كل ما يحتاجه الطالب في جميع الصفوف من أوراق عمل واختبارات ومذكرات، يجده هنا في الروابط التالية لأفضل مواقع تعليمي إماراتي 100 %

تطبيق المناهج الإماراتية	الاجتماعيات	الرياضيات
الصفحة الرسمية على التلغرام	الاسلامية	العلوم
الصفحة الرسمية على الفيسبوك	الانجليزية	
التربية الاخلاقية لجميع الصفوف	اللغة العربية	
التربية الرياضية		

مجموعات التلغرام.	مجموعات الفيسبوك	قنوات تلغرام	قنوات الفيسبوك
الصف الأول	الصف الأول	الصف الأول	الصف الأول
الصف الثاني	الصف الثاني	الصف الثاني	الصف الثاني
الصف الثالث	الصف الثالث	الصف الثالث	الصف الثالث
الصف الرابع	الصف الرابع	الصف الرابع	الصف الرابع
الصف الخامس	الصف الخامس	الصف الخامس	الصف الخامس
الصف السادس	الصف السادس	الصف السادس	الصف السادس
الصف السابع	الصف السابع	الصف السابع	الصف السابع
الصف الثامن	الصف الثامن	الصف الثامن	الصف الثامن
الصف التاسع عام	الصف التاسع عام	الصف التاسع عام	الصف التاسع عام
الصف التاسع متقدم	الصف التاسع متقدم	الصف التاسع متقدم	تاسع متقدم
الصف العاشر عام	الصف العاشر عام	الصف العاشر عام	عاشر عام
الصف العاشر متقدم	الصف العاشر متقدم	الصف العاشر متقدم	عاشر متقدم
الحادي عشر عام	الحادي عشر عام	الحادي عشر عام	حادي عشر عام
الحادي عشر متقدم	الحادي عشر متقدم	الحادي عشر متقدم	حادي عشر متقدم
ثاني عشر عام	الثاني عشر عام	الثاني عشر عام	الثاني عشر عام
ثاني عشر متقدم	الثاني عشر متقدم	ثاني عشر متقدم	ثاني عشر متقدم

Chapter 8: Systems of Particles and Extended Objects

Concept Checks

8.1. b **8.2.** a **8.3.** d **8.4.** b **8.5.** a

Multiple-Choice Questions

8.1. d **8.2.** b **8.3.** d **8.4.** b and d **8.5.** e **8.6.** a **8.7.** b **8.8.** d **8.9.** b **8.10.** e **8.11.** a **8.12.** c **8.13.** a **8.14.** b **8.15.** b **8.16.** b

Conceptual Questions

8.17. It is reasonable to assume the explosion is entirely an internal force. This means the momentum, and hence the velocity of the center of mass remains unchanged. Therefore, the motion of the center of mass remains the same.

8.18. The length of the side of the cube is given as d. If the cubes have a uniform mass distribution, then the center of mass of each cube is at its geometric center. Let m be the mass of a cube. The coordinates of the center of mass of the structure are given by:

$$X_{\text{cm}} = \frac{m\left(\frac{d}{2}+\frac{d}{2}+\frac{d}{2}+\frac{3d}{2}\right)}{4m} = \frac{3d}{4}, \quad Y_{\text{cm}} = \frac{m\left(\frac{d}{2}+\frac{d}{2}+\frac{d}{2}+\frac{3d}{2}\right)}{4m} = \frac{3d}{4} \quad \text{and} \quad Z_{\text{cm}} = \frac{m\left(\frac{d}{2}+\frac{d}{2}+\frac{d}{2}+\frac{3d}{2}\right)}{4m} = \frac{3d}{4}.$$

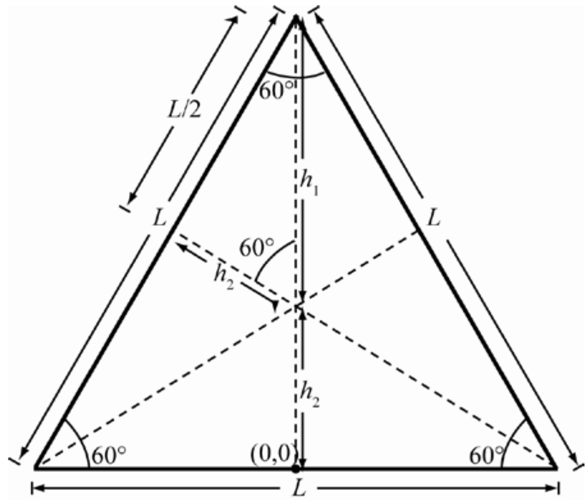
Therefore, the center of mass of the structure is located at $\vec{R} = \left(X_{\text{cm}},\ Y_{\text{cm}},\ Z_{\text{cm}}\right) = \left(\frac{3d}{4},\frac{3d}{4},\frac{3d}{4}\right)$.

8.19. After the explosion, the motion of the center of mass should remain unchanged. Since both masses are equal, they must be equidistant from the center of mass. If the first piece has x-coordinate x_1 and the second piece has x-coordinate x_2, then $\left|X_{\text{cm}} - x_1\right| = \left|X_{\text{cm}} - x_2\right|$. For example, since the position of the center of mass is still 100 m, one piece could be at 90 m and the other at 110 m: $\left|100-90\right| = \left|100-110\right|$.

8.20. Yes, the center of mass can be located outside the object. Take a donut for example. If the donut has a uniform mass density, then the center of mass is located at its geometric center, which would be the center of a circle. However, at the donut's center, there is no mass, there is a hole. This means the center of mass can lie outside the object.

8.21. It is possible if, for example, there are outside forces involved. The kinetic energy of an object is proportional to the momentum squared $\left(K \propto p^2\right)$. So if p increases, K increases.

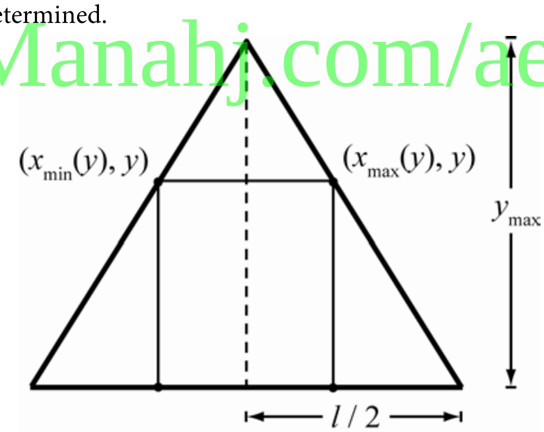
8.22. The intersection of the triangle's altitudes implies the triangle has a uniform mass density, meaning the center of mass is at the geometric center. To show this point by physical reasoning means using geometry to show where it is.



It can be seen that $h_1 \sin 60° = L/2$ and $h_1 \cos 60° = h_2$. Therefore,

$$h_1 = \frac{L}{2\sin 60°} = \frac{L}{2\left(\sqrt{3}/2\right)} = \frac{L}{\sqrt{3}} = \frac{L\sqrt{3}}{3} \Rightarrow h_2 = \frac{L\sqrt{3}}{3}\cos 60° = \frac{L\sqrt{3}}{3}\left(\frac{1}{2}\right) = \frac{L\sqrt{3}}{6}.$$

If the center of the bottom side of the triangle is (0, 0), then the center of mass is located at $\left(0, h_2\right) = \left(0, L\sqrt{3}/6\right)$. To calculate by direct measurement, note that due to symmetry by the choice of origin, the x coordinate of the center of mass is in the middle of the x axis. Therefore, $X_{\text{cm}} = 0$, which means only Y_{cm} must be determined.



Clearly, the x value of a point along the side of the triangle is dependent on the value of y for that point, meaning x is a function of y. When y is zero, x is $L/2$ and when x is zero, y is $y_{\max} = h_1 + h_2 = L\sqrt{3}/2$. The change in x should be linear with change in y, so $x = my + b$, where $m = \frac{\Delta x}{\Delta y} = \frac{\left(L/2\right) - 0}{0 - \left(L\sqrt{3}/2\right)} = -\frac{1}{\sqrt{3}}$.

Therefore, $\frac{L}{2} = -\frac{0}{\sqrt{3}} + b = 0 + b \Rightarrow b = \frac{L}{2}$ and $0 = -\frac{L\sqrt{3}}{2\sqrt{3}} + b = -\frac{L}{2} + b \Rightarrow b = \frac{L}{2}$. The equation for x is then given by $x\left(y\right) = -\frac{y}{\sqrt{3}} + \frac{L}{2}$. Since the mass density is uniform, the geometry of the triangle can be considered. $Y_{\text{cm}} = \frac{1}{A}\iint y\,dA$, where $A = \frac{L^2\sqrt{3}}{4}$ and $dA = dxdy$.

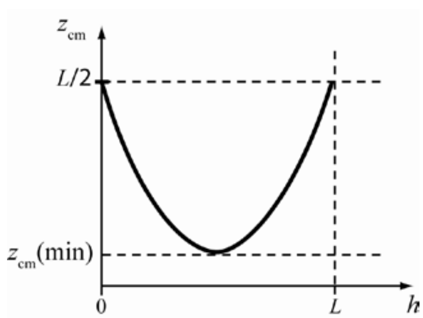
The integral then becomes:

$$Y_{\mathrm{cm}} = \frac{4}{L^2\sqrt{3}} \int_{y_{\min}}^{y_{\max}} y dy \int_{x_{\min}(y)}^{x_{\max}(y)} dx = \frac{4}{L^2\sqrt{3}} \int_0^{\frac{L\sqrt{3}}{2}} y\left(x_{\max}(y) - x_{\min}(y)\right) dy.$$ Due to symmetry, $x_{\max}(y) = -x_{\min}(y)$ and $x_{\max}(y) = x(y)$. Therefore,

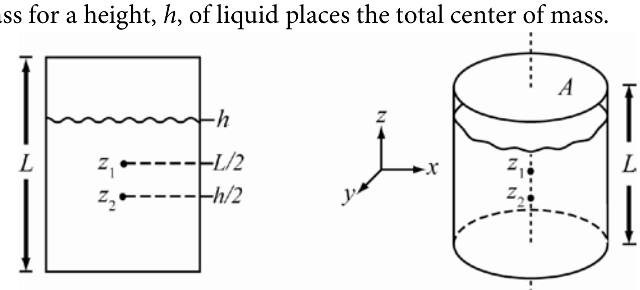
$$\begin{aligned} Y_{\mathrm{cm}} &= \frac{8}{L^2\sqrt{3}} \int_0^{\frac{L\sqrt{3}}{2}} yx(y) dy = \frac{8}{L^2\sqrt{3}} \int_0^{\frac{L\sqrt{3}}{2}} \left(\frac{-y^2}{\sqrt{3}} + \frac{yL}{2} \right) dy \\ &= \frac{8}{L^2\sqrt{3}} \left[\frac{y^2 L}{4} - \frac{y^3}{3\sqrt{3}} \right]_0^{\frac{L\sqrt{3}}{2}} = \frac{8}{L^2\sqrt{3}} \left[\frac{3L^3}{16} - \frac{L^3}{8} \right] = \frac{8}{L^2\sqrt{3}} \left[\frac{L^3}{16} \right] \\ &= \frac{L}{2\sqrt{3}} = \frac{L\sqrt{3}}{6}. \end{aligned}$$

The center of mass is located at $R = (X_{\mathrm{cm}}, Y_{\mathrm{cm}}) = \left(0, \frac{L\sqrt{3}}{6}\right)$. This is consistent with reasoning by geometry. √

8.23. (a) The empty can and the liquid should each have their centers of mass at their geometric centers, so initially the center of mass of both is at the center of the can (assuming the can is filled completely with soda). Assuming the liquid drains out uniformly, only the height changes and the cross sectional area remains constant, so the center of mass is initially at *L*/2 and changes only in height. As liquid drains, its mass *M* will drop by ΔM but the mass of the can, *m*, remains the same. As liquid drains, its center of mass will also fall such that if the liquid is at a height *h*, $0 < h < L$, its center of mass is at *h*/2. As long as $M - \Delta M > m$, the center of mass of both will also fall to some height h', $h/2 < h' < L$. Once $M - \Delta M < m$, the center of mass of both will begin to increase again until $M - \Delta M = 0$ and the center of mass is that of just the can at *L*/2. A sketch of the height of the center of mass of both as a function of liquid height is shown below.



(b) In order to determine the minimum value of the center of mass in terms of *L*, *M* and *m*, first consider where the center of mass for a height, *h*, of liquid places the total center of mass.



Z_1 is the center of mass of the can. Z_2 is the center of mass of the liquid. Notice the center of mass moves along the *z* axis only. *A* is the cross sectional area of the can in the *xy* plane. ρ_M is the density of the liquid. *h* is the height of the liquid.

The coordinate of the center of mass is given by

$$Z_{\text{cm}} = \frac{\frac{mL}{2}+\frac{Mh}{2}}{m+M}.$$

When $h = L$, $Z_{\text{cm}} = L/2$. When $h < L$, $h = \alpha L$, where $0 \le \alpha < 1$. In other words, the height of the liquid is a fraction, α, of the initial height, L. Initially the mass of the liquid is $M = \rho V = \rho AL$. When $h(\alpha) = \alpha L$, the mass of the liquid is $M(\alpha) = \rho Ah(\alpha) = \alpha\rho AL = \alpha M$. This means the center of mass for some value of α is

$$Z_{\text{cm}}(\alpha) = \frac{\frac{mL}{2}+\frac{M(\alpha)h(\alpha)}{2}}{m+M(\alpha)} = \frac{\frac{mL}{2}+\frac{\alpha^2 ML}{2}}{m+\alpha M} = \frac{L}{2}\left(\frac{1+b\alpha^2}{1+b\alpha}\right).$$

where $b = M/m$ and M is the initial mass of the liquid. In order to determine the minimum value of Z_{cm}, $Z_{\text{cm}}(\alpha)$ must be minimized in terms of α to determine where α_{min} occurs and then determine $Z_{\text{cm}}(\alpha_{\text{min}})$.

$$\frac{dZ_{\text{cm}}(\alpha)}{d\alpha} = a\frac{d}{d\alpha}\left(\frac{1+b\alpha^2}{1+b\alpha}\right) = a\left[\frac{b^2\alpha^2+2b\alpha-b}{(1+b\alpha)^2}\right], \text{ where } a = L/2.$$

When $dZ_{\text{cm}}(\alpha)/d\alpha = 0 \Rightarrow b^2\alpha^2 + 2b\alpha - b = 0$. Using the quadratic equation, $\alpha = \dfrac{-1\pm\sqrt{1+b}}{b}$. Since $b > 0$ and $\alpha > 0$, $\alpha_{\text{min}} = \dfrac{-1+\sqrt{1+b}}{b}$. Therefore, $Z_{\text{cm}}(\alpha_{\text{min}}) = a\left(\dfrac{1+b\alpha_{\text{min}}{}^2}{1+b\alpha_{\text{min}}}\right) = 2a\left(\dfrac{1+b-\sqrt{1+b}}{b\sqrt{1+b}}\right)$.

$$Z_{\text{cm}}(\alpha_{\text{min}}) = \frac{L\left(M+m-m\sqrt{1+\frac{M}{m}}\right)}{M\sqrt{1+\frac{M}{m}}}$$

If it is assumed that soda has a similar density to water and the can is made of aluminum, then the ratio of $M/m \approx 30$, giving a minimum Z_{cm} of about $L/6$.

8.24. (a) If the astronaut throws both at the same time, he gains their momentum of them moving at a velocity, v. If he throws one first at a velocity, v, he will recoil back at a velocity, v'. So when he throws the second item, he will gain its momentum at a velocity of $v - v'$, which is less than v. So he gains less momentum from throwing the second item after the first than if he throws both items at the same time. Therefore, he obtains maximum speed when he throws both at the same time.

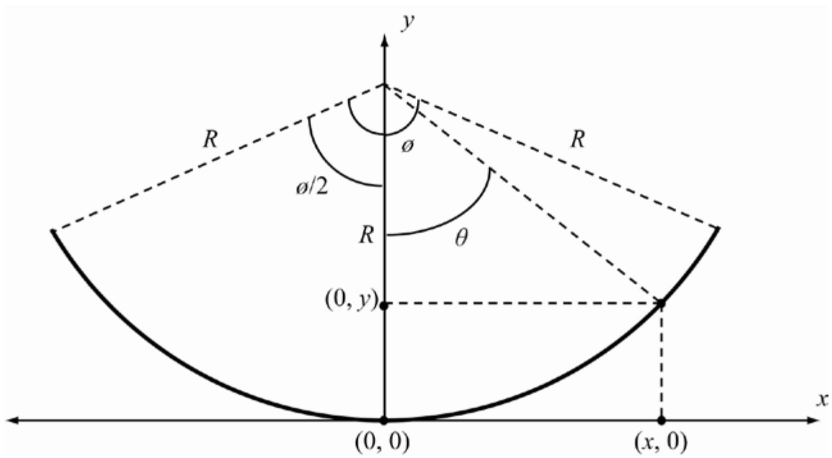
(b) If the astronaut throws the heavier object (tool box) first, it will give the astronaut a large velocity, v', so when he throws the lighter object (hammer), it will have a small velocity of $v - v'$. So its momentum contribution will be very small. However, if he throws the lighter item first, v' will be smaller in this scenario, so the momentum of the box will be dependent on $v - v'$, which is greater and contributes a large amount of momentum to the astronaut, giving him a larger velocity. Therefore, throwing the lighter object first will maximize his velocity.

(c) The absolute maximum velocity is when both items are thrown at the same time. Initially the momentum is zero and after the toss, the astronaut travels with velocity, v' and the box and hammer travel with velocity, v in the opposite direction.

$$\vec{p}_{\text{i}} = \vec{p}_{\text{f}} \Rightarrow 0 = Mv' - \left(\frac{M}{2}+\frac{M}{4}\right)v \Rightarrow v' = \frac{3}{4}v$$

Therefore, the maximum velocity is ¾ of the velocity at which he throws the two items.

8.25. Let the angle θ sweep through from $-\phi/2$ to $\phi/2$. Keeping R constant as θ increases, the length of the rod, $l = R\theta$, increases and in turn the mass, $m = \lambda l$, increases. Since the mass is uniformly distributed, the center of mass should be in the same location. So rather than bending a rod of constant length where θ and R change, keep R constant and change θ and l. Use Cartesian coordinates to determine the center of mass. Since the center of mass is a function of θ, it must be determined how the coordinates change with the angle θ.



$$y = R - R\cos\theta,\quad x = R\sin\theta,\quad m = \lambda R\phi,\quad dm = \lambda R\, d\theta$$

$$X_{\text{cm}} = \frac{1}{m}\int x\,dm = \frac{1}{\lambda R\phi}\int_{-\phi/2}^{\phi/2} R\sin\theta\lambda R d\theta = \frac{R}{\phi}\int_{-\phi/2}^{\phi/2}\sin\theta\, d\theta = \left[-\frac{R}{\phi}\cos\theta\right]_{-\frac{\phi}{2}}^{\frac{\phi}{2}} = -\frac{R}{\phi}\left(\cos\frac{\phi}{2} - \cos\left(-\frac{\phi}{2}\right)\right) = 0$$

$$Y_{\text{cm}} = \frac{1}{m}\int y\,dm = \frac{1}{\lambda R\phi}\int_{-\phi/2}^{\phi/2}(R - R\cos\theta)\lambda R d\theta = \frac{R}{\phi}\int_{-\phi/2}^{\phi/2}(1-\cos\theta)d\theta = \left[\frac{R}{\phi}(\theta - \sin\theta)\right]_{-\frac{\phi}{2}}^{\frac{\phi}{2}}$$

$$= \frac{R}{\phi}\left(\frac{\phi}{2} - \left(-\frac{\phi}{2}\right)\right) - \frac{R}{\phi}\left(\sin\left(\frac{\phi}{2}\right) - \sin\left(-\frac{\phi}{2}\right)\right) = R - \frac{2R\sin\left(\frac{\phi}{2}\right)}{\phi}$$

$$\vec{R}_{\text{cm}} = (X_{\text{cm}}, Y_{\text{cm}}) = \left(0, R - \frac{2R\sin(\phi/2)}{\phi}\right)$$

8.26. As eggs *A*, *B* and/or *C* are removed, the center of mass will shift down and to the left. To determine the overall center of mass, use the center of the eggs as their center position, such that eggs *A*, *B* and *C* are located respectively at

$$\left(\frac{d}{2},\frac{d}{2}\right),\quad \left(\frac{3d}{2},\frac{d}{2}\right),\quad \left(\frac{5d}{2},\frac{d}{2}\right).$$

Since all of the eggs are of the same mass, *m*, and proportional to *d*, *m* and *d* can be factored out of the equations for X_{cm} and Y_{cm}.

(a) $X_{\text{cm}} = \dfrac{md}{11m}\left(2\left(-\dfrac{5}{2}\right)+2\left(-\dfrac{3}{2}\right)+2\left(-\dfrac{1}{2}\right)+\dfrac{1}{2}+2\left(\dfrac{3}{2}\right)+2\left(\dfrac{5}{2}\right)\right) = -\dfrac{d}{22},\ Y_{\text{cm}} = \dfrac{md}{11m}\left(6\left(-\dfrac{1}{2}\right)+5\left(\dfrac{1}{2}\right)\right) = -\dfrac{d}{22}$

$$\vec{R}_{\text{cm}} = \left(-\frac{d}{22}, -\frac{d}{22}\right)$$

(b) $X_{\text{cm}}=\dfrac{md}{11m}\left(2\left(-\dfrac{5}{2}\right)+2\left(-\dfrac{3}{2}\right)+2\left(-\dfrac{1}{2}\right)+2\left(\dfrac{1}{2}\right)+\dfrac{3}{2}+2\left(\dfrac{5}{2}\right)\right)=-\dfrac{3d}{22},\ Y_{\text{cm}}=\dfrac{md}{11m}\left(6\left(-\dfrac{1}{2}\right)+5\left(\dfrac{1}{2}\right)\right)=-\dfrac{d}{22}$

$\vec{R}_{\text{cm}}=\left(-\dfrac{3d}{22},-\dfrac{d}{22}\right)$

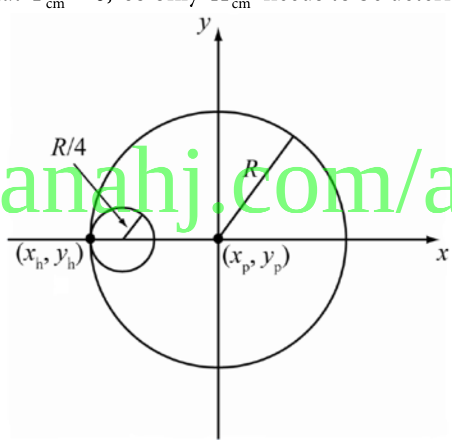
(c) $X_{\text{cm}}=\dfrac{md}{11m}\left(2\left(-\dfrac{5}{2}\right)+2\left(-\dfrac{3}{2}\right)+2\left(-\dfrac{1}{2}\right)+2\left(\dfrac{1}{2}\right)+2\left(\dfrac{3}{2}\right)+\dfrac{5}{2}\right)=-\dfrac{5d}{22},\ Y_{\text{cm}}=\dfrac{md}{11m}\left(6\left(-\dfrac{1}{2}\right)+5\left(\dfrac{1}{2}\right)\right)=-\dfrac{d}{22}$

$\vec{R}_{\text{cm}}=\left(-\dfrac{5d}{22},-\dfrac{d}{22}\right)$

(d) $X_{\text{cm}}=\dfrac{md}{9m}\left(2\left(-\dfrac{5}{2}\right)+2\left(-\dfrac{3}{2}\right)+2\left(-\dfrac{1}{2}\right)+\dfrac{1}{2}+\dfrac{3}{2}+\dfrac{5}{2}\right)=-\dfrac{d}{2},\qquad Y_{\text{cm}}=\dfrac{md}{9m}\left(6\left(-\dfrac{1}{2}\right)+3\left(\dfrac{1}{2}\right)\right)=-\dfrac{d}{6}$

$\vec{R}_{\text{cm}}=\left(-\dfrac{d}{2},-\dfrac{d}{6}\right)$

8.27. The center of the pizza is at $(0,0)$ and the center of the piece cut out is at $(-3R/4,0)$. Assume the pizza and the hole have a uniform mass density (though the hole is considered to have a negative mass). Then the center of mass can be determined from geometry. Also, because of symmetry of the two circles and their *y* position, it can be said that $Y_{\text{cm}}=0$, so only X_{cm} needs to be determined.



$A_{\text{p}}=\pi R^2,\ A_{\text{h}}=\pi\left(\dfrac{R}{4}\right)^2=\pi\dfrac{R^2}{16},\quad (x_{\text{p}},y_{\text{p}})=(0,0),\quad (x_{\text{h}},y_{\text{h}})=\left(-\dfrac{3}{4}R,0\right)$

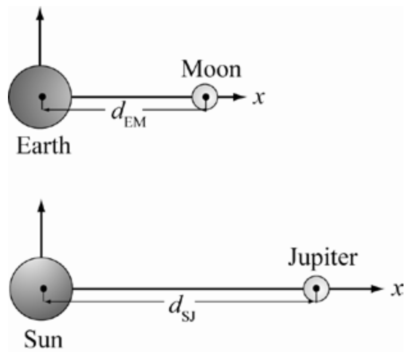
$$X_{\text{cm}}=\frac{x_{\text{p}}A_{\text{p}}-x_{\text{h}}A_{\text{h}}}{A_{\text{p}}-A_{\text{h}}}=\frac{0-\left(-\frac{3}{4}R\right)\left(\frac{\pi R^2}{16}\right)}{\pi R^2-\frac{\pi R^2}{16}}=\frac{R}{20},\quad \vec{R}_{\text{cm}}=\left(\frac{R}{20},0\right)$$

8.28. Since the overall mass of the hourglass does not change and the center of mass must move from the top half to the bottom half, then the center of mass velocity, v_{cm}, must be non-zero and pointing down. As the sand flows from the top part of the hourglass to the lower part, v_{cm} changes with time. The magnitude of v_{cm} is larger when the sand has just started to flow than just before all the sand has flowed through. Thus $dv_{\text{cm}}/dt=a_{\text{cm}}$ must be in the opposite direction from v_{cm}, which is the upward direction. The scale must supply the force required to produce this upward acceleration, so the hourglass weighs more when the sand is flowing than when the sand is stationary. You can find a published solution to a similar version of this problem at the following reference: K.Y. Shen and Bruce L. Scott, American Journal of Physics, **53**, 787 (1985).

Exercises

8.29. **THINK:** Determine (a) the distance, d_1, from the center of mass of the Earth-Moon system to the geometric center of the Earth and (b) the distance, d_2, from the center of mass of the Sun-Jupiter system to the geometric center of the Sun. The mass of the Earth is approximately $m_{\text{E}} = 5.9742 \cdot 10^{24}$ kg and the mass of the Moon is approximately $m_{\text{M}} = 7.3477 \cdot 10^{22}$ kg. The distance between the center of the Earth to the center of the Moon is $d_{\text{EM}} = 384,400$ km. Also, the mass of the Sun is approximately $m_{\text{S}} = 1.98892 \cdot 10^{30}$ kg and the mass of Jupiter is approximately $m_{\text{J}} = 1.8986 \cdot 10^{27}$ kg. The distance between the center of the Sun and the center of Jupiter is $d_{\text{SJ}} = 778,300,000$ km.

SKETCH:



RESEARCH: Determine the center of mass of the two object system from $\vec{R} = \dfrac{\vec{r}_1 m_1 + \vec{r}_2 m_2}{m_1 + m_2}$. By considering the masses on the *x*-axis (as sketched), the one dimensional equation can be used for *x*. Assuming a uniform, spherically symmetric distribution of each planet's mass, they can be modeled as point particles. Finally, by placing the Earth (Sun) at the origin of the coordinate system, the center of mass will be determined with respect to the center of the Earth (Sun), i.e. $d_1 \left(d_2\right) = x$.

SIMPLIFY:

(a) $d_1 = x = \dfrac{x_1 m_{\text{E}} + x_2 m_{\text{M}}}{m_{\text{E}} + m_{\text{M}}} = \dfrac{d_{\text{EM}} m_{\text{M}}}{m_{\text{E}} + m_{\text{M}}}$

(b) $d_2 = x = \dfrac{x_1 m_{\text{S}} + x_2 m_{\text{J}}}{m_{\text{S}} + m_{\text{J}}} = \dfrac{d_{\text{SJ}} m_{\text{J}}}{m_{\text{S}} + m_{\text{J}}}$

CALCULATE:

(a) $d_1 = \dfrac{\left(384,400 \text{ km}\right)\left(7.3477 \cdot 10^{22} \text{ kg}\right)}{\left(5.9742 \cdot 10^{24} \text{ kg}\right) + \left(7.3477 \cdot 10^{22} \text{ kg}\right)} = \dfrac{2.8244559 \cdot 10^{28} \text{ km} \cdot \text{kg}}{6.047677 \cdot 10^{24} \text{ kg}} = 4670.3 \text{ km}$

(b) $d_2 = \dfrac{\left(7.783 \cdot 10^{8} \text{ km}\right)\left(1.8986 \cdot 10^{27} \text{ kg}\right)}{\left(1.98892 \cdot 10^{30} \text{ kg}\right) + \left(1.8986 \cdot 10^{27} \text{ kg}\right)} = 742247.6 \text{ km}$

ROUND:

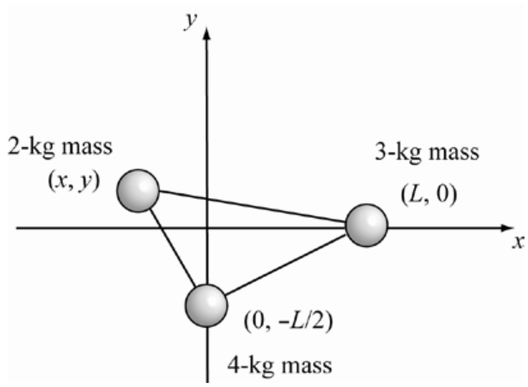
(a) d_{EM} has four significant figures, so $d_1 = 4670.$ km.

(b) d_{SJ} has four significant figures, so $d_2 = 742,200$ km.

DOUBLE-CHECK: In each part, the distance d_1 / d_2 is much less than half the separation distance $d_{\text{EM}} / d_{\text{SJ}}$. This makes sense as the center of mass should be closer to the more massive object in the two body system.

8.30. **THINK:** The center of mass coordinates for the system are $(L/4,-L/5)$. The masses are $m_1=2$ kg, $m_2=3$ kg and $m_3=4$ kg. The coordinates for m_2 are $(L,0)$ and the coordinates for m_3 are $(0,-L/2)$. Determine the coordinates for m_1.

SKETCH:



RESEARCH: The x and y coordinates for m_1 can be determined from the equations for the center of mass in each dimension:

$$X=\frac{1}{M}\sum_{i=1}^{n}x_i m_i \text{ and } Y=\frac{1}{M}\sum_{i=1}^{n}y_i m_i.$$

SIMPLIFY: $X=\dfrac{x_1m_1+x_2m_2+x_3m_3}{m_1+m_2+m_3}\Rightarrow x_1=\dfrac{1}{m_1}\left(X\left(m_1+m_2+m_3\right)-x_2m_2-x_3m_3\right)$

Similarly, $y_1=\dfrac{1}{m_1}\left(Y\left(m_1+m_2+m_3\right)-y_2m_2-y_3m_3\right).$

CALCULATE: $x_1=\left(\dfrac{1}{2\text{ kg}}\right)\left(\dfrac{L}{4}(2\text{ kg}+3\text{ kg}+4\text{ kg})-L(3\text{ kg})-0(4\text{ kg})\right)=-\dfrac{3}{8}L$

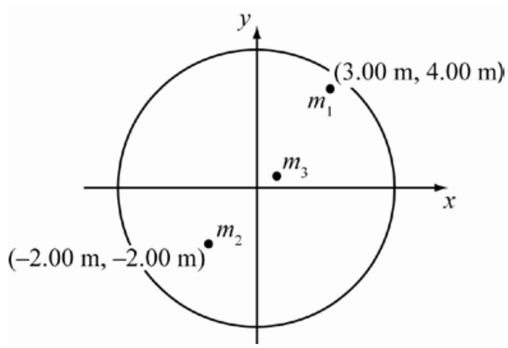
$$y_1=\left(\frac{1}{2\text{ kg}}\right)\left(-\frac{L}{5}(2\text{ kg}+3\text{ kg}+4\text{ kg})-0(3\text{ kg})-\left(-\frac{L}{2}\right)(4\text{ kg})\right)=\frac{1}{10}L$$

ROUND: Rounding is not necessary since the initial values and the results are fractions, so m_1 is located at $(-3L/8,L/10)$.

DOUBLE-CHECK: The coordinates for m_1 are reasonable: since X_{cm} is positive and Y_{cm} is negative and both coordinates have comparatively small values (and thus the center of mass is close to the origin), it makes sense that x will be negative to balance the 3-kg mass and y will be positive to balance the 4-kg mass.

8.31. **THINK:** The mass and location of the first acrobat are known to be $m_1=30.0$ kg and $\vec{r}_1=(3.00\text{ m},4.00\text{ m})$. The mass and location of the second acrobat are $m_2=40.0$ kg and $\vec{r}_2=(-2.00\text{ m},-2.00\text{ m})$. The mass of the third acrobat is $m_3=20.0$ kg. Determine the position of the third acrobat, $\vec{r}_3$, when the center of mass (com) is at the origin.

SKETCH:



RESEARCH: Let M be the sum of the three masses. The coordinates of m_3 can be determined from the center of mass equations for each dimension,

$$X=\frac{1}{M}\sum_{i=1}^{n}x_im_i \text{ and } Y=\frac{1}{M}\sum_{i=1}^{n}y_im_i.$$

SIMPLIFY: Since $X = 0$, $X=\frac{1}{M}\left(x_1m_1+x_2m_2+x_3m_3\right)=0 \Rightarrow x_3=\frac{\left(-x_1m_1-x_2m_2\right)}{m_3}$. Similarly, with $Y = 0$,

$$y_3=\frac{\left(-y_1m_1-y_2m_2\right)}{m_3}.$$

CALCULATE: $x_3=\frac{\left(-\left(3.00\text{ m}\right)\left(30.0\text{ kg}\right)-\left(-2.00\text{ m}\right)\left(40.0\text{ kg}\right)\right)}{20.0\text{ kg}}=-0.500\text{ m},$

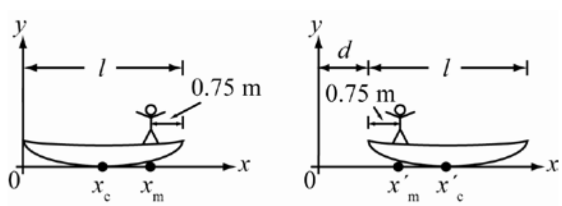
$$y_3=\frac{\left(-\left(4.00\text{ m}\right)\left(30.0\text{ kg}\right)-\left(-2.00\text{ m}\right)\left(40.0\text{ kg}\right)\right)}{20.0\text{ kg}}=-2.00\text{ m}$$

ROUND: $\vec{r}_3=\left(-0.500\text{ m},-2.00\text{ m}\right)$

DOUBLE-CHECK: The resulting location is similar to the locations of the other acrobats.

8.32. **THINK:** The man's mass is $m_{\text{m}}=55\text{ kg}$ and the canoe's mass is $m_{\text{c}}=65\text{ kg}$. The canoe's length is l = 4.0 m. The man moves from 0.75 m from the back of the canoe to 0.75 m from the front of the canoe. Determine how far the canoe moves, d.

SKETCH:



RESEARCH: The center of mass position for the man and canoe system does not change in our external reference frame. To determine d, the center of mass location must be determined before the canoe moves. Then the new location for the canoe after the man moves can be determined given the man's new position and the center of mass position. Assume the canoe has a uniform density such that its center of mass location is at the center of the canoe, $x_{\text{c}}=2.0\text{ m}$. The man's initial position is $x_{\text{m}}=l-0.75\text{ m}=3.25\text{ m}$. After moving, the canoe is located at x_{c}' and the man is located at $x_{\text{m}}'=x_{\text{c}}'+a$. a is the relative position of the man with respect to the canoe's center of mass and $a=-l/2+0.75\text{ m}=-1.25\text{ m}$. Then the distance the canoe moves is $d=x_{\text{c}}'-x_{\text{c}}$.

SIMPLIFY:

$$X = \frac{1}{M}\sum_{i=1}^{n} x_i m_i .$$

The center of mass is $X = \frac{1}{M}\left(x_{\mathrm{m}} m_{\mathrm{m}} + x_{\mathrm{c}} m_{\mathrm{c}}\right)$. After moving,

$X = \frac{1}{M}\left(x'_{\mathrm{m}} m_{\mathrm{m}} + x'_{\mathrm{c}} m_{\mathrm{c}}\right) = \frac{1}{M}\left(\left(x'_{\mathrm{c}} + a\right) m_{\mathrm{m}} + x'_{\mathrm{c}} m_{\mathrm{c}}\right)$. Since X does not change, the equations can be equated:

$$\frac{1}{M}\left(\left(x'_{\mathrm{c}} + a\right) m_{\mathrm{m}} + x'_{\mathrm{c}} m_{\mathrm{c}}\right) = \frac{1}{M}\left(x_{\mathrm{m}} m_{\mathrm{m}} + x_{\mathrm{c}} m_{\mathrm{c}}\right)$$

This implies $x_{\mathrm{m}} m_{\mathrm{m}} + x_{\mathrm{c}} m_{\mathrm{c}} = x'_{\mathrm{c}} m_{\mathrm{m}} + x'_{\mathrm{c}} m_{\mathrm{c}} + a m_{\mathrm{m}} \;\Rightarrow\; x'_{\mathrm{c}} = \dfrac{x_{\mathrm{m}} m_{\mathrm{m}} + x_{\mathrm{c}} m_{\mathrm{c}} - a m_{\mathrm{m}}}{m_{\mathrm{m}} + m_{\mathrm{c}}}$.

CALCULATE: $x'_c = \dfrac{(3.25\text{ m})(55.0\text{ kg}) + (2.00\text{ m})(65.0\text{ kg}) - (-1.25\text{ m})(55.0\text{ kg})}{55.0\text{ kg} + 65.0\text{ kg}} = 3.1458\text{ m}$

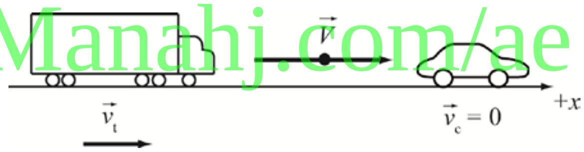
Then $d = 3.1458\text{ m} - 2.00\text{ m} = 1.1458\text{ m}$.

ROUND: As each given value has three significant figures, d = 1.15 m.

DOUBLE-CHECK: This distance is less than the distance traveled by the man (2.5 m), as it should be to preserve the center of mass location.

8.33. **THINK:** The mass of the car is $m_{\mathrm{c}} = 2.00\text{ kg}$ and its initial speed is $v_{\mathrm{c}} = 0$. The mass of the truck is $m_{\mathrm{t}} = 3.50\text{ kg}$ and its initial speed is $v_{\mathrm{t}} = 4.00\text{ m/s}$ toward the car. Determine (a) the velocity of the center of mass, $\vec{V}$, and (b) the velocities of the truck, $\vec{v}'_{\mathrm{t}}$ and the car, $\vec{v}'_{\mathrm{c}}$ with respect to the center of mass.

SKETCH:



RESEARCH:

(a) The velocity of the center of mass can be determined from $\vec{V} = \frac{1}{M}\sum_{i=1}^{n} m_i \vec{v}_i$.

Take $\vec{v}_{\mathrm{t}}$ to be in the positive x-direction.

(b) Generally, the relative velocity, $\vec{v}\,'$, of an object with velocity, $\vec{v}$, in the lab frame is given by $\vec{v}\,' = \vec{v} - \vec{V}$, where $\vec{V}$ is the velocity of the relative reference frame. Note the speeds of the car and the truck relative to the center of mass do not change after their collision, but the relative velocities change direction; that is, $\vec{v}'_{\mathrm{t}}(\text{before collision}) = -\vec{v}'_{\mathrm{t}}(\text{after collision})$ and similarly for the car's relative velocity.

SIMPLIFY:

(a) Substituting $\vec{v}_{\mathrm{c}} = 0$ and $M = m_{\mathrm{c}} + m_{\mathrm{t}}$, $\vec{V} = \frac{1}{M}\left(m_{\mathrm{c}}\vec{v}_{\mathrm{c}} + m_{\mathrm{t}}\vec{v}_{\mathrm{t}}\right)$ becomes $\vec{V} = \dfrac{\left(m_{\mathrm{t}}\vec{v}_{\mathrm{t}}\right)}{\left(m_{\mathrm{c}} + m_{\mathrm{t}}\right)}$.

(b) $\vec{v}'_{\mathrm{t}}$ and $\vec{v}'_{\mathrm{c}}$ before the collision are $\vec{v}'_{\mathrm{t}} = \vec{v}_{\mathrm{t}} - \vec{V}$ and $\vec{v}'_{\mathrm{c}} = \vec{v}_{\mathrm{c}} - \vec{V} = -\vec{V}$.

CALCULATE:

(a) $\vec{V} = \dfrac{(3.50\text{ kg})(4.00\hat{x}\text{ m/s})}{(3.50\text{ kg} + 2.00\text{ kg})} = 2.545\hat{x}\text{ m/s}$

(b) $\vec{v}'_{\mathrm{t}} = (4.00\hat{x}\text{ m/s}) - (2.545\hat{x}\text{ m/s}) = 1.4545\hat{x}\text{ m/s}$, $\vec{v}'_{\mathrm{c}} = -2.545\hat{x}\text{ m/s}$

ROUND: There are three significant figures for each given value, so the results should be rounded to the same number of significant figures.

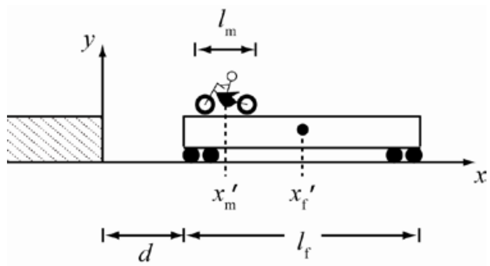
(a) $\vec{V} = 2.55\hat{x}\text{ m/s}$

(b) Before the collision, $\vec{v}_{\rm t}' = 1.45\hat{x}$ m/s and $\vec{v}_{\rm c}' = -2.55\hat{x}$ m/s. This means that after the collision, the velocities with respect to the center of mass become $\vec{v}_{\rm t}' = -1.45\hat{x}$ m/s and $\vec{v}_{\rm c}' = 2.55\hat{x}$ m/s.

DOUBLE-CHECK: $\vec{V}$ is between the initial velocity of the truck and the initial velocity of the car, as it should be.

8.34. **THINK:** The motorcycle with rider has a mass of $m_{\rm m} = 350.$ kg. The flatcar's mass is $m_{\rm f} = 1500.$ kg. The length of the motorcycle is $l_{\rm m} = 2.00$ m and the length of the flatcar is $l_{\rm f} = 20.0$ m. The motorcycle starts at one of end of the flatcar. Determine the distance, *d*, that the flatcar will be from the platform when the motorcycle reaches the end of the flatcar.

SKETCH: After the motorcycle and rider drive down the platform:



RESEARCH: The flatcar-motorcycle center of mass stays in the same position while the motorcycle moves. First, the center of mass must be determined before the motorcycle moves. Then the new location of the flatcar's center of mass can be determined given the center of mass for the system and the motorcycle's final position. Then the distance, *d*, can be determined. Assume that the motorcycle and rider's center of mass and the flatcar's center of mass are located at their geometric centers. Take the initial center of mass position for the motorcycle to be $x_{\rm m} = l_{\rm f} - l_{\rm m}/2$, and the initial center of mass for the flatcar to be $x_{\rm f} = l_{\rm f}/2$. The final position of the center of mass for the motorcycle will be $x_{\rm m}' = d + l_{\rm m}/2$, and the final position for the flatcar will be $x_{\rm f}' = d + l_{\rm f}/2$. Then *d* can be determined from

$$X = \frac{1}{M}\sum_{i=1}^{n} x_i m_i.$$

SIMPLIFY: Originally, $X = \frac{1}{M}\left(x_{\rm m}m_{\rm m} + x_{\rm f}m_{\rm f}\right)$. After the motorcycle moves, $X = \frac{1}{M}\left(x_{\rm m}'m_{\rm m} + x_{\rm f}'m_{\rm f}\right)$.

As the center of mass remains constant, the two expressions can be equated:

$$\frac{1}{M}\left(x_{\rm m}m_{\rm m} + x_{\rm f}m_{\rm f}\right) = \frac{1}{M}\left(x_{\rm m}'m_{\rm m} + x_{\rm f}'m_{\rm f}\right)$$

$$x_{\rm m}m_{\rm m} + x_{\rm f}m_{\rm f} = \left(d + \frac{1}{2}l_{\rm m}\right)m_{\rm m} + \left(d + \frac{1}{2}l_{\rm f}\right)m_{\rm f}$$

$$x_{\rm m}m_{\rm m} + x_{\rm f}m_{\rm f} = d\left(m_{\rm m} + m_{\rm f}\right) + \frac{1}{2}l_{\rm m}m_{\rm m} + \frac{1}{2}l_{\rm f}m_{\rm f}$$

$$d = \frac{\left(x_{\rm m} - \frac{1}{2}l_{\rm m}\right)m_{\rm m} + \left(x_{\rm f} - \frac{1}{2}l_{\rm f}\right)m_{\rm f}}{m_{\rm m} + m_{\rm f}}$$

$x_{\rm m} = l_{\rm f} - \frac{l_{\rm m}}{2}$ and $x_{\rm f} = \frac{l_{\rm f}}{2}$, therefore $d = \frac{\left(l_{\rm f} - l_{\rm m}\right)m_{\rm m}}{m_{\rm m} + m_{\rm f}}$.

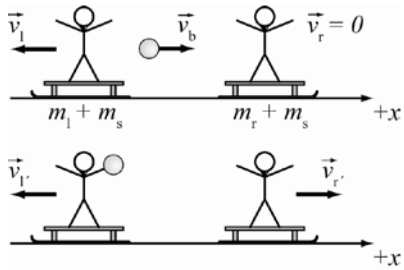
CALCULATE: $d = \frac{\left(20.0\text{ m} - 2.00\text{ m}\right)\left(350.\text{ kg}\right)}{350.\text{ kg} + 1500.\text{ kg}} = 3.4054$ m

ROUND: $m_{\rm m}$ has three significant figures, so the result should be rounded to d = 3.41 m.

DOUBLE-CHECK: It is reasonable that the distance moved is less than length of the flatcar.

8.35. **THINK:** The mass of the sled is $m_s = 10.0$ kg, the mass of the ball is $m_b = 5.00$ kg, and the mass of the student on the left is $m_l = 50.0$ kg. His relative ball-throwing speed is $v_{bl} = 10.0$ m/s. The mass of the student on the right is $m_r = 45.0$ kg and his relative ball-throwing speed is $v_{br} = 12.0$ m/s. Determine (a) the speed of the student on the left, v_l, after first throwing the ball, (b) the speed of the student on the right, v_r, after catching the ball, (c) the speed of the student on the left after catching the pass, v_l', and (d) the speed of the student on the right after throwing the pass, v_r'.

SKETCH:



RESEARCH: Momentum is conserved between each student and ball system. For each step, use $\vec{P}_i = \vec{P}_f$. In addition, the relative velocity of the ball is the difference between its velocity in the lab frame and the velocity of the student in the lab frame who has thrown it. That is, $\vec{v}_{bl} = \vec{v}_b - \vec{v}_l$ and $\vec{v}_{br} = \vec{v}_b - \vec{v}_r$. Recall each student begins at rest.

SIMPLIFY:

(a) Determine v_l after the ball is first thrown:

$$\vec{P}_i = \vec{P}_f \Rightarrow 0 = (m_s + m_l)\vec{v}_l + m_b\vec{v}_b \Rightarrow 0 = (m_s + m_l)\vec{v}_l + m_b(\vec{v}_{bl} + \vec{v}_l) \Rightarrow \vec{v}_l = -\frac{m_b\vec{v}_{bl}}{m_s + m_l + m_b}.$$

(b) Determine $\vec{v}_r$ after the student catches the ball. The velocity of the ball, $\vec{v}_b$, in the lab frame is needed. From part (a), $\vec{v}_l$ is known. Then $\vec{v}_b = \vec{v}_{bl} + \vec{v}_l$. So, $\vec{v}_b$ is known before it is caught. Now, for the student on the right catching the ball,

$$\vec{P}_i = \vec{P}_f \Rightarrow m_b\vec{v}_b = (m_b + m_r + m_s)\vec{v}_r \Rightarrow \vec{v}_r = \frac{m_b\vec{v}_b}{m_b + m_r + m_s}.$$

(c) Now the student on the right throws the ball and the student on the left catches it. To determine $\vec{v}_l'$, the velocity of the ball after it is thrown, $\vec{v}_b'$, is needed. It is known that $\vec{v}_{br} = \vec{v}_b - \vec{v}_r$. Then to determine $\vec{v}_b'$, consider the situation when the student on the right throws the ball. For the student on the right:

$P_i = P_f \Rightarrow (m_s + m_r + m_b)\vec{v}_r = m_b\vec{v}_b' + (m_r + m_s)\vec{v}_r'$, where $\vec{v}_r$ is known from part (b) and $\vec{v}_{br} = \vec{v}_b' - \vec{v}_r' \Rightarrow \vec{v}_r' = \vec{v}_b' - \vec{v}_{br}$. Then, the fact that $(m_s + m_r + m_b)\vec{v}_r = m_b\vec{v}_b' + (m_r + m_s)(\vec{v}_b' - \vec{v}_{br})$ implies $\vec{v}_b' = \dfrac{(m_s + m_r + m_b)\vec{v}_r + (m_r + m_s)\vec{v}_{br}}{m_b + m_r + m_s}$. With $\vec{v}_b'$ known, consider the student on the left catching this ball: $P_i = P_f \Rightarrow m_b\vec{v}_b' + (m_l + m_s)\vec{v}_l = (m_b + m_l + m_s)\vec{v}_l'$. $\vec{v}_l$ is known from part (a) and $\vec{v}_b'$ has just been determined, so $\vec{v}_l' = \dfrac{m_b\vec{v}_b' + (m_l + m_s)\vec{v}_l}{m_b + m_l + m_s}$.

(d) $\vec{v}_{br} = \vec{v}_b' - \vec{v}_r' \Rightarrow \vec{v}_r' = \vec{v}_b' - \vec{v}_{br}$ and $\vec{v}_b'$ has been determined in part (c).

CALCULATE:

(a) $\vec{v}_l = -\dfrac{(5.00 \text{ kg})(10.0 \text{ m/s})}{10.0 \text{ kg} + 50.0 \text{ kg} + 5.00 \text{ kg}} = -0.76923$ m/s

(b) $\vec{v}_b = 10.0 \text{ m/s} - 0.769 \text{ m/s} = 9.231$ m/s, $\vec{v}_r = \dfrac{(5.00 \text{ kg})(9.23077 \text{ m/s})}{5.00 \text{ kg} + 45.0 \text{ kg} + 10.0 \text{ kg}} = 0.76923$ m/s

(c) The ball is thrown to the left, or along the $-\hat{x}$ axis by the student on the right. That is, $\vec{v}_{\text{br}} = -12.0$ m/s.

$$\vec{v}_{\text{b}}' = \frac{(10.0\text{ kg} + 45.0\text{ kg} + 5.00\text{ kg})(0.769\text{ m/s}) + (45.0\text{ kg} + 10.0\text{ kg})(-12.0\text{ m/s})}{5.00\text{ kg} + 45.0\text{ kg} + 10.0\text{ kg}} = -10.23100\text{ m/s}$$

$$\vec{v}_{\text{l}}' = \frac{(5.00\text{ kg})(-10.2310\text{ m/s}) + (50.0\text{ kg} + 10.0\text{ kg})(-0.769\text{ m/s})}{5.00\text{ kg} + 50.0\text{ kg} + 10.0\text{ kg}} = -1.49685\text{ m/s}$$

(d) $\vec{v}_{\text{r}}' = (-10.231\text{ m/s}) - (-12.0\text{ m/s}) = 1.769\text{ m/s}$

ROUND:

(a) $\vec{v}_{\text{l}} = -0.769$ m/s (to the left)

(b) $\vec{v}_{\text{r}} = 0.769$ m/s (to the right)

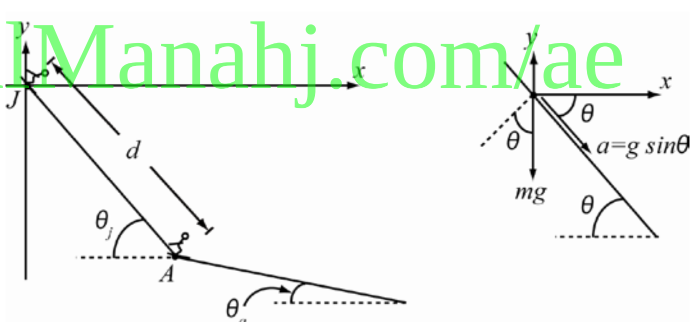
(c) $\vec{v}_{\text{l}}' = -1.50$ m/s (to the left)

(d) $\vec{v}_{\text{r}}' = 1.77$ m/s (to the right)

DOUBLE-CHECK: Before rounding, $|\vec{v}_{\text{l}}'| > |\vec{v}_{\text{l}}| > 0$ (where the initial speed was zero) and $|\vec{v}_{\text{r}}'| > |\vec{v}_{\text{r}}| > 0$, as expected.

8.36. **THINK:** Jack's mass is $m_{\text{J}} = 88.0$ kg. Jack's initial position is taken as $(0,0)$ and the angle of his slope is $\theta_{\text{J}} = 35.0°$. The distance of his slope is d = 100. m. Annie's mass is $m_{\text{A}} = 64.0$ kg. Her slope angle is $\theta_{\text{A}} = 20.0°$. Take her initial position to be $(d\cos\theta_{\text{J}}, -d\sin\theta_{\text{J}})$. Determine the acceleration, velocity and position vectors of their center of mass as functions of time, before Jack reaches the less steep section.

SKETCH:



RESEARCH: To determine the acceleration, velocity and position vectors for the center of mass, the vectors must be determined in each direction. Assuming a constant acceleration, the familiar constant acceleration equations can be used. In addition,

$$\vec{R} = \frac{1}{M}\sum_{i=1}^{n}\vec{r}_i m_i,\quad \vec{V} = \frac{d\vec{R}}{dt} = \frac{1}{M}\sum_{i=1}^{n}\vec{v}_i m_i,\quad \vec{A} = \frac{d\vec{V}}{dt} = \frac{1}{M}\sum_{i=1}^{n}\vec{a}_i m_i,$$

where each equation can be broken into its vector components.

SIMPLIFY: The magnitude of the net acceleration of each skier is $a = g\sin\theta$ down the incline of angle, θ. In the x-direction, $a_{\text{J}x} = (g\sin\theta_{\text{J}})\cos\theta_{\text{J}}$ and $a_{\text{A}x} = (g\sin\theta_{\text{A}})\cos\theta_{\text{A}}$. In the y-direction, $a_{\text{J}y} = -(g\sin\theta_{\text{J}})\sin\theta_{\text{J}} = -g\sin^2\theta_{\text{J}}$ and $a_{\text{A}y} = -(a\sin\theta_{\text{A}})\sin\theta_{\text{A}} = -g\sin^2\theta_{\text{A}}$. Then,

$$A_x = \frac{1}{M}(m_{\text{J}}a_{\text{J}x} + m_{\text{A}}a_{\text{A}x}) = \frac{g}{M}(m_{\text{J}}\sin\theta_{\text{J}}\cos\theta_{\text{J}} + m_{\text{A}}\sin\theta_{\text{A}}\cos\theta_{\text{A}}),\text{ where } M = m_{\text{J}} + m_{\text{A}} \text{ and}$$

$$A_y = \frac{1}{M}(m_{\text{J}}a_{\text{J}y} + m_{\text{A}}a_{\text{A}y}) = -\frac{g}{M}(m_{\text{J}}\sin^2\theta_{\text{J}} + m_{\text{A}}\sin^2\theta_{\text{A}}).$$

Each skier starts from rest. In the x-direction, $v_{\text{J}x} = a_{\text{J}x}t = g\sin\theta_{\text{J}}\cos\theta_{\text{J}}t$ and $v_{\text{A}x} = a_{\text{A}x}t = g\sin\theta_{\text{A}}\cos\theta_{\text{A}}t$. In the y-direction, $v_{\text{J}y} = a_{\text{J}y}t = -g\sin^2\theta_{\text{J}}t$ and $v_{\text{A}y} = a_{\text{A}y}t = -g\sin^2\theta_{\text{A}}t$.

Then,

$$V_x = \frac{1}{M}\left(m_{\text{J}}v_{\text{J}x} + m_{\text{A}}v_{\text{A}x}\right) = \frac{g}{M}\left(m_{\text{J}}\sin\theta_{\text{J}}\cos\theta_{\text{J}} + m_{\text{A}}\sin\theta_{\text{A}}\cos\theta_{\text{A}}\right)t = A_x t \text{ and}$$

$$V_y = \frac{1}{M}\left(m_{\text{J}}v_{\text{J}y} + m_{\text{A}}v_{\text{A}y}\right) = -\frac{g}{M}\left(m_{\text{J}}\sin^2\theta_{\text{J}} + m_{\text{A}}\sin^2\theta_{\text{A}}\right)t = A_y t.$$

The position in the x-direction is given by:

$$x_{\text{J}} = \frac{1}{2}a_{\text{J}x}t^2 + x_{\text{J0}} = \frac{1}{2}g\sin\theta_{\text{J}}\cos\theta_{\text{J}}t^2 \text{ and } x_{\text{A}} = \frac{1}{2}a_{\text{A}x}t^2 + x_{\text{A0}} = \frac{1}{2}g\sin\theta_{\text{A}}\cos\theta_{\text{A}}t^2 + d\cos\theta_{\text{J}}.$$

In the y-direction,

$$y_{\text{J}} = \frac{1}{2}a_{\text{J}y}t^2 + y_{\text{J0}} = -\frac{1}{2}g\sin^2\theta_{\text{J}}t^2 \text{ and } y_{\text{A}} = \frac{1}{2}a_{\text{A}y}t^2 + y_{\text{A0}} = -\frac{1}{2}g\sin^2\theta_{\text{A}}t^2 - d\sin\theta_{\text{J}}.$$

Then,

$$X = \frac{1}{M}\left(m_{\text{J}}x_{\text{J}} + m_{\text{A}}x_{\text{A}}\right) = \frac{1}{M}\left(\frac{1}{2}m_{\text{J}}g\sin\theta_{\text{J}}\cos\theta_{\text{J}}t^2 + \frac{1}{2}m_{\text{A}}g\sin\theta_{\text{A}}\cos\theta_{\text{A}}t^2 + m_{\text{A}}d\cos\theta_{\text{J}}\right) = \frac{1}{2}A_x t^2 + \frac{m_{\text{A}}}{M}d\cos\theta_{\text{J}}$$

$$Y = \frac{1}{M}\left(m_{\text{J}}y_{\text{J}} + m_{\text{A}}y_{\text{A}}\right) = -\frac{1}{M}\left(\frac{1}{2}m_{\text{J}}g\sin^2\theta_{\text{J}}t^2 + \frac{1}{2}m_{\text{A}}g\sin^2\theta_{\text{A}}t^2 + m_{\text{A}}d\sin\theta_{\text{J}}\right) = \frac{1}{2}A_y t^2 - \frac{m_{\text{A}}}{M}d\sin\theta_{\text{J}}.$$

CALCULATE:

$$A_x = \frac{\left(9.81 \text{ m/s}^2\right)}{88.0 \text{ kg} + 64.0 \text{ kg}}\left(\left(88.0 \text{ kg}\right)\sin 35.0°\cos 35.0° + \left(64.0 \text{ kg}\right)\sin 20.0°\cos 20.0°\right) = 3.996 \text{ m/s}^2$$

$$A_y = -\frac{\left(9.81 \text{ m/s}^2\right)}{88.0 \text{ kg} + 64.0 \text{ kg}}\left(\left(88.0 \text{ kg}\right)\sin^2\left(35.0°\right) + \left(64.0 \text{ kg}\right)\sin^2\left(20.0°\right)\right) = -2.352 \text{ m/s}^2$$

$$V_x = \left(3.996 \text{ m/s}^2\right)t, \quad V_y = \left(-2.352 \text{ m/s}^2\right)t$$

$$X = \frac{1}{2}\left(3.996 \text{ m/s}^2\right)t^2 + \frac{64.0 \text{ kg}}{\left(88.0 \text{ kg} + 64.0 \text{ kg}\right)}\left(100. \text{ m}\right)\cos\left(35.0°\right) = \left(1.998 \text{ m/s}^2\right)t^2 + 34.49 \text{ m}$$

$$Y = \frac{1}{2}\left(-2.352 \text{ m/s}^2\right)t^2 - \frac{64.0 \text{ kg}}{\left(88.0 \text{ kg} + 64.0 \text{ kg}\right)}\left(100. \text{ m}\right)\sin\left(35.0°\right) = \left(-1.176 \text{ m/s}^2\right)t^2 - 24.1506 \text{ m}$$

ROUND: Rounding to three significant figures, $A_x = 4.00 \text{ m/s}^2$, $A_y = -2.35 \text{ m/s}^2$, $V_x = \left(4.00 \text{ m/s}^2\right)t$ and $V_y = \left(-2.35 \text{ m/s}^2\right)t$, $X = \left(2.00 \text{ m/s}^2\right)t^2 + 34.5 \text{ m}$ and $Y = \left(-1.18 \text{ m/s}^2\right)t^2 - 24.2 \text{ m}$.

DOUBLE-CHECK: The acceleration of the center of mass is not time dependent.

8.37. **THINK:** The proton's mass is $m_{\text{p}} = 1.6726 \cdot 10^{-27}$ kg and its initial speed is $v_{\text{p}} = 0.700c$ (assumed to be in the lab frame). The mass of the tin nucleus is $m_{\text{sn}} = 1.9240 \cdot 10^{-25}$ kg (assumed to be at rest). Determine the speed of the center of mass, v, with respect to the lab frame.

SKETCH: A sketch is not necessary.

RESEARCH: The given speeds are in the lab frame. To determine the speed of the center of mass use $V = \frac{1}{M}\sum_{i=1}^{n} m_i v_i$.

SIMPLIFY: $V = \frac{1}{m_{\text{p}} + m_{\text{sn}}}\left(m_{\text{p}}v_{\text{p}} + m_{\text{sn}}v_{\text{sn}}\right) = \frac{m_{\text{p}}v_{\text{p}}}{m_{\text{p}} + m_{\text{sn}}}$

CALCULATE: $V = \frac{\left(1.6726 \cdot 10^{-27} \text{ kg}\right)\left(0.700c\right)}{\left(1.6726 \cdot 10^{-27} \text{ kg}\right) + \left(1.9240 \cdot 10^{-25} \text{ kg}\right)} = 0.0060329c$

ROUND: Since v_{p} has three significant figures, the result should be rounded to $V = 0.00603c$.

DOUBLE-CHECK: Since m_{sn} is at rest and $m_{\text{sn}} >> m_{\text{p}}$, it is expected that $V << v_{\text{p}}$.

8.38. **THINK:** Particle 1 has a mass of $m_1 = 2.0\text{ kg}$, a position of $\vec{r}_1 = (2.0\text{ m}, 6.0\text{ m})$ and a velocity of $\vec{v}_1 = (4.0\text{ m/s}, 2.0\text{ m/s})$. Particle 2 has a mass of $m_2 = 3.0\text{ kg}$, a position of $\vec{r}_2 = (4.0\text{ m}, 1.0\text{ m})$ and a velocity of $\vec{v}_2 = (0, 4.0\text{ m/s})$. Determine (a) the position $\vec{R}$ and the velocity $\vec{V}$ for the system's center of mass and (b) a sketch of the position and velocity vectors for each particle and for the center of mass.

SKETCH: To be provided in the calculate step, part (b).

RESEARCH: To determine $\vec{R}$, use $X = \frac{1}{M}(x_1 m_1 + x_2 m_2)$ and $Y = \frac{1}{M}(y_1 m_1 + y_2 m_2)$. To determine $\vec{V}$, use $V_x = \frac{1}{M}(v_{1x} m_1 + v_{2x} m_2)$ and $V_y = \frac{1}{M}(v_{1y} m_1 + v_{2y} m_2)$.

SIMPLIFY: It is not necessary to simplify.

CALCULATE:

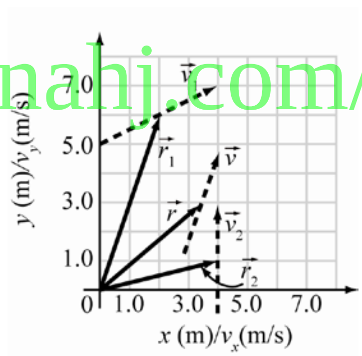
(a) $X = \frac{1}{2.00\text{ kg} + 3.00\text{ kg}}\left((2.00\text{ m})(2.00\text{ kg}) + (4.00\text{ m})(3.00\text{ kg})\right) = 3.20\text{ m}$

$Y = \frac{1}{2.00\text{ kg} + 3.00\text{ kg}}\left((6.00\text{ m})(2.00\text{ kg}) + (1.00\text{ m})(3.00\text{ kg})\right) = 3.00\text{ m}$

$V_x = \frac{1}{2.00\text{ kg} + 3.00\text{ kg}}\left((4.00\text{ m/s})(2.00\text{ kg}) + 0(3.00\text{ kg})\right) = 1.60\text{ m/s}$

$V_y = \frac{1}{2.00\text{ kg} + 3.00\text{ kg}}\left((2.00\text{ m/s})(2.00\text{ kg}) + (4.00\text{ m/s})(3.00\text{ kg})\right) = 3.20\text{ m/s}$

(b)

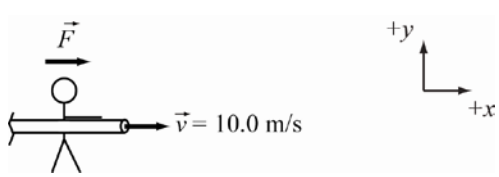


ROUND: Each given value has three significant figures, so the results should be rounded to X = 3.20 m, Y = 3.00 m, $V_x = 1.60\text{ m/s}$ and $V_y = 3.20\text{ m/s}$.

DOUBLE-CHECK: $\vec{R}$ should point between $\vec{r}_1$ and $\vec{r}_2$, and $\vec{V}$ should point between $\vec{v}_1$ and $\vec{v}_2$.

8.39. **THINK:** The radius of the hose is r = 0.0200 m and the velocity of the spray is v = 10.0 m/s. Determine the horizontal force, $\vec{F}_f$, required of the fireman to hold the hose stationary.

SKETCH:



RESEARCH: By Newton's third law, the force exerted by the fireman is equal in magnitude to the force exerted by the hose. The thrust force of the hose can be determined from $\vec{F}_{\text{thrust}} = -\vec{v}_c dm/dt$. To determine dm/dt, consider the mass of water exiting the hose per unit time.

SIMPLIFY: The volume of water leaving the hose is this velocity times the area of the hose's end. That is,

$$\frac{dV_{\text{w}}}{dt} = Av = \pi r^2 v.$$

With $\rho_{\text{w}} = m / V_{\text{w}}$, $\dfrac{dm}{dt} = \rho_{\text{w}} \dfrac{dV_{\text{w}}}{dt} = \rho_{\text{w}} \pi r^2 v$. Now, by Newton's third law, $\vec{F}_{\text{f}} = -\vec{F}_{\text{thrust}}$, so $\vec{F}_{\text{f}} = \vec{v}_{\text{c}} \dfrac{dm}{dt} = \vec{v}_{\text{c}} \rho_{\text{w}} \pi r^2 v$. Since v_{c} is in fact v, $F_{\text{f}} = \rho_{\text{w}} \pi r^2 v^2$.

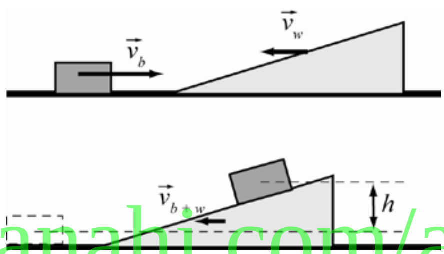
CALCULATE: $F_{\text{f}} = \pi \left(1000 \text{ kg/m}^3\right)\left(0.0200 \text{ m}\right)^2 \left(10.0 \text{ m/s}\right)^2 = 125.7 \text{ N}$

ROUND: Since v has three significant figures, $\vec{F}_{\text{f}} = 126 \text{ N}$ in the direction of the water's velocity.

DOUBLE-CHECK: The result has units of force. Also, this is a reasonable force with which to hold a fire hose.

8.40. **THINK:** The block's mass is $m_{\text{b}} = 1.2$ kg. It has an initial velocity is $\vec{v}_{\text{b}} = 2.5$ m/s (with the positive *x* axis being the right direction). The wedge's mass is m_{w} and its initial velocity is $\vec{v}_{\text{w}} = -1.1$ m/s. Their final velocity when the wedge stops moving is $\vec{v}_{\text{b+w}}$. Determine (a) m_{w}, if the block's center of mass rises by $h = 0.37$ m and (b) $\vec{v}_{\text{b+w}}$.

SKETCH:



RESEARCH: Momentum is conserved. As this is an elastic collision, and there are only conservative forces, mechanical energy is also conserved. Use $P_{\text{i}} = P_{\text{f}}$, $\Delta K + \Delta U = 0$, $K = mv^2/2$ and $U = mgh$ to determine m_{w} and ultimately $\vec{v}_{\text{b+w}}$.

SIMPLIFY: It will be useful to determine an expression for $\vec{v}_{\text{b+w}}$ first:

$$\vec{P}_{\text{i}} = \vec{P}_{\text{f}} \Rightarrow m_{\text{b}}\vec{v}_{\text{b}} + m_{\text{w}}\vec{v}_{\text{w}} = \left(m_{\text{b}} + m_{\text{w}}\right)\vec{v}_{\text{b+w}} \Rightarrow \vec{v}_{\text{b+w}} = \frac{m_{\text{b}}\vec{v}_{\text{b}} + m_{\text{w}}\vec{v}_{\text{w}}}{m_{\text{b}} + m_{\text{w}}}.$$

(a) From the conservation of mechanical energy:

$$\Delta K + \Delta U = K_{\text{f}} - K_{\text{i}} + U_{\text{f}} - U_{\text{i}} = 0 \Rightarrow \frac{1}{2}\left(m_{\text{b}} + m_{\text{w}}\right)\vec{v}_{\text{b+w}}^{\,2} - \frac{1}{2}m_{\text{b}}\vec{v}_{\text{b}}^{\,2} - \frac{1}{2}m_{\text{w}}\vec{v}_{\text{w}}^{\,2} + m_{\text{b}}gh = 0$$

$$\Rightarrow \frac{1}{2}\left(m_{\text{b}} + m_{\text{w}}\right)\frac{\left(m_{\text{b}}\vec{v}_{\text{b}} + m_{\text{w}}\vec{v}_{\text{w}}\right)^2}{\left(m_{\text{b}} + m_{\text{w}}\right)^2} - \frac{1}{2}m_{\text{b}}\vec{v}_{\text{b}}^{\,2} - \frac{1}{2}m_{\text{w}}\vec{v}_{\text{w}}^{\,2} + m_{\text{b}}gh = 0$$

$$\Rightarrow \frac{\left(m_{\text{b}}^2\vec{v}_{\text{b}}^{\,2} + 2m_{\text{b}}m_{\text{w}}\vec{v}_{\text{b}}\vec{v}_{\text{w}} + m_{\text{w}}^2\vec{v}_{\text{w}}^{\,2}\right)}{2\left(m_{\text{b}} + m_{\text{w}}\right)} - \frac{1}{2}m_{\text{b}}\vec{v}_{\text{b}}^{\,2} - \frac{1}{2}m_{\text{w}}\vec{v}_{\text{w}}^{\,2} + m_{\text{b}}gh = 0$$

Multiply the expression by $2\left(m_{\text{b}} + m_{\text{w}}\right)$:

$$m_{\text{b}}^2\vec{v}_{\text{b}}^{\,2} + 2m_{\text{b}}m_{\text{w}}\vec{v}_{\text{b}}\vec{v}_{\text{w}} + m_{\text{w}}^2\vec{v}_{\text{w}}^{\,2} - m_{\text{b}}\vec{v}_{\text{b}}^{\,2}\left(m_{\text{b}} + m_{\text{w}}\right) - m_{\text{w}}\vec{v}_{\text{w}}^{\,2}\left(m_{\text{b}} + m_{\text{w}}\right) + 2m_{\text{b}}gh\left(m_{\text{b}} + m_{\text{w}}\right) = 0$$

$$\Rightarrow m_{\text{b}}^2\vec{v}_{\text{b}}^{\,2} + 2m_{\text{b}}m_{\text{w}}\vec{v}_{\text{b}}\vec{v}_{\text{w}} + m_{\text{w}}^2\vec{v}_{\text{w}}^{\,2} - m_{\text{b}}^2\vec{v}_{\text{b}}^{\,2} - m_{\text{b}}m_{\text{w}}\vec{v}_{\text{b}}^{\,2} - m_{\text{w}}m_{\text{b}}\vec{v}_{\text{w}}^{\,2} + m_{\text{w}}^2\vec{v}_{\text{w}}^{\,2} + 2m_{\text{b}}^2gh + 2m_{\text{b}}m_{\text{w}}gh = 0$$

$$\Rightarrow 2m_{\text{b}}m_{\text{w}}\vec{v}_{\text{b}}\vec{v}_{\text{w}} - m_{\text{b}}m_{\text{w}}\vec{v}_{\text{b}}^{\,2} - m_{\text{b}}m_{\text{w}}\vec{v}_{\text{w}}^{\,2} + 2m_{\text{b}}^2gh + 2m_{\text{b}}m_{\text{w}}gh = 0$$

$$\Rightarrow m_{\text{w}} = -\frac{2m_{\text{b}}^2gh}{2m_{\text{b}}\vec{v}_{\text{b}}\vec{v}_{\text{w}} - m_{\text{b}}\vec{v}_{\text{b}}^{\,2} - m_{\text{b}}\vec{v}_{\text{w}}^{\,2} + 2m_{\text{b}}gh} = \frac{2m_{\text{b}}gh}{\vec{v}_{\text{b}}^{\,2} + \vec{v}_{\text{w}}^{\,2} - 2\vec{v}_{\text{b}}\vec{v}_{\text{w}} - 2gh}.$$

(b) With m_{w} known, $\vec{v}_{\text{b+w}} = \dfrac{m_{\text{b}}\vec{v}_{\text{b}} + m_{\text{w}}\vec{v}_{\text{w}}}{m_{\text{b}} + m_{\text{w}}}$.

CALCULATE:

(a) $$m_{\text{w}} = \frac{2(1.20\text{ kg})(9.81\text{ m/s}^2)0.370\text{ m}}{(2.5\text{ m/s})^2 + (-1.10\text{ m/s})^2 - 2(2.50\text{ m/s})(-1.10\text{ m/s}) - 2(9.81\text{ m/s}^2)(0.370\text{ m})}$$

$$= \frac{8.712\text{ kg}\cdot\text{m}^2/\text{s}^2}{6.25\text{ m}^2/\text{s}^2 + 1.21\text{ m}^2/\text{s}^2 + 5.5\text{ m}^2/\text{s}^2 - 7.2594\text{ m}^2/\text{s}^2} = 1.528\text{ kg}$$

(b) $$\vec{v}_{\text{b+w}} = \frac{(1.20\text{ kg})(2.50\text{ m/s}) + (1.528\text{ kg})(-1.10\text{ m/s})}{1.20\text{ kg} + 1.528\text{ kg}} = 0.4835\text{ m/s}$$

ROUND: Each given value has three significant figures, so the results should be rounded to: $m_w = 1.53\text{ kg}$ and $\vec{v}_{\text{b+w}} = 0.484\text{ m/s}$ to the right.

DOUBLE-CHECK: These results are reasonable given the initial values.

8.41. **THINK:** For rocket engines, the specific impulse is $J_{\text{spec}} = \dfrac{J_{\text{tot}}}{W_{\text{expended fuel}}} = \dfrac{1}{W_{\text{expended fuel}}}\int_{t_0}^{t} F_{\text{thrust}}(t')dt'$.

(a) Determine J_{spec} with an exhaust nozzle speed of v.

(b) Evaluate and compare J_{spec} for a toy rocket with $v_{\text{toy}} = 800.\text{ m/s}$ and a chemical rocket with $v_{\text{chem}} = 4.00\text{ km/s}$.

SKETCH: Not applicable.

RESEARCH: It is known that $\vec{F}_{\text{thrust}} = -v_{\text{c}}dm/dt$. Rewrite $W_{\text{expended fuel}}$ as $m_{\text{expended}}g$. With the given definition, J_{spec} can be determined for a general v, and for v_{toy} and v_{chem}.

SIMPLIFY: $J_{\text{spec}} = \dfrac{1}{m_{\text{expended}}g}\int_{m_0}^{m} -v dm = -\dfrac{v}{m_{\text{expended}}g}(m - m_0)$. Now, $m - m_0 = -m_{\text{expended}}$, so $J_{\text{spec}} = \dfrac{v}{g}$.

CALCULATE: $J_{\text{spec, toy}} = \dfrac{v_{\text{toy}}}{g} = \dfrac{800.\text{ m/s}}{(9.81\text{ m/s}^2)} = 81.55\text{ s}$, $J_{\text{spec, chem}} = \dfrac{v_{\text{chem}}}{g} = \dfrac{4.00\cdot 10^3\text{ m/s}}{(9.81\text{ m/s}^2)} = 407.75\text{ s}$

$$\frac{J_{\text{spec, toy}}}{J_{\text{spec, chem}}} = \frac{v_{\text{toy}}}{v_{\text{chem}}} = \frac{800.\text{ m/s}}{4.00\cdot 10^3\text{ m/s}} = 0.200$$

ROUND:

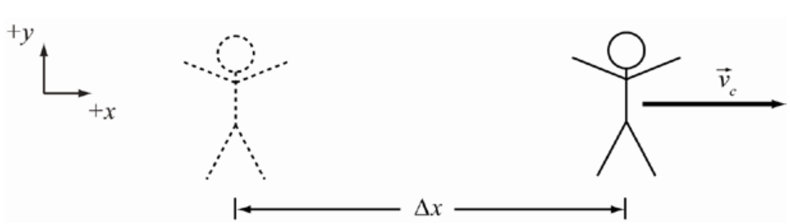
(a) $J_{\text{spec, toy}} = 81.6\text{ s}$

(b) $J_{\text{spec, chem}} = 408\text{ s}$ and $J_{\text{spec, toy}} = 0.200 J_{\text{spec, chem}}$.

DOUBLE-CHECK: The units of the results are units of specific impulse. Also, as expected $J_{\text{spec, toy}} < J_{\text{spec, chem}}$.

8.42. **THINK:** The astronaut's total mass is m = 115 kg. The rate of gas ejection is $dm/dt = 7.00\text{ g/s} = 0.00700\text{ kg/s}$ and the leak speed is $v_{\text{c}} = 800.\text{ m/s}$. After $\Delta t = 6.00\text{ s}$, how far has the astronaut moved from her original position, Δx?

SKETCH:



RESEARCH: Assume that the astronaut starts from rest and the acceleration is constant. Δx can be determined from $\Delta x = (v_{\text{i}} + v_{\text{f}})\Delta t / 2$. To determine v_{f}, use the rocket-velocity equation $v_{\text{f}} - v_{\text{i}} = v_{\text{c}} \ln(m_{\text{i}} / m_{\text{f}})$. The loss of mass can be determined from $\Delta m = \frac{dm}{dt}\Delta t$.

SIMPLIFY: Since $v_{\text{i}} = 0$, $v_{\text{f}} = v_{\text{c}} \ln(m_{\text{i}} / m_{\text{f}})$, where $m_{\text{i}} = m$ and $m_{\text{f}} = m - \Delta m = m - \frac{dm}{dt}\Delta t$. Then,

$$v_{\text{f}} = v_{\text{c}} \ln\left(\frac{m}{m - \frac{dm}{dt}\Delta t}\right) \text{ and } \Delta x = \frac{1}{2} v_{\text{f}} \Delta t.$$

CALCULATE: $v_{\text{f}} = (800.\text{ m/s})\ln\left(\frac{115\text{ kg}}{115\text{ kg} - (0.00700\text{ kg/s})(6.00\text{ s})}\right) = 0.29223\text{ m/s}$

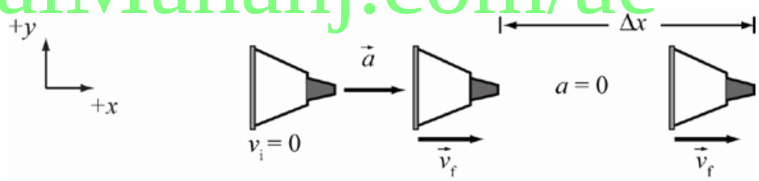
$$\Delta x = \frac{1}{2}(0.29223\text{ m/s})(6.00\text{ s}) = 0.87669\text{ m}$$

ROUND: The problem values have three significant figures, so the results should be rounded to $v_{\text{f}} = 0.292\text{ m/s}$ $\Delta x = 0.877\text{ m}$.

DOUBLE-CHECK: Considering how such a small amount of the total mass has escaped, this is a reasonable distance to have moved.

8.43. **THINK:** The mass of the payload is $m_{\text{p}} = 5190.0\text{ kg}$, and the fuel mass is $m_{\text{f}} = 1.551 \cdot 10^5\text{ kg}$. The fuel exhaust speed is $v_{\text{c}} = 5.600 \cdot 10^3\text{ m/s}$. How long will it take the rocket to travel a distance $\Delta x = 3.82 \cdot 10^8\text{ m}$ after achieving its final velocity, v_{f}? The rocket starts accelerating from rest.

SKETCH:



RESEARCH: The rocket's travel speed, v_{f}, can be determined from $v_{\text{f}} - v_{\text{i}} = v_{\text{c}} \ln(m_{\text{i}} / m_{\text{f}})$. Then Δt can be determined from $\Delta x = v\Delta t$.

SIMPLIFY: $v_{\text{f}} = v_{\text{c}} \ln\left(\frac{m_{\text{p}} + m_{\text{f}}}{m_{\text{p}}}\right)$, and $\Delta t = \Delta x / v_{\text{f}}$.

CALCULATE: $v_{\text{f}} = (5.600 \cdot 10^3\text{ m/s})\ln\left(\frac{5190.0\text{ kg} + 1.551 \cdot 10^5\text{ kg}}{5190.0\text{ kg}}\right) = 19209\text{ m/s}$,

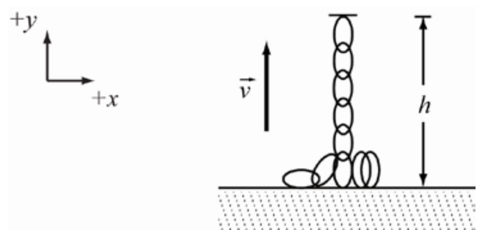
$$\Delta t = \frac{3.82 \cdot 10^8\text{ m}}{19209\text{ m/s}} = 19886\text{ s}$$

ROUND: Δx has three significant figures, so the result should be rounded to $\Delta t = 19{,}886\text{ s} = 5.52\text{ h}$.

DOUBLE-CHECK: This is a reasonable time for a rocket with such a large initial velocity to reach the Moon from the Earth.

8.44. **THINK:** The linear density of the chain is $\lambda = 1.32\text{ kg/m}$, and the speed at which one end of the chain is lifted is $v = 0.47\text{ m/s}$. Determine (a) the net force acting on the chain, F_{net} and (b) the force, F, applied to the end of the chain when $h = 0.15\text{ m}$ has been lifted off the table.

SKETCH:



RESEARCH:

(a) Since the chain is raised at a constant rate, *v*, the net force is the thrust force, $F_{\text{thrust}} = v_c dm/dt$. Since the chain's mass in the air is increasing, $F_{net} = v\,dm/dt$.

(b) The applied force can be determined by considering the forces acting on the chain and the net force determined in part (a): $F_{\text{net}} = \sum F_i$.

SIMPLIFY:

(a) $F_{\text{net}} = v\dfrac{dm}{dt} = v\lambda\dfrac{dh}{dt} = v\lambda v = v^2\lambda$

(b) $F_{\text{net}} = F_{\text{applied}} - mg \Rightarrow F_{\text{applied}} = F_{\text{net}} + mg = v^2\lambda + mg = v^2\lambda + \lambda hg$

CALCULATE:

(a) $F_{\text{net}} = (0.470 \text{ m/s})^2 (1.32 \text{ kg/m}) = 0.2916 \text{ N}$

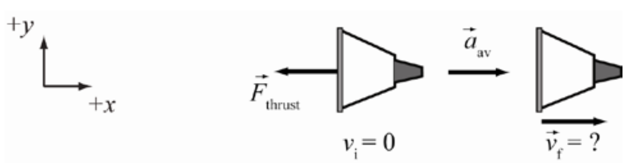
(b) $F_{\text{applied}} = 0.2916 \text{ N} + (1.32 \text{ kg/m})(0.150 \text{ m})(9.81 \text{ m/s}^2) = 0.2916 \text{ N} + 1.942 \text{ N} = 2.234 \text{ N}$

ROUND: *v* and *h* each have three significant figures, so the results should be rounded to $F_{\text{net}} = 0.292 \text{ N}$ and $F_{\text{applied}} = 2.23 \text{ N}$.

DOUBLE-CHECK: These forces are reasonable to determine for this system. Also, $F_{\text{net}} < F_{\text{applied}}$.

8.45. **THINK:** The thrust force is $\vec{F}_{\text{thrust}} = 53.2 \cdot 10^6 \text{ N}$ and the propellant velocity is $v = 4.78 \cdot 10^3 \text{ m/s}$. Determine (a) *dm*/*dt*, (b) the final speed of the spacecraft, v_s, given $v_i = 0$, $m_i = 2.12 \cdot 10^6 \text{ kg}$ and $m_f = 7.04 \cdot 10^4 \text{ kg}$ and (c) the average acceleration, a_{av} until burnout.

SKETCH:



RESEARCH:

(a) To determine *dm*/*dt*, use $\vec{F}_{\text{thrust}} = -v_c dm/dt$.

(b) To determine v_f, use $v_f - v_i = v_c \ln(m_i / m_f)$.

(c) Δv is known from part (b). Δt can be determined from the equivalent ratios, $\dfrac{dm}{dt} = \dfrac{\Delta m}{\Delta t}$, where $\Delta m = m_i - m_f$.

SIMPLIFY:

(a) Since $\vec{F}_{\text{thrust}}$ and $\vec{v}_c$ are in the same direction, the equation can be rewritten as:

$$F_{\text{thrust}} = v_c \frac{dm}{dt} \Rightarrow \frac{dm}{dt} = \frac{F_{\text{thrust}}}{v_c}.$$

(b) $v_i = 0 \Rightarrow v_f = v_c \ln\left(\frac{m_i}{m_f}\right)$

(c) $\frac{dm}{dt} = \frac{\Delta m}{\Delta t} \Rightarrow \Delta t = \frac{\Delta m}{dm/dt},\ a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_f}{\Delta m}\left(\frac{dm}{dt}\right) \quad (v_i = 0)$

CALCULATE:

(a) $\frac{dm}{dt} = \frac{(53.2 \cdot 10^6 \text{ N})}{(4.78 \cdot 10^3 \text{ m/s})} = 11129.7 \text{ kg/s}$

(b) $v_f = (4.78 \cdot 10^3 \text{ m/s})\ln\left(\frac{2.12 \cdot 10^6 \text{ kg}}{7.04 \cdot 10^4 \text{ kg}}\right) = 1.6276 \cdot 10^4 \text{ m/s}$

(c) $a_{av} = \frac{(1.6276 \cdot 10^4 \text{ m/s})}{(2.12 \cdot 10^6 \text{ kg} - 7.04 \cdot 10^4 \text{ kg})}(11129.7 \text{ kg/s}) = 88.38 \text{ m/s}^2$

ROUND: Each given value has three significant figures, so the results should be rounded to $dm/dt = 11100 \text{ kg/s}$, $v_f = 1.63 \cdot 10^4 \text{ m/s}$ and $a_{av} = 88.4 \text{ m/s}^2$.

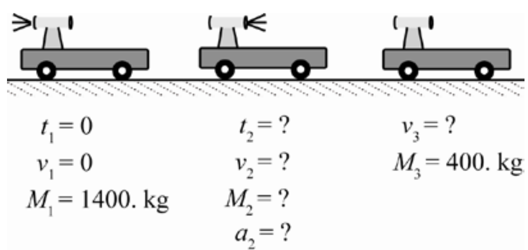
DOUBLE-CHECK: The results all have the correct units. Also, the results are reasonable for a spaceship with such a large thrust force.

8.46. **THINK:** The mass of the cart with an empty water tank is $m_c = 400.$ kg. The volume of the water tank is $V = 1.00 \text{ m}^3$. The rate at which water is ejected in SI units is

$$dV/dt = \left(200. \frac{\text{L}}{\text{min}}\right)\left(\frac{1 \text{ m}^3}{1000 \text{ L}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 0.003333 \text{ m}^3/\text{s}.$$

The muzzle velocity is $v_c = 25.0$ m/s. Determine (a) the time, t_2, to switch from backward to forward so the cart ends up at rest (it starts from rest), (b) the mass of the cart, M_2, and the velocity, v_2, at the time, t_2, (c) the thrust, F_{thrust}, of the rocket and (d) the acceleration, a_2, of the cart just before the valve is switched. Note the mass of the cart increases by 1000. kg when the water tank is full, as $m_w = \rho V = (1000. \text{ kg/m}^3)(1.00 \text{ m}^3)$. That is, the initial mass is $M_1 = 1400.$ kg.

SKETCH:



RESEARCH:

(a) t_2 can be determined from the ratio, $\frac{M_1 - M_2}{t_2 - t_1} = \frac{dm}{dt}$, with $t_1 = 0$. Note that, $dm/dt = \rho dV/dt$. M_2 can be determined from $v_f - v_i = v_c \ln(m_i/m_f)$. When the cart stops moving, the water tank is empty and the total mass is $M_3 = 400$ kg.

(b) Using the mass determined in part (a), v_2 can be determined from $v_f - v_i = v_c \ln(m_i/m_f)$.

(c) Use $\vec{F}_{thrust} = -\vec{v}_c dm/dt$.

(d) Since $\vec{F}_{thrust} = M\vec{a}_{net}$, a_2 can be determined from this equation.

SIMPLIFY:
(a) Consider the first leg of the trip before the valve is switched:

$$v_2 - v_1 = v_c \ln\left(M_1 / M_2\right) \Rightarrow v_2 = v_c \ln\left(M_1 / M_2\right).$$

In the second leg, v_c changes direction, and the similar equation is

$$v_3 - v_2 = -v_c \ln\left(M_2 / M_3\right) \Rightarrow v_2 = v_c \ln\left(M_2 / M_3\right).$$

Then it must be that $\ln\left(M_2 / M_3\right) = \ln\left(M_2 / M_3\right)$, or $M_1 / M_2 = M_2 / M_3$. Then $M_2 = \sqrt{M_3 M_1}$. Now,

$$\frac{M_1 - M_2}{t_2} = \frac{dm}{dt} = \rho \frac{dV}{dt} \Rightarrow t_2 = \frac{M_1 - M_2}{\rho \frac{dV}{dt}} = \frac{M_1 - \sqrt{M_3 M_1}}{\rho \frac{dV}{dt}}.$$

(b) From above, $M_2 = \sqrt{M_3 M_1}$, $v_2 = v_c \ln\left(\frac{M_1}{M_2}\right)$.

(c) $\vec{F}_{\text{thrust}} = -\vec{v}_c \frac{dm}{dt} = -\vec{v}_c \rho \frac{dV}{dt}$

(d) $\vec{a}_2 = \frac{\vec{F}_{\text{thrust}}}{M_2}$

CALCULATE:

(a) $t_2 = \frac{1400.\text{ kg} - \sqrt{(400.\text{ kg})(1400.\text{ kg})}}{\left(1000.\text{ kg/m}^3\right)\left(0.003333\text{ m}^3/\text{s}\right)} = 195.5\text{ s}$

(b) $M_2 = \sqrt{(400.\text{ kg})(1400.\text{ kg})} = 748.33\text{ kg}$, $v_2 = (25.0\text{ m/s})\ln\left(\frac{(1400.\text{ kg})}{(748.33\text{ kg})}\right) = 15.66\text{ m/s}$

(c) Before the valve is switched, v_c is directed backward, i.e. $\vec{v}_c = -25.0\text{ m/s}$. Then $\vec{F}_{\text{thrust}} = -(-25.0\text{ m/s})\left(1000.\text{ kg/m}^3\right)\left(0.003333\text{ m}^3/\text{s}\right) = 83.33\text{ N}$ forward. After the valve is switched, $\vec{F}_{\text{thrust}}$ is directed backward, i.e. $\vec{F}_{\text{thrust}} = -83.33\text{ N}$.

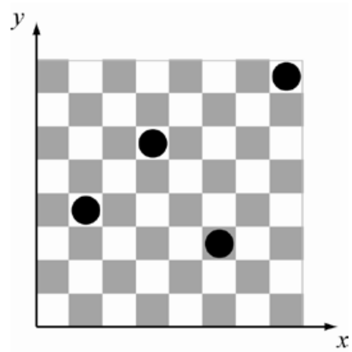
(d) Before the valve is switched, $\vec{a}_2 = \frac{83.33\text{ N}}{748.33\text{ kg}} = 0.111355\text{ m/s}^2$.

ROUND:
Rounding to three significant figures:
(a) $t_2 = 196\text{ s}$
(b) $M_2 = 748\text{ kg}$ and $v_2 = 15.7\text{ m/s}$.
(c) $\vec{F}_{\text{thrust}} = -83.3\text{ N}$
(d) $\vec{a}_2 = 0.111\text{ m/s}^2$

DOUBLE-CHECK: All the units for the results are appropriate. Also, the results are of reasonable orders of magnitude.

8.47. **THINK:** The checkerboard has dimensions 32.0 cm by 32.0 cm. Its mass is $m_b = 100.\text{ g}$ and the mass of each of the four checkers is $m_c = 20.0\text{ g}$. Determine the center of mass of the system. Note the checkerboard is 8 by 8 squares, thus the length of the side of each square is 32.0 cm/8 = 4.00 cm. From the figure provided, the following *x*-*y* coordinates can be associated with each checker's center of mass: $m_1 : (22.0\text{ cm}, 10.0\text{ cm})$, $m_2 : (6.00\text{ cm}, 14.0\text{ cm})$, $m_3 : (14.0\text{ cm}, 22.0\text{ cm})$, $m_4 : (30.0\text{ cm}, 30.0\text{ cm})$. Assuming a uniform density distribution, the checkerboard's center of mass is located at $(x_b, y_b) = (16.0\text{ cm}, 16.0\text{ cm})$.

SKETCH:



RESEARCH: To determine the system's center of mass, use the following equations: $X = \frac{1}{M}\sum_{i=1}^{n} x_i m_i$ and $Y = \frac{1}{M}\sum_{i=1}^{n} y_i m_i$.

SIMPLIFY: $M = m_b + 4m_c$

$X = \frac{1}{M}\left(x_b m_b + m_c\left(x_1 + x_2 + x_3 + x_4\right)\right)$, $Y = \frac{1}{M}\left(y_b m_b + m_c\left(y_1 + y_2 + y_3 + y_4\right)\right)$

CALCULATE: $M = 100.\text{ g} + 4(20.0\text{ g}) = 180.\text{ g}$

$$X = \frac{1}{180.\text{ g}}\left(16.0\text{ cm}(100.0\text{ g}) + 20.0\text{ g}(22.0\text{ cm} + 6.00\text{ cm} + 14.0\text{ cm} + 30.0\text{ cm})\right) = 16.889\text{ cm}$$

$$Y = \frac{1}{180.\text{ g}}\left(16.0\text{ cm}(100.\text{ g}) + 20.0\text{ g}(10.0\text{ cm} + 14.0\text{ cm} + 22.0\text{ cm} + 30.0\text{ cm})\right) = 17.33\text{ cm}$$

ROUND: $X = 16.9$ cm and $Y = 17.3$ cm. The answer is (16.9 cm,17.3 cm).

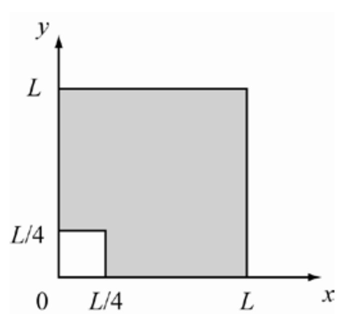
DOUBLE-CHECK: $m_b > m_c$, so it is reasonable to expect the system's center of mass to be near the board's center of mass.

8.48. **THINK:** The total mass of the plate is $M_{tot} = 0.205$ kg. The dimensions of the plate are L by L, $L = 5.70$ cm. The dimensions of the smaller removed plate are $L/4$ by $L/4$. The mass of the smaller removed plate is

$$\frac{M_{tot}}{A_{tot}} = \frac{m_s}{A_s} \Rightarrow m_s = A_s\frac{M_{tot}}{A_{tot}} = \left(\frac{L}{4}\right)^2\frac{M_{tot}}{L^2} = \frac{1}{16}M_{tot}.$$

Determine the distance from the bottom left corner of the plate to the center of mass after the smaller plate is removed. Note the mass of the plate with the void is $m_p = M_{tot} - m_s = 15M_{tot}/16$.

SKETCH:



RESEARCH: The center of mass in each dimension is $X=\frac{1}{M}\sum_{i=1}^{n}x_i m_i$ and $y=\frac{1}{M}\sum_{i=1}^{n}y_i m_i$. The center of mass of the plate with the void, (X_p,Y_p), can be determined by considering the center of mass of the total system as composed of the smaller plate of mass m_s and the plate with the void of mass m_p. Note the center of mass of the total system is at the total plate's geometric center, $(X,Y)=(L/2,L/2)$, assuming uniform density. Similarly, the center of mass of the smaller plate is at its center $(X_\text{s},Y_\text{s})=(L/8,L/8)$. The distance of the center of mass of the plate from the origin is then $d=\sqrt{X_\text{p}^2+Y_\text{p}^2}$.

SIMPLIFY: $X=\frac{1}{M_\text{tot}}\left(X_\text{p}m_\text{p}+X_\text{s}m_\text{s}\right)$, and $X_\text{p}=\frac{\left(XM_\text{tot}-X_\text{s}m_\text{s}\right)}{M_\text{tot}-\frac{1}{16}M_\text{tot}}=\frac{L\left(\frac{1}{2}M_\text{tot}-\frac{1}{8}\left(\frac{1}{16}M_\text{tot}\right)\right)}{\frac{15}{16}M_\text{tot}}=\frac{21}{40}L.$

Similarly, $Y_\text{p}=\frac{\left(YM_\text{tot}-Y_\text{s}m_\text{s}\right)}{m_\text{p}}=\frac{L\left(\frac{1}{2}M_\text{tot}-\frac{1}{8}\left(\frac{1}{16}M_\text{tot}\right)\right)}{\frac{15}{16}M_\text{tot}}=\frac{21}{40}L.$

CALCULATE: $X_\text{p}=Y_\text{p}=\frac{21}{40}(5.70\text{ cm})=2.9925\text{ cm}$

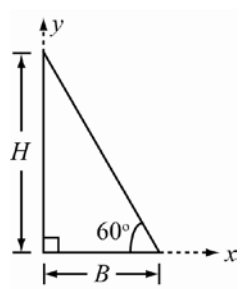
$d=\sqrt{(2.9925\text{ cm})^2+(2.9925\text{ cm})^2}=4.232\text{ cm}$

ROUND: Since L has three significant figures, the result should be rounded to $d=4.23$ cm.

DOUBLE-CHECK: It is expected that the center of mass for the plate with the void would be further from the origin than the center of mass for the total plate.

8.49. **THINK:** The height is H = 17.3 cm and the base is B = 10.0 cm for a flat triangular plate. Determine the x and y-coordinates of its center of mass. Since it is not stated otherwise, we assume that the mass density of this plate is constant.

SKETCH:



RESEARCH: Assuming the mass density is constant throughout the object, the center of mass is given by $\vec{R}=\frac{1}{A}\int_A \vec{r}\,dA$, where A is the area of the object. The center of mass can be determined in each dimension.

The x coordinate and the y coordinate of the center of mass are given by $X=\frac{1}{A}\int_A x\,dA$ and $Y=\frac{1}{A}\int_A y\,dA$, respectively. The area of the triangle is $A=HB/2$.

SIMPLIFY: The expression for the area of the triangle can be substituted into the formulae for the center of mass to get

$$X=\frac{2}{HB}\int_A x\,dA \text{ and } Y=\frac{2}{HB}\int_A y\,dA.$$

In the x-direction we have to solve the integral:

$$\int_A x dA = \int_0^B \int_0^{y_m(x)} x dy dx = \int_0^B x dx \int_0^{y_m(x)} dy = \int_0^B x y_m(x) dx = \int_0^B x H(1 - x/B) dx = H \int_0^B x - \left(x^2/B\right) dx$$

$$= H(\tfrac{1}{2}x^2 - \tfrac{1}{3}x^3/B)\Big|_0^B = \tfrac{1}{2}HB^2 - \tfrac{1}{3}HB^2 = \tfrac{1}{6}HB^2$$

Note that in this integration procedure the maximum for the y–integration depends on the value of x: $y_m(x) = H(1 - x/B)$. Therefore we arrive at

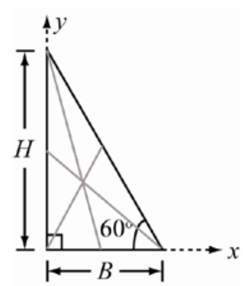
$$X = \frac{2}{HB}\int_A x\, dA = \frac{2}{HB} \cdot \frac{HB^2}{6} = \tfrac{1}{3}B$$

In the same way we can find that $Y = \tfrac{1}{3}H$.

CALCULATE: $X_{\text{com}} = \tfrac{1}{3}(10.0 \text{ cm}) = 3.33333 \text{ cm}$, $Y_{\text{com}} = \tfrac{1}{3}(17.3 \text{ cm}) = 5.76667 \text{ cm}$

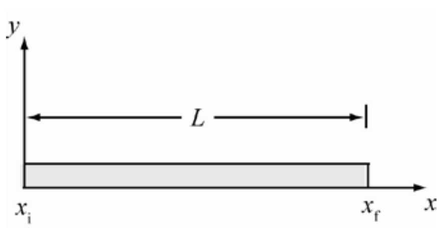
ROUND: Three significant figures were provided in the question, so the results should be written $X = 3.33 \text{ cm}$ and $Y = 5.77 \text{ cm}$.

DOUBLE-CHECK: Units of length were calculated for both X and Y, which is dimensionally correct. We also find that the center of mass coordinates are inside the triangle, which always has to be true for simple geometrical shape without holes in it. Finally, we can determine the location of the center of mass for a triangle geometrically by connecting the center of each side to the opposite corner with a straight line (see drawing). The point at which these three lines intersect is the location of the center of mass. You can see from the graph that this point has to be very close to our calculated result of $(\tfrac{1}{3}B, \tfrac{1}{3}H)$.



8.50. **THINK:** The linear density function for a 1.00 m long rod is $\lambda(x) = 100. \text{ g/m} + 10.0x \text{ g/m}^2$. One end of the rod is at $x_{\text{i}} = 0 \text{ m}$ and the other end is situated at $x_{\text{f}} = 1.00 \text{ m}$. The total mass, M of the rod and the center of mass coordinate are to be determined.

SKETCH:



RESEARCH:

(a) The linear density of the rod is given by $\lambda(x) = dm/dx$. This expression can be rearranged to get $\lambda(x)dx = dm$. An expression for $\lambda(x)$ was given so both sides can be integrated to solve for M.

(b) The center of mass coordinate is given by $X_{\text{com}} = \dfrac{1}{M}\int x dm$.

SIMPLIFY:

(a) Integrate both of sides of the linear density function to get:

$$\int_{x_{\text{i}}}^{x_{\text{f}}} \left(100. \text{ g/m} + 10.0x \text{ g/m}^2\right) dx = \int_0^M dm \Rightarrow \left[100.x \text{ g/m} + 5.0x^2 \text{ g/m}^2\right]_{x_{\text{i}}}^{x_{\text{f}}} = M.$$

(b) Substitute $dm = \lambda(x)dx$ into the expression for X_{com} to get

$$X_{\text{com}} = \frac{1}{M}\int_{x_{\text{i}}}^{x_{\text{f}}} x\lambda(x) dx.$$

The value calculated in part (a) for M can later be substituted. Substitute $\lambda(x)=100\text{ g/m}+10.0x\text{ g/m}^2$ into the expression for X_{com} to get

$$X_{\text{com}}=\frac{1}{M}\int_{x_i}^{x_f}\left(100.x\text{ g/m}+10.0x^2\text{ g/m}^2\right)dx\Rightarrow\left[\frac{1}{M}\left(50.0x^2\text{ g/m}+\frac{10.0}{3}x^3\text{ g/m}^2\right)\right]_{x_i}^{x_f}.$$

CALCULATE:

(a) $M=100.\text{ g/m}(1\text{ m})+5.0\text{ g}\dfrac{(1\text{ m})^2}{\text{m}^2}=105\text{ g}$

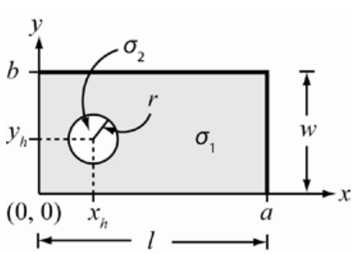
(b) $X_{\text{com}}=\dfrac{1}{105\text{ g}}\left(50.0(1\text{ m})^2\text{ g/m}+\dfrac{10.0}{3}(1\text{ m})^3\text{ g/m}^2\right)=0.50793651\text{ m}$

ROUND:
Rounding to three significant figures
(a) $M=105\text{ g}$
(b) $X_{\text{com}}=0.508\text{ m}$

DOUBLE-CHECK: The correct units were calculated for the mass and the center of mass so the results are dimensionally correct. Our result for the location of the center of mass of the rod, 50.8 cm, is just larger than the geometric center of the rod, 50.0 cm. This makes sense because the density of the rod increases slightly with increasing distance.

8.51. **THINK:** The area density for a thin, rectangular plate is given as $\sigma_1=1.05\text{ kg/m}^2$. Its length is a = 0.600 m and its width is b = 0.250 m. The lower left corner of the plate is at the origin. A circular hole of radius, r = 0.0480 m is cut out of the plate. The hole is centered at the coordinates $x_{\text{h}}=0.068\text{ m}$ and $y_{\text{h}}=0.068\text{ m}$. A round disk of radius, r is used to plug the hole. The disk, D, has a uniform area density of $\sigma_2=5.32\text{ kg/m}^2$. The distance from the origin to the modified plate's center of mass, R, is to be determined.

SKETCH:



RESEARCH: The center of mass, R, of an object can be defined mathematically as $R=\dfrac{1}{M}\sum_{i=1}^{n}\vec{r}_i m_i$ (1). In this equation, M is the total mass of the system. The vector $\vec{r}_i$ denotes the position of the i^{th} object's center of mass and m_{i} is the mass of that object. To solve this problem, the center of mass of the plate, R_{p}, and the center of mass of the disk, $R_{\text{D,}}$ must be determined. Then equation (1) can be used to determine the distance from the origin to the modified center of mass, R. First, consider the rectangular plate, P, which has the hole cut in it. The position of the center of mass, R_{p}, is not known. The mass of P can be denoted m_{p}. Consider the disk of material, d, that was removed (which has a uniform area density of σ_1), and denote its center of mass as R_{d} and its mass as m_{d}. Next, define S as the system of the rectangular plate, P, and the disc of removed material, d. The mass of S can be denoted $m_{\text{s}}=m_{\text{p}}+m_{\text{d}}$. The center of mass of S is $R_{\text{s}}=(a/2)\hat{x}+(b/2)\hat{y}$. m_{p} and m_{d} are not known but it is known that they have uniform area density of σ_1. The uniform area density is given by $\sigma=m/A$. Therefore, $m_{\text{p}}=\sigma_1 A_{\text{p}}$ and

$m_d = \sigma_1 A_d$, where A_p is the area of the plate minus the area of the hole and A_d is the area of the disk, *d*. The expressions for these areas are $A_p = ab - \pi r^2$ and $A_d = \pi r^2$. Substituting these area expressions into the expressions for m_p and m_d gives $m_p = \sigma_1\left(ab - \pi r^2\right)$ and $m_d = \sigma_1 \pi r^2$. So the center of mass of the system is given by:

$$\vec{R}_s = \frac{\left(x_h \hat{x} + y_h \hat{y}\right) m_d + \vec{R}_p m_p}{\sigma_1\left(ab - \pi r^2\right) + \sigma_1 \pi r^2} \quad (2).$$

Now, consider the disk, *D*, that is made of the material of uniform area density, σ_2. Define its center of mass as $\vec{R}_D = x_h \hat{x} + y_h \hat{y}$. Also, define its mass as $m_D = \sigma_2 \pi r^2$.

SIMPLIFY: Rearrange equation (2) to solve for $\vec{R}_p$:

$$\vec{R}_p m_p = \vec{R}_s \sigma_1 ab - \left(x_h \hat{x} + y_h \hat{y}\right) m_d \Rightarrow \vec{R}_p = \frac{\vec{R}_s \sigma_1 ab - \left(x_h \hat{x} + y_h \hat{y}\right) m_d}{m_p}.$$

Now, substitute the values for $\vec{R}_s$, m_d and m_p into the above equation to get:

$$\vec{R}_p = \frac{\left(\frac{a}{2}\hat{x} + \frac{b}{2}\hat{y}\right)\sigma_1 ab - \left(x_h \hat{x} + y_h \hat{y}\right)\sigma_1 \pi r^2}{\sigma_1\left(ab - \pi r^2\right)}.$$

Once $\vec{R}_p$ is solved, it can be substituted into the expression for $\vec{R}$ to get $\vec{R} = \dfrac{\vec{R}_p m_p + \vec{R}_D m_D}{m_p + m_D}$. Use the distance formula $R = \sqrt{R_x^{\,2} + R_y^{\,2}}$.

CALCULATE:

$$\vec{R}_p = \frac{\left(\frac{0.600}{2}\hat{x} + \frac{0.250}{2}\hat{y}\right)\left(\left(1.05 \text{ kg/m}^3\right)\left(0.600 \text{ m}\right)\left(0.250 \text{ m}\right)\right) - \left(0.068\hat{x} + 0.068\hat{y}\right)\left(1.05 \text{ kg/m}^3\right)\pi\left(0.0480 \text{ m}\right)^2}{\left(1.05 \text{ kg/m}^3\right)\left(\left(0.600 \text{ m}\right)\left(0.250 \text{ m}\right) - \pi\left(0.0480 \text{ m}\right)^2\right)}$$

$$= \left(0.31176\hat{x} + 0.12789\hat{y}\right) \text{ m}$$

$$\vec{R} = \frac{\left(0.31176\hat{x} + 0.12789\hat{y}\right) \text{ m}\left(0.1499 \text{ kg}\right) + \left(0.068\hat{x} + 0.068\hat{y}\right) \text{ m}\left(0.038507 \text{ kg}\right)}{0.1499 \text{ kg} + 0.038507 \text{ kg}}$$

$$= \left(0.26194\hat{x} + 0.11565\hat{y}\right) \text{ m}$$

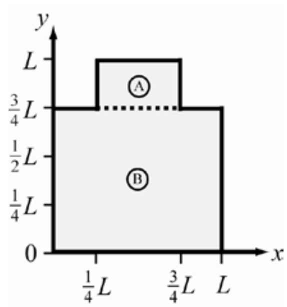
Then, the distance to the origin is given by $R = \sqrt{\left(0.26194 \text{ m}\right)^2 + \left(0.11565 \text{ m}\right)^2} = 0.28633 \text{ m}$.

ROUND: Densities are given to three significant figures. For dimensions the subtraction rule applies, where all dimensions are known to three decimal places. The result should be rounded to $R = 0.286$ m.

DOUBLE-CHECK: The position of the center of mass for the modified system is shifted closer to the position of the disk, *D*, which has an area density of 5.32 kg/m^2. This is reasonable because the disk has a much higher area density than the rest of the plate. Also, the results are reasonable considering the given values.

8.52. **THINK:** The object of interest is a uniform square metal plate with sides of length, L = 5.70 cm and mass, m = 0.205 kg. The lower left corner of the plate sits at the origin. Two squares with side length, $L/4$ are removed from each side at the top of the square. Determine the *x*-coordinate and the *y*-coordinate of the center of mass, denoted X_{com} and Y_{com}, respectively.

SKETCH:



RESEARCH: Because the square is uniform, the equations for X_{com} and Y_{com} can be expressed by

$$X_{\text{com}} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i \text{ and } Y_{\text{com}} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i.$$

M is the total mass of the system. In this problem it will be useful to treat the system as if it were made up of two uniform metal rectangles, R_A and R_B.

(a) The center of mass x-coordinate for rectangle A is $x_{\text{A}} = (L/2)\hat{x}$. The center of mass x-coordinate for rectangle B is $x_{\text{B}} = (L/2)\hat{x}$.

(b) The center of mass y-coordinate for rectangle A is $y_{\text{A}} = (7L/8)\hat{y}$. The center of mass y-coordinate for rectangle B is $y_{\text{B}} = (3L/8)\hat{y}$. Both rectangles have the same uniform area density, σ. The uniform area density is given by $\sigma = m_{\text{A}}/A_{\text{A}} = m_{\text{B}}/A_{\text{B}}$. Therefore, $m_{\text{A}} = m_{\text{B}}A_{\text{A}}/A_{\text{B}}$. The areas are given by the following expressions:

$$A_{\text{A}} = \left(\frac{L}{4}\right)\left(\frac{L}{2}\right) = \frac{L^2}{8} \text{ and } A_{\text{B}} = \left(\frac{3L}{4}\right)L = \frac{3L^2}{4}.$$

SIMPLIFY:

(a) $X_{\text{com}} = \dfrac{x_{\text{A}}m_{\text{A}} + x_{\text{B}}m_{\text{B}}}{m_{\text{A}} + m_{\text{B}}}$

Substitute the expression for m_{A} into the above equation to get:

$$X_{\text{com}} = \frac{x_{\text{A}}m_{\text{B}}\dfrac{A_{\text{A}}}{A_{\text{B}}} + x_{\text{B}}m_{\text{B}}}{m_{\text{B}}\dfrac{A_{\text{A}}}{A_{\text{B}}} + m_{\text{B}}} = \frac{x_{\text{A}}\left(\dfrac{A_{\text{A}}}{A_{\text{B}}}\right) + x_{\text{B}}}{\dfrac{A_{\text{A}}}{A_{\text{B}}} + 1}.$$

Then substitute the expressions for x_{A}, x_{B}, A_{A} and A_{B} to get:

$$X_{\text{com}} = \frac{\dfrac{L}{2}\left(\dfrac{L^2/8}{3L^2/4}\right) + \dfrac{L}{2}}{\dfrac{L^2/8}{3L^2/4} + 1} = \frac{\dfrac{L}{12} + \dfrac{L}{2}}{\dfrac{1}{6} + 1} = \frac{\dfrac{7L}{12}}{\dfrac{7}{6}} = \frac{1}{2}L.$$

(b) The same procedure can be used to solve for the y-coordinate of the center of mass:

$$Y_{\text{com}} = \frac{y_{\text{A}}\left(\dfrac{A_{\text{A}}}{A_{\text{B}}}\right) + y_{\text{B}}}{\dfrac{A_{\text{A}}}{A_{\text{B}}} + 1} = \frac{\dfrac{7L}{8}\left(\dfrac{1}{6}\right) + \dfrac{3L}{8}}{\dfrac{7}{6}} = \frac{\dfrac{7L}{48} + \dfrac{18L}{48}}{\dfrac{7}{6}} = \frac{25L}{48}\left(\frac{6}{7}\right) = \frac{25L}{56}.$$

CALCULATE:

(a) $X_{\text{com}} = \dfrac{1}{2}(5.70 \text{ cm}) = 2.85 \text{ cm}$

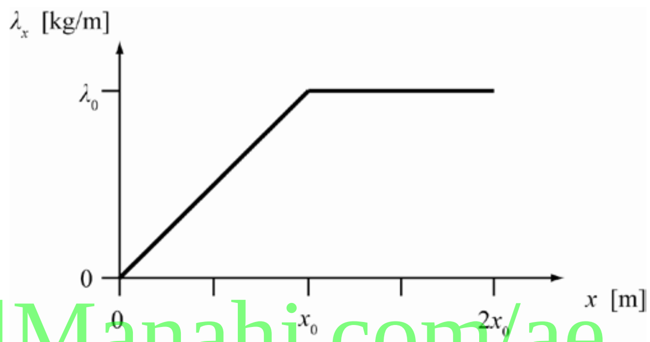
(b) $Y_{\text{com}} = \frac{25}{56}(5.70 \text{ cm}) = 2.5446 \text{ cm}$

ROUND: Three significant figures were provided in the question, so the results should be rounded to $X_{\text{com}} = 2.85 \text{ cm}$ and $Y_{\text{com}} = 2.54 \text{ cm}$.

DOUBLE-CHECK: Units of distance were calculated, which is expected when calculating the center of mass coordinates. The squares were removed uniformly at the top of the large square, so it makes sense that the *x*-coordinate of the center of mass stays at *L*/2 by symmetry and the *y*-coordinate of the center of mass is shifted slightly lower.

8.53. **THINK:** The linear mass density, $\lambda(x)$, is provided in the graph. Determine the location for the center of mass, X_{com}, of the object. From the graph, it can be seen that $\lambda(x) = \begin{Bmatrix} \frac{\lambda_0}{x_0}x, & 0 \le x < x_0 \\ \lambda_0, & x_0 \le x \le 2x_0 \end{Bmatrix}$.

SKETCH:



RESEARCH: The linear mass density, $\lambda(x)$, depends on *x*. To determine the center of mass, use the equation $X_{\text{com}} = \frac{1}{M}\int_L x\lambda(x)dx$. The mass of the system, M, can be determined using the equation $M = \int_L \lambda(x)dx$. In order to evaluate the center of mass of the system, two separate regions must be considered; the region from $x = 0$ to $x = x_0$ and the region from $x = x_0$ to $x = 2x_0$. The equation for X_{com} can be expanded to $X_{\text{com}} = \frac{1}{M}\int_0^{x_0} x\frac{\lambda_0}{x_0}xdx + \frac{1}{M}\int_{x_0}^{2x_0} \lambda_0 xdx$. The equation for M is $M = \int_0^{x_0} \frac{\lambda_0}{x_0}xdx + \int_{x_0}^{2x_0} \lambda_0 dx$.

SIMPLIFY: Simplify the expression for *M* first and then substitute it into the expression for X_{com}.

$$M = \int_0^{x_0} \frac{\lambda_0}{x_0}xdx + \int_{x_0}^{2x_0} \lambda_0 dx = \left[\frac{1}{2}\frac{\lambda_0}{x_0}x^2\right]_0^{x_0} + \left[x\lambda_0\right]_{x_0}^{2x_0} = \frac{1}{2}\lambda_0 x_0 + 2x_0\lambda_0 - x_0\lambda_0 = \frac{3}{2}x_0\lambda_0.$$

Substitute the above expression into the equation for X_{com} to get:

$$X_{\text{com}} = \frac{2}{3x_0\lambda_0}\left[\int_0^{x_0} x^2\frac{\lambda_0}{x_0}dx + \int_{x_0}^{2x_0} \lambda_0 xdx\right] = \frac{2}{3x_0\lambda_0}\left[\frac{1}{3}\lambda_0 x_0^{\ 2} + 2\lambda_0 x_0^{\ 2} - \frac{1}{2}\lambda_0 x_0^{\ 2}\right] = \frac{2}{3x_0\lambda_0}\left[\lambda_0 x_0^{\ 2}\left(\frac{2}{6} + \frac{12}{6} - \frac{3}{6}\right)\right]$$

$$= \frac{2}{3x_0\lambda_0}\left(\frac{11}{6}\lambda_0 x_0^{\ 2}\right) = \frac{11x_0}{9}.$$

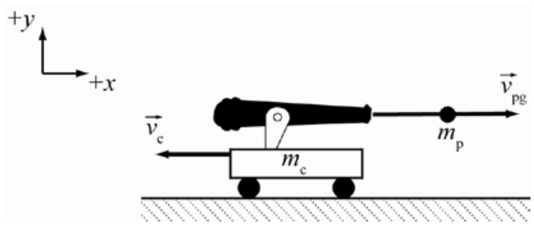
CALCULATE: This step does not apply.

ROUND: This step does not apply.

DOUBLE-CHECK: The units for the result are units of length, so the answer is dimensionally correct. It is reasonable that the calculated value is closer to the denser end of the object.

8.54. **THINK:** The mass of the cannon is $m_c = 750$ kg and the mass of the projectile is $m_p = 15$ kg. The total mass of the cannon and projectile system is $M = m_c + m_p$. The speed of the projectile is $v_p = 250$ m/s with respect to the muzzle just after the cannon has fired. The cannon is on wheels and can recoil with negligible friction. Determine the speed of the projectile with respect to the ground, v_{pg}.

SKETCH:



RESEARCH: The problem can be solved by considering the conservation of linear momentum. The initial momentum is $\vec{P}_i = 0$ because the cannon and projectile are both initially at rest. The final momentum is $\vec{P}_f = m_c\vec{v}_c + m_p\vec{v}_{pg}$. The velocity of the recoiling cannon is v_c. The equation for the conservation of momentum is $\vec{P}_i = \vec{P}_f$. The velocity of the projectile with respect to the cannon's muzzle can be represented as $\vec{v}_p = \vec{v}_{pg} - \vec{v}_c$. Take $\vec{v}_{pg}$ to be in the positive *x*-direction.

SIMPLIFY: Rearrange the above equation so that it becomes $\vec{v}_c = \vec{v}_{pg} - \vec{v}_p$. Then substitute this expression into the conservation of momentum equation:

$$P_i = P_f \Rightarrow 0 = m_c v_c + m_p v_{pg} \Rightarrow 0 = m_c\left(v_{pg} - v_p\right) + m_p v_{pg} \Rightarrow v_{pg}\left(m_c + m_p\right) = m_c v_p \Rightarrow v_{pg} = \frac{m_c v_p}{\left(m_c + m_p\right)}.$$

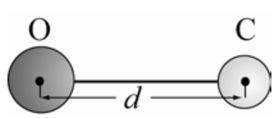
CALCULATE: $v_{pg} = \dfrac{(750 \text{ kg})(250 \text{ m/s})}{(750 \text{ kg} + 15 \text{ kg})} = 245.098$ m/s

ROUND: The least number of significant figures provided in the question is three, so the result should be rounded to $v_{pg} = 245$ m/s.

DOUBLE-CHECK: The units of speed are correct for the result. The velocity calculated for the projectile with respect to the ground is slower than its velocity with respect to the cannon's muzzle, which is what is expected.

8.55. **THINK:** The mass of a carbon atom is $m_C = 12.0$ u and the mass of an oxygen atom is $m_O = 16.0$ u. The distance between the atoms in a CO molecule is $d = 1.13 \cdot 10^{-10}$ m. Determine how far the center of mass, X_{com}, is from the carbon atom. Denote the position of the carbon atoms as X_C and the position of the oxygen atom as X_O.

SKETCH:



RESEARCH: The center of mass of a system is given by $X_{com} = \dfrac{1}{M}\sum_{i=1}^{n} m_i x_i$.

The total mass of the system is $M = m_C + m_O$. It is convenient to assign the position of the oxygen atom to be at the origin, $X_O = 0$. Then the center of mass becomes

$$X_{com} = \frac{(0)m_O + m_C d}{m_O + m_C} = \frac{m_C d}{m_O + m_C}.$$

Once X_{com} is determined, then the distance from it to the carbon atom can be determined using the equation $X_{\text{dC}} = X_{\text{C}} - X_{\text{com}}$, where X_{dC} is the distance from the center of mass to the carbon atom.

SIMPLIFY: Substitute the expression $X_{\text{com}} = (m_{\text{C}} d)/(m_{\text{O}} + m_{\text{C}})$ into the expression for X_{dC} to get $X_{\text{dC}} = X_{\text{C}} - \dfrac{m_{\text{C}} d}{m_{\text{O}} + m_{\text{C}}}$. Substitute $X_C = d$ to get $X_{\text{dC}} = d - \dfrac{m_{\text{C}} d}{m_{\text{O}} + m_{\text{C}}}$.

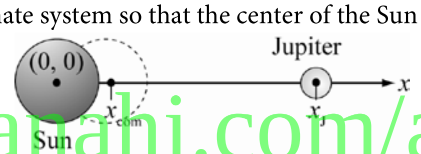
CALCULATE: $X_{\text{dC}} = (1.13 \cdot 10^{-10}\text{ m}) - \left(\dfrac{12.0\text{ u}}{28.0\text{ u}}\right)(1.13 \cdot 10^{-10}\text{ m}) = 6.4571 \cdot 10^{-11}\text{ m}$

ROUND: Three significant figures were provided in the problem so the answer should be rounded to $X_{\text{dC}} = 6.46 \cdot 10^{-11}$ m.

DOUBLE-CHECK: The center of mass of the system is closer to the more massive oxygen atom, as it should be.

8.56. **THINK:** The system to be considered consists of the Sun and Jupiter. Denote the position of the Sun's center of mass as X_{S} and the mass as m_{S}. Denote the position of Jupiter's center of mass as X_{J} and its mass as m_{J}. Determine the distance that the Sun wobbles due to its rotation about the center of mass. Also, determine how far the system's center of mass, X_{com}, is from the center of the Sun. The mass of the Sun is $m_{\text{S}} = 1.98892 \cdot 10^{30}$ kg. The mass of Jupiter is $m_{\text{J}} = 1.8986 \cdot 10^{27}$ kg. The distance from the center of the Sun to the center of Jupiter is $X_{\text{J}} = 7.78 \cdot 10^{8}$ km.

SKETCH: Construct the coordinate system so that the center of the Sun is positioned at the origin.



RESEARCH: The system's center of mass is given by $X_{\text{com}} = \dfrac{1}{M}\sum_{i=1}^{n} m_i x_i$.

The total mass of the system is $M = m_{\text{S}} + m_{\text{J}}$. The dashed line in the sketch denotes the Sun's orbit about the system's center of mass. From the sketch it can be seen that the distance the sun wobbles is twice the distance from the Sun's center to the system's center of mass.

SIMPLIFY: $X_{\text{com}} = \dfrac{m_{\text{S}} X_{\text{S}} + m_{\text{J}} X_{\text{J}}}{m_{\text{S}} + m_{\text{J}}}$. The coordinate system was chosen in such a way that $X_{\text{S}} = 0$. The center of mass equation can be simplified to $X_{\text{com}} = \dfrac{m_{\text{J}} X_{\text{J}}}{m_{\text{S}} + m_{\text{J}}}$. Once X_{com} is determined, it can be doubled to get the Sun's wobble.

CALCULATE: $X_{\text{com}} = \dfrac{(1.8986 \cdot 10^{27}\text{ kg})(7.78 \cdot 10^{8}\text{ km})}{1.98892 \cdot 10^{30}\text{ kg} + 1.8986 \cdot 10^{27}\text{ kg}} = 741961.5228\text{ km}$

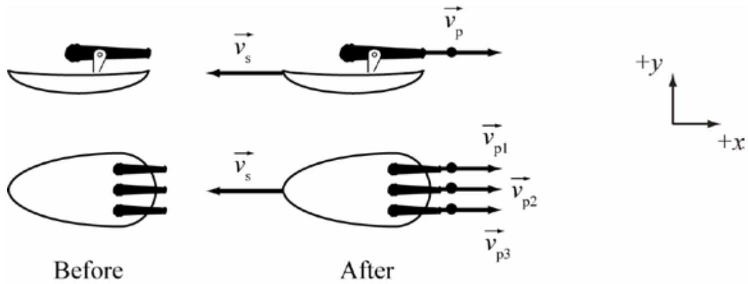
The Sun's wobble is $2(741961.5228\text{ km}) = 1483923.046\text{ km}$.

ROUND: Rounding the results to three figures, $X_{\text{com}} = 7.42 \cdot 10^{5}$ km and the Sun's wobble is $1.49 \cdot 10^{6}$ km.

DOUBLE-CHECK: It is expected that the system's center of mass is much closer to the Sun than it is to Jupiter, and the results are consistent with this.

8.57. **THINK:** The mass of the battleship is $m_{\text{s}} = 136{,}634{,}000$ lbs. The ship has twelve 16-inch guns and each gun is capable of firing projectiles of mass, $m_{\text{p}} = 2700.$ lb, at a speed of $v_{\text{p}} = 2300.$ ft/s. Three of the guns fire projectiles in the same direction. Determine the recoil velocity, v_{s}, of the ship. Assume the ship is initially stationary.

SKETCH:



RESEARCH: The total mass of the ship and projectile system is $M = m_s + \sum_{i=1}^{n} m_{pi}$.

All of the projectiles have the same mass and same speed when they are shot from the guns. This problem can be solved considering the conservation of momentum. The equation for the conservation of momentum is $\vec{P}_i = \vec{P}_f$. $\vec{P}_i$ is the initial momentum of the system and $\vec{P}_f$ is the final momentum of the system. Assume the ship carries one projectile per gun. $\vec{P}_i = 0$ because the battleship is initially at rest and $\vec{P}_f = -(m_s + 9m_p)v_s + 3m_p v_p$.

SIMPLIFY: $\vec{P}_i = \vec{P}_f \Rightarrow 0 = -(m_s + 9m_p)v_s + 3m_p v_p \Rightarrow v_s = \dfrac{3m_p v_p}{(m_s + 9m_p)}$

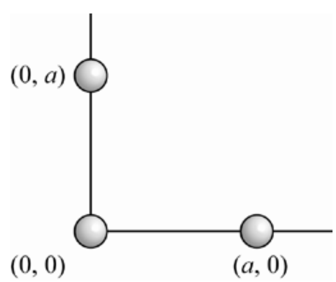
CALCULATE: $v_s = \dfrac{3(2700.\text{ lb})(2300.\text{ ft/s})}{(136{,}634{,}000\text{ lb} + 9(2700.\text{ lb}))} = 0.136325\text{ ft/s}$

ROUND: The values for the mass and speed of the projectile that are given in the question have four significant figures, so the result should be rounded to $v_s = 0.1363$ ft/s. The recoil velocity is in opposite direction than the cannons fire.

DOUBLE-CHECK: The mass of the ship is much greater than the masses of the projectiles, so it reasonable that the recoil velocity is small because momentum depends on mass and velocity.

8.58. **THINK:** The system has three identical balls of mass *m*. The *x* and *y* coordinates of the balls are $\vec{r}_1 = (0\hat{x}, 0\hat{y})$, $\vec{r}_2 = (a\hat{x}, 0\hat{y})$ and $\vec{r}_3 = (0\hat{x}, a\hat{y})$. Determine the location of the system's center of mass, *R*.

SKETCH:



RESEARCH: The center of mass is a vector quantity, so the *x* and *y* components must be considered separately. The *x*- and *y*-components of the center of mass are given by

$$X_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i \text{ and } Y_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i.$$

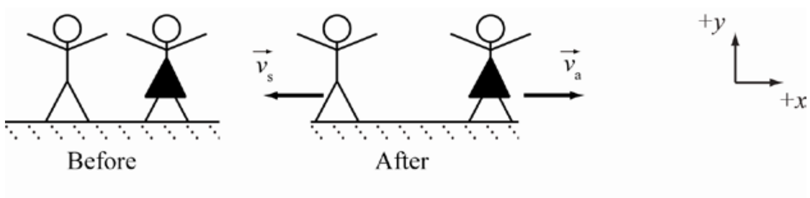
For this system, the equations can be rewritten as

$$X_{com} = \frac{m(0) + ma\hat{x} + m(0)}{3m} = \frac{a}{3}\hat{x} \text{ and } Y_{com} = \frac{m(0) + m(0) + ma\hat{y}}{3m} = \frac{a}{3}\hat{y}.$$

SIMPLIFY: The x and y components of the center of mass are known, so $\vec{R}_{com} = \frac{a}{3}\hat{x} + \frac{a}{3}\hat{y}$.

CALCULATE: This step is not necessary.
ROUND: This step is not necessary.
DOUBLE-CHECK: Considering the geometry of the system, the results are reasonable. In the x-direction we would expect the center of mass to be twice as far from the mass on the right as from the two on the left, and in the y-direction we would expect the center of mass to be twice as far from the upper mass as from the two lower ones.

8.59. **THINK:** Sam's mass is $m_S = 61.0$ kg and Alice's mass is $m_A = 44.0$ kg. They are standing on an ice rink with negligible friction. After Sam pushes Alice, she is moving away from him with a speed of $v_A = 1.20$ m/s with respect to the rink. Determine the speed of Sam's recoil, v_S. Also, determine the change in kinetic energy, ΔK, of the Sam-Alice system.
SKETCH:



RESEARCH:
(a) To solve the problem, consider the conservation of momentum. The equation for conservation of momentum can be written $\vec{P}_i = \vec{P}_f$. $\vec{P}_i$ is the initial momentum of the system and $\vec{P}_f$ is the final momentum of the system. $\vec{P}_i = 0$ because Sam and Alice are initially stationary and $\vec{P}_f = -m_S\vec{v}_S + m_A\vec{v}_A$.
(b) The change in kinetic energy is $\Delta K = K_f - K_i = \left(m_S v_S^2\right)/2 + \left(m_A v_A^2\right)/2$.

SIMPLIFY:

(a) $\vec{P}_i = \vec{P}_f \Rightarrow 0 = -m_S\vec{v}_S + m_A\vec{v}_A \Rightarrow v_S = \dfrac{m_A\vec{v}_A}{m_S}$

(b) The expression determined for v_S in part (a) can be substituted into the equation for ΔK to get

$$\Delta K = \frac{1}{2}m_S\left(\frac{m_A\vec{v}_A}{m_S}\right)^2 + \frac{1}{2}m_A v_A^{\ 2}.$$

CALCULATE:

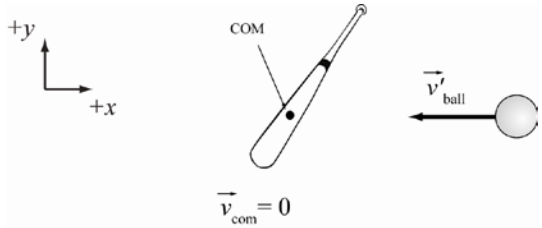
(a) $v_S = \dfrac{(44.0 \text{ kg})(1.20 \text{ m/s})}{61.0 \text{ kg}} = 0.8656$ m/s

(b) $\Delta K = \dfrac{1}{2}(61.0 \text{ kg})\left(\dfrac{(44.0 \text{ kg})(1.20 \text{ m/s})}{61.0 \text{ kg}}\right)^2 + \dfrac{1}{2}(44.0 \text{ kg})(1.20 \text{ m/s})^2 = 54.53$ J

(c) Sam did work on Alice when he pushed her. The work that Sam did was the source of the kinetic energy. Sam was able to do this work by converting chemical energy that was stored in his body into mechanical energy. The energy stored in Sam's body was provided by food that he ate and his body processed.
ROUND: Three significant figures were provided in the problem so the results should be rounded accordingly to $v_S = 0.866$ m/s and $\Delta K = 55$ J.
DOUBLE-CHECK: Sam's mass is greater than Alice's so it reasonable that his recoil speed is slower than her sliding speed. The change in kinetic energy is reasonable considering the masses and velocities given.

8.60. **THINK:** The mass of the bat is m_{bat} and the mass of the ball is m_{ball}. Assume that the center of mass of the ball and bat system is essentially at the bat. The initial velocity of the ball is $\vec{v}_{\text{ball,i}} = -30.0$ m/s and the initial velocity of the bat is $\vec{v}_{\text{bat}} = 35.0$ m/s. The bat and ball undergo a one-dimensional elastic collision. Determine the speed of the ball after the collision.

SKETCH:



RESEARCH: In the center of mass frame, $\vec{v}_{\text{com}} = 0$. Since the collision is elastic, in the center of mass frame the final velocity of the ball, $\vec{v}_{\text{ball,f}}$, will be equal to the negative of the ball's initial velocity, $\vec{v}_{\text{ball,i}}$. This statement can be written mathematically as $\vec{v}_{\text{ball,i}} = -\vec{v}_{\text{ball,f}}$. Since the center of mass is in the bat, the $\vec{v}_{\text{com}}$ in the lab reference frame equals $\vec{v}_{\text{bat}}$. The following relationships can be written for this system: $\vec{v}'_{\text{ball,i}} = \vec{v}_{\text{ball,i}} - \vec{v}_{\text{com}}$ (1) and $\vec{v}'_{\text{ball,f}} = \vec{v}_{\text{ball,f}} - \vec{v}_{\text{com}}$ (2).

SIMPLIFY: Recall that $\vec{v}'_{\text{ball,i}} = -\vec{v}'_{\text{ball,f}}$. Therefore, the following equality can be written:

$$\vec{v}_{\text{ball,i}} - \vec{v}_{\text{com}} = -\left(\vec{v}_{\text{ball,f}} - \vec{v}_{\text{com}}\right) \Rightarrow \vec{v}_{\text{ball,f}} = 2\vec{v}_{\text{com}} - \vec{v}_{\text{ball,i}}.$$

Recall that $\vec{v}_{\text{com}}$ is equal to $\vec{v}_{\text{bat}}$, so the above expression can be rewritten as $\vec{v}_{\text{ball,f}} = 2\vec{v}_{\text{bat}} - \vec{v}_{\text{ball,i}}$.

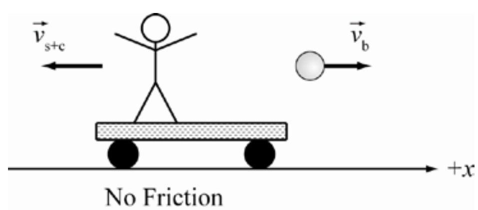
CALCULATE: $\vec{v}_{\text{ball,f}} = 2(35.0 \text{ m/s}) - (-30.0 \text{ m/s}) = 100.0$ m/s

ROUND: Rounding to three significant figures: $\vec{v}_{\text{ball,f}} = 100.$ m/s

DOUBLE-CHECK: The initial velocities of the bat and ball are similar, but the bat is much more massive than the ball, so the speed of the ball after the collision is expected to be high.

8.61. **THINK:** The student's mass is $m_{\text{s}} = 40.0$ kg, the ball's mass is $m_{\text{b}} = 5.00$ kg and the cart's mass is $m_{\text{c}} = 10.0$ kg. The ball's relative speed is $v'_{\text{b}} = 10.0$ m/s and the student's initial speed is $v_{\text{si}} = 0$. Determine the ball's velocity with respect to the ground, $\vec{v}_{\text{b}}$, after it is thrown.

SKETCH:



RESEARCH: $\vec{v}_{\text{b}}$ can be determined by considering the conservation of momentum, $\vec{P}_{\text{i}} = \vec{P}_{\text{f}}$, where $p = mv$. Note the ball's relative speed is $\vec{v}'_{\text{b}} = \vec{v}_{\text{b}} - \vec{v}_{\text{s+c}}$, where $\vec{v}_{\text{b}}$ and $\vec{v}_{\text{s+c}}$ are measured relative to the ground.

SIMPLIFY: $\vec{P}_{\text{i}} = \vec{P}_{\text{f}} \Rightarrow 0 = \left(m_{\text{s}} + m_{\text{c}}\right)\vec{v}_{\text{s+c}} + m_{\text{b}}\vec{v}_{\text{b}} \Rightarrow 0 = \left(m_{\text{s}} + m_{\text{c}}\right)\left(\vec{v}_{\text{b}} - \vec{v}'_{\text{b}}\right) + m_{\text{b}}\vec{v}_{\text{b}} \Rightarrow \vec{v}_{\text{b}} = \dfrac{\vec{v}'_{\text{b}}\left(m_{\text{s}} + m_{\text{c}}\right)}{m_{\text{s}} + m_{\text{c}} + m_{\text{b}}}$

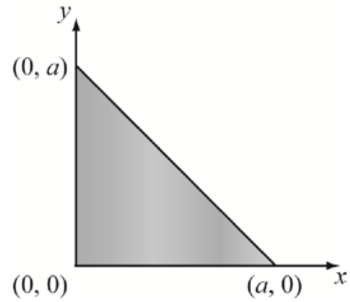
CALCULATE: $\vec{v}_{\text{b}} = \dfrac{(10.0 \text{ m/s})(40.0 \text{ kg} + 10.0 \text{ kg})}{(40.0 \text{ kg} + 10.0 \text{ kg} + 5.00 \text{ kg})} = 9.0909$ m/s

ROUND: $\vec{v}_{\text{b}} = 9.09$ m/s in the direction of $\vec{v}'_{\text{b}}$ (horizontal)

DOUBLE-CHECK: It is expected that $v_{\text{b}} < v'_{\text{b}}$ since the student and cart move away from the ball when it is thrown.

8.62. **THINK:** Determine the center of mass of an isosceles triangle of constant density σ.

SKETCH:



RESEARCH: To determine the center of mass of a two-dimensional object of constant density σ, use $X = \frac{1}{A}\int_A \sigma x dA$ and $Y = \frac{1}{A}\int_A \sigma y dA$.

SIMPLIFY: Note the boundary condition on the hypotenuse of the triangle, $x + y = a$. First, determine X. As x varies, take $dA = ydx$. Then the equation becomes $X = \frac{\sigma}{A}\int_0^a xydx$. From the boundary condition, $y = a - x$. Then the equation can be rewritten as $X = \frac{\sigma}{A}\int_0^a x(a-x)dx = \left[\frac{\sigma}{A}\left(\frac{1}{2}ax^2 - \frac{1}{3}x^3\right)\right]_0^a = \frac{a^3\sigma}{6A}$.

Similarly for Y, take $dA = xdy$ and $x = a - y$ to get $Y = \frac{\sigma}{A}\int_0^a y(a-y)dy = \left[\frac{\sigma}{A}\left(\frac{1}{2}ay^2 - \frac{1}{3}y^3\right)\right]_0^a = \frac{a^3\sigma}{6A}$, with $A = \int \sigma dA = \sigma \cdot \frac{bh}{2} = \frac{a^2\sigma}{2}$ we get $X = Y = \frac{2}{a^2\sigma}\cdot\frac{a^3\sigma}{6A} = \frac{a}{3}$.

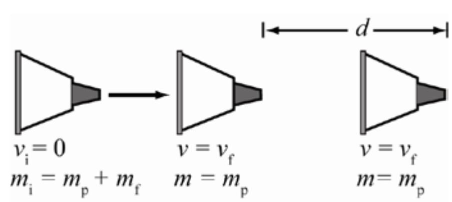
CALCULATE: This step is not applicable.

ROUND: This step is not applicable.

DOUBLE-CHECK: The center of mass coordinates that we obtained are contained within the isosceles triangle, as expected for a solid object.

8.63. **THINK:** The payload's mass is $m_p = 4390.0$ kg and the fuel mass is $m_f = 1.761\cdot 10^5$ kg. The initial velocity is $v_i = 0$. The distance traveled after achieving v_f is $d = 3.82\cdot 10^8$ m. The trip time is $t = 7.00 \text{ h} = 2.52\cdot 10^4$ s. Determine the propellant expulsion speed, v_c.

SKETCH:



RESEARCH: v_c can be determined from $v_f - v_i = v_c \ln(m_i / m_f)$. First, v_f must be determined from the relationship $v = d/t$.

SIMPLIFY: First, determine v_f from $v_f = d/t$. Substitute this expression and $v_i = 0$ into the above equation to determine v_c:

$$v_c = \frac{v_f}{\ln\left(\frac{m_i}{m_f}\right)} = \frac{d}{t\ln\left(\frac{m_i}{m_f}\right)} = \frac{d}{t\ln\left(\frac{m_p + m_f}{m_p}\right)}.$$

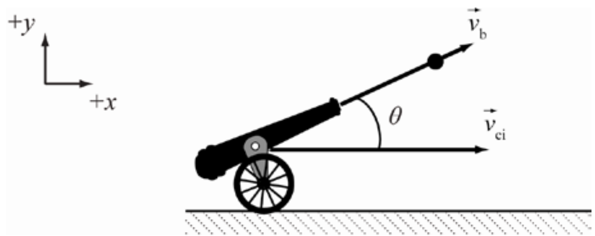
CALCULATE: $$v_c = \frac{3.82\cdot 10^8 \text{ m}}{\left(2.52\cdot 10^4 \text{ s}\right)\ln\left(\dfrac{4390.0 \text{ kg}+1.761\cdot 10^5 \text{ kg}}{4390.0 \text{ kg}}\right)} = 4.079\cdot 10^3 \text{ m/s}$$

ROUND: Since t has three significant figures, the result should be rounded to $v_c = 4.08$ km/s.

DOUBLE-CHECK: This expulsion velocity is reasonable.

8.64. **THINK:** The cannon's mass is M = 350 kg. The cannon's initial speed is $v_{ci} = 7.5$ m/s. The ball's mass is m = 15 kg and the launch angle is $\theta = 55°$. The cannon's final velocity after the shot is $v_{cf} = 0$. Determine the velocity of the ball relative to the cannon, $\vec{v}'_b$.

SKETCH:



RESEARCH: Use conservation of momentum, $\vec{P}_i = \vec{P}_f$, where $\vec{P} = m\vec{v}$. To determine the relative velocity, $\vec{v}'_b$, with respect to the cannon, use $\vec{v}'_b = \vec{v}_b - \vec{v}_c$, where $\vec{v}_b$ is the ball's velocity in the lab frame. Finally, since the cannon moves only in the horizontal (x) direction, consider only momentum conservation in this dimension. Take $\vec{v}_{ci}$ to be along the positive x-direction, that is $v_{ci} = +7.5$ m/s. With v_{bx} known, find v_b from the expression $v_{bx} = v_b\cos\theta$ and then v'_b can be determined.

SIMPLIFY: $P_{xi} = P_{xf} \Rightarrow (m_b + m_c)v_{ci} = m_c v_{cf} + m_b v_{bx}$. Note since v_{cf} is zero, $v_{bx} = v'_{bx}$, that is, the ball's speed relative to the cannon is the same as its speed in the lab frame since the cannon has stopped moving.

Rearranging the above equation gives $v_{bx} = \dfrac{(m_b + m_c)v_{ci}}{m_b} \Rightarrow v_b = \dfrac{(m_b + m_c)v_{ci}}{m_b\cos\theta}$.

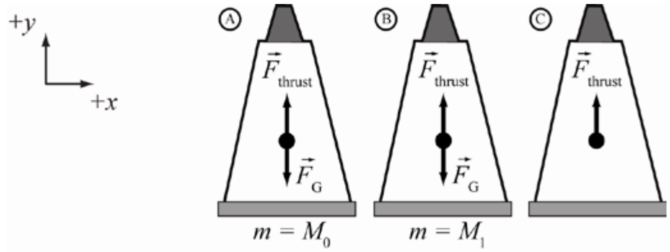
CALCULATE: $$v_b = \frac{(15.0 \text{ kg} + 350 \text{ kg})(7.50 \text{ m/s})}{(15.0 \text{ kg})\cos(55.0°)} = 318.2 \text{ m/s}$$

ROUND: Each given value has three significant figures, so the result should be rounded to $v_b = 318$ m/s.

DOUBLE-CHECK: This is a reasonable speed at which to launch a cannonball. The component of the momentum of the cannon/cannon ball system in the x-direction before the ball is shot is $p_{x,\text{before}} = (350 \text{ kg} + 15 \text{ kg})(7.5 \text{ m/s}) = 2737.5$ kg m/s. The component of the momentum of the cannon/cannon ball system in the x-direction after the ball is shot is $p_{x,\text{after}} = (15 \text{ kg})(318.2 \text{ m/s})\cos(55°) = 2737.68$ kg m/s. These components agree to within three significant figures.

8.65. **THINK:** The rocket's initial mass is $M_0 = 2.80\cdot 10^6$ kg. Its final mass is $M_1 = 8.00\cdot 10^5$ kg. The time to burn all the fuel is $\Delta t = 160.$ s. The exhaust speed is $v = v_c = 2700.$ m/s. Determine (a) the upward acceleration, a_0, of the rocket as it lifts off, (b) its upward acceleration, a_1, when all the fuel has burned and (c) the net change in speed, Δv in time Δt in the absence of a gravitational force.

SKETCH:



RESEARCH: To determine the upward acceleration, all the vertical forces on the rocket must be balanced. Use the following equations: $\vec{F}_{\text{thrust}} = -\vec{v}_{\text{c}}\dfrac{dm}{dt}$, $\vec{F}_{\text{g}} = m\vec{g}$, $\dfrac{dm}{dt} = \dfrac{\Delta m}{\Delta t}$. The mass of the fuel used is $\Delta m = M_0 - M_1$. To determine Δv in the absence of other forces (other than $\vec{F}_{\text{thrust}}$), use $v_{\text{f}} - v_{\text{i}} = v_{\text{c}}\ln\left(m_{\text{i}} / m_{\text{f}}\right)$.

SIMPLIFY:

(a) $\dfrac{dm}{dt} = \dfrac{M_0 - M_1}{\Delta t}$

Balancing the vertical forces on the rocket gives

$$F_{\text{net}} = F_{\text{thrust}} - F_{\text{g}} = ma \Rightarrow M_0 a_0 = v_{\text{c}}\frac{dm}{dt} - M_0 g \Rightarrow a_0 = \frac{v_{\text{c}}}{M_0}\left(\frac{M_0 - M_1}{\Delta t}\right) - g \Rightarrow a_0 = \frac{v_{\text{c}}}{\Delta t}\left(1 - \frac{M_1}{M_0}\right) - g.$$

(b) Similarly to part (a):

$$F_{\text{net}} = F_{\text{thrust}} - F_{\text{g}} = ma \Rightarrow M_1 a_1 = v_{\text{c}}\frac{dm}{dt} - M_1 g \Rightarrow a_1 = \frac{v_{\text{c}}}{M_1}\left(\frac{M_0 - M_1}{\Delta t}\right) - g \Rightarrow a_1 = \frac{v_{\text{c}}}{\Delta t}\left(\frac{M_0}{M_1} - 1\right) - g.$$

(c) In the absence of gravity, $F_{\text{net}} = F_{\text{thrust}}$. The change in velocity due to this thrust force is $\Delta v = v_{\text{c}}\ln\left(M_0 / M_1\right)$.

CALCULATE:

(a) $a_0 = \left(\dfrac{2700.\text{ m/s}}{160\text{ s}}\right)\left(1 - \dfrac{8.00 \cdot 10^5\text{ kg}}{2.80 \cdot 10^6\text{ kg}}\right) - 9.81\text{ m/s}^2 = 2.244\text{ m/s}^2$

(b) $a_1 = \left(\dfrac{2700.\text{ m/s}}{160.\text{ s}}\right)\left(\dfrac{2.80 \cdot 10^6\text{ kg}}{8.00 \cdot 10^5\text{ kg}} - 1\right) - 9.81\text{ m/s}^2 = 32.38\text{ m/s}^2$

(c) $\Delta v = \left(2700.\text{ m/s}\right)\ln\left(\dfrac{2.80 \cdot 10^6\text{ kg}}{8.00 \cdot 10^5\text{ kg}}\right) = 3382\text{ m/s}$

ROUND:

(a) $a_0 = 2.24\text{ m/s}^2$

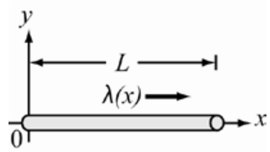
(b) $a_1 = 32.4\text{ m/s}^2$

(c) $\Delta v = 3380\text{ m/s}$

DOUBLE-CHECK: It can be seen that $a_1 > a_0$, as it should be since $M_1 < M_0$. It is not unusual for Δv to be greater than v_{c}.

8.66. **THINK:** The rod has a length of L and its linear density is $\lambda(x)=cx$, where c is a constant. Determine the rod's center of mass.

SKETCH:



RESEARCH: To determine the center of mass, take a differentially small element of mass: $dm=\lambda dx$ and use $X=\frac{1}{M}\int_L x\cdot dm=\frac{1}{M}\int_L x\lambda(x)dx$, where $M=\int_L dm=\int_L \lambda(x)dx.$

SIMPLIFY: First, determine M from $M=\int_0^L cxdx=\left[c\frac{1}{2}x^2\right]_0^L=\frac{1}{2}cL^2$. Then, the equation for the center of mass becomes:

$$X=\frac{1}{M}\int_0^L x(cx)dx=\frac{1}{M}\int_0^L cx^2dx=\frac{1}{M}c\left[\frac{1}{3}x^3\right]_0^L=\frac{1}{3M}cL^3.$$

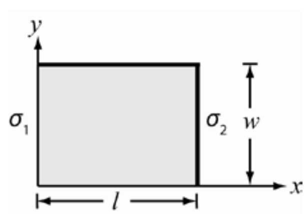
Substituting the expression for M into the above equation gives:

$$X=\frac{cL^3}{3\left(\frac{1}{2}cL^2\right)}=\frac{2}{3}L.$$

CALCULATE: This step is not applicable.
ROUND: This step is not applicable.
DOUBLE-CHECK: X is a function of L. Also, as expected, X is closer to the denser end of the rod.

8.67. **THINK:** The length and width of the plate are l = 20.0 cm and w = 10.0 cm, respectively. The mass density, σ, varies linearly along the length; at one end it is $\sigma_1=5.00\text{ g/cm}^2$ and at the other it is $\sigma_2=20.0\text{ g/cm}^2$. Determine the center of mass.

SKETCH:



RESEARCH: The mass density does not vary in width, i.e. along the y-axis. Therefore, the Y coordinate is simply $w/2$. To determine the X coordinate, use

$$X=\frac{1}{M}\int_A x\sigma(\vec{r})dA,\text{ where } M=\int_A\sigma(\vec{r})dA.$$

To obtain a functional form for $\sigma(\vec{r})$, consider that it varies linearly with x, and when the bottom left corner of the plate is at the origin of the coordinate system, σ must be σ_1 when x = 0 and σ_2 when $x = l$. Then, the conditions are satisfied by $\sigma(\vec{r})=\sigma(x)=\frac{(\sigma_2-\sigma_1)}{l}x+\sigma_1.$

SIMPLIFY: First determine M from $M=\int_A\sigma(\vec{r})dA=\int_0^l\int_0^w\sigma(x)dydx=\int_0^w dy\int_0^l\left(\frac{(\sigma_2-\sigma_1)}{l}x+\sigma_1\right)dx$. y is not dependent on x in this case, so

$$M = \left[y\right]_0^w \left[\frac{1}{2}\frac{(\sigma_2-\sigma_1)}{l}x^2+\sigma_1 x\right]_0^l = w\left(\frac{1}{2}\frac{(\sigma_2-\sigma_1)}{l}l^2+\sigma_1 l\right) = wl\left(\frac{1}{2}(\sigma_2-\sigma_1)+\sigma_1\right) = \frac{wl}{2}(\sigma_2+\sigma_1).$$

Now, reduce the equation for the center of mass:

$$X = \frac{1}{M}\int_A x\sigma(x)dA = \frac{1}{M}\int_0^l\int_0^w x\left(\frac{(\sigma_2-\sigma_1)}{l}x+\sigma_1\right)dydx = \frac{1}{M}\int_0^w dy\int_0^l\left(\frac{(\sigma_2-\sigma_1)}{l}x^2+\sigma_1 x\right)dx$$

$$= \frac{1}{M}\left[y\right]_0^w\left[\frac{(\sigma_2-\sigma_1)}{3l}x^3+\frac{1}{2}\sigma_1 x^2\right]_0^l = \frac{1}{M}w\left(\frac{(\sigma_2-\sigma_1)}{3l}l^3+\frac{1}{2}\sigma_1 l^2\right) = \frac{1}{M}wl^2\left(\frac{1}{3}(\sigma_2-\sigma_1)+\frac{1}{2}\sigma_1\right)$$

$$= \frac{1}{M}wl^2\left(\frac{1}{3}\sigma_2+\frac{1}{6}\sigma_1\right)$$

Substitute the expression for M into the above equation to get

$$X = \frac{l\left(\frac{1}{3}\sigma_2+\frac{1}{6}\sigma_1\right)}{\frac{1}{2}(\sigma_2+\sigma_1)} = \frac{2l\left(\frac{1}{3}\sigma_2+\frac{1}{6}\sigma_1\right)}{\sigma_2+\sigma_1}.$$

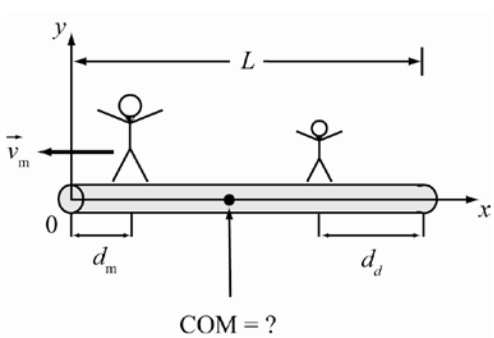
CALCULATE: $X = \frac{2(20.0 \text{ cm})\left(\frac{1}{3}(20.0 \text{ g/cm}^2)+\frac{1}{6}(5.00 \text{ g/cm}^2)\right)}{20.0 \text{ g/cm}^2+5.00 \text{ g/cm}^2} = 12.00 \text{ cm}, \quad Y = \frac{1}{2}(10.0 \text{ cm}) = 5.00 \text{ cm}$

ROUND: The results should be written to three significant figures: $X = 12.0$ cm and $Y = 5.00$ cm. The center of mass is at (12.0 cm, 5.00 cm).

DOUBLE-CHECK: It is expected that the center of mass for the *x* coordinate is closer to the denser end of the rectangle (before rounding).

8.68. **THINK:** The log's length and mass are $L = 2.50$ m and $m_1 = 91.0$ kg, respectively. The man's mass is $m_m = 72$ kg and his location is $d_m = 0.220$ m from one end of the log. His daughter's mass is $m_d = 20.0$ kg and her location is $d_d = 1.00$ m from the other end of the log. Determine (a) the system's center of mass and (b) the initial speed of the log and daughter, v_{l+d}, when the man jumps off the log at a speed of $v_m = 3.14$ m/s.

SKETCH:



RESEARCH: In one dimension, the center of mass location is given by $X = \frac{1}{M}\sum_{i=1}^{n} x_i m_i$. Take the origin of the coordinate system to be at the end of log near the father. To determine the initial velocity of the log and girl system, consider the conservation of momentum, $\vec{p}_i = \vec{p}_f$, where $\vec{p} = m\vec{v}$. Note that the man's velocity is away from the daughter. Take this direction to be along the $-\hat{x}$ direction, so that $\vec{v}_m = -3.14 \text{ m/s}\,\hat{x}$.

SIMPLIFY:

(a) $$X = \frac{1}{M}\left(x_{\text{m}}m_{\text{m}} + x_{\text{d}}m_{\text{d}} + x_{\text{l}}m_{\text{l}}\right) = \frac{\left(d_{\text{m}}m_{\text{m}} + \left(L - d_{\text{d}}\right)m_{\text{d}} + \frac{1}{2}Lm_{\text{l}}\right)}{m_{\text{m}} + m_{\text{d}} + m_{\text{l}}}$$

(b) $$\vec{p}_{\text{i}} = \vec{p}_{\text{f}} \;\Rightarrow\; 0 = m_{\text{m}}\vec{v}_{\text{m}} + \left(m_{\text{d}} + m_{\text{l}}\right)\vec{v}_{\text{d+l}} \;\Rightarrow\; \vec{v}_{\text{d+l}} = -\frac{m_{\text{m}}\vec{v}_{\text{m}}}{\left(m_{\text{d}} + m_{\text{l}}\right)}$$

CALCULATE:

(a) $$X = \frac{\left((0.220\text{ m})(72.0\text{ kg}) + (2.50\text{ m} - 1.00\text{ m})(20.0\text{ kg}) + \frac{1}{2}(2.50\text{ m})(91.0\text{ kg})\right)}{72.0\text{ kg} + 20.0\text{ kg} + 91.0\text{ kg}} = 0.8721\text{ m}$$

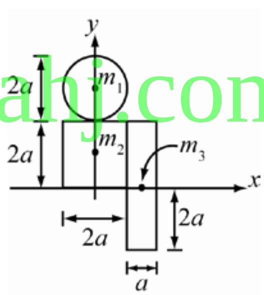
(b) $$\vec{v}_{\text{d+l}} = -\frac{(72.0\text{ kg})\left(-3.14\text{ m/s }\hat{x}\right)}{\left(20.0\text{ kg} + 91.0\text{ kg}\right)} = 2.0368\text{ m/s }\hat{x}$$

ROUND: To three significant figures, the center of mass of the system is X = 0.872 m from the end of the log near the man, and the speed of the log and child is $v_{\text{d+l}} = 2.04$ m/s.

DOUBLE-CHECK: As it should be, the center of mass is between the man and his daughter, and $v_{\text{d+l}}$ is less than v_{m} (since the mass of the log and child is larger than the mass of the man).

8.69. **THINK:** Determine the center of mass of an object which consists of regularly shaped metal of uniform thickness and density. Assume that the density of the object is ρ.

SKETCH:



RESEARCH: First, as shown in the figure above, divide the object into three parts, m_1, m_2 and m_3. Determine the center of mass by using $\vec{R} = \frac{1}{M}\sum_{i=1}^{3} m_i\vec{r}_i$, or in component form $X = \frac{1}{M}\sum_{i=1}^{3} m_i x_i$ and $Y = \frac{1}{M}\sum_{i=1}^{3} m_i y_i$. Also, use $m = \rho At$ for the mass, where A is the area and t is the thickness.

SIMPLIFY: The center of mass components are given by:

$$X = \frac{m_1x_1 + m_2x_2 + m_3x_3}{M} \text{ and } Y = \frac{m_1y_1 + m_2y_2 + m_3y_3}{M}$$

The masses of the three parts are $m_1 = \rho\pi a^2 t$, $m_2 = \rho(2a)^2 t$ and $m_3 = \rho 4a^2 t$. The center of mass of the three parts are $x_1 = 0$, $y_1 = 3a$, $x_2 = 0$, $y_2 = a$, $x_3 = 3a/2$ and $y_3 = 0$. The total mass of the object is $M = m_1 + m_2 + m_3 = \rho\pi a^2 t + 4\rho a^2 t + 4\rho a^2 t = \rho a^2 t(8 + \pi)$.

CALCULATE: The center of mass of the object is given by the following equations:

$$X = \frac{0 + 0 + 4\rho a^2 t(3a/2)}{\rho a^2 t(8 + \pi)} = \left(\frac{6}{8 + \pi}\right)a;$$

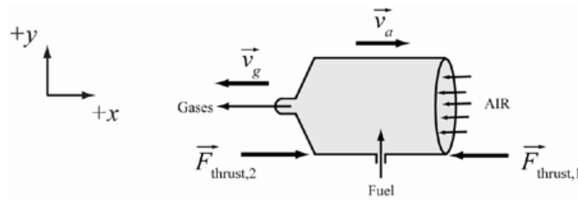
$$Y = \frac{\rho\pi a^2 t(3a) + 4\rho a^2 t(a) + 0}{\rho a^2 t(8 + \pi)} = \left(\frac{4 + 3\pi}{8 + \pi}\right)a.$$

ROUND: Rounding is not required.

DOUBLE-CHECK: The center of mass of the object is located in the area of m_2. By inspection of the figure this is reasonable.

8.70. **THINK:** A jet aircraft has a speed of 223 m/s. The rate of change of the mass of the aircraft is $(dM/dt)_{\text{air}} = 80.0$ kg/s (due to the engine taking in air) and $(dM/dt)_{\text{fuel}} = 3.00$ kg/s (due to the engine taking in and burning fuel). The speed of the exhaust gases is 600. m/s. Determine the thrust of the jet engine.

SKETCH:



RESEARCH: The thrust is calculated by using $\vec{F}_{\text{thrust}} = -\vec{v}\,dM/dt$, where $\vec{v}$ is the velocity of the gases or air, relative to the engine. There are two forces on the engine. The first force, $F_{\text{thrust,1}}$, is the thrust due to the engine taking in air and the second force, $F_{\text{thrust,2}}$, is the thrust due to the engine ejecting gases.

$$\vec{F}_{\text{thrust,1}} = -\vec{v}_{\text{a}}\left(\frac{dM}{dt}\right)_{\text{air}}, \quad \vec{F}_{\text{thrust,2}} = -\vec{v}_{\text{g}}\left[\left(\frac{dM}{dt}\right)_{\text{air}} + \left(\frac{dM}{dt}\right)_{\text{fuel}}\right]$$

The net thrust is given by $\vec{F}_{\text{thrust}} = \vec{F}_{\text{thrust,1}} + \vec{F}_{\text{thrust,2}}$.

SIMPLIFY: Simplification is not required.

CALCULATE: $\vec{F}_{\text{thrust,1}} = -(223 \text{ m/s } \hat{x})(80.0 \text{ kg/s}) = -17840 \text{ N } \hat{x}$,

$\vec{F}_{\text{thrust,2}} = -(600. \text{ m/s } (-\hat{x}))[80.0 \text{ kg/s} + 3.00 \text{ kg/s}] = 49800 \text{ N } \hat{x}$,

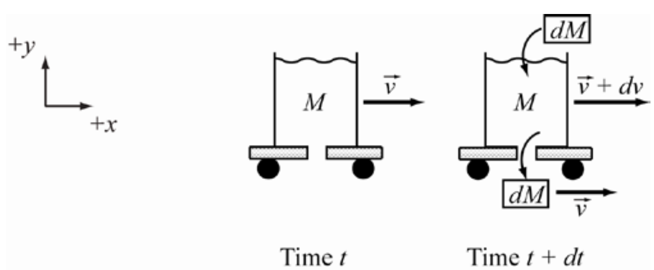
$\vec{F}_{\text{thrust}} = -17840 \text{ N } \hat{x} + 49800 \text{ N } \hat{x} = 31960 \text{ N } \hat{x}$

ROUND: To three significant figures, the thrust of the jet engine is $\vec{F}_{\text{thrust}} = 32.0 \text{ kN } \hat{x}$.

DOUBLE-CHECK: Since the $\hat{x}$ direction is in the forward direction of the aircraft, the plane moves forward, which it must. A jet engine is very powerful, so the large magnitude of the result is reasonable.

8.71. **THINK:** The solution to this problem is similar to a rocket system. Here the system consists of a bucket, a skateboard and water. The total mass of the system is M = 10.0 kg. The total mass of the bucket, skateboard and water remains constant at $\lambda = dM/dt = 0.100$ kg/s since rain water enters the top of the bucket at the same rate that it exits the bottom. Determine the time required for the bucket and the skateboard to reach a speed of half the initial speed.

SKETCH:



RESEARCH: To solve this problem, consider the conservation of momentum, $\vec{p}_{\text{i}} = \vec{p}_{\text{f}}$. The initial momentum of the system at time t is $p_{\text{i}} = Mv$. After time $t + dt$, the momentum of the system is $p_{\text{f}} = vdM + M(v + dv)$.

SIMPLIFY: $p_{\text{i}} = p_{\text{f}} \Rightarrow Mv = vdM + Mv + Mdv \Rightarrow Mdv = -vdM$

Dividing both sides by dt gives

$$M\frac{dv}{dt} = -v\frac{dM}{dt} = -v\lambda \text{ or } \frac{1}{v}\frac{dv}{dt} = -\frac{\lambda}{M} \Rightarrow \frac{1}{v}dv = -\frac{\lambda}{M}dt.$$

Integrate both sides to get

$$\int_{v=v_0}^{v}\frac{1}{v}dv = \int_{t=0}^{t}-\frac{\lambda}{M}dt \Rightarrow \ln v - \ln v_0 = -\frac{\lambda}{M}t \Rightarrow \ln\left(\frac{v}{v_0}\right) = -\frac{\lambda}{M}t.$$

Determine the time such that $v = v_0/2$. Substituting $v = v_0/2$ into the above equation gives

$$\ln\left(\frac{v_0/2}{v_0}\right) = -\frac{\lambda}{M}t \Rightarrow t = -\frac{M}{\lambda}\ln\left(\frac{1}{2}\right) = \frac{M}{\lambda}\ln(2).$$

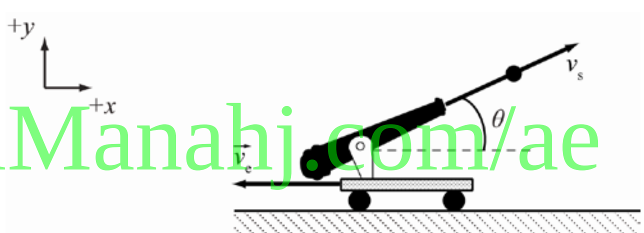
CALCULATE: $t = \dfrac{(10.0 \text{ kg})\ln(2)}{0.100 \text{ kg/s}} = 69.3147 \text{ s}$

ROUND: To three significant figures, the time for the system to reach half of its initial speed is $t = 69.3$ s.

DOUBLE-CHECK: It is reasonable that the time required to reduce the speed of the system to half its original value is near one minute.

8.72. **THINK:** The mass of a cannon is $M = 1000.$ kg and the mass of a shell is $m = 30.0$ kg. The shell is shot at an angle of $\theta = 25.0°$ above the horizontal with a speed of $v_s = 500.$ m/s. Determine the recoil velocity of the cannon.

SKETCH:



RESEARCH: The momentum of the system is conserved, $p_i = p_f$, or in component form, $p_{xi} = p_{xf}$ and $p_{yi} = p_{yf}$. Use only the x component of the momentum.

SIMPLIFY: p_{xi} is equal to zero since both the cannon and the shell are initially at rest. Therefore,

$$p_{xi} = p_{xf} \Rightarrow mv_s\cos\theta + Mv_c = 0 \Rightarrow v_c = -\frac{m}{M}v_s\cos\theta.$$

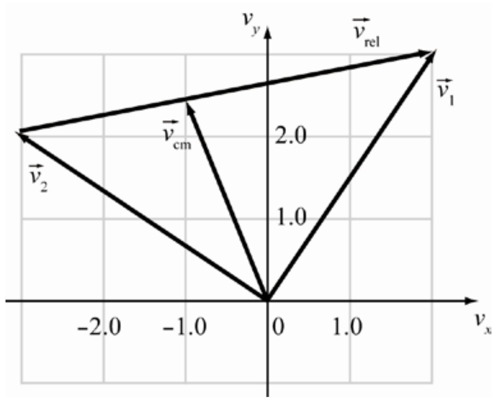
CALCULATE: $v_c = -\dfrac{(30.0 \text{ kg})(500. \text{ m/s})\cos(25.0°)}{1000. \text{ kg}} = -13.595 \text{ m/s}$

ROUND: To three significant figures: $v_c = -13.6$ m/s

DOUBLE-CHECK: The direction of the recoil is expected to be in the opposite direction to the horizontal component of the velocity of the shell. This is why the result is negative.

8.73. **THINK:** There are two masses, $m_1 = 2.0$ kg and $m_2 = 3.0$ kg. The velocity of their center of mass and the velocity of mass 1 relative to mass 2 are $\vec{v}_{cm} = (-1.00\hat{x} + 2.40\hat{y})$ m/s and $\vec{v}_{rel} = (5.00\hat{x} + 1.00\hat{y})$ m/s. Determine the total momentum of the system and the momenta of mass 1 and mass 2.

SKETCH:



RESEARCH: The total momentum of the system is $\vec{p}_{\text{cm}} = M\vec{v}_{\text{cm}} = m_1\vec{v}_1 + m_2\vec{v}_2$. The velocity of mass 1 relative to mass 2 is $\vec{v}_{\text{rel}} = \vec{v}_1 - \vec{v}_2$.

SIMPLIFY: The total mass M of the system is $M = m_1 + m_2$. The total momentum of the system is given by $\vec{p}_{\text{cm}} = M\vec{v}_{\text{cm}} = (m_1 + m_2)\vec{v}_{\text{cm}} = m_1\vec{v}_1 + m_2\vec{v}_2$. Substitute $\vec{v}_2 = \vec{v}_1 - \vec{v}_{\text{rel}}$ into the equation for the total momentum of the system to get $M\vec{v}_{\text{cm}} = m_1\vec{v}_1 + m_2(\vec{v}_1 - \vec{v}_{\text{rel}}) = (m_1 + m_2)\vec{v}_1 - m_2\vec{v}_{\text{rel}}$. Therefore, $\vec{v}_1 = \vec{v}_{\text{cm}} + \dfrac{m_2}{M}\vec{v}_{\text{rel}}$. Similarly, substitute $\vec{v}_1 = \vec{v}_2 + \vec{v}_{\text{rel}}$ into the equation for the total momentum of the system to get $M\vec{v}_{\text{cm}} = m_1\vec{v}_{\text{rel}} + (m_1 + m_2)\vec{v}_2$ or $\vec{v}_2 = \vec{v}_{\text{cm}} - \dfrac{m_1}{M}\vec{v}_{\text{rel}}$. Therefore, the momentums of mass 1 and mass 2 are $\vec{p}_1 = m_1\vec{v}_1 = m_1\vec{v}_{\text{cm}} + \dfrac{m_1 m_2}{M}\vec{v}_{\text{rel}}$ and $\vec{p}_2 = m_2\vec{v}_2 = m_2\vec{v}_{\text{cm}} - \dfrac{m_1 m_2}{M}\vec{v}_{\text{rel}}$.

CALCULATE:

(a)

$$\vec{p}_{\text{cm}} = (2.00\text{ kg} + 3.00\text{ kg})(-1.00\hat{x} + 2.40\hat{y})\text{ m/s} = (-5.00\hat{x} + 12.0\hat{y})\text{ kg m/s}$$

$$\vec{p}_{\text{cm}} = (2.0\text{ kg} + 3.0\text{ kg})(-1.0\hat{x} + 2.4\hat{y})\text{ m/s} = (-5.0\hat{x} + 12\hat{y})\text{ kg m/s}$$

(b) $$\vec{p}_1 = (2.00\text{ kg})(-1.00\hat{x} + 2.40\hat{y})\text{ m/s} + \frac{(2.00\text{ kg})(3.00\text{ kg})}{2.00\text{ kg} + 3.00\text{ kg}}(5.00\hat{x} + 1.00\hat{y})\text{ m/s}$$

$$= (-2.00\hat{x} + 4.80\hat{y})\text{ kg m/s} + (6.00\hat{x} + 1.20\hat{y})\text{ kg m/s} = (4.00\hat{x} + 6.00\hat{y})\text{ kg m/s}$$

(c) $$\vec{p}_2 = (3.00\text{ kg})(-1.00\hat{x} + 2.40\hat{y})\text{ m/s} - \frac{(2.00\text{ kg})(3.00\text{ kg})}{2.00\text{ kg} + 3.00\text{ kg}}(5.00\hat{x} + 1.00\hat{y})\text{ m/s}$$

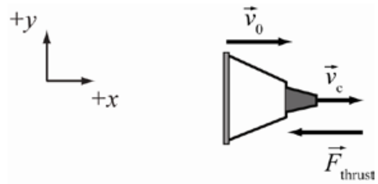
$$= (-3.00\hat{x} + 7.20\hat{y})\text{ kg m/s} - (6.00\hat{x} + 1.20\hat{y})\text{ kg m/s} = (-9.00\hat{x} + 6.00\hat{y})\text{ kg m/s}$$

ROUND: The answers have already been rounded to three significant figures.

DOUBLE-CHECK: It is clear from the results of (a), (b) and (c) that $\vec{p}_{\text{cm}} = \vec{p}_1 + \vec{p}_2$.

8.74. **THINK:** A spacecraft with a total initial mass of $m_s = 1000.\text{ kg}$ and an initial speed of $v_0 = 1.00\text{ m/s}$ must be docked. The mass of the fuel decreases from 20.0 kg. Since the mass of the fuel is small compared to the mass of the spacecraft, we can ignore it. To reduce the speed of the spacecraft, a small retro-rocket is used which can burn fuel at a rate of $dM/dt = 1.00\text{ kg/s}$ and with an exhaust speed of $v_E = 100.\text{ m/s}$.

SKETCH:



RESEARCH:

(a) The thrust of the retro-rocket is determined using $F_{\text{thrust}} = v_c dM/dt$.

(b) In order to determine the amount of fuel needed, first determine the time to reach a speed of $v = 0.0200$ m/s. Use $v = v_0 - at$. By Newton's Second Law the thrust is also given by $\vec{F}_{\text{thrust}} = m_s\vec{a}$.

(c) The burn of the retro-rocket must be sustained for a time sufficient to reduce the speed to 0.0200 m/s, found in part (b).

(d) Use the conservation of momentum, $\vec{p}_i = \vec{p}_f$.

SIMPLIFY:

(a) $\vec{F}_{\text{thrust}} = -\vec{v}_c \dfrac{dM}{dt}$

(b) $t = \dfrac{v_0 - v}{a}$

The acceleration is given by $a = F_{\text{thrust}}/m_s$. Substitute this expression into the equation for t above to get $t = \dfrac{(v_0 - v)m_s}{F_{\text{thrust}}}$. Therefore, the mass of fuel needed is $m_F = \left(\dfrac{dM}{dt}\right)t = \left(\dfrac{dM}{dt}\right)\dfrac{(v_0 - v)m_s}{F_{\text{thrust}}}$.

(c) $t = \dfrac{(v_0 - v)m_s}{F_{\text{thrust}}}$

(d) $m_s\vec{v} = (M + m_s)\vec{v}_f \Rightarrow \vec{v}_f = \dfrac{m_s}{M + m_s}\vec{v}$, where M is the mass of the space station.

CALCULATE:

(a) The thrust is $\vec{F}_{\text{thrust}} = -(100.\text{ m/s})(1.00\text{ kg/s})\hat{v}_E = -100.0\text{ N}\,\hat{v}_c$, or 100.0 N in the opposite direction to the velocity of the spacecraft.

(b) $m_F = (1.00\text{ kg/s})\dfrac{(1.00\text{ m/s} - 0.0200\text{ m/s})1000.\text{ kg}}{100.0\text{ N}} = 9.800\text{ kg}$

(c) $t = \dfrac{(1.00\text{ m/s} - 0.0200\text{ m/s})1000.\text{ kg}}{100.0\text{ N}} = 9.800\text{ s}$

(d) $\vec{v}_f = \dfrac{1000.\text{ kg}(0.0200\text{ m/s})}{5.00\cdot 10^5\text{ kg} + 1000.\text{ kg}}\hat{v} = 3.992\cdot 10^{-5}\text{ m/s}\,\hat{v}$; that is, in the same direction as the spacecraft is moving.

ROUND: The answers should be expressed to three significant figures:

(a) $\vec{F}_{\text{thrust}} = -100.\text{ N}\,\hat{v}_c$

(b) $m_F = 9.80\text{ kg}$

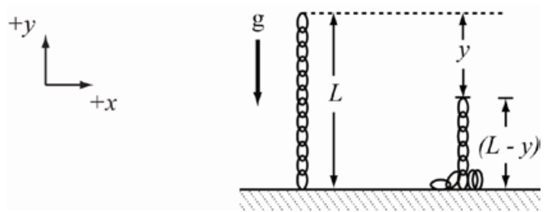
(c) $t = 9.80\text{ s}$

(d) $\vec{v}_f = 3.99\cdot 10^{-5}\text{ m/s}\,\hat{v}$

DOUBLE-CHECK: It is expected that the speed of the combined mass will be very small since its mass is very large.

8.75. **THINK:** A chain has a mass of 3.00 kg and a length of 5.00 m. Determine the force exerted by the chain on the floor. Assume that each link in the chain comes to rest when it reaches the floor.

SKETCH:



RESEARCH: Assume the mass per unit length of the chain is $\rho = M / L$. A small length of the chain, dy has a mass of dm, where $dm = Mdy / L$. At an interval of time dt, the small element of mass dm has reached the floor. The impulse caused by the chain is given by $J = F_J dt = \Delta p = vdm$. Therefore, the force F_J is given by $F_J = v\frac{dm}{dt} = v\frac{dm}{dy}\frac{dy}{dt}$.

SIMPLIFY: Using $dm / dy = M / L$ and $v = dy / dt$, the expression for force, F_J is

$$F_J = v^2 \frac{M}{L}.$$

For a body in free fall motion, $v^2 = 2gy$. Thus, $F_J = 2Mgy / L$. There is another force which is due to gravity. The gravitational force exerted by the chain on the floor when the chain has fallen a distance y is given by $F_g = Mgy / L$ (the links of length y are on the floor). The total force is given by

$$F = F_J + F_g = \frac{2Mgy}{L} + \frac{Mgy}{L} = \frac{3Mgy}{L}.$$

When the last link of the chain lands on the floor, the force exerted by the chain is obtained by substituting $y = L$, that is, $F = \frac{3Mgy}{L} = 3Mg$.

CALCULATE: $F = 3(3.0 \text{ kg})(9.81 \text{ m/s}^2) = 88.29 \text{ N}$

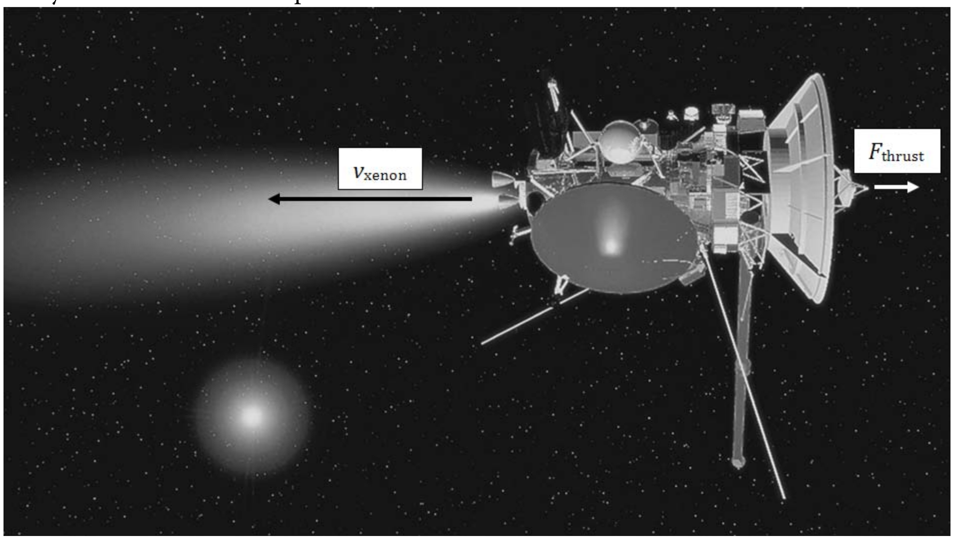
ROUND: To three significant figures, the force exerted by the chain on the floor as the last link of chain lands on the floor is $F = 88.3$ N.

DOUBLE-CHECK: F is expected to be larger than Mg due to the impulse caused by the chain as it falls.

Multi-Version Exercises

8.76. **THINK:** This question asks about the fuel consumption of a satellite. This is an example of rocket motion, where the mass of the satellite (including thruster) decreases as the fuel is ejected.

SKETCH: The direction in which the xenon ions are ejected is opposite to the direction of the thrust. The velocity of the xenon with respect to the satellite and the thrust force are shown.



RESEARCH: The equation of motion for a rocket in interstellar space is given by $\vec{F}_{\text{thrust}} = -\vec{v}_c \dfrac{dm}{dt}$. The velocity of the xenon ions with respect to the shuttle is given in km/s and the force is given in Newtons, or $\text{kg}\cdot\text{m}/\text{s}^2$. The conversion factor for the velocity is given by $\dfrac{1000 \text{ m/s}}{1 \text{ km/s}}$.

SIMPLIFY: Since the thrust and velocity act along a single axis, it is possible to use the scalar form of the equation, $F_{\text{thrust}} = -v_c \dfrac{dm}{dt}$. The rate of fuel consumption equals the change in mass (the loss of mass is due to xenon ejected from the satellite), so solve for $\dfrac{dm}{dt}$ to get $\dfrac{dm}{dt} = -\dfrac{F_{\text{thrust}}}{v_c}$.

CALCULATE: The question states that the speed of the xenon ions with respect to the rocket is $v_c = v_{\text{xenon}} = 21.45$ km/s. The thrust produced is $F_{\text{thrust}} = 1.187 \cdot 10^{-2}$ N. Thus the rate of fuel consumption is:

$$\begin{aligned}
\frac{dm}{dt} &= -\frac{F_{\text{thrust}}}{v_c} \\
&= -\frac{1.187 \cdot 10^{-2} \text{ N}}{21.45 \text{ km/s} \cdot \dfrac{1000 \text{ m/s}}{1 \text{ km/s}}} \\
&= -5.533799534 \cdot 10^{-7} \text{ kg/s} \\
&= -1.992167832 \text{ g/hr}
\end{aligned}$$

ROUND: The measured values are all given to four significant figures, and the final answer should also have four significant figures. The thruster consumes fuel at a rate of $5.534 \cdot 10^{-7}$ kg/s or 1.992 g/hr.

DOUBLE-CHECK: Because of the cost of sending a satellite into space, the weight of the fuel consumed per hour should be pretty small; a fuel consumption rate of 1.992 g/hr is reasonable for a satellite launched from earth. Working backwards, if the rocket consumes fuel at a rate of $5.534 \cdot 10^{-4}$ g/s, then the thrust is

$$-21.45 \text{ km/s} \cdot \left(-5.534 \cdot 10^{-4} \text{ g/s}\right) = 0.01187 \text{ km} \cdot \text{g/s}^2 = 1.187 \cdot 10^{-2} \text{ N}$$

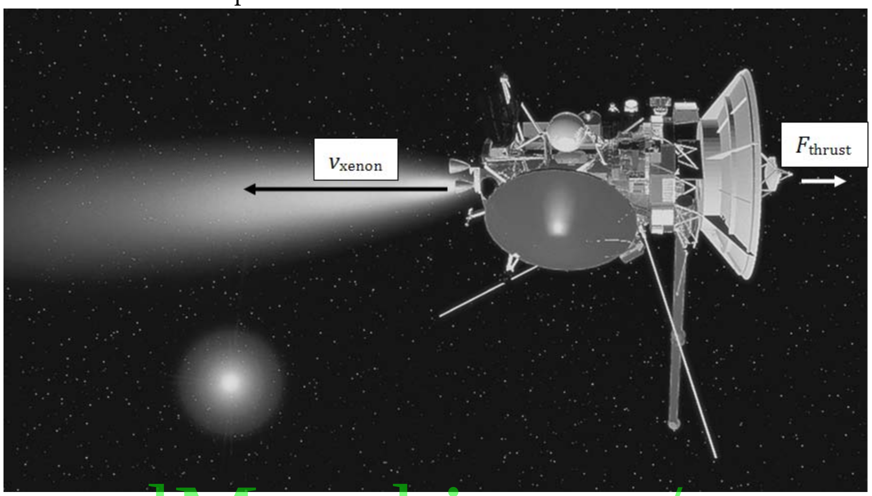
(the conversion factor is $1 \text{ km}\cdot\text{g/s}^2 = 1 \text{ kg}\cdot\text{m/s}^2$). So, this agrees with the given thrust force of $1.187 \cdot 10^{-2}$ N.

8.77. $F = v_c \dfrac{dm}{dt} = \left(23.75\cdot 10^3 \text{ m/s}\right)\left(5.082\cdot 10^{-7} \text{ kg/s}\right) = 1.207\cdot 10^{-2} \text{ N}$

8.78. $v_c = \dfrac{F}{dm/dt} = \dfrac{1.299\cdot 10^{-2} \text{ N}}{4.718\cdot 10^{-7} \text{ kg/s}} = 26.05 \text{ km/s}$

8.79. **THINK:** This question asks about the speed of a satellite. This is an example of rocket motion, where the mass of the satellite (including thruster) decreases as the fuel is ejected.

SKETCH: The direction in which the xenon ions are ejected is opposite to the direction of the thrust. The velocity of the xenon with respect to the satellite and the thrust force are shown.



RESEARCH: Initially, the mass of the system is the total mass of the satellite, including the mass of the fuel: $m_{\text{i}} = m_{\text{satellite}}$ After all of the fuel is consumed, the mass of the system is equal to the mass of the satellite minus the mass of the fuel consumed: $m_{\text{f}} = m_{\text{satellite}} - m_{\text{fuel}}$. The change in speed of the satellite is given by the equation $v_{\text{f}} - v_{\text{i}} = v_c \ln\left(m_{\text{i}} / m_{\text{f}}\right)$, where v_c is the speed of the xenon with relative to the satellite.

SIMPLIFY: To make the problem easier, choose a reference frame where the initial speed of the satellite equals zero. Then $v_{\text{f}} - v_{\text{i}} = v_{\text{f}} - 0 = v_{\text{f}}$, so it is necessary to find $v_{\text{f}} = v_c \ln\left(m_{\text{i}} / m_{\text{f}}\right)$. Substituting in the masses of the satellite and fuel, this becomes $v_{\text{f}} = v_c \ln\left(m_{\text{satellite}} / \left[m_{\text{satellite}} - m_{\text{fuel}}\right]\right)$.

CALCULATE: The initial mass of the satellite (including fuel) is 2149 kg, and the mass of the fuel consumed is 23.37 kg. The speed of the ions with respect to the satellite is 28.33 km/s, so the final velocity of the satellite is:

$$\begin{aligned} v_{\text{f}} &= v_c \ln\left(m_{\text{satellite}} / \left[m_{\text{satellite}} - m_{\text{fuel}}\right]\right) \\ &= \left(28.33 \text{ km/s}\right)\ln\left(\frac{2149 \text{ kg}}{2149 \text{ kg} - 23.37 \text{ kg}}\right) \\ &= 3.0977123\cdot 10^{-1} \text{ km/s} \end{aligned}$$

ROUND: The measured values are all given to four significant figures, and the weight of the satellite minus the weight of the fuel consumed also has four significant figures, so the final answer will have four figures. The change in the speed of the satellite is $3.098\cdot 10^{-1}$ km/s or 309.8 m/s.

DOUBLE-CHECK: Alhough the satellite is moving quickly after burning all of its fuel, this is not an unreasonable speed for space travel. Working backwards, if the change in speed was $3.098\cdot 10^{-1}$ km/s, then the velocity of the xenon particles was $v_c = \dfrac{\Delta v_{\text{satellite}}}{\ln(m_{\text{i}} / m_{\text{f}})}$, or

$$v_c = \frac{3.098\cdot 10^{-1} \text{km /s}}{\ln(2149 \text{ kg} / \left[2149 \text{ kg} - 23.37 \text{ kg}\right])} = 28.33 \text{ km /s}.$$

This agrees with the number given in the question, confirming that the calculations are correct.

8.80.

$$\Delta v = v_c \ln\left(\frac{m_i}{m_f}\right)$$

$$\frac{\Delta v}{v_c} = \ln\left(\frac{m_i}{m_f}\right)$$

$$e^{\frac{\Delta v}{v_c}} = \frac{m_i}{m_f}$$

$$m_f = m_i e^{-\frac{\Delta v}{v_c}}$$

$$m_{fuel} = m_i - m_f = m_i - m_i e^{-\frac{\Delta v}{v_c}} = m_i\left(1 - e^{-\frac{\Delta v}{v_c}}\right)$$

$$m_{fuel} = (2161 \text{ kg})\left(1 - e^{-\frac{236.4 \text{ m/s}}{20.61\cdot 10^3 \text{ m/s}}}\right) = 24.65 \text{ kg}$$

8.81.

$$\Delta v = v_c \ln\left(\frac{m_i}{m_f}\right)$$

$$\frac{\Delta v}{v_c} = \ln\left(\frac{m_i}{m_f}\right)$$

$$e^{\frac{\Delta v}{v_c}} = \frac{m_i}{m_f}$$

$$m_i = m_f e^{\frac{\Delta v}{v_c}}$$

$$m_f = m_i - m_{fuel}$$

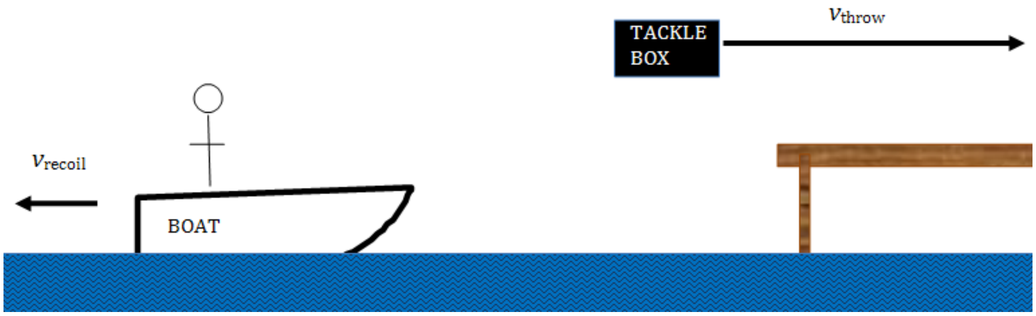
$$m_i = (m_i - m_{fuel})e^{\frac{\Delta v}{v_c}} = m_i e^{\frac{\Delta v}{v_c}} - m_{fuel} e^{\frac{\Delta v}{v_c}}$$

$$m_i e^{\frac{\Delta v}{v_c}} - m_i = m_{fuel} e^{\frac{\Delta v}{v_c}}$$

$$m_i = \frac{m_{fuel} e^{\frac{\Delta v}{v_c}}}{e^{\frac{\Delta v}{v_c}} - 1} = m_{fuel}\frac{1}{1 - e^{-\frac{\Delta v}{v_c}}} = (25.95 \text{ kg})\frac{1}{1 - e^{-\frac{275.0 \text{ m/s}}{22.91\cdot 10^3 \text{ m/s}}}} = 2175 \text{ kg}$$

8.82. **THINK:** The fisherman, boat, and tackle box are at rest at the beginning of this problem, so the total momentum of the fisherman, boat, and tackle box before and after the fisherman throws the tackle box must be zero. Using the principle of conservation of momentum and the fact that the momentum of the tackle box must cancel out the momentum of the fisherman and boat, it is possible to find the speed of the fisherman and boat after the tackle box has been thrown.

SKETCH: The sketch shows the motion of the tackle box, boat, and fisherman after the throw:



RESEARCH: The total initial momentum is zero, because there is no motion with respect to the dock. After the fisherman throws the tackle box, the momentum of the tackle box is $p_{\text{box}} = m_{\text{box}} v_{\text{box}} = m_{\text{box}} v_{\text{throw}}$ towards the dock. The total momentum after the throw must equal the total momentum before the throw, so the sum of the momentum of the box, the momentum of the boat, and the momentum of the fisherman must be zero: $p_{\text{box}} + p_{\text{fisherman}} + p_{\text{boat}} = 0.$ The fisherman and boat both have the same velocity, so $p_{\text{fisherman}} = m_{\text{fisherman}} v_{\text{fisherman}} = m_{\text{fisherman}} v_{\text{recoil}}$ away from the dock and $p_{\text{boat}} = m_{\text{boat}} v_{\text{boat}} = m_{\text{boat}} v_{\text{recoil}}$ away from the dock.

SIMPLIFY: The goal is to find the recoil velocity of the fisherman and boat. Using the equation for momentum after the tackle box has been thrown, $p_{\text{box}} + p_{\text{fisherman}} + p_{\text{boat}} = 0$, substitute in the formula for the momenta of the tackle box, boat, and fisherman: $0 = m_{\text{box}} v_{\text{throw}} + m_{\text{fisherman}} v_{\text{recoil}} + m_{\text{boat}} v_{\text{recoil}}$. Solve for the recoil velocity:

$$\begin{aligned} m_{\text{box}} v_{\text{throw}} + m_{\text{fisherman}} v_{\text{recoil}} + m_{\text{boat}} v_{\text{recoil}} &= 0 \\ m_{\text{fisherman}} v_{\text{recoil}} + m_{\text{boat}} v_{\text{recoil}} &= -m_{\text{box}} v_{\text{throw}} \\ v_{\text{recoil}} \left(m_{\text{fisherman}} + m_{\text{boat}} \right) &= -m_{\text{box}} v_{\text{throw}} \\ v_{\text{recoil}} &= -\frac{m_{\text{box}} v_{\text{throw}}}{m_{\text{fisherman}} + m_{\text{boat}}} \end{aligned}$$

CALCULATE: The mass of the tackle box, fisherman, and boat, as well as the velocity of the throw (with respect to the dock) are given in the question. Using these values gives:

$$\begin{aligned} v_{\text{recoil}} &= -\frac{m_{\text{box}} v_{\text{throw}}}{m_{\text{fisherman}} + m_{\text{boat}}} \\ &= -\frac{13.63 \text{ kg} \cdot 2.911 \text{ m/s}}{75.19 \text{ kg} + 28.09 \text{ kg}} \\ &= -0.3841685709 \text{ m/s} \end{aligned}$$

ROUND: The masses and velocity given in the question all have four significant figures, and the sum of the mass of the fisherman and the mass of the boat has five significant figures, so the final answer should have four significant figures. The final speed of the fisherman and boat is –0.3842 m/s towards the dock, or 0.3842 m/s away from the dock.

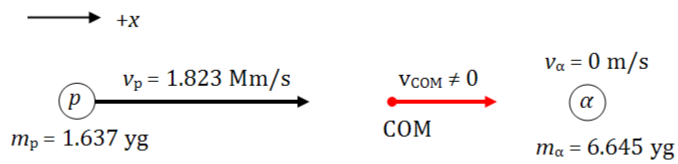
DOUBLE-CHECK: It makes intuitive sense that the much more massive boat and fisherman will have a lower speed than the less massive tackle box. Their momenta should be equal and opposite, so a quick way to check this problem is to see if the magnitude of the tackle box's momentum equals the magnitude of the man and boat. The tackle box has a momentum of magnitude 13.63 kg · 2.911 m/s = 39.68 kg·m/s after it is thrown. The boat and fisherman have a combined mass of 103.28 kg, so their final momentum has a magnitude of 103.28 kg · 0.3842 m/s = 39.68 kg·m/s. This confirms that the calculations were correct.

8.83. $$v_{\text{box}} = \frac{m_{\text{man}} + m_{\text{boat}}}{m_{\text{box}}} v_{\text{boat}} = \frac{77.49 \text{ kg} + 28.31 \text{ kg}}{14.27 \text{ kg}} (0.3516 \text{ m/s}) = 2.607 \text{ m/s}$$

8.84. $$\begin{aligned} &\left(m_{\text{man}} + m_{\text{boat}} \right) v_{\text{boat}} = m_{\text{box}} v_{\text{box}} \\ &m_{\text{man}} v_{\text{boat}} + m_{\text{boat}} v_{\text{boat}} = m_{\text{box}} v_{\text{box}} \\ &m_{\text{man}} = \frac{m_{\text{box}} v_{\text{box}} - m_{\text{boat}} v_{\text{boat}}}{v_{\text{boat}}} = m_{\text{box}} \frac{v_{\text{box}}}{v_{\text{boat}}} - m_{\text{boat}} \\ &m_{\text{man}} = (14.91 \text{ kg}) \frac{3.303 \text{ m/s}}{0.4547 \text{ m/s}} - 28.51 \text{ kg} = 79.80 \text{ kg} \end{aligned}$$

8.85. **THINK:** The masses and initial speeds of both particles are known, so the momentum of the center of mass can be calculated. The total mass of the system is known, so the momentum can be used to find the speed of the center of mass.

SKETCH: To simplify the problem, choose the location of the particle at rest to be the origin, with the proton moving in the $+x$ direction. All of the motion is along a single axis, with the center of mass (COM) between the proton and the alpha particle.



RESEARCH: The masses and velocities of the particles are given, so the momenta of the particles can be calculated as the product of the mass and the speed $p_\alpha = m_\alpha v_\alpha$ and $p_p = m_p v_p$ towards the alpha particle. The center-of-mass momentum can be calculated in two ways, either by taking the sum of the momenta of each particle ($P_{COM} = \sum_{i=0}^{n} p_i$) or as the product of the total mass of the system times the speed of the center of mass ($P_{COM} = M \cdot v_{COM}$).

SIMPLIFY: The masses of both particles are given in the problem, and the total mass of the system M is the sum of the masses of each particle, $M = m_\text{p} + m_\alpha$. The total momentum $P_{COM} = \sum_{i=0}^{n} p_i = p_\alpha + p_p$ and $P_{COM} = M \cdot v_{COM}$, so $M \cdot v_{COM} = p_\alpha + p_p$. Substitute for the momenta of the proton and alpha particle (since the alpha particle is not moving, it has zero momentum), substitute for the total mass, and solve for the velocity of the center of mass:

$$M \cdot v_{COM} = p_\alpha + p_p \Rightarrow$$

$$v_{COM} = \frac{p_\alpha + p_p}{M} = \frac{m_\alpha v_\alpha + m_p v_p}{m_\alpha + m_p} = \frac{m_\alpha \cdot 0 + m_p v_p}{m_\alpha + m_p} = \frac{m_p v_p}{m_\alpha + m_p}$$

CALCULATE: The problem states that the proton has a mass of $1.673 \cdot 10^{-27}$ kg and moves at a speed of $1.823 \cdot 10^{6}$ m/s towards the alpha particle, which is at rest and has a mass of $6.645 \cdot 10^{-27}$ kg. So the center of mass has a speed of

$$v_\text{COM} = \frac{m_\text{p} v_\text{p}}{m_{\text{p}\alpha} + m} = \frac{\left(1.823 \cdot 10^{6} \text{ m/s}\right)\left(1.673 \cdot 10^{-27} \text{ kg}\right)}{1.673 \cdot 10^{-27} \text{ kg} + 6.645 \cdot 10^{-27} \text{ kg}} = 3.666601346 \cdot 10^{5} \text{ m/s}$$

ROUND: The masses of the proton and alpha particle, as well as their sum, have four significant figures. The speed of the proton also has four significant figures. The alpha particle is at rest, so its speed is not a calculated value, and the zero speed does not change the number of figures in the answer. Thus, the speed of the center of mass is $3.667 \cdot 10^{5}$ m/s, and the center of mass is moving towards the alpha particle.

DOUBLE-CHECK: To double check, find the location of the center of mass as a function of time, and take the time derivative to find the velocity. The distance between the particles is not given in the problem, so call the distance between the particles at an arbitrary starting time $t = 0$ to be d_0. The positions of each particle can be described by their location along the axis of motion, $r_\alpha = 0$ and $r_\text{p} = d_0 + v_\text{p} t$.

Using this, the location of the center of mass is

$$R_{\text{COM}} = \frac{1}{m_{\text{p}} + m_\alpha}\left(r_{\text{p}} m_{\text{p}} + r_\alpha m_\alpha\right).$$

Take the time derivative to find the velocity:

$$\begin{aligned}
\frac{d}{dt} R_{\text{COM}} &= \frac{d}{dt}\left[\frac{1}{m_{\text{p}} + m_\alpha}\left(r_{\text{p}} m_{\text{p}} + r_\alpha m_\alpha\right)\right] \\
&= \frac{1}{m_{\text{p}} + m_\alpha}\frac{d}{dt}\left[\left(d_0 + v_{\text{p}} t\right) m_{\text{p}} + 0 \cdot m_\alpha\right] \\
&= \frac{1}{m_{\text{p}} + m_\alpha}\frac{d}{dt}\left(d_0 m_{\text{p}} + v_{\text{p}} m_{\text{p}} t + 0\right) \\
&= \frac{1}{m_{\text{p}} + m_\alpha}\frac{d}{dt}\left(d_0 m_{\text{p}} + v_{\text{p}} m_{\text{p}} t\right) \\
&= \frac{1}{m_{\text{p}} + m_\alpha}\left(0 + v_{\text{p}} m_{\text{p}}\right) \\
&= \frac{v_{\text{p}} m_{\text{p}}}{m_{\text{p}} + m_\alpha} \\
&= \frac{\left(1.823 \cdot 10^{6} \text{ m/s}\right)\left(1.673 \cdot 10^{-27} \text{ kg}\right)}{1.673 \cdot 10^{-27} \text{ kg} + 6.645 \cdot 10^{-27} \text{ kg}} \\
&= 3.666601346 \cdot 10^{5} \text{ m/s}
\end{aligned}$$

This agrees with the earlier result.

8.86. $(m_p + m_\alpha) v_{\text{cm}} = m_p v_p + m_\alpha v_\alpha$

Since $v_\alpha = 0$,

$$v_p = \frac{m_p + m_\alpha}{m_p} v_{\text{cm}} = \frac{1.673 \cdot 10^{-27} \text{ kg} + 6.645 \cdot 10^{-27} \text{ kg}}{\left(1.673 \cdot 10^{-27} \text{ kg}\right)}\left(5.509 \cdot 10^{5} \text{ m/s}\right) = 2.739 \cdot 10^{6} \text{ m/s}$$