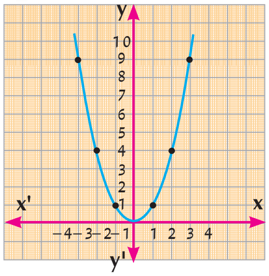
Solution

Identify some of the ordered pairs (x , f (x)) which belong to the function f where $x \in R$ and that the interval is [- 3 , 3] gives some possible values the variable x.

$f(-3) = 9$, $f(-2) = 4$, $f(-1) = 1$, $f(0) = 0$, $f(1) = 1$, $f(2) = 4$, $f(3) = 9$

Put these ordered pairs in a table as follows:

x	3	2	1	0	-1	-2	-3
y = f(x)	9	4	1	0	1	4	9



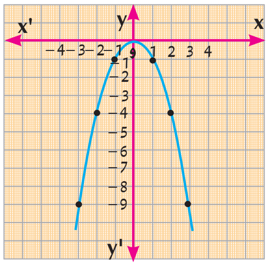
Identify in the cartesian plane the points which represent these ordered pairs, then draw a curve passing through these points.

Notice that:

1. The curve of the function f is symmetrical about the y-axis and the equation of the symmetrical axis is x = 0
2. The coordinate of the vertex of the curve is (0, 0), and the minimum value of the function = 0

Example 4

Represent graphically the quadratic function f where:
$f(x) = -x^2, x \in R$ where $x \in [-3, 3]$



Solution

Repeat the previous solution steps:

x	3	2	1	0	-1	-2	-3
y = f(x)	-9	-4	-1	0	-1	-4	-9

From the previous drawing, we notice:

1. The curve of the function f is symmetrical about the y-axis, thus, the equation of the symmetrical axis is x = 0
2. The coordinate of the vertex of the curve is (0, 0) and the maximum value of the function = 0

Exercises 1-4

First: Complete the following:

1. The linear function given by the rule y= 2 x - 1 is represented graphically by a straight line intersecting the y-axis at the point ....
2. The linear function given by the rule y = 3 x + 6 is represented graphically by a straight line intersecting the x - axis at the point ....
3. If the point (a, 3) is located on the straight line which represents the function f : R → R where f(x) = 4 x - 5 then a equals ....

Second: 1 If f: R → R , mention the degree of f then find f (-2) , f (0) , $f(\frac{1}{2})$ when:

A $f(x) = 3$ B $f(x) = 3 - 2x$ C $f(x) = x^2 - 4$

2. Represent grapically the following linear functions and find the points of intersection of the straight line by the two coordinate axes:

   A) $f(x) = 2x$ B) $f(x) = -\frac{1}{2}x$ C) $f(x) = 2x + 1$

   D) $f(x) = 2 - x$ E) $f(x) = 3x - 1$ F) $f(x) = -2x + 3$

3. Represent graphically each of the following functions and from the drawing deduce the coordinate of the vertex of the curve, and the equation of the symmetry axis and the minimum and the maximum value of the function.

   A) $f(x) = x^2 - 2$ where $x \in [-3, 3]$ B) $f(x) = (x - 2)^2$ where $x \in [-1, 5]$

   C) $f(x) = x^2 + 2x + 1$ where $x \in [-4, 2]$ D) $f(x) = 2 - x^2$ where $x \in [-3, 3]$

Connecting with technology

Using computer programs:

There are many free computer programs to draw the curves and solve the equations. It is available on the worldwide web, such as the free program (GeoGebra) its url is: http://www.geogebra.org, the program supports Arabic language.

By using the program, represent graphically each of the following functions:

1. $f(x) = 2x + 1$ 2. $f(x) = 5 - 3x$

3. $f(x) = x^2 - 3x + 2$ 4. $f(x) = 4 - 3x - x^2$

Activity

1. A Pavement company gets paid 100.000 pounds (fixed fee) then 30 pounds for each meter. If X (the length of the paved road in meters) and **Y** is (the total cost that the company recieves).

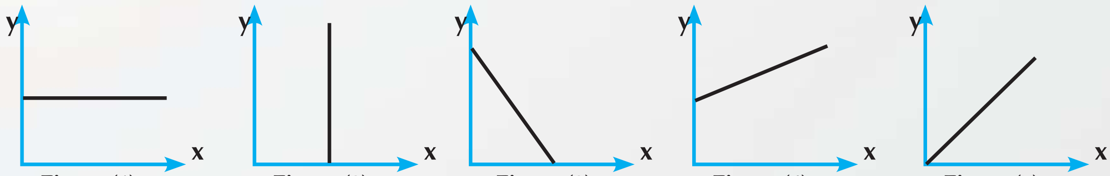


Figure (1) Figure (2) Figure (3) Figure (4) Figure (5)

First: The figure that represents the relation between x and y is the figure number ....

Second: Which of the following relations represents the previous infrmation:

A) $y = 30x$ B) $y = 30x + 100000$ C) $y = 100000x + 30$ D) $y = 3000000x$

Third: Write an essay about the great efforts of our country to improve and pave the roads to be faster and safer. Discuss what you should follow such obeying traffic laws and keeping the roads clean and safe.

Unit Test

1 If X = {0, 1, 4, 7}, Y = {1, 3, 5, 6}, R is a relation from X to Y where a R b means: «$a + b < 6$» for each of $a \in X$, $b \in Y$, write R and represent it by an arrow diagram and by cartesian diagram. Is R a function? Tell the reason.

2 Represent graphically each of the following functions:

A $f(x) = 3x - 1$

B $f(x) = -2x$

C $f(x) = x^2 - 3$ where $x \in [-3, 3]$

D $f(x) = 1 - 3x + x^2$ where $x \in [-1, 4]$

3 While Karim was reading a book, he found that after 3 hours 50 pages remained, and after 6 hours, 20 pages remained. If the relation between time (t) and the number of pages (b) is a linear relation.

A Represent graphically the relation between t and b then find the algebraic relation between the two variables.

B What is the time that should be taken to finish the book?

C What are the number of pages remaining when Karim began to read?

4 **The opposite figure :** Represents the curve of the function f. where:

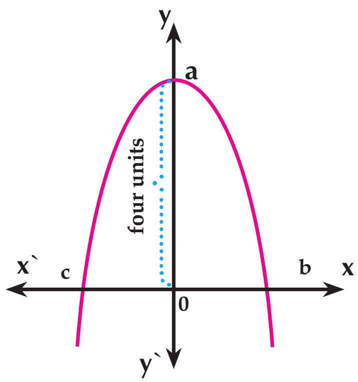
$f(x) = m - x^2$, If a o = 4 units

Find:

A The value of m.

B The coordinates of b and c

C The area of the triangle with vertices a , b and C.



Algebra

Unit 2: Ratio, proportion, Direct Variation and Inverse Variation

Do you Know ?

The weight of a human body on the surface of the moon equals $\frac{1}{6}$ of the weight on the surface of Earth.

Imagine you are going to a trip on the moon: What will your weight be?

2 - 1

Ratio

What you'll learn

- ☆ Ratio.
- ☆ Properties of ratio.

Key Terms

- ☆ Antecedent.
- ☆ Consequent.
- ☆ The two terms of the ratio.

Think and Discuss

We have learned in the previous phases the subject of ratio and that ratio is: a comparison between two quantaties.

for example: If there are 4 boys and 3 girls so the ratio between the number of boys to the number of girls can be written as 4 to 3 or $\frac{4}{3}$ Generally, if a and b are two real numbers

Then, the ratio between the two numbers a and b

Can be written as a to b or a:b or $\frac{a}{b}$.

a will be called an antecedant and b is consequent and a and b together are the two terms of ratio.

Complete and answer the questions:

1. Is the ratio changed if each of its two terms is multiplied in a fixed amount not equalling to zero?

$$\frac{3}{5} \overset{?}{=} \frac{3 \times ....}{5 \times ....}$$

2. Is the ratio changed if you add a real number to each of its two terms?

$$\frac{2}{3} \overset{?}{=} \frac{2 + ...}{3 + ...}$$

3. If $\frac{a}{b} = \frac{3}{5}$, Is a = 3, b = 5 for the values of a and b?

Example

Find the number which if added to the two terms of ratio 7 : 11 it will be 2 : 3

Solution

Consider the number is x.

$\therefore \frac{x+7}{x+11} = \frac{2}{3}$ $\therefore 3(x+7) = 2(x+11)$

$\therefore 3x + 21 = 2x + 22$ $\therefore 3x - 2x = 22 - 21$

$\therefore x = 1$

Drill

Find the positive number which if we add its square to each of the two terms of ratio 5 : 11 it becomes 3 : 5.

Exercises (2-1)

1. Two integer numbers, the ratio between them is 3 : 7 and if subtracted 5 from each term, the ratio between each of them becomes 1 : 3. Find the two numbers?

2. Two integer numbers, the ratio between them is 2:3, if you add to the first 7 and substract from the second 12, the ratio between them becomes 5 : 3, find the two numbers?

3. ***Find*** the number that if subtracted thrice of it from each of the two terms of ratio $\frac{49}{69}$ the ratio becomes $\frac{2}{3}$.

4. ***Find*** the number which if its square is added to each of the two terms of the ratio 7:11 it becomes 4:5.

2 - 2 Proportion

What you'll learn

- ☆ Proportion
- ☆ Properties of proportion
- ☆ Continued properties

Key Terms

- ☆ Proportion
- ☆ First proportional
- ☆ Second proportional
- ☆ Third proportional
- ☆ Fourth proportional
- ☆ Extremes
- ☆ Means

If $\frac{a}{b} = \frac{c}{d}$ then it's said that a, b, c and d are in proportion.

If a, b, c and d are in proportion, **then** $\frac{a}{b} = \frac{c}{d}$

> **Definition:**
> **The proportion is the equality of two ratios or more.**

In ratio $\frac{a}{b} = \frac{c}{d}$

So, *a* is called the **first proportional**, *b* is called the **second proportional**, *c* is called the **third proportional**, and *d* is called the **fourth proportional**.

a and d are called extremes, b and c are called means

The proporties of proportion

first: if $\frac{a}{b} = \frac{c}{d}$ *then:*

1. $a = m\,c$, $b = md$ **where** $m \in R^*$
2. $a\,d = b\,c$ **(product of the extremes equals product of the means)**
3. $\frac{a}{c} = \frac{b}{d}$

Check the previous properties by giving numerical examples of your own

Second: If: ad = bc *then :* $\frac{a}{b} = \frac{c}{d}$

$\frac{a}{c} = \frac{b}{d}$

Check the proporties in the following numeric example:

You know that: $4 \times 8 = 2 \times 16$

then: $\frac{4}{2} = \frac{.....}{.....}$, $\frac{4}{16} = \frac{.....}{.....}$

Example 1

If $\frac{x}{y} = \frac{2}{3}$ find the value of the ratio: $\frac{3x+2y}{6y-x}$

Solution

Consider x = 2m, y = 3m (where m constant ≠ zero)

$$\therefore \frac{3x+2y}{6y-x} = \frac{3\times 2m + 2\times 3m}{6\times 3m - 2m} = \frac{12m}{16m} = \frac{3}{4}$$

Another Solution

Divide the numerator and denominator on y, then substitute for the value of $\frac{x}{y}$

$$\therefore \text{The expression} = \frac{3\times\frac{x}{y}+2}{6-\frac{x}{y}} = \frac{3\times\frac{2}{3}+2}{6-\frac{2}{3}} \longrightarrow \textit{Complete} = \frac{\dots}{\frac{\dots}{\dots}} = \frac{\dots}{\dots}$$

Example 2

Find the fourth proportional for the numbers 4, 12, 16

Solution

Consider the fourth proportional to be x

$\frac{4}{12} = \frac{16}{x}$

$\therefore 4 \times x = 12 \times 16$ *[product of the extremes = product of the means]*

$\therefore x = \frac{12\times 16}{4} = 48$ $\therefore$ The fourth proportional = 48

Example 3

Find the number that if added to the numbers 3, 5, 8 and 12 it becomes proportional .

Solution

Consider the number is x i.e. 3 + x, 5 + x, 8 + x, 12 + x **are in proportional**

$\therefore \frac{3+x}{5+x} = \frac{8+x}{12+x}$ $\therefore (5+x)(8+x) = (3+x)(12+x)$

$\therefore 40 + 13x + x^2 = 36 + 15x + x^2$ $\therefore 15x - 13x = 40 - 36$

$\therefore 2x = 4$ $\therefore x = 2$

Drill

1. **A** Find the second proportional of the numbers 2, ... , 4, 6

   B Find the third proportional of the numbers 8, 6, ... , 12

2. If $\frac{a}{b} = \frac{3}{5}$ find the value of $7a + 9b : 4a + 2b$

Third: If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = ..., m_1, m_2, m_3, ... \in R^*$

then: $\frac{a m_1 + c m_2 + e m_3 + ...}{b m_1 + d m_2 + f m_3 + ...}$ = one of the ratios

For example: If: $\frac{a}{2} = \frac{b}{3} = \frac{c}{4}$ multiply the first two terms of the first ratio by 2, multiply the two terms of the second ratio by -5 and multiplying the two terms of the third ratio by 3, then

$\frac{2a - 5b + 3c}{2 \times 2 - 3 \times 5 + 3 \times 4}$ = one of these ratios

i.e.: $2a - 5b + 3c$ = one of these ratios

Example 4

If: a, b, c and d are proportional quantities, ***then prove that :*** $\frac{3a - 2c}{5a + 3c} = \frac{3b - 2d}{5b + 3d}$

Solution

$\because$ If a ,b , c and d are proprotional quantities $\therefore \frac{a}{b} = \frac{c}{d}$

Multiply the first two means by five and the second means by 3, then the sum of antecedents and the sum of consequents = one of these ratios .

$\therefore \frac{5a + 3c}{5b + 3d}$ = one of these ratios **(1)**

Multiply the two terms of ratio by 3 and the second by -2 then the sum of antecendents : the sum of consequents = one of these ratios .

$\therefore \frac{3a - 2c}{3b - 2d}$ = one of these ratios **(2)**

from (1), (2) $\therefore \frac{5a + 3c}{5b + 3d} = \frac{3a - 2c}{3b - 2d}$ $\therefore \frac{3a - 2c}{5a + 3c} = \frac{3b - 2d}{5b + 3d}$ **(Q.E.D)**

Another Solution

Consider $\frac{a}{b} = \frac{c}{d}$ = m where m is a constant expression

a = b m , c = d m and substitute in both sides.

Drill

If $\frac{a}{b} = \frac{c}{d}$ *prove that :*

First: $\frac{a+b}{b} = \frac{c+d}{d}$ **Second:** $= \frac{a-b}{b} = \frac{c-d}{d}$

Hint: Consider $\frac{a}{b} = \frac{c}{d}$ = m where m is a constant expression ≠ zero and complete or in any other way.

Continued proportional

2, 6, 18 are three numbers. Compare between the proportions $\frac{2}{6}, \frac{6}{18}$

1. Is there a relation between $(6)^2$ and the product of 2×18?
2. If you replace the number 6 with (-6) is there a relation between $(-6)^2$ and the product of 2×18?

Definition:

The quantities a, b and c are said to be in continued proportional if:
$\frac{a}{b} = \frac{b}{c}$ a is called the first proportional , b is called the middle proportional, and c is called the third proportional, where : $b^2 = ac$ or $b = \pm\sqrt{ac}$

Example 5

Find the middle proportional between 3, 27

Solution

The middle proportional $= \pm\sqrt{3\times27} = \pm 9$

Example 6

If b is a middle proportional between a and c, prove that : $\frac{a^2 + b^2}{b^2 + c^2} = \frac{a}{c}$

Solution

b is middle proportional between a and c *i.e.* a, b, c in continued proportional

Consider $\frac{a}{b} = \frac{b}{c} = m$ $\therefore$ b = c m

$a = b\,m = c\,m \times m = c\,m^2$

L.H.S $= \frac{a^2 + b^2}{b^2 + c^2} = \frac{c^2 m^4 + c^2 m^2}{c^2 m^2 + c^2}$

$= \frac{c^2 m^2 (m^2+1)}{c^2 (m^2+1)} = m^2$ **(1)**

R.H.S $= \frac{a}{c} = \frac{c\,m^2}{c} = m^2$ **(2)**

From **(1)**, **(2)** we get $\frac{a^2 + b^2}{b^2 + c^2} = \frac{a}{c}$

Another Solution

Consider : $\frac{a}{b} = \frac{b}{c} = m$ $\therefore \frac{a^2}{b^2} = \frac{b^2}{c^2} = m^2$

From the first ratio and the second ratio $m^2 = \frac{a^2 + b^2}{b^2 + c^2}$ = L.H.S

$m^2 = \frac{a}{b} \times \frac{b}{c} = \frac{a}{c}$ = R.H.S

From **(1)**, **(2)** $\therefore \frac{a^2 + b^2}{b^2 + c^2} = \frac{a}{c}$

Drill

If a , b , c and d are in continued proportional . Prove that ; $\frac{a - 2b}{b - 2c} = \frac{3b + 4c}{3c + 4d}$

Hint Let $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$

then $c = dm$, $b = dm^2$, $a = dm^3$ complete

Exercises (2-2)

1. If: $\frac{y}{x-z}=\frac{x}{y}=\frac{x+y}{z}$ **Prove that each ratio is equal to** 2 (unless: $x+y=0$) *then Find* $x:y:z$

2. If $\frac{a}{2}=\frac{b}{3}=\frac{c}{4}=\frac{2a-b+5c}{3x}$ *then find* the value of x.

3. **If $a:b:c=5:7:3$ and $a+b=27.6$** *then find* **the value of a, b and c**

4. **If x, y, z and ℓ are proportional quantities then prove that :**

   A $\left(\frac{x+y}{z+\ell}\right)^2=\frac{2x^2-3y^2}{2z^2-3\ell^2}$

   B $\sqrt[3]{\frac{5x^3-3z^3}{5y^3-3\ell^3}}=\frac{x+z}{y+\ell}$

5. If $\frac{x}{3}=\frac{y}{4}=\frac{z}{5}$ *then prove that:*

   A $\frac{2y-z}{3x-2y+z}=\frac{1}{2}$

   B $\sqrt{3x^2+3y^2+z^2}=2x+y$

6. **If** a, b, c and d are four real proportional quantities.

   then prove that:

   A $\frac{ac}{bd}=\left(\frac{a-c}{b-d}\right)^2$

   B $\sqrt[3]{\frac{5a^3-3c^3}{5b^3-3d^3}}=\frac{a+c}{b+d}$

7. **If** b is the middle proportional between a and c, then prove that:

   A $\frac{a+b+c}{a^{-1}+b^{-1}+c^{-1}}=b^2$

   B $\frac{2c^2-3b^2}{2b^2-3a^2}=\frac{c}{a}=\frac{c^2}{b^2}$

8. **If** a , b, c and are d in continued proportional, *then prove that*:

   A $\frac{ab-cd}{b^2-c^2}=\frac{a-c}{b}$

   B $\frac{a^2-3c^2}{b^2-3d^2}=\frac{b}{d}$

   C $\frac{a}{b+d}=\frac{c^3}{c^2d+d^3}$

   D $\frac{c^2-d^2}{a-c}=\frac{bd}{a}$

9. **If** : 5a, 6b, 7c and 8d are positive quantities in continued proportional .

   Prove that : $\sqrt[3]{\frac{5a}{8d}}=\sqrt{\frac{5a+6b}{7c+8d}}$

2 - 3 Direct Variation and Inverse Variation

What you'll learn

- Direct variation
- Inverse variation
- Difference between direct variation and inverse variation.

Key Terms

- Variation
- Direct variation
- Inverse variation

First: Direct variation

Think and Discuss (1)

A car moves at a uniform velocity (V) 15 m/sec. If the covered distance (d) in meter in a time (t) per second to give the relation: $d = v\ t$.

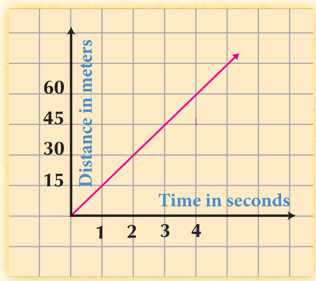
t	1	2	3	4
d	15	30	45	60

A Represent the relation between d and t graphically.

B Does the graphical representation pass through the origin point (0, 0)?

C Find $\frac{d}{t}$ in each case, what do you notice?

We notice from the above :



$\frac{d}{t}$ equals a constant expression which is 15

i.e.: $d = 15\ n$ and is said to be directly due to n and written symbolically $d \propto n$.

Definition:

y is said to be varies directly with x and is written as $y \propto x$ and written $y = m\ x$ (where m constant $\neq 0$). If the variable x takes the two values x_1, x_2 and the variable y takes the two variables y_1, y_2 respectively , then: $\frac{y_1}{y_2} = \frac{x_1}{x_2}$

From the previous, we conclude:

1. The previous relation is a linear relation between x and y and the two variables x and y, and is represented by a straight line passing through the origin point.
2. If $y \propto x$ then $y = m\,x$
   and if $y = m\,x$ then $y \propto x$.

Example 1

If $y \propto x$ then $y = 14$ when $x = 42$, ***then find***
first: the relation between x and y **second:** find the value of y when $x = 60$

Solution

First: $\because y \propto x$ $\therefore y = m\,x$ (where m constant $\neq 0$)

subtitute for the values of x and y in the relation

$\therefore 14 = 42 \times m$ $\therefore m = \frac{14}{42} = \frac{1}{3}$ $\therefore$ the relation is: $y = \frac{1}{3}\,x$

Second: when $x = 60$ $\therefore y = \frac{1}{3} \times 60 = 20$

notice : You can find the relation $\frac{y_1}{y_2} = \frac{x_1}{x_2}$ to find the value of y in the second requirement

Second: Inverse variation

If the area of the rectangle m and one of both dimensions x and the other dimension y, then:

A. ***Write*** the relation between m, x and y.

B. If the area of the rectangle is constant and equal to 30 cm^2 ***complete*** the following table:

x	3	5	6	10
y	………	………	………	………

C. ***Find*** x y in each case . What do you notice?

From the previous , we notice that:

$x\,y = 30$ *i.e.:* $y = \frac{30}{x}$ *i.e.* y inversely changes with Y and Written symbolically $y \propto \frac{1}{x}$

Similiarly: $x = \frac{30}{y}$ *i.e.:* x inverserly changes with Y and Written symbolically $x \propto \frac{1}{y}$

Definition:

y is said to be changed inversely with x and written $y \propto \frac{1}{x}$ if $x\,y = m$ (where m constant $\neq 0$)

and if the variable x takes the two values x_1, x_2 accordingly, the variable y takes the two values y_1, y_2 respectively: $\frac{y_1}{y_2} = \frac{x_2}{x_1}$

From the previous, we conclude that :

1. The previous relation is not a linear relation between the two variables x and y and is not represented by a straight line .

2. If y inversely changes with x ***then:*** $y = \frac{m}{x}$ (where m constant $\neq 0$)

   and if $y = \frac{m}{x}$ then $y \propto \frac{1}{x}$.

Example 2

If $y \propto \frac{1}{x}$ and y = 3 when x = 2

first: ***find*** the relation between x and y. **second:** ***find*** the value of y when x = 1.5.

Solution

$\because y \propto \frac{1}{x}$ $\therefore y = \frac{m}{x}$ (where m constant $\neq 0$)

substitute for the two values of x and y in the relation

$\therefore 3 = \frac{m}{2}$ $\therefore m = 2 \times 3 = 6$

$\therefore$ the relation is : $y = \frac{6}{x}$

when x = 1.5 $\therefore y = \frac{6}{1.5} = 4$

Note: you can find the value of y from the relation $\frac{y_1}{y_2} = \frac{x_2}{x_1}$

Drill

Show **which of the following tables represents the direct variation and which represents the inverse variation and which does not represent the direct variation or inverse variation while mentioning the reason in each case:**

x	y
3	20
5	12
4	15
6	10

x	y
2	9
4	18
12	54
16	72

x	y
5	9
10	18
15	27
25	45

x	y
3	6
-2	-9
-18	1
9	-2

Example 3

Connecting with Physics : If the relation between velocity (v) in (m/sec) and time t (sec) is $v = 9.8\,t$

First: *determine* the kind of variation between v and t .

Second: A *Find* the values of v when $t = 2$ seconds , $t = 4$ seconds

B *Find* the value of t when $v = 24.5$ m/sec

Solution

First: $\because$ $v = \text{constant} \times t$ *i.e.* $v \propto t$ *i.e.* v directly changes with **t**.

Second: A when $t = 2$ then $v = 9.8 \times 2 = 19.6$ m/s

when $t = 4$ then $v = 9.8 \times 4 = 39.2$ m/s

B When $V = 24.5$ then $24.5 = 9.8 \times t$ $\therefore t = \frac{24.5}{9.8} = 2.5$ seconds.

Example 4

Connecting with Geometry: If the height of a right constant cylinder (constant volume) is (h) varies inversely as the square of its radius length r. If the (h) is = 27 cm, when the radius = 10.5 cm, Find (h) when r = 15.75 cm.

Solution :

$\because v \propto \frac{1}{r^2}$ $\therefore v = m \times \frac{1}{r^2}$ (Where m constant $\neq 0$)

$v = 27$ when $r = 10.5$

$\therefore 27 = m \times \frac{1}{(10.5)^2}$ $\therefore m = 27 \times (10.5)^2$ **(1)**

Substitute $\therefore v = 27 \times (10.5)^2 \times \frac{1}{r^2}$ from **(1)**

when $r = 15.75$ cm $\therefore v = 27 \times (10.5)^2 \times \frac{1}{(15.75)^2} = 12$ cm

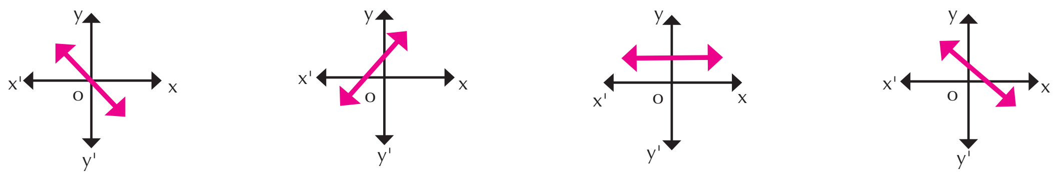
Use the calculator to find the last step as follows:

27 | × | 10,5 | x^2 | - | 10,75 | x^2 | =

Exercises (2-3)

First: ***choose*** **the correct answers from the given answers:**

1 **The graphical form represents the direct variation between x and y is :**



A B C D

2 **The relation represents the direct variation between the two variables x and y which is:**

A $x\,y = 5$ B $y = x + 3$ C $\frac{x}{3} = \frac{4}{y}$ D $\frac{x}{5} = \frac{y}{2}$

3 **If y varies inversely with x , and $x = \sqrt{3}$ when $y = \frac{2}{\sqrt{3}}$ then the constant proportional equals:**

A $\frac{1}{2}$ B $\frac{2}{3}$ C 2 D 6

Second: (Mental Math): From the data of the following table, answer the following questions:

x	2	4	6
y	6	3	2

A Show the kind of variation between y and x B Find the constant proportion

C Find the value of y when x = 3 D Find the value of x when $y = 2\frac{2}{5}$

General Exercises

1. **If the total cost of a trip is (y), some of it is constant (a), and the other is directly proportional with the number of participants (x) then choose the correct answer :**

   A $y = a\,x$ B $y = \frac{a}{x}$

   C $y = a + \frac{m}{x}$ (m constant $\neq 0$) D $y = a + m\,x$ (m constant $\neq 0$)

2. **If $y \propto x$ and $y = 40$ when $x = 14$,** ***then find*** **x when $y = 80$.**

3. **A car moves with a uniform velocity where the distance varies directly with time. If the car covers 150 km in 6 hours, find the distance covered by that car in 10 hours?**

4. **If the weight of a body on the moon (W) is directly proportional with its weight on the ground (R) if the body weights 84 kg on the ground and its weight on the moon is 14 kg. What will be its weight on the moon if its weight on the ground is 144 kg?**

5. **If y changes inversely with x and $y = 2$ when $x = 4$. Then find the value of y when $x = 16$**

6. **If $y \propto x$** ***prove that*** **$y^2 + x^2 \propto y^2 - x^2$**

   If a, b, c, and d are in a continued proportional, then prove that:

   A $\frac{a^2 - 3c^2}{b^2 - 3d^2} = \frac{b}{d}$ B $\frac{2a + 3d}{3a - 4d} = \frac{2a^3 + 3b^3}{3a^3 - 4b^3}$

7. **If** $\frac{x}{2a + b} = \frac{y}{2b - c} = \frac{z}{2c - a}$ ***then prove that*** $\frac{2x + y}{4a + 4b - c} = \frac{2x + 2y + Z}{3a + 6b}$.

8. ***Connecting with Geometry:*** x, y, z are three proportional sides in a triangle and $x + y = 15$ cm, $y + z = 22.5$ cm: ***find*** x : y.

9. ***Life application:*** Through the interest of the Egyptian authorities with the villages, a budget of 1.85×10^6 pounds was set for one of the villages to build a school, a medical unit and a youth center. If the costs of the school is $\frac{3}{2}$ of the cost of the medical unit and the cost of the medical unit is $\frac{5}{6}$ of the costs of the youth center, what is the cost of each of them?

10. ***Life application:*** If the needed hours to fulfill a work (t) is proportionaly inverse with the number of workers (x) who do the work, If 6 workers fullfiled the work in four hours, what is the time needed for 8 workers to fullfil this work?

Activity

1. *(Mental Math)* **From the data in the following table answer the following questions :**

x	3	8	6	12
y	8	3	4	2

A *Show* and tell the reason why the variation between x and y is an inverse variation.

B *Write* the constant of variation

C *Write* the relation between x and y

D *Find* the value of y when x = 48

E *Find* the value of x when y = 12

2. **If the rate of success in one of the governrates of the third preporatory is 83% and the rate of success for boys is 79% and the rate of success of girls is 89%. *Find* the rate of success between the number of boys to the number of girls in this governrate .**

Unit Test

1. **If** $\frac{a+b}{3} = \frac{b+c}{6} = \frac{c+a}{5}$ *then prove that:* $\frac{a+b+c}{a} = 7$.

2. **If** $y = a - 9$ **and** $y \propto \frac{1}{x^2}$ **and** $a = 18$ **when** $x = \frac{2}{3}$ *then find* **the relation between y and x ,** *then deduce* **the value of y when x = 1.**

3. **If** $\frac{21x - y}{7x - z} = \frac{y}{z}$ *then prove that* $y \propto z$.

4. **If** $x^4 y^2 - 14 x^2 y + 49 = 0$ *then prove that* $y \propto \frac{1}{x^2}$.

5. *Connecting with Astronomy:* **If a weight of a body on Earth (R) directly changes with its weight on the moon (W) , if** $R_1 = 182$ **kg,** $W_1 = 35$ **kg,** *then find* W_2 **and** $R_2 = 312$ **kg.**

6. *Connecting with Physics:* **If the speed of expression v of water to pass through a hose nuzzle inversely changes with the square of the hose nuzzle radius length r and v = 5 cm /s when r = 3 cm. Find z when r = 2.5 cm.**

Statistics

Unit 3: Statistics

Ice Cream stores produce different kinds of ice cream. The manager conducted a survey on the favorite ice cream the consumers prefer.

Statistics helps you select the sample representing the consumers.

3 - 1

Collecting Data

What you'll learn

- ★ Resources of collecting data
- ★ Methods of collecting data
- ★ How to select a sample
- ★ Types of samples

Key terms

- ☆ Primary resources
- ☆ Secondary resources
- ☆ Method of mass population
- ☆ Method of sample
- ☆ Biased choice
- ☆ Random choice sample
- ☆ Random sample
- ☆ Layer sample

Think and Discuss

The method of collecting data is considered one of the most important phases that statistical research mainly depends on. Collecting data in such scientific methods will lead to get accurate outcomes when doing operations of statistical inferrence and proper decision making.

1. What are the resources of collecting data?
2. How is the method of collecting data identified?

Resources of collecting data

1 *Primary resources (Field resources):*

These are the resources which we originally get data through interviewing or questionaires (survey). This type is distinguished by accuracy. However, it needs time and efforts beside it is highly expensive to conduct such a type.

2 *Secondary resources (historical resources):*

We can get our data from authorities and agancies formally work such as central agency for mobilization and statistics , internet and media This type is a good type of resources such that it saves time and money.

The method of collecting data

The method of collecting data is determined according to the aim and the size of the statistical society under study.

For example: The students of a school represent a statistical society whose value is the student .

First : *Method of mass population :*

It means to collect the data related to the phenomenon of the statistical society. It's used to include all the society such as the population. This type is including all the values and it's unbiased in addition the outcomes are so accurate.

The disadvantages of such a method are ; it needs long time and great efforts. Further more, it costs much money.

Second: ***Mehods of samples:***

It mainly depends upon selecting a sample from the statistical society that it represents.

We conduct reseaches on the sample. The outcomes we get are generalized on the whole society.

Advantages of using methods of samples:

1. It saves time, efforts and money.
2. The only way to collect data about gigantic societies (like fish).
3. The only method to study some limited societies such as:
   - A Check the patient blood by getting a sample (checking the whole blood leads to death).
   - B Check the production of a factory productinng electric lamps to determine the validity of the lamp. (Know for how long the lamp can be used before getting burned).

Some of the disadvantages of the sample methods are : the outcomes of such type are not accurate if the selected sample doesn't represent all the society well in such a case the sample is **called biased**.

How we select samples and the conditions must be found in getting a sample:

First: the biased selction (samples are not randomly selected)

It means that we select the sample in a way to satisfy the objectives of the research. This is called as the sample delibrate. For example, when we want to know how the students understood a lesson in mathematics we must analyze the outcomes of the test by considering the outcomes of a group of students studied the same topic without the other students this is not a random selection.

Second: Random seclecion (random samples)

It means to select a sample such that the chance of getting any value from the society is equal.

Of the most important types of the random samples :

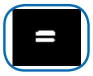
1. Simple random sample:
   Is the simplest type of samples and it can be get from the homogeneous socities where their selection is related to the size and number of units in the society.

A **If the size of the society is small:**

When we choose 5 students of a 40-student class, then we can prepare a card for each student on which their names or numbers are written, where all the cards are identical, put them back again in the box and draw a card from the box randomly and return the ball back again. Repeat this experiment till you get the sample needed.

B If the size of the society is big:

suppose we want to select the sample (5 students) from all the students whose numbers 800. The process of selection will be difficult to be done. So, we number the students from 1 to 800, then use the calculator or excel program to give 10 random digits in the field from 0.000 to 0.999 and take out the decimal point to make the field from zero to 999 you can take out the decimal digits which are more than 800 as follows:

Repeat pressing on  the appearence of numbers will be successive.

2 5 digits unrepreated are enough to give the digits of the sample for the students.

Layer random sample:

When the society needed to be examined is heterogeneous or made up of qualitative sets that are different in characteristics, the society is divided into homogeneous sets according to the characteristics forming it. Each set is called a layer and the researcher selects a random sample which each layer is represented according to its size in the society, such as a sample is called the layer sample .

For example: when we want to study an educational level of a society of 400 persons where the ratio of males to females is 3:2 and we want to select a sample of 50 persons, we must select 30 persons from the male layer and 20 persons from the female layer randomly.

Exercises (3-1)

1. ***Compare between*** **the mass population and samples showing the advantages and disadvantages .**

2. **The adminstration of a hotel wanted to conduct a survey to 300 customers on the service level produced. Every customer got a digit from 201 to 500. 10% of them were selected as a random sample to question them about the service level. Determine using the calculator the digits of the marked customers in this sample.**

3. **At a faculty, there are 4000 university students in the first grade, 3000 in the second grade, 2000 in the third grade and 1000 in the fourth grade if we want to draw a layer sample of 500 students, where each layer is represented in this sample according to the size. Calculate the number of students in each layer in the sample.**

3 - 2 Dispersion

Think and Discuss

You have previously learned the central tendancy (mean - domain - mode) and you used them to calculate a set of data to identify one value describing the trend of these data in centeralization around this value.

If the weekly wages in pounds of two sets of workers A and B in a factory are as follows:

Set A: 170, 180, 180, 230, 240
Set B: 50, 180, 180, 190, 400

1. ***Find*** the mean to the wages of the two sets A and B.
2. ***Compare*** the wages of the two sets A and B. **What do you deduce?**

What you'll learn

- Dispersions (Range- standard deviation)

Key term

- Central tendency
- Mean
- Dispersion
- Range
- Standard deviation

You know that The mean = $\dfrac{\text{Total of these values}}{\text{Their number}}$

then:

the mean of wages for set A = $\dfrac{170 + 180 + 180 + 230 + 240}{5}$

$= \dfrac{1000}{5} =$ LE 200

The mean of wages of set B = $\dfrac{50 + 180 + 180 + 190 + 400}{5}$

$= \dfrac{1000}{5} =$ LE 200

Compare the wages of the two sets A and B to find :

1. The mean of wages for set A = the mean of wages of set B

   = LE 200

2. The median of wages = the mode wage = LE 180 for each set A and B

We notice that :

(1) The wages of the two sets are different but both have the same measures of central tendency.

(2) The wages of set A are close so the values are included between 170 and 240 pounds where the wages of set B are divergent so the values are included between 50 and 400 pounds.

i.e. **The wages of set B is more divergent than the wages of set A**.

So When we compare two sets, we must consider the dispersion of the values of both sets and being divergent from each other .

Dispersion: **to any set of values means divergent or the differences between its values. The dispersion is small if the difference between the values are little whereas th dispersion is great if the difference between the values are very big (if the difference between the values are great). When the dispersion is zero, then all the values are equal.**

i.e. the dispersion is a measure that express how much the sets are homogenous

From the previous, we deduce:

To compare two sets of data or more, we must have a measure to the central tendency and another for dispersion for each set.

Dispersions measurements

1 *Range: (The simplest measure of dispersions)*

It is the diference between the greatest value and the smallest value in the set.
Compare the two sets above :

First set: 51, 53, 55, 57, 58, 60

Second set : 42, 45, 47, 49, 52, 92

We find that the range of the first set = 60 - 51 = 9

the range of the second set = 92 - 42 = 50

So the second set is more divergent than the first set

Notice that :

(1) The range is the simplest and easiest method of measuring dispersion.

(2) The range is influenced greatly by the outlier. it is clear that the values of the second set disparses in a range of 50 when we remove the last value (92) from and the range $= 52 - 42 = 10$ or $\frac{1}{5}$ of the previous range .

(3) Since the range doesn't influence by any value in the set except the greatest and smallest values, it doesn't give a clear picture to the dispersion of the set.

2 *Standard deviation :*

Is the commonest measure of dispersions and the most accurate (under certain conditions) which is the positive square root to the average of **squares deviations of values from the mean.**

i.e.:

$$\text{Standard deviation } \sigma = \sqrt{\frac{\sum (x - \bar{x})^2}{n}}$$

where σ denotes to: (sigma) to tell the standard deviation to the society of data.

$\bar{x}$ (x Bar) denotes the mean of the values of society.

n denotes the number of values .

$\sum$ denotes addition.

First : *calculating the standard deviation to a set of data :*

Example

Calculate the standard deviation for the values : 12, 13, 16, 18, 21

Solution

To calculate the standard deviation , form the table opposite the mean of a set of values

$$\bar{x} = \frac{\text{Total of these values}}{\text{Their numbers}}$$

$$\therefore \bar{x} = \frac{\sum x}{n}$$

$$\bar{x} = \frac{12 + 13 + 16 + 18 + 21}{5} = \frac{80}{5} = 16$$

$$\because \text{ The standard deviation } \sigma = \sqrt{\frac{\sum (x - \bar{x})^2}{n}}$$

	x	$x - \bar{x}$	$(x - \bar{x})^2$
	12	12 - 16 = -4	16
	13	13 - 16 = -3	9
	16	16 - 16 = 0	zero
	18	18 - 16 = 2	4
	21	21 - 16 = 5	25
Sum	80		54

$$\therefore \text{ The standard deviation } \sigma = \sqrt{\frac{54}{5}} = \sqrt{10.8} = \simeq 3.286$$

Second: *Calculating the standard deviation to a frequency distribution :*

For any frequency distribution :

the standard deviation $\sigma = \sqrt{\dfrac{\sum (x-\bar{x})^2 k}{\sum k}}$

where : **x** represents the value or the center of the set ,

k represents the frequency of the value or the set

$\sum k$ is the total of frequency , $\bar{x}$ is the mean $= \dfrac{\sum x k}{\sum K}$

Example

The following are the frequency distribution for a number of defective units which found in 100 boxes of manufactured units :

Number of defective units	zero	1	2	3	4	5
Number of boxes	3	16	17	25	20	19

Find the standard deviation to the defective units .

Solution

Consider the number of defetive units **(x)** and the number of the corresponding boxes **(k)** to calculate the standard deviation to the defective units form the following table **:**

Number of defective units	Number of boxes k	$x \times k$	$x - \bar{x}$	$(x - \bar{x})^2$	$(x - \bar{x})^2 k$
zero	3	zero	-3	9	27
1	16	16	-2	4	64
2	17	34	-1	1	17
3	25	75	zero	zero	zero
4	20	80	1	1	20
5	19	95	2	4	76
Total	100	300			204

The mean $\bar{x}$

$= \dfrac{\sum x \times k}{\sum k} = \dfrac{300}{100} = 3$

The standard variation σ

$= \sqrt{\dfrac{\sum (x-\bar{x})^2 k}{\sum k}}$

$= \sqrt{\dfrac{204}{100}} \simeq 1.428$ units

Drill

The following frequency distribution shows the goals scored in a number of football matches:

Number of goals	Zero	1	2	3	4	5	6
Number of matches	1	4	6	9	5	3	2

Find the standard deviation for the numbers of goals.

Example

The following frequency distribution shows the marks of 40 students in an exam:

Sets	0-	4-	8-	12-	16-20	Total
Frequency	2	5	8	15	10	40

Find the standard deviation for this distribution.

Solution

1. Find the centers of sets x

   Then: The center of the first set $= \frac{0+4}{2} = 2$

   The center of the second set $= \frac{4+8}{2} = 6$

   and then record them in the third column.

2. Multiply the centers of sets × its corresponding frequencies: i.e. **x** × **k** and record in the fourth column. Then find the mean $\overline{x} = \frac{\sum x\,k}{\sum k}$

3. Find the deviation of the center of each set (x) from the mean i.e. find $(x - \overline{x})$

4. Find squares of deviations of the center of each set from the mean: i.e $(x - \overline{x})^2$

5. Find the product of the square deviation of the center of each set from the mean × frequency of this set; i.e. $(x - \overline{x})^2 \times k$

6. Calculate the standard deviation $\sigma = \sqrt{\frac{\sum (x-\overline{x})^2\,k}{\sum k}}$

Sets	Frequency (k)	Center of sets (x)	$x \times k$	$x - \bar{x}$	$(x - \bar{x})^2$	$(x - \bar{x})^2 k$
0-	2	2	4	- 10.6	112.36	224.72
4-	5	6	30	- 6.6	43.56	217.80
8-	8	10	80	- 2.6	6.76	54.08
12-	15	14	210	1.4	1.96	29.40
16-20	10	18	180	5.4	29.16	291.60
Sets	40		504			817.6

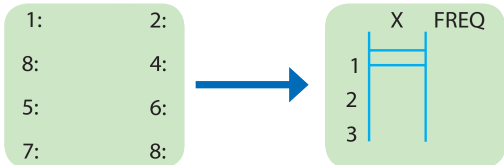
The mean $\bar{x} = \frac{504}{40} = 12.6$

The standard deviation $\sigma = \sqrt{\frac{817.6}{40}} = \sqrt{20.44} \simeq 4.52$ marks

You can use the calculator [Fx-82ES, Fx-83ES, Fx-85ES, Fx-300ES, Fx-350ES] to check the standard deviation.

First: State the calculator on statistical system to enter data

On Mode 2 (Stat) 1 (1-VAR)

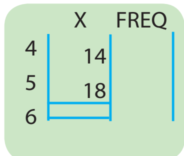


Second: Calculate the standard deviation to the frequency distribution (Example 2)

1 Enter the centers of sets 2, 6, 10, 14, 18

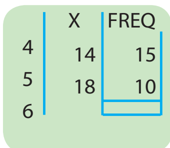
On Mode 2 (Stat) 1 (1-VAR)

2 = 6 = 1 0 = 1 4 = 1 8 =



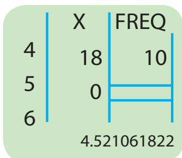
2 Go to the initial of the second column (FREQ) and enter the coresponding frequency for each set 2, 5, 8, 15, 10

▶ ▲ 2 = 5 = 8 = 1 5 = 1 0 =



3 Recall sum (standard deviation) then $\sigma \simeq 4.521$

Shift 1 5 (VAR) 3 (X σn) = ▲



4 Go back to the original system and switch off the calculator.

Notice that :

(1) The standard deviation is affected by the deviations of all the values and its value is affected by the outlier.

(2) The standard deviation has the same measuring units of the original data , so it is used to compare the dispersion of sets which have the same measuring units when the mean is equal in the mean . The set which contains more standard deviation is more dispersion.

Drill

The two frequency tables represent the marks of students of two classes A and B in third prepartory in an exam:

Class A

Sets of marks	0-	10-	20-	30-	40-50	Sum
Number of students	2	5	11	15	7	40

Class B

Sets of marks	0-	10-	20-	30-	40-50	Sum
Number of students	2	3	18	7	10	40

1. *Represent* both distribution using the frequency polygon in one figure.
2. *Find* the mean and standard deviation for both frequency distributions.
3. Which class is more homogeneous in getting marks?

Exercises (3-2)

1. ***Calculate*** **the standard deviation for the next data:**

   A 16, 32, 5, 20, 27 B 72, 53, 61, 70, 59

   C 15, -12, -9, 27, -6 D 22, 20, 20, 20, 18

2. **If the standard deviation of a set of data = zero,** ***what do you*** **infer?**

3. **The following frequency distribution shows the number of childern of some families in a new city:**

Number of children	Zero	1	2	3	4
Number of families	8	16	50	20	6

Calculate **the mean and standard deviation to the number of children.**

1 The following frequency distribution shows the weights of 200 students in a school:

Weight in kg	35-	45-	55-	65-	75-85	Total
Number of students	20	55	80	30	15	200

Find: A the mean of students weights.

B The standard deviation of students weights.

General Exercises

1 *Tell* the proper method for collecting the data in each of the following:

A Check the quality of wheat before buying.

B Check the salt degree of seawater.

C Check the validity of gas pipes before distribution.

2 There is a need to draw a layer sample to represent all the layers according to their sizes of a total 40000 values divided into three layers as follows:

Number of layer	1	2	3
number of values in layer	12000	20000	8000

If the number of values in the first layer is 240, calculate the size of the whole sample.

3 *Calculate* the mean and standard deviation to the following data:

23, 12, 17, 13, 15, 16, 8, 9, 37, 10.

4 The following frequency distribution shows the ages of 10 students:

Ages in year	5	8	9	10	12	Total
Number of children	1	2	3	3	1	10

Calculate the standard deviation to ages in years.

5 The following distribution table shows the amount of gasoline a set of cars consumes:

Number of kilometers per litre	5-	7-	9-	11-	13-	15-17	Total
number of cars	3	6	10	12	5	4	40

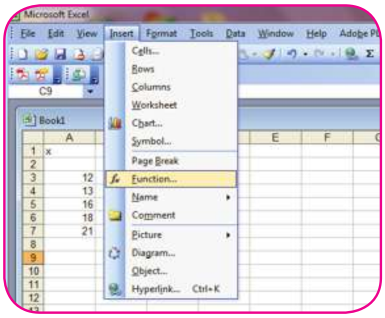
Find the standard deviation to the number of kilometers per litre.

Connecting with technology

Use the computer to calculate the standard deviation.

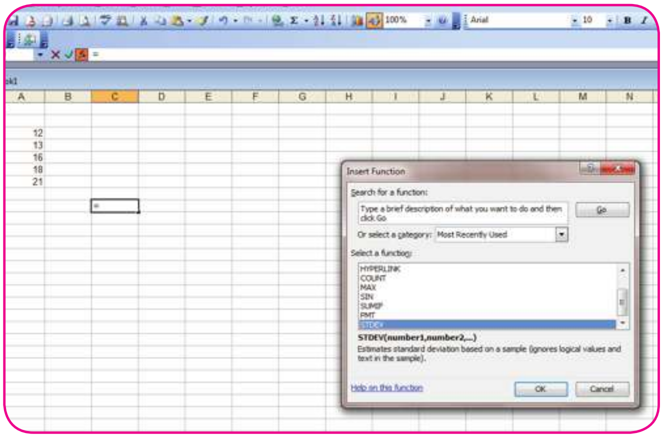
First: (Start) then (programs) (Excel) the following screen appears:

1



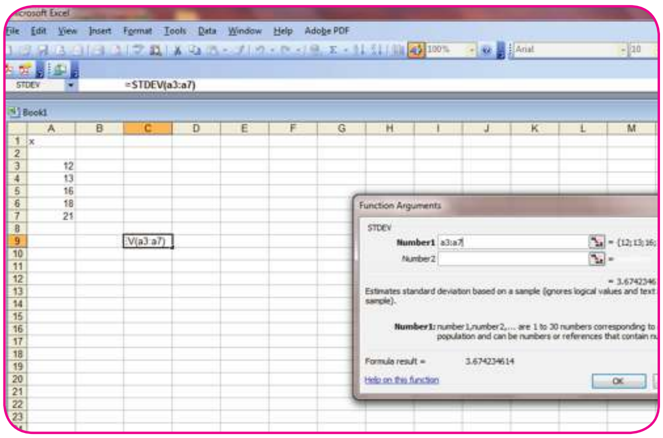
- Enter data of example (1) in the range (A3, A7) as shown
- From (insert) select function (fx) then enter

2



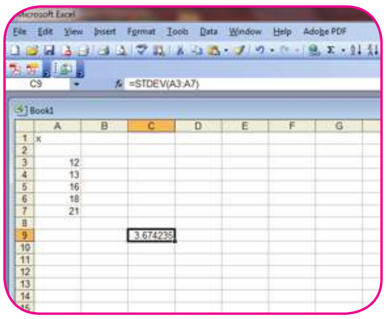
- From the square of searching for data, select the function STDEVP then enter

3



- To calculate the standard deviation to the society of data , determine the range of the veriable (A3, A7) then enter

4



- Notice that the standard deviation to the society of data = 3.286335 is the same as the result in the previous example which the calculator is used

Activity

1. Use the method of samples to select a random sample from your classmates of 10 values. Measure their heights in centimeters and find the average height of your classmates .

Comapre your results and your classmates. Explain your answer.

2. The table opposite shows the temperature in some cities.

   A. Calculate the mean and standard deviation to the maximum temperature.

   B. Calculate the mean and the standard deviation to the minimum temperature .

City	Max	Min
Ismailia	25	11
Suez	26	12
Arish	24	10
Nekhl	24	6
Taba	22	7
Tore	26	16
Hurghada	27	15
Rafah	26	11

(You can follow the daily weather reports and calculate the standard deviation and add it to your portfolio)

Unit test

1. ***Explain*** **briefly the simple random sample explaining how it can be selected.**

2. ***Calculate*** **the mean and the standard deviation for the following data:**

   A. 65, 61, 70, 64, 70, 76, 70

   B. 39, 85, 46, 91, 88, 50, 77

   ***Which set is more homogeneous*?**

3. **Calculate the mean and the standard deviation for the following frequency distribution:**

Set	Zero	4-	8-	12-	16-20	Total
Frequency	3	4	7	2	9	25

4. **200 employees were surveyed about their favorite food during break time. Every one was given a digit numbered from 1 to 200 then a sample represent s 10% was selected to be interviewed about their favorite food :**

   A. Hot drinks
   B. light meals
   C. soft drinks

   Determine using your calculator the digits of target employees in this sample.

Unit 4 : Trigonometry

Trigonometry is a branch of mathematics that concerned with studying relationships among sides and angles of triangles. Ancient Egyptians were the first to apply the rules of trigonometry in constructing their immortal pyramids and temples as well as applying in astronomy and in calculating geographical distances. Further more Babylonians had also measured the angles in degrees, minutes and seconds. Abou Alryhan Albyrony had settled a table for tangents of angles . Al tousi had deduced that the cosinese of the angles are in proportion with the legs opposite. West civilization learned about what Arab and Muslims wrote through translating the Arab astronomy books by the German Scientist Yohan Muller

Abou Alrayhan Albyrony
Was a great scientist born in
Algorithm in 973 and died in
1048 AD

4 - 1

The main trigonometrical ratios of the acute angle

What you'll learn

- ☆ Ratios of the acute angle in the right angled triangle.

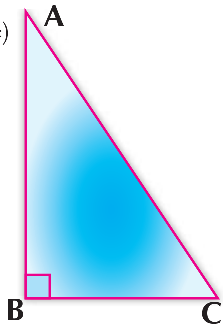
Key Terms

- ☆ Circular measure
- ☆ Sine angle
- ☆ Cosine angle
- ☆ Tangent angle

Think and Discuss

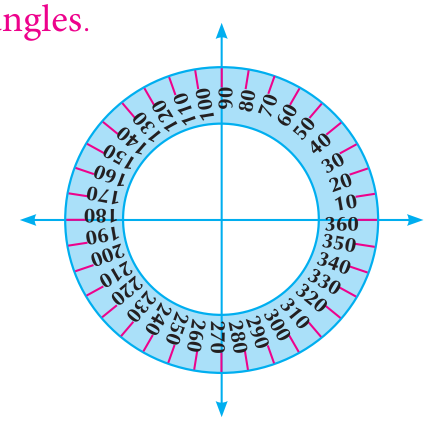
Use the right angled triangle a, b and c shown in the figure opposite ,

Complete using one of these symbols (> or < or =)



1. If m(∠C) > m (∠A) then AB ...... BC
2. $\frac{AB}{AC}$ 1
3. $\frac{AC}{BC}$ 1
4. $\frac{AB}{AC} \div \frac{BC}{AC}$ ...... $\frac{AB}{BC}$
5. $\frac{AB}{AC} + \frac{BC}{AC}$ 1
6. $\frac{(AB)^2}{(AC)^2} + \frac{(BC)^2}{(AC)^2}$ 1

Circular measure of the angles.



We studied that the product of the accummlative angles around a point equals 360°, if you divide the angles into four equal quadrants then a quadrant includes 90° (right angle); and a degree is the circular measuring unit. Similarly, parts of a degree are as follows:

degree = 60 minutes , minute = 60 seconds

35 degrees , 24 minutes ,42 seconds written

as the follows : **35°, 24` ,42``** you can convert minutes and seconds into parts of the degree in one of the following two ways:

First: Convert **24`** to minutes **24`** $= \frac{24}{60} = 0{,}4$, and convert **42``** first into minutes then into degrees : **42``** $= \frac{42}{60} = 0{,}7`$

$$0{,}7` = \frac{0{,}7}{60} = 0{,}0116667$$

then the sum is 35° 24` 42`` = 35 + 0,4 + 0,0116667 = 35,4116667°

Second: Use the calculator as follows :

The sum is : 35,4116667° **equals** 35 [° ' ''] 24 [° ' ''] 42 [° ' '']

Similarly, convert the fractions of degree into minutes and seconds.

For example: 54. 36° You can convert into degrees , minutes and seconds by using the following keys:

The sum is : 54° 21` 36`` [° ' ''] [SHIFT] [=] 54,36

Drill

1. **Write each of the following angles in degrees:**

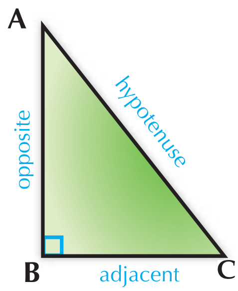
   A 76° 16`  B 45° 3` 56``  C 85° 38` 8`` D 65° 26` 43``

2. **Write each of the following angles in degrees, minutes and seconds.**

   A 34,6° B 78,08° C 56,18° D 83,246°

The main trigonometrical ratios of the acute angles:

The figure opposite:



The triangle ABC represents the right angled triangle at B where A and C are two complementary acute angles, the side opposite angle C is called leg opposite, the side adjacent to angle C is called adjacent and the side opposite to the right angle is called hypotenuse.

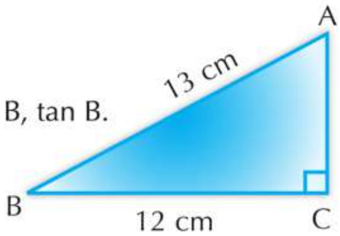
We will know the trigonometrical ratios of the acute angles as the following :

1. **Sine angle:** is denoted by the symbol [sin] .
2. **Cosine angle:** is denoted by the symbol [cos] .
3. **Tangential angle:** is denoted by the symbol [tan] .

sin C	=	$\frac{\text{opposite}}{\text{hypotenuse}}$	=	$\frac{AB}{AC}$
cos C	=	$\frac{\text{adjacent}}{\text{hypotenuse}}$	=	$\frac{BC}{AC}$
Tan C	=	$\frac{\text{opposite}}{\text{adjacent}}$	=	$\frac{AB}{BC}$

Example

1 ABC is a right angled triangle at C, A B = 13 cm, B C = 12 cm



A ***Find*** the length $\overline{AC}$

B ***Find each of the following:*** sin A, cos A, tan A, sin B, cos B, tan B.

C ***Prove that :*** sin A cos B + cos A sin B = 1

D ***Find :*** $1 + \tan^2 A$

Solution

A $\because$ ABC is a right angled triangle at C $\quad\therefore (AC)^2 = (AB)^2 - (BC)^2$

$\therefore (AC)^2 = (13)^2 - (12)^2 = (13 + 12)(13 - 12) = 25$

$\therefore AC = 5$ cm

B $\text{Sin } A = \frac{12}{13}$, $\text{Cos } A = \frac{5}{13}$, $\tan A = \frac{12}{5}$, $\sin B = \frac{5}{13}$, $\cos B = \frac{12}{13}$, $\tan B = \frac{5}{12}$

C **The right side =** sin A cos B + cos A sin B

$$\frac{12}{13} \times \frac{12}{13} + \frac{5}{13} \times \frac{5}{13} = \frac{144}{169} + \frac{25}{169} = \frac{144 + 25}{169} = 1$$

D $1 + \tan^2 A = 1 + (\frac{12}{5})^2 = 1 + \frac{144}{25} = \frac{169}{25}$

Drill

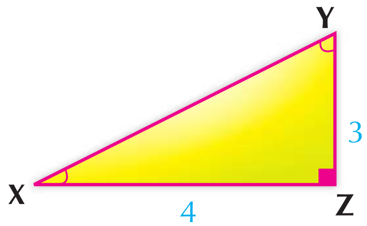
ABC is a triangle in which AB = AC =10 cm , BC = 12 cm, drawn $\overrightarrow{AD} \perp \overline{BC}$, $\overrightarrow{AD} \cap \overline{BC} = \{D\}$

First: find the value of sin (CAD) , cos (CAD) , tan (CAD)

Second: Prove that : **A** $\sin^2 C + \sin^2 C = 1$ **B** $\sin B + \cos C > 1$

Exercises (4-1)

1 **In the figure opposite : Complete**



A sin X = ...   B cos X = ...

C tan X = ...   D cos Y = ...

E tan Y = ...   F sin Y = ...

2 If the ratio between two measures of complementary angles as a ratio of 3 : 5, ***find*** the value of each one by circular measure .

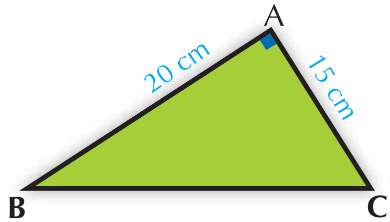
3 If the ratio between two measures of supplementary angles as a ratio 3 : 5, ***find*** the value of each one by circular measure.

4 If the ratio between the measures of the triangle as a ratio 3 : 4 : 7 ***find*** the circular measure for each angle.

5 A B C is a right angle triangle in B, A B = 8 cm, B C= 15 cm. Write what each trignometric ratios equal to the following: sin C , cos A , cos C , tan C .

6 ABC is a right angled triangle in B , if 2 A B = $\sqrt{3}$ AC, ***find*** the main trigonometrical of the angle C .

7 **In figure opposite :**



A B C is a triangle, m (∠A) = 90°, AC = 15 cm , AB = 20 cm

Prove that : cos C cos B - sin C sin B = zero

8 X Y Z is right angled triangle at Y, where X Y = 5 cm , XZ = 13 cm

Find **the value of :** A tan X + tan Z   B cos X cos Z - sin X cos Z

C sin X cos Z + cos X sin Z

9 X Y Z is a right angled triangle at Z where X Z = 7 cm, X Y = 25 cm.

Find the value of each of the following :

A tan X × tan Y   B $\text{Sin}^2 X + \sin^2 Y$

10 A B C D is an isosceles trapezoid $\overline{AD}$ // $\overline{BC}$, A D = 4 cm, A B = 5 cm where B C = 12 cm   **Prove that :** $\frac{5 \tan B \cos C}{\sin^2 C + \cos^2 B} = 3$

4 - 2 The main trigonometrical ratios of some angles

What you'll learn

- Finding the trigonometric ratios of angles
- (30° , 45° , 60°)

Key Terms

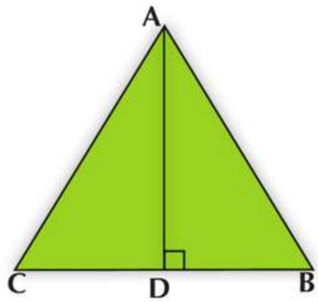
- Trigonometric ratios
- Special angles

Think and Discuss

1 In the figure opposite :

A B C is an a equilateral triangle of side length 2L, is and $\overline{AD} \perp \overline{BC}$

Complete:



1. m (∠B) = ...°
2. m (∠B A D) = ...°
3. B D = ... and A D = ... (by L)
4. B D : A B : A D = ... : ...... : ......

From the previous, we notice that :

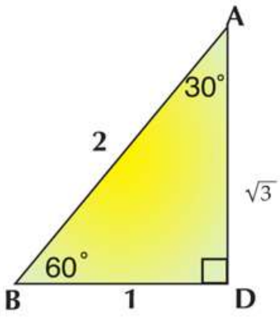
the triangle A B C is 30° , 60° and the ratio between the lengths of the triangle sides are BD : AB : AD = 1 : 2 : $\sqrt{3}$ So you can find the basic trigonometric ratios of the angles 30° , 60° **as follows:**

$\sin 30° = \frac{BD}{AB} = \frac{1}{2}$ and $\cos 30° = \frac{AD}{AB} = \frac{\sqrt{3}}{2}$

$\tan 30° = \frac{BD}{AD} = \frac{1}{\sqrt{3}}$

$\sin 60° = \frac{AD}{AB} = \frac{\sqrt{3}}{2}$

$\cos 60° = \frac{BD}{AB} = \frac{1}{2}$ and $\tan 60° = \frac{AD}{BD} = \sqrt{3}$



Drill

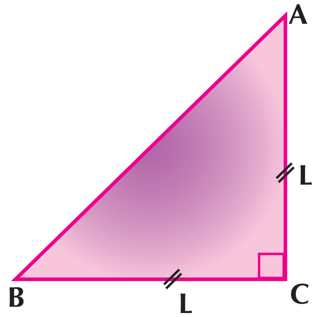
Complete: sin 30° = cos ......°, tan 30° = $\frac{1}{...}$,cos 30° = sin ....°

Think and Discuss

1 In the figure opposite:

ABC is an isosceles triangle and a right angled triangle at C. The length of each leg is L.

Complete:



1. $m(\angle A) = ...^\circ$, $m(\angle B) = ...^\circ$
2. $\because (AB)^2 = (AC)^2 + ...$ $\therefore (AB)^2 = L^2 + ...$
3. $AC : BC : AB = ... : .... : ....$
   $\therefore (AB)^2 = 2L^2$ $\therefore AB = \sqrt{2}\ L$

From the previous, we notice that :

ABC is a triangle in which $m(\angle A) = m(\angle B) = 45^\circ$ and the ratio between the lengths of its sides are $AC : BC : AB = 1 : 1 : \sqrt{2}$ So you can find the trigonometrical ratios of the angle 45° **as follows:**

$\sin 45^\circ = \frac{AC}{AB} = \frac{1}{\sqrt{2}}$ and $\cos 45 = \frac{BC}{AB} = \frac{1}{\sqrt{2}}$, $\tan 45^\circ = \frac{AC}{BC} = 1$

You can put the previous trigonometrical ratios in the following table:

m angle / ratio	30°	60°	45°
Sin	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$
Cos	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$
Tan	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$	1

Remarks:

1. **From the previous, we find that :** **(sine)** any angle equals **(cosine)** the supplementary angle of this angle and vice versa .

   for example: sin 30° = cos 60°, cos 30° = sin 60° and sin 45° = cos 45°.

2. For any angle A : $\tan A = \frac{\sin A}{\cos A}$.

Example

1 **Find the value of the following :**

A $\cos 60^\circ \sin 30^\circ - \sin 60^\circ \tan 60^\circ + \cos^2 30^\circ$

B $\dfrac{\cos^2 60^\circ + \cos^2 30^\circ + \tan^2 45^\circ}{\sin 60^\circ \tan 60^\circ - \sin 30^\circ}$

Solution

A The expression = $\cos 60^\circ \sin 30^\circ - \sin 60^\circ \tan 60^\circ + \cos^2 30^\circ$

$$= \frac{1}{2} \times \frac{1}{2} - \frac{\sqrt{3}}{2} \times \sqrt{3} + \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} - \frac{3}{2} + \frac{3}{4} = -\frac{1}{2}$$

B The expression = $\dfrac{\cos^2 60^\circ + \cos^2 30^\circ + \tan^2 45^\circ}{\sin 60^\circ \tan 60^\circ - \sin 30^\circ} = \dfrac{(\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2 + (1)^2}{\frac{\sqrt{3}}{2} \times \sqrt{3} - (\frac{1}{2})} = \dfrac{\frac{1}{4} + \frac{3}{4} + 1}{\frac{3}{2} - \frac{1}{2}} = \dfrac{1+1}{1} = 2$

Drill

Prove that:

A $\sin^2 30^\circ = 5 \cos^2 60^\circ - \tan^2 45^\circ$

B $\tan^2 60^\circ - \tan^2 30^\circ = (1 + \tan 60^\circ \tan 30^\circ) \div \cos^2 30^\circ$

Example

2 **Find the following trigonometrical ratios :**

sin 43° , cos 53° 28` , tan 64° 37` 49``

Roundig the sum to the nearest four decimal numbers .

Solution

Start → sin 43 = $\qquad$ $\sin 43^\circ \simeq 0{,}6820$

Start → cos 53 °’’’ 28 °’’’ = $\qquad$ cos 53° 28` $\simeq 0{,}5953$

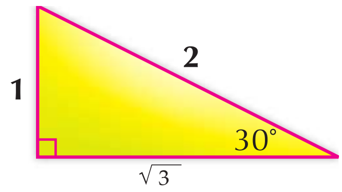
Start → tan 64 °’’’ 37 °’’’ 49 °’’’ = $\qquad$ Tan 64° 37` 49`` $\simeq 2{,}1089$

Finding the angle given its trigonometrical ratio :

You learned that if you have a given angle, you can find its trigonometrical ratios.

For example: If the measure of an angle is 30° then sin 30° $= \frac{1}{2}$ and similarly, if the angle measure is 33°, then sin 33° = 0,544639035

sin 33° = 0,544639035



Now, we want to identify the angle given its trigonometerical ratio.

for example: If cos C = 0,544639035 find the value of C .

Use the calculator as follows :

Start → SHIFT sin 0,544639035 = °’’’ 33°

Example

3 **Find m (∠E) in each of the following :**

sin E = 0,6 , cos E = 0,6217 , tan E= 1,0823

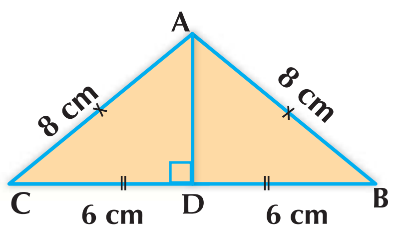
Solution

∵ sin E = 0,6	∴ m (∠E) = 36° 52` 12``	°’’’	= 0,6 sin SHIFT
∵ cos E = 0,6217	∴ m (∠E) = 51° 33` 35``	°’’’	= 0,6217 cos SHIFT
∵ tan E = 1,0823	∴ m (∠E) = 47° 15` 48``	°’’’	= 1,0823 tan SHIFT

4 **Connecting with Geometry:** ABC is an isosceles triangle in which AB = AC = 8 cm and BC = 12 cm .

Find : First: m (∠B)

Second: The area of the surface of the triangle to the nearest two decimal numbers.



Solution

Draw $\overrightarrow{AD} \perp \overline{BC}$

∵ The triangle ABC is an isosceles triangle.

$\therefore$ D the midpoint of $\overline{BC}$ and BD = CD = 6 cm

$\because$ $\cos B = \frac{6}{8} = \frac{3}{4} = 0{,}75$

Using the calculator :

SHIFT cos 0.75 = °’’

$\therefore$ $m(\angle B) = 41^\circ\ 24'\ 35''$ *(Q.E.D 1)*

To find the surface area of the triangle : find AD **(From Phythegoran's theorem)**

$\because$ $(AD)^2 = (AB)^2 - (BD)^2$

$\therefore$ $(AD)^2 = 64 - 36 = 28$ $\therefore$ $AD = 2\sqrt{7}$

$\because$ The area of the triangle ABC $= \frac{1}{2} \times BC \times AD = \frac{1}{2} \times 12 \times 2\sqrt{7}$

$= 12\sqrt{7}\ cm^2 \simeq 31{,}75 cm^2$ *(Q.E.D. 2)*

Another solution for the second part:

$\because$ $\sin B = \frac{AD}{AB}$ $\therefore$ $\sin B = \frac{AD}{8}$

$\therefore$ $AD = 8 \sin(41^\circ\ 24'\ 35'')$ (1)

The area of the triangle ABC $= \frac{1}{2} \times BC \times AD$ **substitute from (1) in this relation**

$\therefore$ The area of the triangle ABC $= \frac{1}{2} \times 12 \times 8 \sin(41^\circ\ 24'\ 35'') \simeq 31{,}75 cm^2$.

Use the calculator as follows :

start → 1 ÷ 2 × 12 × 8 × sin 41 °’’ 24 °’’ 35 °’’ =

Drill

Complete the following :

1. If $\sin X = \frac{1}{2}$ where X is an acute angle then $m(\angle X) =$ ....
2. If $\sin \frac{X}{2} = \frac{1}{2}$ where X is an acute angle then $m(\angle X) =$ ....
3. $\sin 60^\circ + \cos 30^\circ - \tan 60^\circ =$ ....
4. If $\tan(X + 10) = \sqrt{3}$ where X . IS an acute angle then $m(\angle X) =$ ....
5. If $\tan 2X = \sqrt{3}$ where X is an acute angle then $m(\angle X) =$ ....

Exercises (4-2)

1 **Find the value of the following :**

$\sin 45^\circ \cos 45^\circ + \sin 30^\circ \cos 60^\circ - \cos^2 30^\circ$

2 **Prove that :**

A $\cos 60^\circ = 2\cos^2 30^\circ - 1$

B $\tan^2 60^\circ - \tan^2 45^\circ = \cos^2 60^\circ + \sin^2 60^\circ + 2\sin 30^\circ$

3 ***Find*** **X**

$4X = \cos^2 30^\circ \tan^2 30^\circ \tan^2 45^\circ$

4 ***Find*** **angle E , where E is an acute angle**

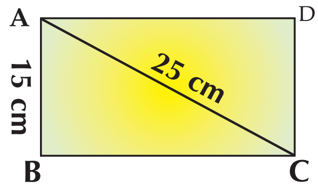
$\sin E = \sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$

5 **Connecting with Geometry:** in the figure opposite:

ABCD is a rectangle in which AB = 15 cm and AC = 25 cm .

Find : First: m(∠ACB)

Second : The surface area of the rectangle ABCD .



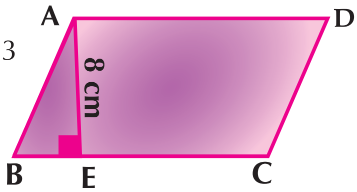
6 **Connecting with Geometry :** in the figure opposite:

ABCD is a parallelogram of surface area 96 cm^2 , BE : EC = 1 : 3

$\overline{AE} \perp \overline{BC}$ and AE = 8cm

Find: First: The length of $\overline{AD}$ Second: m(∠B)

Third: The length of $\overline{AB}$ to the nearest decimal number *(Use more than one way)*



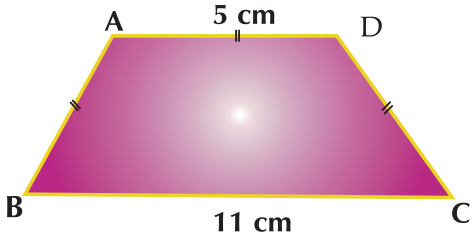
7 **Connecting with Geometry:** in the figure opposite :

ABCD is an isoceles trapezoid in which

AB =AD= DC = 5 cm and BC = 11cm.

Find : First: m (∠B) and m (∠ A)

Second: The area of the isosceles trapezoid ABCD.



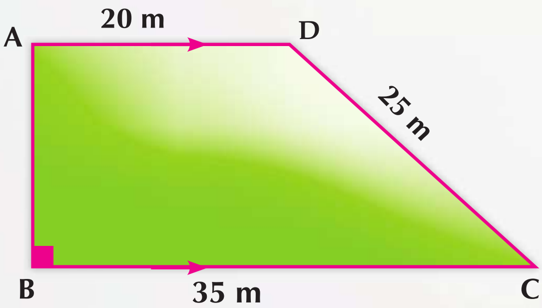
Activity

A trapezoid shaped piece of land ABCD in which $\overline{AD}$ // $\overline{BC}$, m(∠B) = 90°, AD = 20 meters

BC = 35 meters and DC = 25meters

R.T.P. : A Find the length of $\overline{AB}$.

B m(∠C).



C If the land owner made a circular shaped fountain inside it; What is the largest possible area for the fountain? Find the area of the remaining part of the land. (π = 3,14)

Unit test

1 **Prove each of the following equalities :**

A sin 60° = 2 sin 30° cos 30°

B $\tan 60° = \frac{2 \tan 30°}{1 - \tan^2 30°}$

2 **Without using the calculator find the value of X (where X is an acute angle) satisfies each of:**

A tan X = 4 cos 60° sin 30°

B 2 sin X = sin 30° cos 60° + cos 30° sin 60°

3 ABC is an isosceles triangle in which AB = AC = 12,6 cm and m(∠c) = 84° 24`.

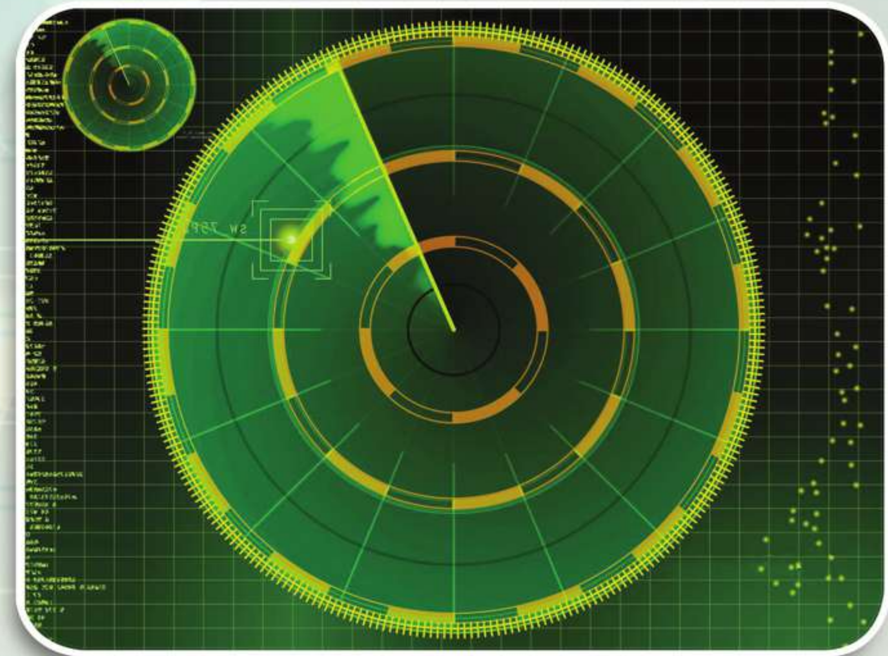
Find the length of $\overline{BC}$ to the nearest decimal number.

4 A B C D is a trapezoid in which $\overrightarrow{AD}$ // $\overline{BC}$, m(∠B) = 90°, If AB = 3 cm, AD = 6 cm and BC = 10 cm , **prove that :** cos (DCB) - tan (ACB) = $\frac{1}{2}$

5 A ladder $\overline{AB}$ of length 6 meters, its upper edge A lies on a vertical wall and its other edge B on a horizontal floor. If C is the projection of point A on the surface of the floor and its angle of slope on the surface of the floor was 60°, then find the length of $\overline{AC}$.

Coordinate geometry

Unit 5: Coordinate geometry



The Radar is used for identifying the range, height, direction and velocity of moving objects like airplanes and ships.

The radar tower receives the reflected waves. The radar screens can determine the coordinates of the target's location (airplane-ship- ....).

5 - 1 Distance between two points

What you'll learn

☆ Finding the distance between two points by using the distance rule.

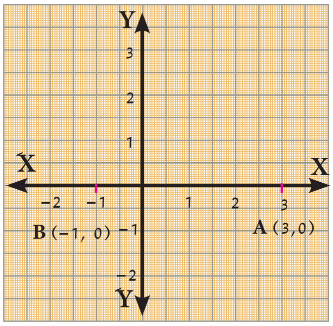
Key terms

☆ Coordinate plane
☆ Ordered pair
☆ Distance between two points.

Think and Discuss

You represented the ordered pair on the coordinate plane.
Now can you find the distance between the pairs of the following points?

1. A (3, 0) , B (-1, 0)
2. C (0, -3), D (0, -1)
3. M (3, 2), N (7, 5)

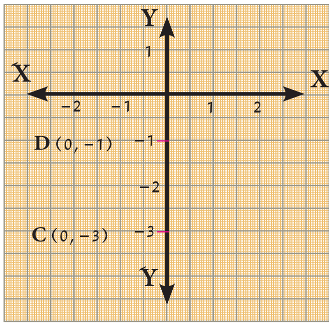


From the previous, we notice that :

1. The two points A (3, 0), B (-1, 0) are both located on x - axis, so :

   A B = |-1 -3 | = | -4|

   So A B = 4 unit length .



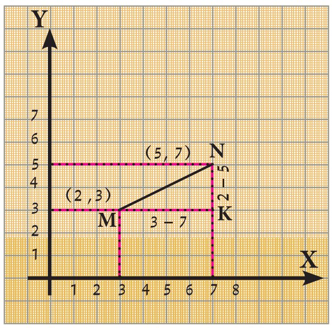
2. The two points C (0, -3), D (0, -1) are both located in the y - axis, so;

   C D = |-3 - (-1)|

   = |-3 +1| = | -2|

   C D = 2 unit length .

3. The two point M (3, 2), N (7, 5) can be represented graphically as in the following figure opposite. To find The length of $\overline{MN}$ we find;



   M K = |7 -3 |= 4 unit length,

   N K = |5 -2| = 3 unit length .

   △ M K N is right angle at K

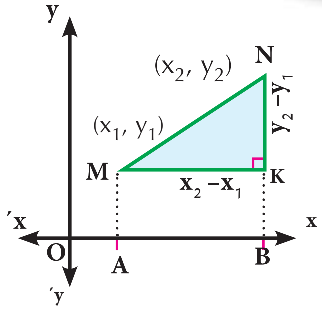
   ∴ $(MN)^2 = (MK)^2 + (KN)^2$

   (Pythagoren theory)

   $(MN)^2 = (3)^2 + (4)^2$ $(LM)^2 = 9 + 16$

   $(MN)^2 = 25$ ∴ (M N) = 5 unit length

In general :



If M (x_1, y_1), N (x_2, y_2) are two points on the coordinate plane

then: K M = |O B - O A|

$= |x_2 - x_1|$

K N = |N B - K B| = $|y_2 - y_1|$

$\because$ $\triangle$ N K M is a right angle in K **(pythagoean theory)**

$\therefore (MN)^2 = (KM)^2 + (KN)^2$

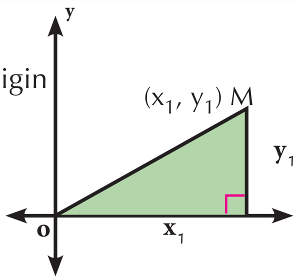
$= (x_2 - x_1)^2 + (y_2 - y_1)^2$

$\therefore MN = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

The distance between two points (x_1,y_1), $(x_2,y_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

The distance between two points = $\sqrt{\text{square difference in the x - axis + square differance in y - axis}}$

Remark:



In the figure opposite the distance of a point M (x_1, y_1) fom the origin point O (0, 0) , O M = $\sqrt{x^2_1 + y^2_1}$

Drill

If A , B , C and D are four given points in the perpendicular coordinate plane, mention the conditions which make those points vertices for each of the following geometrical shapes:

1. Paralleogram
2. Rectangle
3. rhombus
4. Square

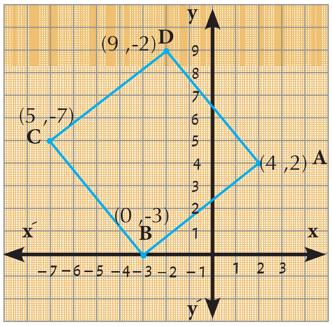
Example

1. ABCD is a quadrilateral where, A (2, 4), B (-3, 0), C(-7, 5) and D(-2, 9). Prove that ABCD is a square.

Solution

$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{[-3-2]^2 + [0-4]^2} = \sqrt{(-5)^2 + (-4)^2} = \sqrt{41}$

$B\,C = \sqrt{[-7-(-3)]^2 + [5-0]^2} = \sqrt{(-4)^2 + (5)^2} = \sqrt{41}$



$C\,D = \sqrt{[-2-(-7)]^2 + [9-5]^2} = \sqrt{(5)^2 + (4)^2} = \sqrt{41}$

$D\,A = \sqrt{[2-(-2)]^2 + [4-9]^2} = \sqrt{(4)^2 + (-5)^2} = \sqrt{41}$

$\because A\,B = BC = DC = DA = \sqrt{41}$

$\therefore$ Figure A B C D whether a square ar arhombus

to prove that the figure A B C D is a square, find the lengths of the two diagonal $\overline{AC}, \overline{BD}$

$AC = \sqrt{[-7-2]^2 + [5-4]^2} = \sqrt{(-9)^2 + 1} = \sqrt{82}$

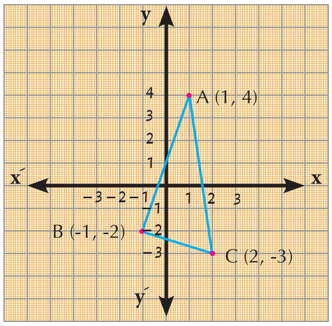
$BD = \sqrt{[-2-(-3)]^2 + [9-0]^2} = \sqrt{(-1)^2 + (9)^2} = \sqrt{82}$

$\because AC = BD = \sqrt{82}$ and the sides of the figure A B C D is equal in length

$\therefore$ Figure A B C D is a square.

2 Prove that the triangle of the vertices A (1, 4), B (- 1, -2), C (2, -3) is a right angle. Find its surface area.

Solution



$(A\,B)^2 = (-1-1)^2 + (-2-4)^2 = 4 + 36 = 40$

$(B\,C)^2 = [2-(-1)]^2 + [-3-(-2)]^2 = 9 + 1 = 10$

$(A\,C)^2 = (2-1)^2 + (-3-4)^2 = 1 + 49 = 50$

$(A\,B)^2 + (B\,C)^2 = 40 + 10 = 50$, $(A\,C)^2 = 50$

$\because (A\,C)^2 = (A\,B)^2 + (B\,C)^2$

$\therefore M(\angle B) = 90^\circ$ **(The converse to the pytheogeran theary)**

$\therefore M(\triangle ABC) = \frac{1}{2} AB \times BC = \frac{1}{2} \times \sqrt{40} \times \sqrt{10} = \frac{1}{2} \times 2\sqrt{10} \times \sqrt{10} = 10$ square units

3 Prove that the points A (3, -1), B (- 4, 6) and C (2, -2), are located in circle whose center is the point M (-1, 2), , then find the circumference of the circle.

Solution

$A\,M = \sqrt{(-1-3)^2 + [2-(-1)]^2} = \sqrt{(-4)^2 + (3)^2} = \sqrt{25} = 5$

$B\,M = \sqrt{[-1-(-4)]^2 + [2-6]^2} = \sqrt{(3)^2 + (-4)^2} = \sqrt{25} = 5$

$C\,M = \sqrt{(-1-2)^2 + [2-(-2)]^2} = \sqrt{(-3)^2 + (4)^2} = \sqrt{25} = 5$

$\because A\,M = B\,M = C\,M = 5$ $\therefore$ A , B and c are located in a circle whose center is M.

Drill

Prove that the points: A (4, 3.), B(1, 1.) and C (-5, -3.) are collinear.

Complete :

$A B = \sqrt{(1-4)^2 + (1-3)^2} =$ ....

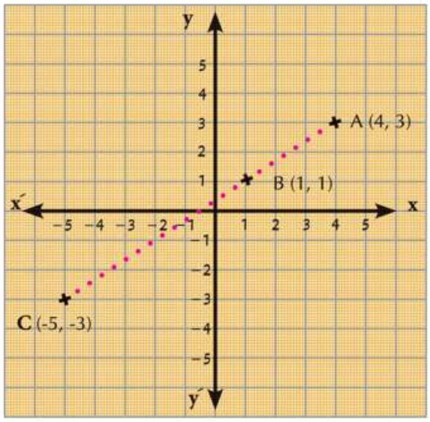
$B C = \sqrt{(-5-1)^2 + (-3-1)^2} =$ ....

$A C = \sqrt{(-5-4)^2 + (-3-3)^2} =$ ....

$\therefore$ A B + B C = ... + ... = ...

$\because$ A B + ... = A C

$\therefore$ The points A , B and C are collinear.



Exercises (5-1)

First: Complete the following :

1. The distance between the point (-3, 4) and the point of origin equals ....
2. The distance between the two points (- 5, 0), (0, 12) equals ....
3. The distance between two points (15, 0), (6, 0) equals ....
4. The radius length of the circle of center (7, 4) passing through the point (3, 1) equals ....
5. If the distance between two points (a, 0), (0, 1) is unit length, then a = ...

Second: Choose the correct answer from the given answers :

1. The points (0, 0), (6, 0), (0, 8) :
   - A form an obtuse triangle
   - B form an acute triangle
   - C form a right triangle.
   - D are collinear.
2. A circle its center is the origin point and radius length 2 units. Which of the following points belongs to the circle?
   - A (1, 2)
   - B (- 2, 1)
   - C $(\sqrt{3}, 1)$
   - D $(\sqrt{2}, 1)$
3. Show which of the following sets of points are colliner :
   - A (1, 4), (3, - 2), (- 3, 16)
   - B (7, 0), (- 3, - 3), (22, 9)
   - C (- 1, - 4), (1, 0), (0, - 2)
   - D (- 1, - 4), (1, 0), (0, - 2)

Third: Answer the following questions.

1. Find the value of a in each of following cases :

   A. If the distance between the two points (a, 7), (-2, 3) equals 5

   B. If the distance between the two points (a, 7), (3 a -1, -5) equals 13

2. If A (x, 3), B (3, 2), C (5, 1) and A B = B C: then find the value of x.

3. If the distance of the point (x, 5) from the point (6, 1) equals $2\sqrt{5}$, then find the value of x.

4. Tell the kind of each of the following triangles with respect to its angles :

   A. A (3, 10), B (8, 5), C (5, 2)

   B. A (1, -1), B (2, 1), C (-3, -2)

   C. A (3, 3), B (4, -1), C (1, 1)

5. State the kind of triangle whose vertices are the points A (- 2, 4), B (3, - 1), C (4, 5) with respect to its sides .

6. Prove that the triangle whose vertices A (5, -5), B (- 1, 7), C (15, 15) is right angle in B , then find its area .

7. ABCD is a quadrilateral, where points A (5, 3), B (6, -2), C (1, -1), D (0, 4). Prove that ABCD is a rhumbus, then find its area.

8. Prove that the points A (-2, 5), B (3, 3), C (-4, 2) are non collinear, and if D (-9, 4) Prove that the figure A B C D is a parallelogram. .

9. **In the figure opposite :**

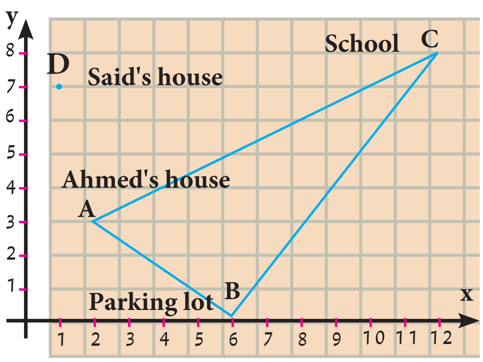
A. Find the coordinate points which represent the location of Ahmed's house, Saids' house and the parking lot .

B. The distance of Ahmed's house from the school .

C. The distance of Said's house from the school.

D. Which is closer to school : Ahmed's house or Said's house?.

E. Are the two ways of $\overline{AB}$, $\overline{BC}$ perpendcular ? Give the reason.



5 - 2

The Two Coordinates of the midpoint segment

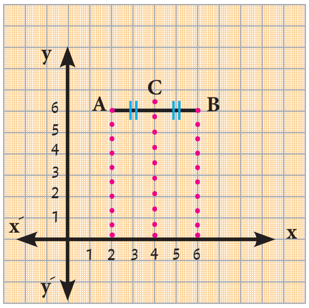
Think and Discuss

On a perpendicular coordinate plane, find the two coordinates of the midpoint on C straight segment $\overline{AB}$:

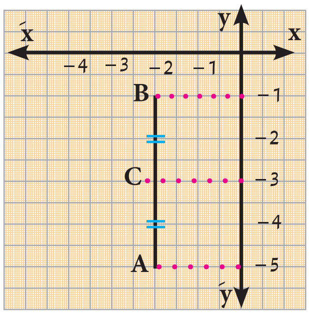
First : A (2, 6) and B (6, 6)

Second : A (-2 , - 5) and B (- 2 , - 1),

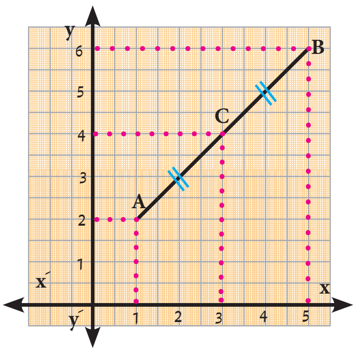
Third : A (1, 2) and B (5, 6)



First: The line segement, which its end are the two points (2, 6), B (6, 6), is parallel to the x-axes and the two coordinate of the point of its midpoint C (4, 6)



Second : The straight segment with the two ends A (-2, -5), B (-2, -1) is parallel to the y-coordinate. The two coordinates of its midpoints C are (-2, -3) .



Third : In the figure opposite :

Consider that the C is the midpoint of the straight segment with the two ends A(1, 2), B (5, 6) from the drawing, we find that the two coordinates of C are (3, 4).

i. e C $(\frac{1+5}{2}, \frac{2+6}{2})$ i. e. C (3, 4)

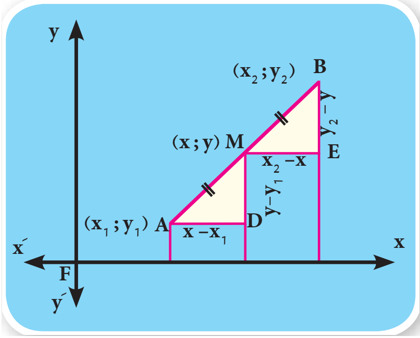
What you'll learn

- Finding the two coordinates of the midpoint of a straight segment .

Key terms

- The two ends of the line segment
- The two coordinates of the midpoint of a straight segment .

In general. you can duduce the law of the coordinate of the midpoint of a straight segment as follows.



If A (x_1, y_1), B (x_2, y_2), M (x, y) where M in the midpoint of $\overline{AB}$ that: $\triangle$ B E M , $\triangle$ M D A are cogruent

we find: that A D = M E

$\therefore x - x_1 = x_2 - x$

$\therefore 2x = x_1 + x_2 \qquad \therefore x = \frac{x_1 + x_2}{2}$

Similarly: M D = B E $\quad \therefore y - y_1 = y_2 - y$

$\therefore 2y = y_1 + y_2 \qquad \therefore y = \frac{y_1 + y_2}{2}$

$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Example : If C is the midpoint of $\overline{AB}$ and A (3, - 7) , B (- 5, - 3)

Then the coordinates of midpoint of $\overline{AB}$ are $\left(\frac{3 - 5}{2}, \frac{-7 - 3}{2}\right)$ i.e. (- 1, - 5)

Drill

Calculate the coordinates of point C the midpoint of $\overline{AB}$ in the following cases :

1. A(2, 4), B (6, 0)
2. A (7, -5), B (-3, 5)
3. A (-3, 6), B (3, -6)
4. A (7, -6), B (-1, 0)

Examples

1. If C (6, -4) is the midpoint of $\overline{AB}$ where: A (5, -3) then find the coordinates of a point B .

Solution

Consider that B (x_2, y_2), A (5, -3), and the midpoint of $\overline{AB}$ is the point C (6 - 4)

$\because x = \frac{x_1 + x_2}{2}, \; y = \frac{y_1 + y_2}{2}$

$\therefore 6 = \frac{5 + x_2}{2} \qquad \therefore 5 + x_2 = 12 \qquad \therefore x_2 = 12 - 5 = 7$

$-4 = \frac{-3 + y_2}{2} \qquad \therefore -3 + y_2 = -8$

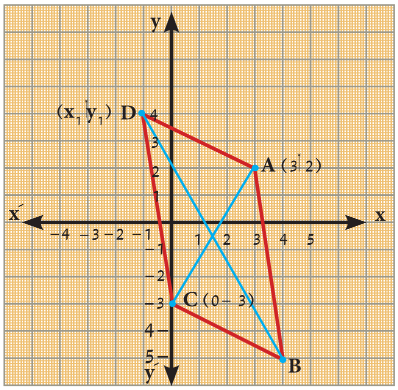
$y_2 = -8 + 3 \qquad y_2 = -5 \qquad \therefore$ B (7, -5)

2 A B C D is a parallorogram, A (3, 2), B (4, -5), C (0- 3) - Find the two coordinates of the point at which the two diagonals intersect. Then find the coordinates of point D.

Solution

The figure A B C D is a parallelogram , M is the intersection point of its diagonal.

consider D (x_1 , y_1)



$\because$ M is the mid of $\overline{AC}$ $\quad \therefore M(\frac{3+0}{2}, \frac{2-3}{2})$

$\therefore M(\frac{3}{2}, -\frac{1}{2})$

$\therefore M(\frac{4+x_1}{2}, \frac{-5+y_1}{2})$

$\therefore \frac{3}{2} = \frac{4+x_1}{2}$ $\quad \therefore 3 = 4 + x_1$

$\therefore x_1 = -1$

$, -\frac{1}{2} = \frac{-5+y_1}{2}$ $\quad \therefore -1 = -5 + y_1$

$\therefore y_1 = 4$

$\therefore$ The coordinates of the point D are (-1, 4)

Exercises (5-2)

First : Complete

A If the point of the origin is the midpoint of a straight segment $\overline{AB}$, where A (5, -2) then the coordinates of the point B are ....

B If A B = B C = C D, A (1, 3), C (5, 1) where A, B, C, and D are collinear Find :

First : the coordinates of the point B are (........., ...)

Second : The coordinates of the point D are (........., ...)

C $\overline{AD}$ is the median in $\triangle$ A B C, M is the midpoint of $\overline{AD}$

Where A (0, 8), B (3, 2), C (- 3, 6) Find :

First : The coordinates of the point D are (........., ...)

Second : The coordinates of the point M are (........., ...)

Verify by determining the coordinates of the points.

Second:

1. If C in the midpoint of $\overline{AB}$, then find x , y , in each of the following cases:

   A. A (1, 5) B (3, 7) C (x, y)

   B. A (-3, y) B (9, 11) C (x, -3)

   C. A (x, -6) B (9, -11) C (-3, y)

   D. A (x, 3) B (6, y) C (4, 6)

2. If A (1, - 6), B (9, 2) , then find. the coordinates of the points which divide $\overline{AB}$ into four equal parts in length.

3. Prove that the points A (6, 0), B (2, -4), C (-4, 2) are the vertices of the right angled triangled at B, then find the coordinates of the point D that make the figure A B C D a rectangle .

4. If the points A (3, 2), B (4, -3), C (-1, -2), D (-2, 3) are vertices of the rhombus. Find: :

   A. The coordinates of the the point where the two diagonals intersect the two diagonals .

   B. The area of the rhombus A B C D.

5. Prove that the points A, (-3, 0), B (3, 4) and C (1, -6) are the vertices of an isosceles triangle of vertex A, then find the length of the drawn straight segment from A perpendicular on $\overline{BC}$.

6. If A (-1, -1), B (2, 3), C (6, 0) and D (3, -4) are four points in perpendicular coordinates plane. **Prove that** $\overline{AC}$ and $\overline{BD}$ bisect each other, then identify the type of the figure.

7. **Prove that** the points A (5, 3), B (3, -2), C (-2, -4) are the vertices of the obtuse triangle at B, then find the coordinates of the point D that makes the figure A B C D a rhombus, and find its surface area.

8. A B C D is a parallelogram where , A (3, 4), B (2, -1), C (-4, -3) : Find the coordinates of D . Take E $\in \overrightarrow{AD}$ where A E = 2 A D . What are the coordinates of the point E?

5 - 3

The slope of the straight line

What you'll learn

- The relation between the slope of two parallel straight lines.
- The relation between the slope of two perpendicular, straight lines.

Key terms

- A Positive measure of the angle
- A negative measure of the angle
- The slope of the straight line
- Two parallel straight lines
- Two perpendicular straight lines.

You know that the slope of the straight line passing through two points (x_1 , y_1), (x_2 , y_2) equals $\frac{y_2 - y_1}{x_2 - x_1}$

Think and Discuss

Find the slope of the straight line passing through each pair of the following ordered pairs :

First: (3, 1), (4, 2)

Second: (4, 0), (2, 2)

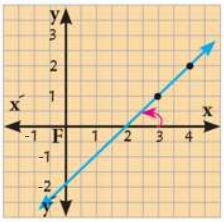
Third: (-1, 3), (2, 3)

Fourth: (2, -1), (2, 3)

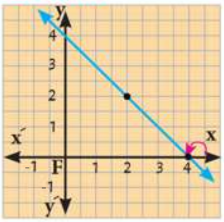
What do you notice?

From the previous, you can draw the straight lines passing through the previous pairs of points in the perpendicular coordinate plane as in the following figure:

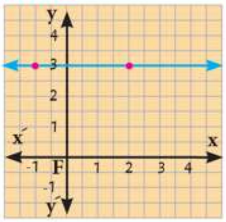
First:



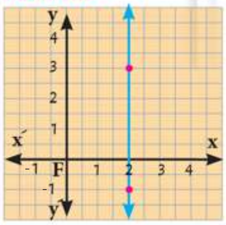
Second:



Third:



Fourth:



The positive and the negative measure of the angle :

An angle is positive when it is formed by a counter anticlockwise rotation and it is negative when it is formed by a clockwise rotation.

From the previous figures, we deduce that:

The figure number	The slope $\left(\frac{y_2 - y_1}{x_2 - x_1}\right)$,	The type of the positive angle that the straight line makes in. the positive direction to the x-coordinates	The slope of the straight line
1	$m = \frac{2-1}{4-3} = 1$	acute	Larger than zero
2	$m = \frac{2-0}{2-4} = -1$	obtuse	Smaller than zero
3	$m = \frac{3-3}{2+1} = 0$	zero	equal to zero
4	$m = \frac{3+1}{2-2}$ (unidentified)	right	unidentified

We can deduce the slope of the straight line as follows:

Slope of the straight line is the tangent of the positive angle which the straight line makes with the positive direction to x axis.

i.e slope of a straight line = tan E, where E is the positive angle that the straight line makes with the positive direction of the x axis.

Examples

1. Find the slope of the straight line which makes an angle of a measure 56° 12\` 48\`\` in the positive direction to the x-axes.
2. Find the measure of the positive angle that the straight line makes to the x - axis if m =1.4865 (where m is the slope) .

Solution

1. ∵ m = tan E   ∴ m = Tan 56° 12\` 48\`\` = 1.494534405

   Start → tan 56 °‚‚ 12 °‚‚ 48 °‚‚ =

2. ∵ m = Tan E   ∴ Tan E = 1.4865   ∴ m (∠ E) = 56° 4\` 13\`\`

   Start → SHIFT tan 1,4865 = °‚‚

Drill

1. Find the slope of the straight line that makes a positive angle in the positive direction of to the x - axis, its measure:

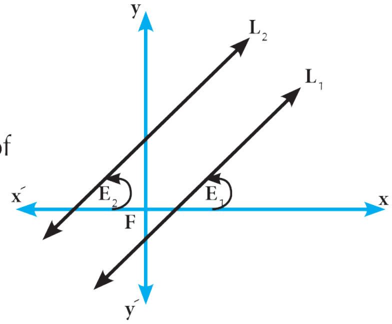
   A 30°   B 45°   C 60°

2. Using the calculator, find the measure of the positive angle made by the straight line of slope (m) in the positive direction of x-axis in the following cases :

   A m = 0.3673   B m = 1.0246   C m = 3.1648

The relation between the slope of the two parallel straight lines.

Think and Discuss



The figure opposite: Represents two parallel straight lines L_1, L_2 with two slpoes m_1, m_2, making two positive angles of measures E_1, E_2 in the positive direction of the x-axes.

Complete the following :

1. m ($\angle$ E_1) = m ($\angle$ E_2) because ....
2. Tan E_1 .... Tan E_2
3. m_1 .... m_2

from the previous, we deduce that :

IF L_1 // L_2 **then** $m_1 = m_2$

i.e.: If two lines are parallel, then their slopes are equal and vice versa .

Thus If $m_1 = m_2$ then L_1 // L_2

i.e.: If two lines have equal slopes, then the two lines are parallel.

Examples

1. Prove that the straight line passing through two points (-3, -2), (4, 5) is parallel to the straight line that makes with the positive direction to the x-axes an angle of 45 ° measure

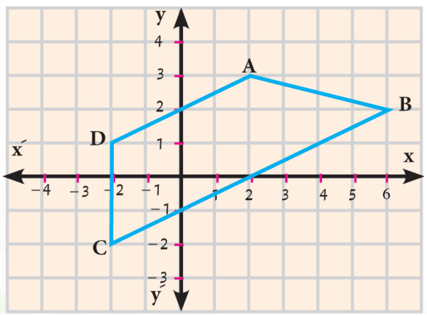
Solution

The slope of the first straight line (m_1) = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - (-2)}{4 - (-3)} = \frac{7}{7} = 1$

The slope of the second straight line (m_2) = Tan 45° = 1 $\quad \because m_1 = m_2$

$\therefore$ The two straight lines are parallel.

2. Represent graphically the points A (2, 3), B (6, 2) C (-2, -2) and D (- 2, 1), in the coordinate plane then **prove that** the figure A B C D is trapezoid .



Solution

From the drawing, we find that : $\overline{AD}$ // $\overline{BC}$

To prove that anaclytically, we find the slope of each of both: $\overleftrightarrow{AD}$, $\overleftrightarrow{BC}$.

The slope of $\overline{AD}$ (Let it be m_1)

$\because m = \frac{y_2 - y_1}{x_2 - x_1}$ $\therefore m_1 = \frac{3-1}{2+2} = \frac{2}{4} = \frac{1}{2}$

and the slope of $\overline{BC}$ (Let it be M_2)

$m_2 = \frac{2+2}{6+2} = \frac{4}{8} = \frac{1}{2}$ $\because m_1 = m_2$ $\therefore \overline{AD} // \overline{BC}$

$\therefore$ The figure A B C D is a trapezoid unles the points A, B, C, D are collinear ..... (1)

$\because$ The slope of $\overline{AB} = \frac{3-2}{2-6} = \frac{1}{-4}$, , the slope of $\overline{CD} = \frac{2+1}{-2+2}$ .... (unknown)

$\therefore$ The two straight lines are not parallel............ (2)

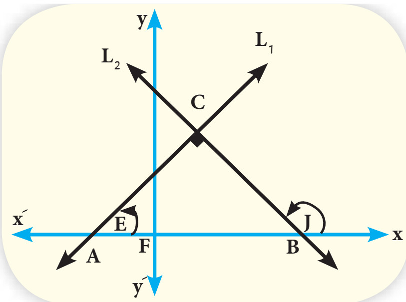
From (1), (2) $\therefore$ The figure A B C D is a trapezoid .

Drill

1. Prove that the straight line passsing through the two points (2, 3), (0, 0) is parallel to the straight line passing through the two points (- 1, 4), (1, 7).
2. Prove that the straight line passing through the two points (2, -1), (6, 3) is parallel to the straight line that makes an angle its of 45° measure with the positive direction to the x-axis.
3. If the straight line $\overleftrightarrow{AB}$ // the y-axis where A (x, 7), B (3, 5) , then find the value of x.
4. If the straight line $\overleftrightarrow{CD}$ // the x-axis where C (4, 2), D (- 5,y) then find the value of y.

The relation between the slope of the two perpendicular straight lines.

Think and Discuss



The figure opposite : represents the two straight lines L_1, L_2 which their two slopes are m_1, m_2 where $L_1 \perp L_2$.

Find the relation between ∠ E , ∠ J)

Then complete the following table :

Values of E	20°	40°	.....	.....	....
Values of J	......	.....	140°	150°	.....
$\tan E_1 \times \tan J_2$	.....	.....	.....	.....	.....

From the previous table, we deduce that :

$\tan E_1 \times \tan J_2 = -1$ **i. e :** $m_1 \times m_2 = -1$

If L_1, L_2 are two straight lines of slopes m_1, m_2 , where m_1, $m_2 \in R^*$

If $L_1 \perp L_2$ then $m_1 \times m_2 = -1$

i. e: The product of multiplying the slopes of the two perpendicular straight, lines = -1 and vice versa, if $m_1 \times m_2 = -1$, then $L_1 \perp L_2$

i. e: If the product of multiplying the slopes of two straights lines = -1, then the two straight lines are perpendiculars.

Examples

1 Prove that the straight line passing through the two points $(4, 3\sqrt{3})$, $(5, 2\sqrt{3})$ is perpendicular on the straight line that makes with the positive direction to the x-axes to an angle of 30° measure.

Solution

Consider that the slope of first straight line is m_1 and the slope of the second straight line is m_2.

$\because m = \frac{y_2 - y_1}{x_2 - x_1}$

$\because m = \tan E$

$\therefore m_1 \times m_2 = -\sqrt{3} \times \frac{1}{\sqrt{3}} = -1$

$\therefore m_1 = \frac{3\sqrt{3} - 2\sqrt{3}}{4 - 5} = -\sqrt{3}$

$\therefore m_2 = \text{Tan } 30° = \frac{1}{\sqrt{3}}$

$\therefore$ The two straight line are perpendicular .

2 If a triangle with vertices y(4, 2),x (3, 5), Z (-5, A) is right angle at y then find the value of A.

Solution

Find the slope of $\overleftrightarrow{xy}$ thus $m_1 = \frac{5-2}{3-4} = \frac{3}{-1} = -3$, find the slope of thus $m_2 = \frac{A-2}{-5-4} = \frac{A-2}{-9}$

$\because \triangle xyz$ is a right angle at y

$\therefore -3 \times \frac{A-2}{-9} = -1$

$\therefore A - 2 = -3$

$\therefore m_1 \times m_2 = -1$

$\therefore \frac{(A-2)}{3} = -1$

$\therefore A = 2 - 3$

$\therefore A = -1$

Drill

Find the slope of the perpendicular straight line on the straight line through the two points (3, -2), (5, 1) .

Exercises (5-3)

First : Complete the following

1. If $\overleftrightarrow{AB} // \overleftrightarrow{CD}$ and the slope of $\overleftrightarrow{AB} = \frac{2}{3}$ then the slope of $\overleftrightarrow{CD}$ equals ....
2. If $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ and the slope of $\overleftrightarrow{AB} = \frac{1}{2}$ then the solpe of $\overleftrightarrow{CD}$ equals ....
3. The slope of straight line which is parallel to the straight line passing through the two points (2, 3), (-2, 3) equals ....
4. If the straight line $\overleftrightarrow{AB}$ is parallel to x-axis, where A (8, 3), B (2, K) then K = ...
5. If the straight line $\overleftrightarrow{CD}$ is parallel to the y-axis where C (M, 4), D (-5, 7) then M equals
6. A B C is a right angled triangle in B ,A (1, 4), B (-1, -2) then the slope of $\overleftrightarrow{BC}$ equals .....
7. If the straight, line passing throught the two points (A, 0), (0, 3) and the straight line that makes a triangle its measure is 30° with the positive direction to the x-axes are perpendicular then A = ...

Second :

1. Prove that the straight line passing through the two points A (- 3, 4), C (- 3, - 2)is perpendicular on the straight line passing throught the two points B (1, 2), D (- 3, 2) .
2. If A (- 1, - 1), B (2, 3), C (6, 0) prove that the traingle A B C is right angled triangle in B.
3. If the straight line L_1 passes the two points (3, 1), (2, K) and the straight line L_2 makes with the positive direction to the x-axes a triangle its measure is 45° , then find K. if the two straightline L_1, L_2 :

   A Parallel B Perpendicular
4. If the points (0, 1), (A, 3), (2, 5) are located on one straight line. Then find the value of A.
5. Prove that the points A (- 1, 1), B (0, 5), C (4, 2), D (5, 6) are the vertices of the parallelogram.
6. Prove by using the slope that the points A (- 1, 3), B (5, 1), C (6, 4), D (0, 6) are the vertices of the rectangle .
7. In the figure drawn :
   A B C D is trapezoid, $\overline{AB} // \overline{CD}$,
   A (9, -2), B (3, 2), C (x, -x),
   D (4, -3), Find the coordinates of the point C.

8. Prove that the points A (4 , 3), B (7, 0), C (1, -2) are vertices of the triangle, and if the point of D (1, 2) then prove that the figure A B C D is trapezoid and find the ratio between A D , B C.

5 - 4

The Equation of the straight line given its slope and its y - intercept

Think and Discuss

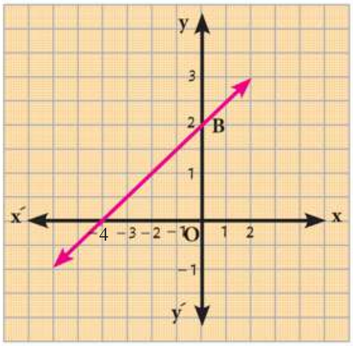
You learned the linear relation between two variables x , y, it is :

A x + B y+ C = 0 where A, B **(each of both)** ≠ 0

Is represented graphically by a straight line .

Example

Represent the relation :
x -2y+ 4 = 0 graphically .
From the graphical figure, calculate:



A The slope of the straight line .

B The lenght of the vertical part included between the origin point and the intesection point of the straight line with y - axis.

Solution to make the drawing easier, select the intersection point of the 2 axes: as follows :

y= 0	∴x + 4 = 0	
∴x = -4	(-4 , 0)	**satisfies the relation.**
x = 0	∴ -2y+ 4 = 0	
∴ 2y= 4	(0, 2)	**satisfies the relation**

From the drawing we find that the slope of the straight line

(m) > 0 ..... **(why?)** thus, $m = \frac{......}{......} = \frac{......}{......}$

The distance between the 2 points o and B are called the y - intercept .

intercept and is equal to 2 unit lenght and is denoted by the symbol (b).

The previous equation is written as: y = m x + b

thus, 2y = x + 4 and by dividing both sides by 2

$\therefore y = \frac{1}{2} x + 2$

We notice in this form that:

The slope the straight line (m) which is the coefficient of x equals $\frac{1}{2}$, and the lenght of y- intercept b = 2 and these are the same results we got the previous drawing .

What you'll learn

- Finding the equation of the straight line with given the slope and the intersected part from the y - axis.
- Finding the equation of the straight line given its slope and its Y-intercept.

Key terms

- Equation of straight line.
- Slope of a straight line.
- y - intercept .

Defintion

The equation of the straight line with respect to its slope (m) and the y - intercept (b).

Is $y = m x + b$ where $m \in R$

Notice that : The equation of the straight line is written: $ax + by + c = \text{zero}, b \neq 0$

In the fromula: $y = m x + b$ as the following :

$ax + by + c = \text{zero}$ thus $by = ax - c$

$\therefore y = -\frac{a}{b} x - \frac{c}{b}$ and it is in the formula: $y = m x + c$

Where $m = \frac{-a}{b} = \frac{-\text{ Coefficient } x}{\text{Coefficient } y}$ Where c is the lenght of the y - intercept .

Examples

1 Find the slope of the straight line $3x + 4y - 5 = \text{zero}$ in two different methods then find the lenght of the y intercept .

Solution

$\because$ The equation of the straight line in the formula of $ax + by + c = 0, b \neq 0$

$\therefore$ The slope of the straight line $= \frac{-a}{b}$ $\therefore$ The slope of the straight line $= \frac{-3}{4}$

or : it is written in the formula of $y = mx + c$

$\therefore 4y = -3x + 5$ $y = \frac{-3}{4} x + \frac{5}{4}$

$\therefore$ The slope of the straight line $= \frac{-3}{4}$ $\therefore$ The length of y - intercept $= \frac{5}{4}$

2 Find the equation of the straight line passing through the point (1, 2) and perpendicular on the straight line passing through the two points A (2, -3), B (5, -4) .

Solution

$\because$ The slope of the straight line passing through the two points a, b $= \frac{-4-(-3)}{5-2} = \frac{-4+3}{5-2} = \frac{-1}{3}$

thus, the slope of the straight line is perpendicular on = 3

$\therefore$ The equation of the straight line is written in the formula: $y = 3x + c$

$\because$ The straight line passes through the point (1, 2) so, it satisfies the equation .

$\therefore 2 = 3 \times 2 + c$

$\therefore c = 2 - 6 = -4$

$\therefore$ The equation of the straight is written in this formula : $y = 3x - 4$

3 If A (- 3, 4), B (5, - 1), C (3, 5) find the equation of the straight line passing through the vertex A and bisecting $\overline{BC}$.

Solution

The midpoint of $\overline{BC} = (\frac{3+5}{2}, \frac{5-1}{2}) = (\frac{8}{2}, \frac{4}{2}) = (4, 2)$

∴ The slope of the required straight line $= \frac{2-4}{4+3} = \frac{-2}{7}$

∵ $y = mx + c$ ∴ $y = \frac{-2}{7}x + c$

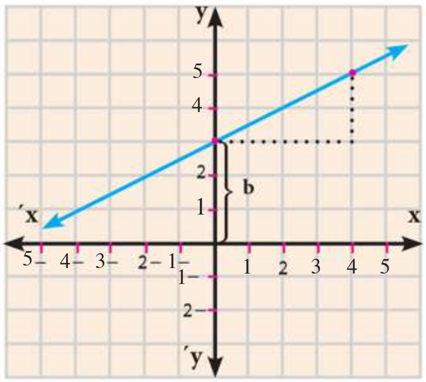
∵ The point of A (-3, 4) passes through the straight line, so it satisfies the equation.

∴ $4 = \frac{-2}{7} \times -3 + c$ ∴ $4 = \frac{6}{7} + c$ ∴ $c = \frac{22}{7}$

∴ The equation of the straight line is written as in the formula:$y = \frac{-2}{7}x + \frac{22}{7}$ and by the multiyipling two sides in 7

∴ $7y = -2x + 22$ **i . e** the equation is : $2x + 7y - 22 = 0$

Drill



1 **In the figure opposite, find the following :**

- **A** The slope of the straight line (m) .
- **B** The length of the y - intercept (c).
- **C** The equation of the straight line with given m and c.
- **D** The length of the x intercept.
- **E** The area of the identified triangle by the x - y receprocal.

Exercises (5-4)

1 If y = m x + b represents the equation of straight line with its given slope and the y- intercept. then complete the following :

- **A** The equation of the straight line, when m = 1, c = 3 is in the form of ....
- **B** The equation of the straight line, when m = -2, c = 1 is in the form of ....
- **C** The equation of the straight line m = 3, c = 0, is in the form of ....

2 Find the slope of the straight line and the length of the y - intercept in each of the following :

A $2x - 3y - 6 = 0$ **B** $5x + 4y - 10 = 0$ **C** $\frac{x}{2} + \frac{y}{3} = 1$

3 **Find the equation of the straight line in the following cases:**

- **A** When its slope is 2 and intersects a positive part from the y-axis that is equals 7 units.

B When its slope is equal to slope of the straight line $\frac{y-1}{x} = \frac{1}{3}$ and intersects a part from the negative direction 3

C Passes by the two points (2, -1) , (1, 1) .

D The equation of the straight line where m = Zero, c = Zero .

4 Draw the straight line in each of the following:

A Its slope equals $\frac{-1}{2}$ and intersects a positive part of the y - axis that is equal to one unit.

B Its slope equals 2 and intersects a negative part of the y - axis equals 3 units.

C Cuts from the two positive parts of the x - y axes two parts, both lenght are 2 , 3 of the units respectively.

5 The following table represents linear relation:

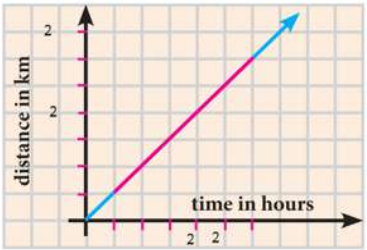
x	1	2	3
y= f (x)	1	3	A

A Find the equation of the straight line.

B Find the length of the intersected part from the y - axis.

C Find the value of A.

6 **In the figure opposite** : The relation between distance the car covers is d in (kilometers), and time the car covers in is t in hour, Find:



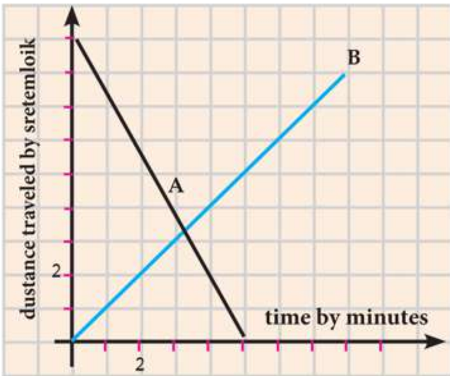
A The distance traveled in 90 minutes.

B The time which in the car traveled 150 kilometers.

C The velocity of the car.

D The equation of the straight line which converts the relation between d and t.

7 The figure opposite represents the distance traveled (D) in kilometers and the time (T) in minutes of the two objects A and B.



A If A, B move at the same time?

B After how many minutes did A, B intersect?

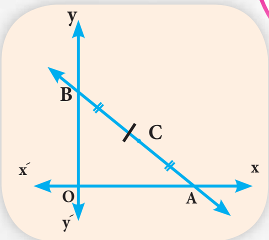
C What is the velocity of A?

D Write the equation of the straight line that represents the relation between the distance and the velocity to the movement of the object B?

Activity

1 **In the figure opposite :**

The point C in the midpoint of $\overline{AB}$ where C (4, 3).



First : Complete the following :

A O A = ... unit length

B O B = ... unit length

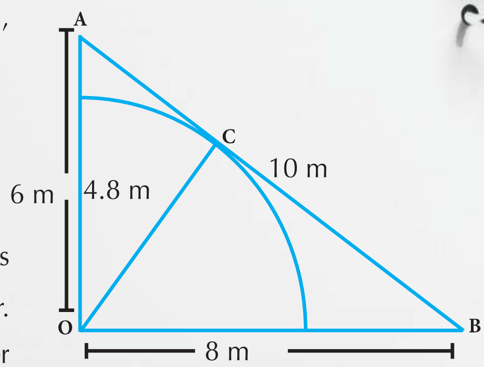
Second : Match between A and B:

Group (A)	Group (B)
A Slope of $\overleftrightarrow{AB}$	-1
B Slope of $\overleftrightarrow{OC}$	$\frac{-3}{4}$
C Slope of $\overleftrightarrow{OA}$	zero
D Slope of $\overleftrightarrow{OB}$	$\frac{3}{4}$
H Slope of $\overleftrightarrow{OB}$ ×	1
Slope of $\overleftrightarrow{OA}$	unknown

Third : Find the coordinates of the points A , B and O then find the equation of $\overleftrightarrow{AB}$, and the equation of $\overleftrightarrow{CO}$.

Four : Find the length of each of $\overline{CA}$, $\overline{CB}$, $\overline{CO}$

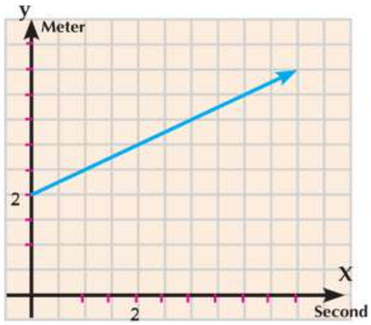
Fifth: Prove in more then or way that C is the center of the circle passing through the points A, O and B.



2 A caw is tied to a point O with rope of 4.8 meters length. If the area is O A B planted with clover. Calculate the area of the cultivated land with clover in which the cow cannot eat, to the nearest meter.

Unit test

1 **The figure opposite :** In the figure opposite a particle moves with a constant speed (v) where the distance (d) is measured by meter and time (t) by second. Find the following:



- **A** The distance at the beginning of moving.
- **B** The velocity of the particle.
- **C** The equation of the straight line which represented the movement of the particle.
- **D** The traveled distance after 4 seconds from the beginning of the movement .
- **E** The time in which the particle covers in distance of 3. 5 meters from the beginning of the movement.

2 Choose the correct answer from the given answers :

1 The slope of the straight line whose equation is $2x - 3y - 6 = 0$:

A -6 **B** -2 **C** $\frac{2}{3}$ **D** 2

2 If the two straight lines $3x - 4y - 3 = 0$ and $ky + 4x - 8 = 0$ are both perpendicular then k equals:

A -4 **B** -3 **C** 3 **D** 4

3 If the two straight lines $x + y = 5$ and $kx + 2y = 0$ are both parallel, then K equals :

A -2 **B** -1 **C** 1 **D** 2

4 The area of the traingle in square unit, identified by straight lines $3x - 4y = 12$, $x = 0$, $y = 0$ equals:

A 6 **B** 7 **C** 12 **D** 12

5 $\overleftrightarrow{AB}$ is a straight line passes through the two points (2, 5) and (5, 2) which of the following points $\in \overleftrightarrow{AB}$

A (1, 6) **B** (2, 3) **C** (0, 0) **D** (3, -4)

6 If A (3, 5), B (2, -1) and C (x,y) then the coordinates of the point C. That makes the triangle A B C a right angle triangle at B is:

A (6, -1) **B** (-4, 5) **C** (3, -2) **D** (8, -2)

3 A (5, -6), B (3, 7) and C (1, -3). Then find the equation of the straight line passes through point A and the midpoint of $\overline{BC}$.

4 Find The equation of the straight line perpendicular to $\overline{AB}$ from its midpoint C where A (1, 3) and B (3, 5) .

5 Find the equation of the straight line passing through the point (3, - 5) and parallel to the straight line $x + 2y - 7 = 0$.

6 Find the equation of the straight line passing through the two points (4, 2) and (-2, -1). Then prove that it passes through the origin point.

7 Find the equation of the straight line which intersects from the x - y axes two positive parts both lengths are 4 and 9 respectively.

8 A B C is a triangle where A (1, 2), B (5, - 2), C (3, 4), D is the midpoint of $\overline{AB}$, drawn $\overleftrightarrow{DE}$ // $\overleftrightarrow{BC}$ and intersects $\overline{AC}$ in E, find the equation of the straight line $\overleftrightarrow{DE}$.

Model tests

Model tests in algebra and statistics

Model 1

First: Choose the correct answer from the given :

1. The point (-3, 4) lies in .... quadrant:
   - A first
   - B second
   - C third
   - D fourth

2. The positive square root of mean of the squares of deviations of values from its arithmetic mean is called.
   - A The range
   - B the arithmetic mean
   - C The standard deviation
   - D the mode

3. If 3a = 4b , then a: b = ...
   - A 3:4
   - B 4:3
   - C 3:7
   - D 4:7

4. If $n(x)= 2$, $n(y^2) = 9$, then $n(x \times y) =$ ....
   - A 6
   - B 18
   - C 11
   - D 7

5. The range of the set of the values 7, 3, 6, 9 and 5 = ...
   - A 3
   - B 4
   - C 6
   - D 12

6. If $y \propto x$ and y = 2 when x = 8, then y = 3 when x = ...
   - A 16
   - B 12
   - C 24
   - D 6

Second:

A If $X \times Y = \{ (2, 2), (2, 5), (2, 7) \}$. Find

First: Y . **Second:** $Y \times X$

B If a, b, c and d are proportional **prove that:**

$$\frac{a}{b-a} = \frac{c}{d-c}$$

Third:

A If X = {2, 3, 5} , Y = { 4, 6, 8, 10} and R is a relation form X to Y where a R b means "2a = b" for all a ∈x, b ∈Y.

First: Write R and represent it by an arrow diagram.

Second: Show that R is a function.

B Find the number that If we add to each terms of the ratio 7:11 it becomes 2:3.

TEST

Fourth:

A If X = { 1, 3, 5} and R is a function on X , where R = { (a, 3) , (b, 1) , (1, 5) }. **Find:**

First: The range of the relation .

Second: The value of a + b.

B If Y $\propto \frac{1}{x}$ and y = 3 when x = 2 . **Find:**

First: The relation between x, y.

Second: The value of y when x = 1.5.

Fifth:

A Represent graphically the function $f(x) = (x-3)^2$, $X \in [0, 6]$ from the graph deduce the vertex of the curve , minimum value of the function , equation of the axis of symmetry.

B Calculate the arithmetic mean and the standard deviation of the set of values 8, 9, 7, 6 and 5.

Model 2

First: Choose the correct answer from the given :

1. The point (3 , 4) lies in .... quadrant:

   A first B second C third D fourth

2. .... is one of the measures of the dispersions.

   A The median B The arithemetic mean C The srandard deviation D The mode

3. The third proportation of the two numbers 3 and 6 is ....

   A $\frac{1}{2}$ B 9 C 2 D 12

4. If n(x) = 2 , n(Y × x) = 6 , then $n(y^2)$ = ...

   A 4 B 9 C 16 D 12

5. The range of the set of the values 7, 3, 6, 9 and 5 = ...

   A 3 B 4 C 6 D 12

6. If xy = 7 , then y∝ = ...

   A $\frac{1}{x}$ B x - 7 C x D x + 7

Second:

A If $x = \{2, 5\}$, $Y = \{1, 2\}$, $Z = \{3\}$. **Find:**

First: $n(X \times Z)$. **Second:** $(Y \cap X) \times Z$.

B If b is a middle proporational between a and c **prove that:**

$$\frac{a - b}{a - c} = \frac{b}{b + c}$$

Third:

A If $X = \{1, 3, 4, 5\}$, $Y = \{1, 2, 3, 4, 5, 6\}$ and R is a relation from X to Y where a R b means $a + b = 7$ For all $a \in X$, $6 \in Y$.

First: Write R and represent it by an arrow diagram.

Second: Show that R is a function.

B If $5a = 3b$. Find the value of: $\frac{7a + 9b}{4a + 2b}$

Fourth:

A If $f(x) = 4x + b$ and $f(3) = 15$ find the value of b.

B If $Y \propto X$, $y = 6$ when $x = 3$. **Find:**

First: The relation between X, Y.

Second: The value of y when $X = 5$.

Fifth:

A Represent graphically the function $f(x) = 4 - X^2$, $X \in [-3, 3]$ from the graph deduce the vertex of the curve, maximum value of the function , equation of the axis of symmetry.

B **The following frequency distribution shows the number of childern of some families in a new city:**

Number of children	0	1	2	3	4	sum
Number of families	6	15	40	25	14	100

Calculate **the mean and the standard deviation to the number of children.**

Model 3

(Merge Student's)

First: Complete:

1. The point (5, 3) lies in .... quadrant
2. $n(x) = X^3 + 8$ is called a polynomail of .... degree
3. The range of the set of the values 4, 14, 25, and 34 is ....

4. If y = 2x , then y ∝ ....

5. If X = {2, 4, 6} , then $n(x^2)$ = ...

6. If (a, 3) = (6, b) , then a +b= ....

Second: Choose the correct answer:

1. If xy = 7, then y∝ ....

   A $\frac{1}{x}$ B x - 7 C x D x + 7

2. If 2, 3, 6 and X are proportional , then x = ...

   A 9 B 18 C 12 D 3

3. If 2a = 5 b , then $\frac{a}{b}$ = ...

   A $\frac{-5}{2}$ B $\frac{-2}{5}$ C $\frac{2}{5}$ D $\frac{5}{2}$

4. .... is one of the measures of the dispersions

   A the arithemetic mean B The range

   C the mode D The median

5. If n(x) = 5 , n(x × Y) =10 , then n(Y) = ...

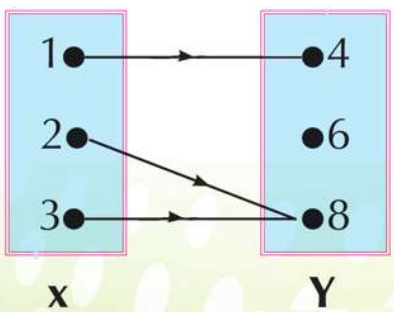
   A 4 B 3 C 2 D 1

6. If x = {1}, then x^2 = ...

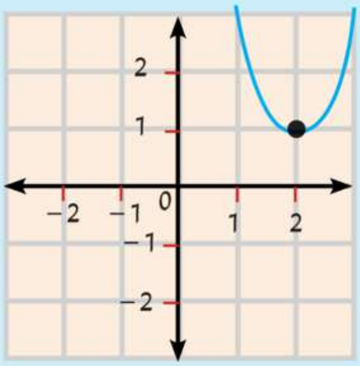
   A 1 B (1,1) C {(1,1)} D {1}

Third: Put (✓) or (✗):

1. If the relation of f = { (1, 3) , (2, 4) , (3, 3)}, then the domain of the function {1, 2, 3} ( )
2. If y∝x and y = 6 when x = 3 , then y = 2 when x = 4 ( )
3. If $\Sigma(x-\overline{x})^2$ = 36 for a set of values whose number equals 9 , then σ 4 ( )
4. The intersection point of the straight line f(x) = x +2 with x-axis is the point (-2, 0) ( )
5. If f:x ⟶ Y then x is called the domain of this function ( )
6. The arrow diagram from X to Y is a function ( )



Fourth: join from Column (A) to Column (B):

A	B
(1) If (1, 4) ∈ { 2, x} X { 1, 4 },then X=	6
(2) If The Function f Which f(X) = X - 4 is represented graphically By a Straight Line Passes therough the Point (a , 2),then a = ...	1
(3) $\frac{1}{2} = \frac{3}{6} = \frac{4}{8} = \frac{...}{16}$	10
(4) If f (x) = 5 , then f (5) + f(-5) = ...	±6
(5) The third proprational of the two numbers 4 and 9 is	2
(6) In the opposite figure the equation of the line of symmetry is x = ... 	8

Model tests on gemoetry and Trigonometry

Model 1

First: Choose the correct answer:

1. Tan 45º = ...
   - A) 1
   - B) $2\sqrt{2}$
   - C) $\frac{1}{2}$
   - D) $\sqrt{2}$

2. If Sin x = $\frac{1}{2}$, X is an acute angle, then m (∠ X) = ...
   - A) 45
   - B) 60
   - C) 30
   - D) 90

3. The distance between the two points (3, 0) , (0, -4) = ...
   - A) 4
   - B) 5
   - C) 6
   - D) 7

4. If X + y = 5 , Kx + 2y = 0 are perpendicular , than K = ...
   - A) -2
   - B) -1
   - C) 1
   - D) 2

5. If A (5, 7) , B (1, -1) , then the mid-point $\overline{AB}$ is ....
   - A) (2, 3)
   - B) (3, 3)
   - C) (3, 2)
   - D) (3, 4)

6. The equation of the sraight line which passes through the point (3, -5) and parallel to Y-axis is..........
   - A) x= 3
   - B) y = -5
   - C) y = 2
   - D) x = -5

Second:

A) Whithout using calculator prove that sin 60º = 2 sin 30º Cos 30º

B) Prove that the points A (-3, -1) , B (6, 5) , C(3, 3) are collinear

Third:

A) If 4 cos 60º sin 30º = tan x . Find the value of x, then x is an acute angle.

B) If the mid-point of $\overline{AB}$ is c(6, -4) then A (5, -3) Find the point B.

Fourth:

A If the straight line L_1 passes through the points (3, 1), (2, K) and the straight line L_2 makes with the positive direction of the x--axis anhgle of measure 45° . Find the value of K if $L_1 // L_2$

B ABC is a right angled triangle at , AC = 6cm , BC = 8cm find

First: Cos A cos B - sin A sin B .

Second: m (∠B).

Fifth:

A Find the equation of the straight line which slope is 2 and passes through the point (1, 0).

B Prove that the points A(3, -1) , B (-4, 6) , C (2, -2) which belong to an orthogonal certesian co-ordinates plane lie on the circle whose centre M(-1, 2) . Find the circumference of the circle.

Model 2

First: Choose the correct answer:

1. 2 sin 30° tan 60° = ...:

   A $\sqrt{3}$ B 3 C $\frac{\sqrt{3}}{3}$ D $\frac{1}{2}$

2. The equation of the straight line which passes through the point (-2,-3) and parallel to x-axis is ....

   A x = -2 B x = -3 C y = −2 D y = −3

3. If Cos x = $\frac{\sqrt{3}}{2}$, X is acute angle , then sin 2x = ...

   A 1 B $\frac{\sqrt{3}}{2}$ C -2 D $\frac{1}{\sqrt{3}}$

4. A circle of centre at the origin point and its redius is 2 unit length which of the following points belongs to the circle ?

   A (1,-2) B (-2,$\sqrt{5}$) C ($\sqrt{3}$,1) D (0,1)

5. The perpendicular distance between the two straight lines x - 2 = 0 , x + 3 = 0 equals .....

   A 1 B 5 C 2 D 3

Test

6 If $\frac{-3}{2}$, $\frac{6}{k}$ are the slopes of two parallel straight lines then k = ...

A 6 B -4 C $\frac{3}{2}$ D 2

Second:

A If cos E tan 30° = Cos ^{2} 45° find m(∠E) , E is a cute angle

B Show the type of the triangle whose vertices A(3,3) m B (1, 5) , C(1, 3) due to its side lengths.

Third:

A Find the equation of straight line which passes through the points (1, 3) , (-1, -3) and prove that it is passing through the origin point.

B If the point (3, 1) is the mid-point of (1, y) , (x, 3) find the point of (x, y).

Fourth:

A Find the equation of the straight line which intercepts two axes . Two positive parts of length 1 and 4 for x and y axes respectively and find its slope

B ABC is a right - angled triangle at B AC = 10cm BC = 8cm , prove that $\sin^2 A + 1 = 2\cos^2 C + \cos^2 A$

Fifth:

A prove that the straight line which passes through the points (-1, 3) , (2, 4) parallel to the straight line 3y - X - 1 = 0

B ABCD is a trapezium , $\overline{AD}$ // $\overline{BC}$ m(∠B) = 90°, AB = 3cm , BC = 6cm , AD = 2cm . Find the length of $\overline{DC}$ and the value of Cos ∠BCD

Model 3

(Merge Student's)

First: Put (✓) or (✗):

1. The distance between the point (9,0), (4,0) = 5 ( )
2. If Tan E = 1 , then: m ($\angle$ E) = 45° ( )
3. The straight line y = 2×+1 intercepts a part of length - 1 for y - axis ( )
4. If $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$, then the slope of $\overleftrightarrow{AB}$ × The slope of $\overleftrightarrow{CD}$ = 1 (both of $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ aren`t parallal any axes) ( )
5. Tan 60° = $\frac{1}{\sqrt{3}}$ ( )
6. If A (1, 2) , B (3, 4) , then the coordinates of the midpoint of $\overline{AB}$ is (2, 3) ( )

Second: Choose the correct answer form given:

1. The distance between the point (4,3) and x - axis is ....
   - A -3
   - B 3
   - C 4
   - D -4
2. 4 cos 30° Tan 60° = ...
   - A 3
   - B $2\sqrt{3}$
   - C 6
   - D 12
3. If X + y = 5 , kx + 2y = 0 are parallel , than k ....
   - A -2
   - B -1
   - C 1
   - D 2
4. The points (0, 1) , (3, 0), (0, 4) ....
   - A from a right angled triangle
   - B from a acute angled triangle
   - C from an obtuse angled triangle
   - D are collinear
5. If $\overleftrightarrow{AB}$ // $\overleftrightarrow{CD}$ and the slope of $\overleftrightarrow{AB}$ = $\frac{2}{3}$, then the slope of $\overleftrightarrow{CD}$ = ...
   - A $\frac{2}{3}$
   - B $\frac{3}{2}$
   - C $\frac{-2}{3}$
   - D $\frac{-3}{2}$
6. If sin x = $\frac{1}{2}$, x acute angle, then sin 2x = ...
   - A 1
   - B $\frac{1}{4}$
   - C $\frac{\sqrt{3}}{2}$
   - D $\frac{1}{\sqrt{3}}$

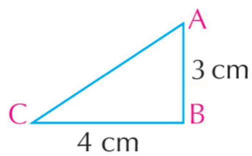
Test

Third: Join From column (A) to column (B):

A	B
1 The slope of the straight line which parallel to x - axis is	10
2 $\sin^2 30^\circ + \cos^2 30^\circ$ = ...	0
3 If ABCD is a rectangle A (-1, -4) , C (5, 4) then the length of $\overline{BD}$ = ... unit lenght	1
4 The equation of the straight line which passes through the origin point and its slope is 2 is Y = ... x	-3
5 The equation of the straight Line which passes through the point (2, -3) , parallel x - axis y = ...	2
6 The value of $\frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ}$ = ...	$\frac{\sqrt{3}}{2}$

Fourth: Complete the following

1. If $\overline{AB}$ // $\overline{CD}$ and the slope of $\overline{AB}$ = $\frac{1}{2}$, then
   The slope of $\overline{CD}$ = ...
2. The opposite figure: ABC is a right angle
   at B, AB = 3 cm , BC = 4cm, then
   Sin C = ...



3. If the point (0, a) belongs to straight line
   3x - 4y = -12 , then a = ...
4. If X cos 60° = tan 45°, then x = ...
5. The distance between the point (4, 3) and the origin point in the coorinate plane = ...
6. If the origin point is the mid - point of $\overline{AB}$, A(5, -2), then B (......, ...)

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