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Mechanics



Third secondary

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Student Book

غير مصرح بتداول هذا الكتاب خارج وزارة التربية والتعليم والتعليم الفني

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Introduction

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

We are pleased to introduce this book to show the philosophy on which the academic content has been prepared. This philosophy aims at:

Today's world is witnessing an ongoing scientific development. The future generation is required to get qualified using the developing methods of the future in order to keep matching with the gigantic progress of the different sciences. In regard to apply such a principle, the Ministry of Education is doing the best to improve the curricula through having the learner in the position of the explorer to the scientific facts. In addition, the Ministry of Education provides the proper training for the students in the scientific research fields throughout thinking in order to enable them to use their minds as tools of the scientific thinking. As we present the book of Calculus for the third secondary to be a helping for lightening the minds of the students to trace the scientific thinking and to motivate them to search and explore.

In the light of what previously mentioned, the following details have been considered:

- ★ The book has been divided into related and integrated units. Each unit has an introduction illustrating the learning outcomes, the unit planning guide, and the related key terms. In addition, the unit is divided into lessons where each lesson shows the objective of learning it through the title You will learn. Each lesson starts with the main idea of the lesson content. It is taken into consideration to introduce the content gradually from easy to hard. The lesson includes some activities, which relate Math to other school subjects and the practical life. These activities suit the students' different abilities, consider the individual differences throughout Discover the error to correct some common mistakes of the students, confirm the principle of working together and integrate with the topic. Furthermore, this book contains some issues related to the surrounding environment and how to deal with.
- ★ Each lesson contains examples starting gradually from easy to hard and containing various levels of thoughts accompanied with some exercises titled Try to solve. Each lesson ends in Exercises that contain various problems related to the concepts and skills that the students learned through the lesson.
- ★ Each unit ends in Unit summary containing the concepts and the instructions mentioned and General exams containing various problems related to the concepts and skills, which the student learned through the unit.
- ★ Each unit ends in an Accumulative test to measure some necessary skills to be gained to fulfill the learning outcome of the unit.
- ★ The book ends in General tests including some concepts and skills, which the student learned throughout the term.

Last but not least. We wish we had done our best to accomplish this work for the benefits of our dear youngsters and our dearest Egypt.

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Introduction

Friction force is found since the early years of life. The ancient Egyptians had depended on it in regard to the scientific and geometric methods available on that time. Ancient workers had used a various group of tools to cut the stone masses used in building the pyramids they had used to draw such masses from a place to another using lubricated trusses to reduce the friction force between the masses and the trusses. At the age of Roman empire, the military engineers had greased the military equipment during the siege to reduce the friction force among the parts of the vehicles. At the age of the renaissance, the Italian scientist Leonardo da Vinci (1462 - 1519) was the first to state the scientific foundation to the friction science. He had defined the concept of the friction as a value to the friction force. From the scientific experiments conducted by the scientists, it had been noticed that the friction force of the bodies is greater than the friction force of the bodies in motion we can observe this matter in our practical life you can notice that the person needs more power at the beginning to move a wood box on the ground but when the box moves, you can notice that the power needed has become less than before. This is because the body is in motion and in turn, the friction force decreases. As a result, the friction can be divided into two types; static friction and kinetic friction. In this unit, you are going to learn the concept and properties of the friction and the condition of the equilibrium of a body on a horizontal rough plane and another on inclined rough plane this unit will be ended up with some life application on the friction static friction.

Unit objectives

By the end of this unit and by doing all the activities involved, the student should be able to:

- ⊕ Distinguish the smooth surfaces and rough surfaces.
- ⊕ Identify the concept and properties of friction
- ⊕ Identify the friction force and the limiting friction force
- ⊕ Determine the coefficient of the friction, angle of friction and the relation between them.
- ⊕ Determine the conditions of equilibrium of a body on a rough horizontal plane.
- ⊕ Determine the conditions of equilibrium of a body on a rough inclined plane.
- ⊕ Deduce the relation between the measure of the angle of friction and the measure of the angle of inclination of the plane on the horizontal as a body is placed on a rough inclined plane on a condition the body is about to slide under the effect of its weight only.
- ⊕ Solve life applications on the friction.

Key terms

- Friction
- Smooth Surface
- Rough Surface
- Reaction Normal
- Static Frictional force
- Kinetic Frictional force

- Limiting Static Friction
- Resultant Reaction
- Angle of Friction
- Rough horizontal plane
- Rough inclined plane

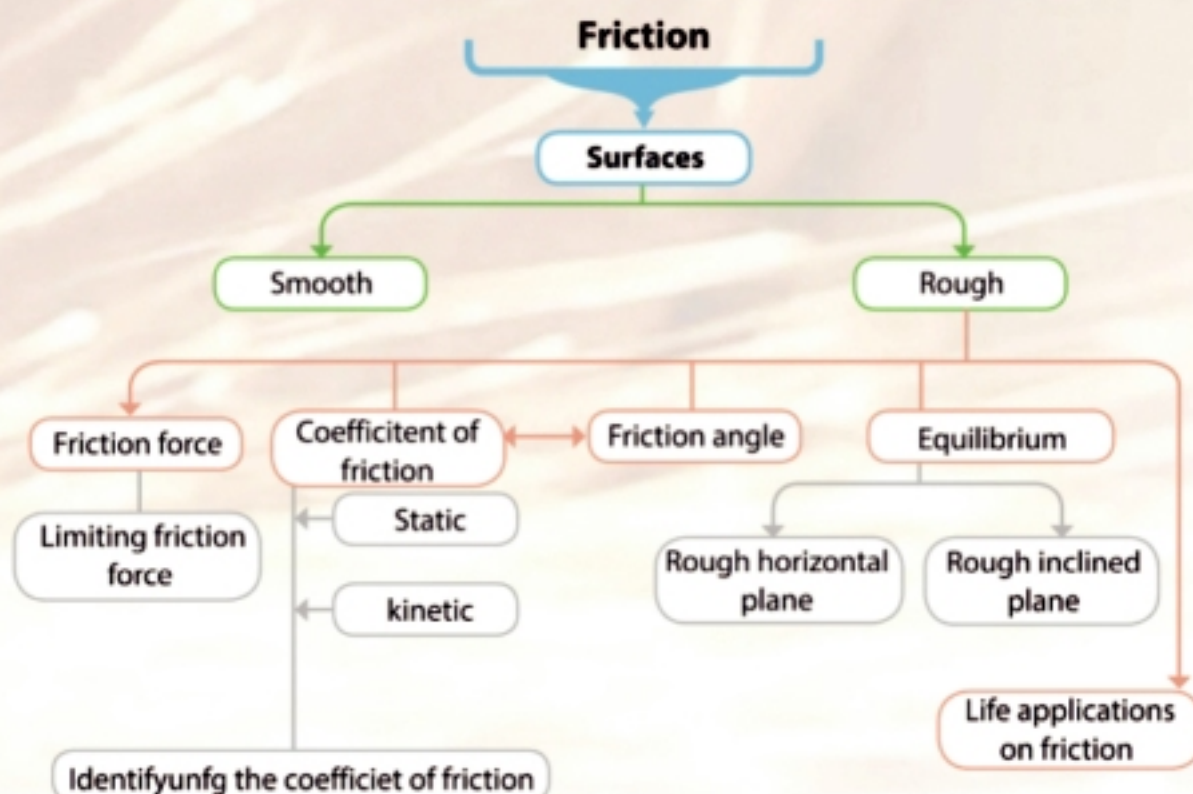
Lessons of the unit

- (2 -1): The equilibrium of a body on a rough horizontal plane.
- (2 -2): The equilibrium of a body on a rough inclined plane.

Materials

Scientific calculator .

Unit planning guide



Equilibrium of a body on a horizontal rough plane

You will learn

- Smooth surfaces and rough surfaces.
- The concept of the friction
- The force of the static friction
- The force of kinetic friction
- The relation between the coefficient of the friction and the tangent of the angle of the friction
- Properties of the friction
- The equilibrium of a body on a rough horizontal plane.

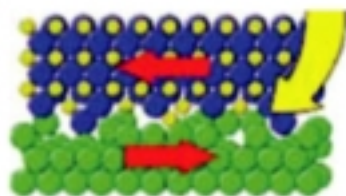
Key terms

- Friction
- Smooth Surface
- Rough Surface
- Normal Reaction
- Static Friction
- Kinetic Friction
- Limiting Static Friction
- Resultant Reaction
- Angle of Friction
- Rough horizontal plane

Materials

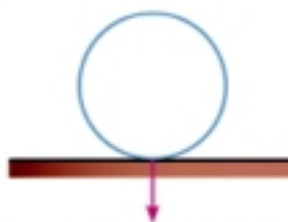
- Scientific calculator

What would happen if the friction disappeared in a moment in the world? if the friction disappeared, we would find cars, trains and all the means of transportation could not move since they move relying on the friction between the ground and their wheels. If these machines moved, they would not stop since the brakes depend mainly up on the friction. Furthermore, people would not walk or stand properly because they seem to stand on icy land . People would not also catch the different objects since the objects would slide away from their hands. Without friction, mountain would break down and the soil cover would no longer cover them, building would tear down and tied ropes would go part. Briefly speaking, life would be impossible without friction. As a result, the friction has many benefits it makes car's wheels move on the roads and the train's wheels stick to the rail road it also allows the conveyor belt to rotate the pulley without sliding you also would not walk without the friction to keep your shoes from getting slided on the sidewalk in other words, it is extremely difficult to walk on the snow where the surface is smooth and cannot cause friction and the shoes slipped. Friction does not allow your shoes to slip on ice, but helps to fix the soil on the mountains and to fix and makes the plants straightly stand and keeps the tied ropes fixed in addition to other benefits.

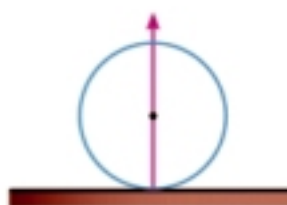


Reaction:

We have previously learned that there is a type of force generated when two bodies touch and it is called reaction . If you place a ball horizontal table, then the ball affects on the table by a force pressure (p) equal the ball weight in this case and the Newton's third law then the table affects the ball by force reaction (r) and equal the pressure ball on the table i.e $r = p$.



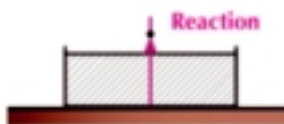
The pressure affecting the table
Figure (1)



The reaction affecting the ball
Figure (2)

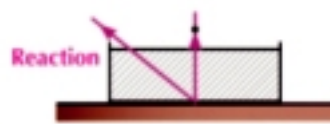
Smooth and Rough Surfaces

Scientists relate the friction forces among bodies to the presence of microscopic cavities and projections in the surfaces of the bodies whatever their smoothness is. The overlapping of such projections and cavities of the two surfaces in contact produces what is called the friction force. As a result, we can find resistance as we try to move one of the two surfaces against the other. The coefficient of the friction a good scale to measure the roughness degree of the surfaces. If the value of the coefficient of the friction increases, the roughness increases and vice versa. If the coefficient of the friction equals zero, the friction forces are not existed totally. The reaction between the two bodies in contact depends upon the nature of the two bodies and upon the other forces acting on the body in case of the smooth surfaces the reaction is normal to the common tangent plane to the surfaces of the two bodies in contact. On the contrary, when the two bodies are rough, the reaction would have a component in the direction of the tangent surface which is called the static friction. Besides the reaction has a normal component on the tangent surface which is called the normal reaction.



Reaction in case of smooth surfaces

figure (3)



Reaction in case of rough surfaces

figure (4)

Practical experiment:

Put a piece of wood on a horizontal table and attach the piece to a string passing over a smooth pulley at the edge of the table such that the string suspends vertically to end in a scale pan as shown in the figure place proper weight on the piece of wood, then place a bit weight in the scale pan you notice that the piece of wood doesn't move. This means that the friction force acting on the wood piece was enough.

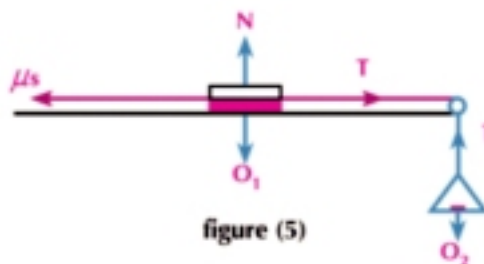


figure (5)

To hinder the motion despite the presence of the tension T in the string. As it is known, this tension is of course equal to the scale pan and the weights it holds.

As we increase the weights in the pan gradually, we observe that the wood piece starts to move on the table when the weights placed in the scale pan reach a certain limit.

This means that the static friction force increases by the increasing of tension until it reaches a certain limit which it does not exceed. If the tension increases to exceed such a limit, then the friction force cannot balance it and then the body starts to move. It is observed that as we increase the weights placed on the wood piece, we require to increase the weight placed in the scale pan until the wood piece is about to move.

The properties of the static friction force:

- (1) The static friction force (F) acts in opposing the slide it is in the opposite direction to the direction which the body tends to slide
- (2) The static friction force (F) is only equal to the tangential force which tends to move the body so that it can't be more than such a force and remain equal to the force as long as the body balanced.
- (3) The static friction force (F) increases, whenever the tangential force which cause the motion increases until you arrive up to a certain limit which it doesnot exceed it . At such a limit, the body is about to slide in this case, the friction is called the limiting static friction and it is denoted by the symbol (F_s).
- (4) The ratio between the limiting static friction and the normal reaction N is constant and this ratio depends up on the nature of the two bodies in contact but not up to their shape or mass. This ratio is called the coefficient of the static friction and is denoted by the symbol (μ_s) .

i.e. $\mu_s = \frac{F_s}{N}$ where F_s the limiting static friction

it is noticed that the static friction coefficient often have $0 < \mu_s < 1$ but in some special cases it may be more than one.

Kinetic Friction force

if a body moves upon a rough surface , it is subjected to the kinetic friction force (F_k) and its direction is opposite to the direction of its motion and its value is given by the relation: $F_k = \mu_k N$: where μ_k is the kinetic friction coefficient and R the normal reaction.

i.e.: the kinetic friction force equals the product of the kinetic friction coefficient multiplied by the normal reaction force

Hence, the kinetic friction coefficient can be defined as the ratio between the kinetic friction force and the normal reaction force .

i.e.: $\mu_k = \frac{F_k}{N}$ where F_k is the kinetic friction force

Resultant Reaction (R')

in case of the rough surfaces the resultant reaction is inclined on the tangent surface since it expresses the resultant of the normal reaction and the static friction force. It is called the resultant reaction.

Definition

The resultant reaction (\vec{R}') is the resultant of the normal reaction \vec{N} and the static friction force \vec{F}

Angle of Friction

Notes that the measure of the angle included between the normal reaction and the resultant reaction increases as the magnitude of the friction force increases [suppose that the normal reaction is constant] and this value is limiting λ when the friction becomes limiting and this angle in this case called the angle of friction.

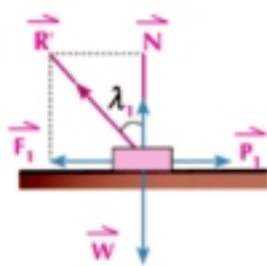


figure (6)

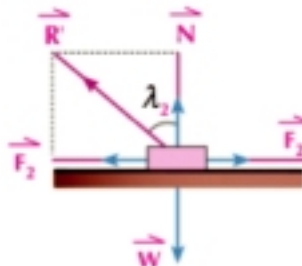


figure (7)

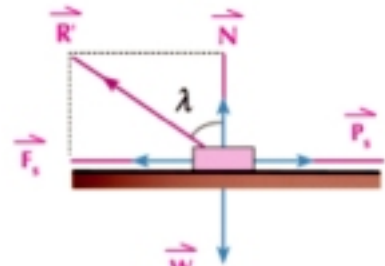


figure (8)

In fig (1) and fig (2) we find: the vector resultant \vec{R}' is the resultant of the normal reaction \vec{N} and the friction force \vec{F} , its magnitude is given by $R' = \sqrt{N^2 + F^2}$

and in fig (3) when the friction force is limiting we have :

$$\therefore R' = \sqrt{N^2 + F_s^2} \quad \because F_s = \mu_s N \quad \therefore R' = \sqrt{N^2 + N^2 \mu_s^2} \quad \therefore R' = N \sqrt{1 + \mu_s^2}$$

The relation between coefficient of friction and angle of friction :

in case the friction is limiting as in shown fig (8) :

$$\text{we find : } \tan \lambda = \frac{F_s}{N} \text{ where } \frac{F_s}{N} = \mu_s$$

$$\text{i.e. : } \mu_s = \tan \lambda$$

i.e : In the case of limiting friction , the coefficient of friction is equal to the tangent of the angle of friction

Critical thinking: compare between the static and kinetic angle of friction.

Equilibrium of a body on a rough horizontal plane

If a body of weight (w) is in equilibrium on a horizontal rough plane and acted upon by a force p inclined by an angle of measure θ with the horizontal fig (9) the body is equilibrium under the action of :

- 1) The weight \vec{w} which is directed vertically downward
- 2) The resultant reaction \vec{R}' and its magnitude is R'
- 3) The given force \vec{P} with magnitude P .

by resolve \vec{P} into two components in the horizontal and vertical, direction then their magnitudes are $P \cos \theta$, $P \sin \theta$.

and by resolve \vec{R}' into two perpendicular components which are the normal reaction \vec{N} and its magnitude N , and the friction force \vec{F} and its magnitude F as shown in fig (10).

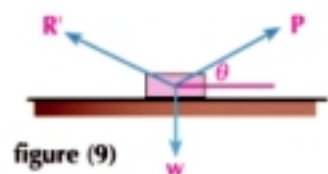


figure (9)

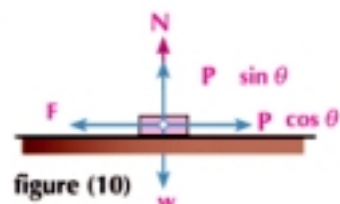


figure (10)

The equations for equilibrium are : $F = P \cos \theta$, $N + P \sin \theta = w$

Example The acting force on a body

- 1 Karim pushes a box full of books towards his car on the using horizontal, if the weight of the box and books together 124 Newton and coefficient of friction between the road and the box 0.45 then find the magnitude of the force horizontal required by karim to push the box to make it about to move.

Solution

Suppose that $w = 124$ newton , $\mu_s = 0.45$

From the conditions of equilibrium of the body in the horizontal plane :

$$N = w \quad \text{i.e. : } N = 124 \quad (1)$$

$$F = \mu_s N \quad \text{and from (1) then : } F = 0.45 \times 124 = 55.8 \text{ newton}$$

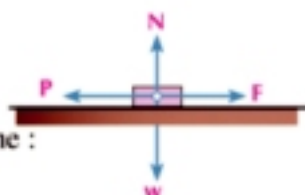


figure (11)

Try to solve

- 1 A mass of weight 32 newton is put on a horizontal rough plane and act on it a horizontal force magnitude P until the mass becomes about to move.
- If $P = 8$ newton, find the coefficient of the static friction between the mass and the plane
 - If $\mu_s = 0.4$ find P

Example Friction force

- 2 A body of weight 8 kg.wt is placed on a horizontal table, and is connected by a string passing over a smooth pulley at the edge, to a weight of magnitude 1.5 kg.wt which is hanging freely and the body is in equilibrium, find the friction force. If the coefficient of friction between the body and the table is $\frac{1}{4}$. State whether or not the body is about to move.

Solution

From the equilibrium of the body hanging vertically we find $T = 1.5$ kg.wt and from the equilibrium is placed on a horizontal table then $N = W$.

$$\therefore N = 8 \text{ kg.wt}$$

$$\therefore \text{Friction force } F = T \therefore T = 1.5 \text{ kg.wt}$$

To know whether the body is about to move or not determine the limiting static friction F_s

$$\therefore F_s = \mu_s N \quad \therefore F_s = \frac{1}{4} \times 8 = 2 \text{ kg.wt.}$$

$\therefore F < \mu_s N$ the friction is not limiting and the body is not about to move.

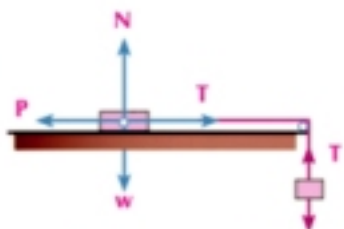


figure (12) 1.5 kg.wt

P Try to solve

- 2 A particle of weight 20 newton is placed on a horizontal rough plane, if the static friction coefficient between the particle and the plane $\frac{1}{4}$ find:
- The required horizontal force which enough to make the particle is about to move .
 - The inclined force which makes an angle measure 30° to the plane and makes the particle is about to move .

Example Angle of friction

- 3 A body of weight 12 kg.wt is placed on a horizontal rough plane, two forces act on the body of magnitudes 4 , 4 kg.wt and include an angle of measure 60° where the two horizontal forces are on the same horizontal plane. If the body is about to move, find the coefficient of friction between the body and the plane also find angle of friction.

Solution

\therefore The body is about to move:

the body in equilibrium

$$\therefore N = W \qquad \therefore N = 12 \text{ kg.wt.}$$

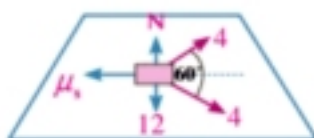


figure (14)

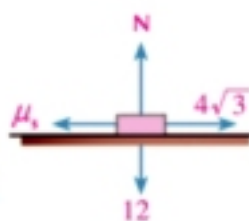


figure (13)

The resultant of the forces 4, 4 kg.wt = limiting friction forces

$$\therefore F = \sqrt{F_1^2 + F_2^2 + 2F_1 F_2 \cos \alpha} \qquad \text{the resultant of two forces law}$$

$$\therefore F = \sqrt{4^2 + 4^2 + 2 \times 4 \times 4 \times \frac{1}{2}} = 4\sqrt{3} \text{ kg.wt}$$

$$\mu_s N = F \qquad \therefore 12\mu_s = 4\sqrt{3}$$

$$\therefore \mu_s = \frac{4\sqrt{3}}{12} = \frac{\sqrt{3}}{3} \qquad \therefore \mu_s = \frac{1}{\sqrt{3}}$$

$$\therefore \mu_s = \tan \lambda \qquad \therefore \tan \lambda = \frac{\sqrt{3}}{3} \qquad \therefore \lambda = 30^\circ$$

P Try to solve

- 3 A particle of weight 6 newton is placed on a horizontal rough plane and two forces in the same plane of magnitudes 2 and 4 newton include an angle of measure 120° act on it, the particle kept at rest prove that the measure of the angle of friction (λ) between the body and the plane must not less than 30° .
and if $\lambda = 45^\circ$ and the directions of the two forces unchange , and the force of magnitude 4 newton without change. Determine the magnitude of the other force for the particle to be about to move.

Example Theoretical demonstrate

- 4 A body of weight (w) is placed on a horizontal rough plane and the measure of the angle of friction between the body and the plane is λ located in the vertical plane passing through body weight, the body is attached by a force inclined the horizontal by angle of measure θ the body is about to move. Prove that the magnitude of this force = $\frac{w \sin \lambda}{\cos (\theta - \lambda)}$, then find the magnitude of the smallest value of this force and condition happening.

Solution

\therefore the measure of angle of friction = λ

\therefore Coefficient of static friction (μ_s) = $\tan \lambda = \frac{\sin \lambda}{\cos \lambda}$

\therefore The value of the limiting friction force $F_s = \mu_s N = N \times \frac{\sin \lambda}{\cos \lambda}$

and resolve the force \vec{P} into two perpendicular components, which are $P \cos \theta$, $P \sin \theta$

\therefore equations of equilibrium are : $P \cos \theta = \mu_s R$

$\therefore P \cos \theta = R \times \frac{\sin \lambda}{\cos \lambda}$ (1), (because $\mu_s = \tan \lambda$),

$R + P \sin \theta = w$ (2)

from (1) $R = \frac{P \cos \theta \cos \lambda}{\sin \lambda}$ substituting in (2)

$\therefore \frac{P \cos \theta \cos \lambda}{\sin \lambda} + P \sin \theta = w$

$\therefore P \cos \theta \cos \lambda + P \sin \theta \sin \lambda = w \sin \lambda$

$\therefore P (\cos \theta \cos \lambda + \sin \theta \sin \lambda) = w \sin \lambda \quad \therefore P \cos (\theta - \lambda) = w \sin \lambda$

$\therefore P = \frac{w \sin \lambda}{\cos (\theta - \lambda)}$ and the required is finding the smallest value of this force, then $\cos (\theta - \lambda)$

must have the greatest possible i.e. $\cos (\theta - \lambda) = 1$

\therefore the smallest value of this force $F = w \sin \lambda$

when $\cos (\theta - \lambda) = 1$ i.e. : $\theta - \lambda = 0^\circ$ i.e. : $\theta = \lambda$

\therefore The condition must be satisfy is the measure of the angle of inclination of the force to the horizontal is equal to the measure of the angle of friction

Solution

$\therefore R'$ is the resultant of the two forces N , F_s :

\therefore The body is in equilibrium under the action of three forces which are: \vec{P} , \vec{W} , \vec{R}'

and by using Lami's rule:

$\therefore \frac{P}{\sin (180^\circ - \lambda)} = \frac{W}{\sin [90^\circ - (\theta - \lambda)]}$

$\therefore \frac{P}{\sin \lambda} = \frac{W}{\cos (\theta - \lambda)}$

$\therefore P = \frac{w \sin \lambda}{\cos (\theta - \lambda)}$

\therefore the required force is smallest value of this force \vec{F} , then the value of $\cos (\theta - \lambda)$ is a maximum possible

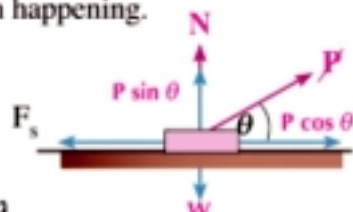


figure (15)

Remember

$$\begin{aligned} \cos (\theta - \lambda) &= \cos \theta \cos \lambda + \sin \theta \sin \lambda \\ \cos (\theta + \lambda) &= \cos \theta \cos \lambda - \sin \theta \sin \lambda \end{aligned}$$

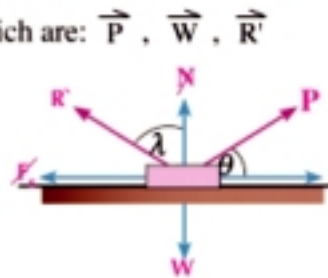


Figure (16)

- $\therefore \cos(\theta - \lambda) = 1$ $\therefore F = w \sin \lambda$ and the condition which must satisfy is :
 $\cos(\theta - \lambda) = \cos 0$ $\therefore \theta - \lambda = 0$ $\therefore \theta = \lambda$
 \therefore The condition which must be satisfy is the measure of the angle of inclination of the force to the horizontal equal the measure of the angle of friction

Try to solve

- 4 A body of weight (w) kg.wt is placed on a rough horizontal plane, the measure of the angle of friction between the body and the plane is (λ), the body is attached by a force inclined the horizontal by an angle of measure (2λ) upwards located in the vertical plane passing through body weight the body is about to move. Prove that the magnitude of this force equal $w \tan \lambda$.



Exercise 1 - 1



First : Complete each of the following :

- 1 The force resulted from sliding two rough surfaces by the force of _____
- 2 The friction force vanished and the coefficient of friction equal zero in the _____ planes
- 3 When the static friction force is limiting, then the body is _____
- 4 The kinetic friction force equal the product of coefficient of kinetic friction at _____
- 5 The resultant of the normal reaction force and the limiting static friction force is called _____
- 6 The static friction force less than or equal the product of the coefficient of static friction at _____
- 7 If the static friction coefficient between a mass of magnitude 40 kg and the ground surface equal 0.45 then the magnitude of the horizontal force which act on the mass and make it about to move equal _____
- 8 If a body of weight 6 newton is placed on a rough horizontal plane and the static friction force was 4 newton, then the coefficient of static friction is _____

Second : Answer the following questions

- 9 A boy pushes a stone of weight 56 newton by a horizontal force of magnitude 42 newton on a ridge, the stone was about to move. Find the coefficient of static friction between the stone and the ridge.
- 10 A body of weight 240 kg.wt is placed on a horizontal rough plane and wanted to attached by a wire inclined to the horizontal by angle of measure 30° , if the coefficient of static friction equal $\frac{\sqrt{3}}{3}$ find the required tension in the wire for the body about to move.
- 11 A body of weight 39 kg.wt is placed on a horizontal rough plane two forces of magnitudes, 7 and 8 kg.wt and include an angle of measure 60° act on the body so it become about to move. Find the static coefficient friction.

- 12 A body of weight 12 newton is placed on a horizontal table and tied with a horizontal string passing over a smooth pulley fixed at the end of the table edge and the other end of the string is tied by a weight of magnitude 4 newton . If the body is in equilibrium on the table , find the friction force. If it is known that the coefficient of static friction between the body and the table equal $\frac{1}{3}$. is the body about to move? Explain your answer .
- 13 A box of weight (w) kg.wt is placed on a rough horizontal plane and attached by two horizontal strings the tensions on it are 6 , 8 kg.wt and include an angle of measure 90° if the coefficient of the static friction between the box and the plane equal $\frac{1}{4}$, find the weight of the box (w) if the box is about to move .
- 14 A body of weight 39 newton is placed on a rough horizontal plane and the tangent of the static friction angle between the body and the plane is $\frac{1}{3}$, the body is attached by force inclined the horizontal by an angle , whose sine is $\frac{4}{5}$ makes the body is about to move. Find :

first : The magnitude of the tension .

second : The magnitude of the static friction

Equilibrium of a body on an Inclined rough plane

Unit One

1 - 2

We will study in this lesson equilibrium of a body on a rough inclined plane..

We consider the body is in equilibrium on a rough horizontal inclined plane with an angle of measure θ .

The body become in equilibrium under the action of two forces:

- (1) The weight of the body \vec{w} that acts vertically down wards and its magnitude is (w)
- (2) The resultant reaction and its magnitude (R')

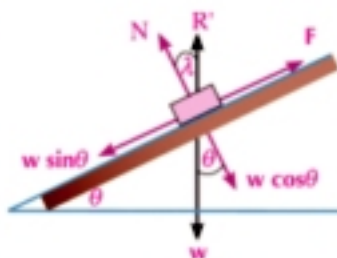


figure (1)

And from the conditions of equilibrium we find that :

The resultant reaction force act vertically up.

$$\text{and : } R' = w \quad \text{-----} \quad (1)$$

we can designate two forces, the friction force and the normal reaction considering the resultant reaction resolved into two directions one of them is parallel to the plane and the other perpendicular to it as shown in the figure (1) .

Friction force .

$$F = w \sin \theta \quad \text{-----} \quad (2)$$

and this force act on opposite direction of the expected motion, this means that is parallel to the line of the greatest slope upwards .

Normal Reaction .

$$N = w \cos \theta \quad \text{-----} \quad (3)$$

The relation between the static friction angle and the measure of the angle of inclination of the plane to the horizontal.

If we put a body on a rough inclined plane and the body is about to slipping, then the measure of the static friction force equal the measure of the angle of inclination of the plane to the horizontal.

You will learn

- The conditions of equilibrium of a body on an inclined rough plane.
- The relation between the measure of the angle of friction and the measure of the angle of inclination of the plane to the horizontal.
- Identifying the coefficient of friction between two contact surfaces (Activity)

Key terms

- Inclined rough plane
- Normal Reaction
- Resultant Reaction
- Angle of Friction
- Coefficient of Friction

Materials

- Scientific calculator

Proof:

- ∴ The friction is limiting
- ∴ The resultant reaction force makes an angle with the normal to the plane of measure equal the measure of the static friction force, and its measure is (λ).

and from the previous figure, we find : $\theta = \lambda$

and we can put the equation in terms of coefficient of friction as follow :

$$\tan \lambda = \mu_s$$

or

$$\mu_s = \tan \theta$$

For example :

If a body is placed on a rough inclined plane and it was about to move under its weight only, when the angle of inclination of the plane on the horizontal is 30° , then the static coefficient friction $\mu_s = \tan 30^\circ = \frac{1}{\sqrt{3}}$

**Activity****Determination of the coefficient of friction between two contacting surfaces****The aim of activity :**

To determine the coefficient of friction between two surfaces, in contact, using the inclined plane.

Equipment required for the experiment :

A rough plane - a wooden block with one face plain on the opposite face a rectangular hole - chab is tan clam P with a pivot - a protractor - a plumb line.

Method:

- (1) Clamp the pivot to the stand and affix the plane to it
- (2) Affix the protractor is the plane so that its straight edge is levelled with the edge of the plane so shown in fig (2).
- (3) Hang the plumb line from the center of the protector so that the line is in alignment with the midline of the protractor when the plane is horizontal.
- (4) Set the plane in a horizontal position and place the block with its plane face resting on it, then put a suitable weight in the hollow on the opposite face of the block.
- (5) Gradually till the plane until the block begins to slip down with help of slight tapping.
- (6) Find the angle of inclination of the plane to the horizontal indicated by the plumb line on the protractor.
- (7) Repeat the experiment for different weights in the hollow and each time record the angle made by the plane to the horizontal it will be noticed that

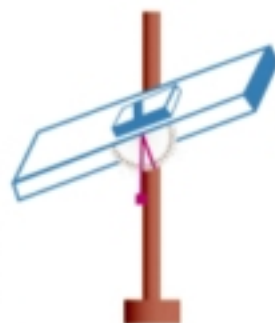


Figure (2)

and we can deduce that :

The recorded angles are nearly the same

- The angle of friction is the average value of these measurements .
- The coefficient of friction is found by calculating the tangent of the angle of friction .

Example

- 1) A body of weight 3 newtons is placed on a plane inclined the horizon by angle of measure 30° and the static coefficient friction between the body and the plane equals $\frac{2}{3}$. A force of magnitude 2N act on the body and in the direction of the line of the greatest slope upwards, if the body is in equilibrium determine the friction force state wether or not the motion is about to begin

Solution

By resolve the weight \vec{W} into two components in the direction of the plane and the normal to it.

1) The tangential component in the direction of the line of the greatest slope downwards and of magnitude $w \sin \theta = 3 \sin 30^\circ = \frac{3}{2}$ newtons

2) The perpendicular component of magnitude $w \cos \theta = 3 \cos 30^\circ = \frac{3\sqrt{3}}{2}$ newtons and by comparing the tangential component $w \sin \theta = \frac{3}{2}$ newton , and the magnitude of the force which acts on the body in the direction of the line of the greatest slope upwards = 2 newton we find: $P > w \sin \theta$.

So, the body tends to move upwards, so the friction force must be in opposite direction in the line of greatest slope down wards \vec{F} :

$$P = F + w \sin \theta \quad \therefore 2 = F + \frac{3}{2}$$

$$\therefore F = \frac{1}{2} \text{ newtons}$$

$$N = w \cos \theta \quad \therefore N = 3 \cos 30^\circ$$

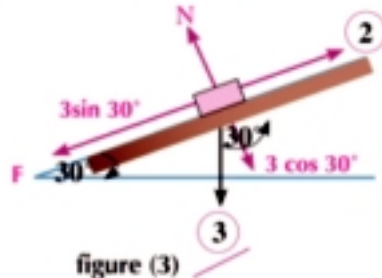
$$\therefore N = \frac{3}{2} \sqrt{3} \text{ newtons}$$

the magnitude of the friction = $\frac{1}{2}$ newton acts on the line of the greatest slope down and to know whether the body is about to move or not

We find the $F_s = \mu_s N = \frac{2}{3} \times \frac{3}{2} \sqrt{3} = \sqrt{3}$ newtons

then: $F < F_s$ then the friction is not limiting

\therefore the body is not about to move .



Try to solve

- 1) A particle of weight 2 kg.wt is placed on a plane inclined the horizon by an angle of measure 30° and the static friction coefficient is $\frac{\sqrt{3}}{2}$. a force of magnitude 2.5 kg.wt acts on the body and in the line of the greatest slope up wards if the body is in equilibrium. Determine the friction force and show whether the body is about to move or not?

Critical thinking: If a body is placed on an inclined plane makes with the horizontal an angle if measure (θ), if the measures of static friction angle between the body and the plane is (λ) what is your expectation for the body if:

- a) $\theta < \lambda$ b) $\theta > \lambda$

Example

- 2 A body of weight 10 kg.wt is placed on a rough inclined plane. A force \vec{P} acts on it in the direction of the line of the greatest slope up. If it is known that the body is about to move upwards the plane when $P = 6$ kg.wt and about to move downwards the plane when $P = 4$ kg.wt. Find:
- The angle of inclination of the plane to the horizontal.
 - The static friction coefficient.

Solution

When $P = 6$ kg.wt the body is about to move up the plane and the static friction is limiting and acts, down the plane.

$\therefore N_1 = 10 \cos \theta$, $6 = 10 \sin \theta + \mu_s N_1$ eliminating N_1 from the two equations :

$$\therefore 10 \sin \theta + 10 \mu_s \cos \theta = 6 \quad \text{--- (1)}$$

when $P = 4$ kg.wt the body is about to move down the plane and the static friction is limiting and acts up the plane.

$\therefore N_2 = 10 \cos \theta$, $4 + \mu_s N_2 = 10 \sin \theta$ eliminating N_2

from the two equations :

$$\therefore 10 \mu_s \cos \theta = 10 \sin \theta - 4 \quad \text{--- (2)}$$

from (1), (2) :

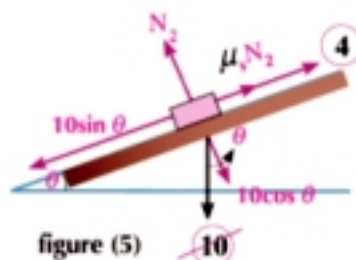
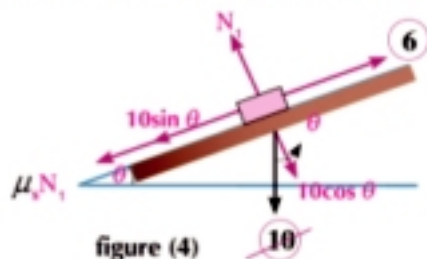
$$\therefore 6 = 10 \sin \theta + 10 \sin \theta - 4 \quad \therefore 20 \sin \theta = 10$$

$$\therefore \sin \theta = \frac{1}{2} \quad \therefore \theta = 30^\circ$$

by substituting in (2) $\therefore 10 \mu_s \cos 30^\circ = 10 \sin 30^\circ - 4$

$$\therefore \frac{\sqrt{3}}{2} \times 10 \mu = 5 - 4 \quad \therefore \mu_s = \frac{1}{5\sqrt{3}} = \frac{\sqrt{3}}{15}$$

the static coefficient friction between the body and the plane = $\frac{\sqrt{3}}{15}$



Try to solve

- 2 A body of weight 30 newtons is placed on a rough inclined plane, it is noticed that the body is about to move. If the plane is inclined to the horizontal by an angle of measure 30° , if the inclination of the plane to the horizontal is increased to 60° , then find:
- The least force which acts on the body parallel to the line of the greatest slope and prevents the body from slipping.
 - The force which acts on the body parallel to the line of the greatest slope and makes it about to move up the plane.

Example

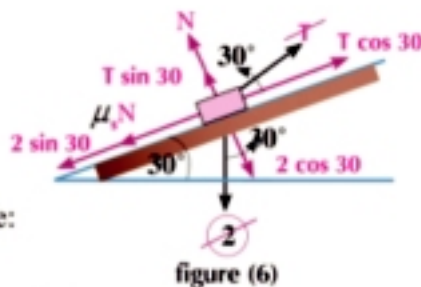
- 3 A body of weight 2 kg.wt is placed on a horizontal rough plane, then the plane is inclined step by step so that the body become about to move downwards the plane when the measure of the angle of inclination is 30° find the static coefficient friction between the body and the plane , and if the body is tied by a string and the string is tensioned in a direction makes an angle of measure 60° to the horizontal until the body become a bout to move upwards the plane, find:
- a The magnitude of the tension force b The magnitude of static friction force

Solution

First: \therefore the body is about to move downwards under the action of its weight only

$$\therefore \mu_s = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

Second: \therefore the body is about to move upwards the plane:



The equation of equilibrium are :

$$N + T \sin 30^\circ = 2 \cos 30^\circ \quad \therefore N = \sqrt{3} - \frac{1}{2}T \quad \text{..... (1) ,}$$

$$T \cos 30^\circ = 2 \sin 30^\circ + \frac{1}{\sqrt{3}} N \quad \therefore 3T = 2\sqrt{3} + 2N \quad \text{..... (2)}$$

from (1) , (2)

$$3T = 2\sqrt{3} + 2(\sqrt{3} - \frac{1}{2}T) \quad \therefore 4T = 4\sqrt{3} \quad \therefore T = \sqrt{3} \text{ kg.wt}$$

by substituting in (1) $\therefore N = \sqrt{3} - \frac{1}{2}\sqrt{3} = \frac{1}{2}\sqrt{3} \text{ kg.wt}$

$$\therefore \text{the static friction force} = \mu_s N = \frac{1}{\sqrt{3}} \times \frac{1}{2}\sqrt{3} = \frac{1}{2} \text{ kg.wt}$$

Try to solve

- 3 A body of weight 30 newtons is placed on a rough inclined plane, it is notice that the body is about to move if sine the angle of inclination of the plane to the horizontal = $\frac{5}{13}$ and if the inclination of the plane increased such that the sine of the angle of inclination to the horizontal became = $\frac{3}{5}$:
- a Find the magnitude of the least force which acts on the body parallel to the line of the greatest slope of the plane to prevent it form sliding
- b The force which makes it about to move upwards the plane and parallel to the line of the greatest slope.

Exercise 1 - 2

First: put the sign (✓) or (X):

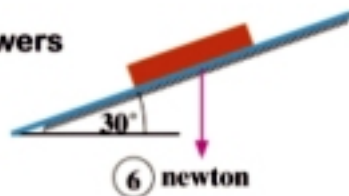
- 1 The coefficient of friction between two bodies depends on their shapes and their masses.
- 2 The ratio between the magnitudes of the limiting static friction force and the normal reaction is called the coefficient of friction.

- 3 The tangent of the angle of the static friction equal the ratio between the limiting friction force and the normal reaction
- 4 If a body is placed on a rough inclined plane and was about to move, then the static coefficient friction between the body and the plane equals the measure of the angle of inclination to the horizontal.
- 5 if a body is placed on a rough inclined plane and was about to move, then the measure of the angle of friction equal the measure of the angle of inclination of the plane to the horizontal.
- 6 The angle of friction is the angle included between the limiting friction force and the resultant reaction.

Second: Choose the correct answer from the given answers

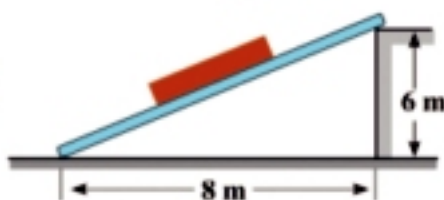
- 7 In the opposite figure: if the body is about to move downwards, then the limiting friction equal:

- a 3 b $2\sqrt{3}$
c $3\sqrt{3}$ d 9



- 8 In the opposite figure: The body is about to move downwards, then the measure of the angle of the static friction equal:

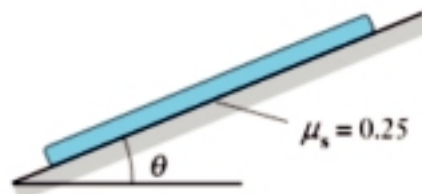
- a 36.87° b 41.41°
c 48.59° d 53.13°



- 9 In the opposite figure:

the body is about to move downwards, then $\lambda =$

- a 14.04° b 14.48°
c 75.52° d 75.87°



Third : Answer the following questions

- 10 A body of weight 400 gm.wt is placed on a plane inclined by 30° to the horizontal and the coefficient of friction between it and the body equal $\frac{\sqrt{3}}{4}$. A force of magnitude 50 gm.wt acts on it in the direction of the line of the greatest slope upwards. If the body is in equilibrium, determine the friction force and show whether the body is about to move or not.
- 11 A body of mass 4 kg is placed on a rough inclined plane makes an angle of measure 30° to the horizontal and the coefficient of friction between it and the plane is $\frac{\sqrt{3}}{2}$. Show whether the body sliding on the plane or about to move or the friction is not limiting and find the magnitude and direction of the friction force where upon, then find the magnitude of the force which acts on the body in the direction of the line of the greatest slope such that the body is about to move upwards the plane.
- 12 A body of weight $20\sqrt{3}$ N is placed on a rough inclined plane makes an angle of measure 30° with the horizontal, then the body is attached upwards by a string lies on the vertical plane passing by the line of the greatest slope and make an angle of measure 30° with the plane, if the coefficient of friction equal $\frac{\sqrt{3}}{6}$, find the least magnitude of tension in the string prevent the body from moving down the plane.

- 13 A body of weight (w) is placed on a rough plane inclined to the horizontal by an angle of measure (θ), it is found that the force which parallel to the line of the greatest slope of the plane and makes the body is about to move upwards the plane equals $2w \sin \theta$ prove that:
- a the measure of the angle of friction = θ b the magnitude of the resultant reaction = w
- 14 A body of weight 25 kg.wt is placed on a rough inclined plane a magnitude force P acts on it in the direction of the line of the greatest slope upwards the plane. If its known that the body is about to move up the plane when $P = 15$ kg.wt, and it is about to move downwards the plane if $P = 10$ kg.wt find :
- a the measure of the angle of inclination of the plane to the horizontal
b the static coefficient friction
- 15 A body of weight (w) is placed on a rough inclined plane makes an angle of $\sin^{-1} \frac{5}{13}$ with the horizontal the body is attached by a horizontal force of magnitude 22 newtons lies in the vertical plane which passes through the line of the greatest slope makes the body is about to move upwards the plane, if the static coefficient friction between the body and the plane is $\frac{1}{2}$, then find of the weight (w).
- 16 A body of weight 8 kg.wt is placed on a horizontal rough plane, then the plane incline gradually until the body becomes about to move downwards the plane when the measure of the angle of inclination to the horizontal is 30° . Find the coefficient friction between the body and the plane, and if the body is tied by a string and the string is attached in the direction makes an angle of measure 30° with the plane until the body becomes about to move upwards the plane, find:
- a the magnitude of the tension force b magnitude of the normal reaction
- 17 A body of weight 3 kg.wt is placed on a rough plane inclined to the horizontal by an angle of measure 60° and the coefficient friction between the body and the plane was $\frac{\sqrt{3}}{9}$, show giving reason that the body can not be at rest, then find the value of the greatest and smallest horizontal force [lie in the vertical plane passing by the line of the greatest slope] acts on the body for being in equilibrium.
- 18 3 and 5 are two masses connected by a light string and are placed on a rough inclined plane and the coefficient friction between the plane and the two bodies $\frac{2}{3}$, $\frac{4}{5}$ respectively. Show which of the bodies is placed bottom the other so that the two bodies move together, then prove that the tangent of the angle of inclination of the plane to the horizontal when the two bodies are about to move is $\frac{3}{4}$
- 19 **Creative thinking:** A body of weight (w) is placed on a rough inclined plane makes an angle of measure (θ) with the horizontal and the friction angle between the body and the plane is of measure (λ). A force of magnitude (P) acts on the body and inclined to the plane upwards by an angle of measure (θ) find the magnitude of the force (P) which makes the body is about to move upwards the plane

UNIT SUMMARY

The smooth surfaces: the friction force is completely vanished and the coefficient of friction = zero is a virtual surfaces.

The rough surfaces: the friction force is appear and the coefficient of friction equal a real positive number.

Reaction:

⇒ **In case the smooth planes** the reaction is to be perpendicular on the surface of the common tangent of the touching bodies.

⇒ **In case of rough planes** the direction of the reaction is unspecified and it is depended on the natural of the touching surfaces and the other forces which acts on the body .

The static friction force: appears when try moving the body on the rough surface and its direction is in opposite direction the direction in which the body tends to the movement in it and it is given by the inequality $0 \leq F \leq \mu_s N$ where μ_s is the static coefficient friction

The limiting static friction force: when the force of friction become the limiting static friction (F_s) the body is about to move (without moving) and the friction is limiting and we symbolize it by (F_s).

and : $F_s = \mu_s N$

Kinetic friction force: If a body moves on a rough plane, then it is subject to a kinetic friction force its direction is opposite to its motion and the value of its is given by the relation : $F_k = \mu_k N$ where μ_k is the kinetic coefficient friction .

Notes on coefficients of static and kinetic friction:

⇒ μ_s , μ_k depend on the nature of the two bodies touching and not depended on the areas of the contiguous surfaces of the mass of the moving body.

⇒ Coefficient of static friction (μ_s) > Coefficient of kinetic friction (μ_k)

The resultant reaction: the resultant reaction (R) is the resultant of the normal reaction \vec{R} and the friction force limiting F_s

Angle of friction: the angle which included between the force normal reaction and the force resultant reaction .

The relation between coefficient of friction and angle of friction : then the coefficient of friction equal the tangent of the angle of friction

The relation between the measure of angle of friction and the measure of the angle of inclination of the plane to the horizon: If a body is placed on a rough inclined plane and the body is about to under effect weight only move, then the measure of the angle of inclination of the plane to the horizontal equal to the measure of the angle of friction.

GENERAL EXERCISES

Choose the correct answer from the given :

- 1 The angle of friction is :
 - a The included angle between the resultant reaction and the normal reaction incase the friction is limiting.
 - b The included angle between the resultant reaction and the limiting friction force .
 - c The ratio between the normal reaction and the limiting friction force.
 - d The ratio between the static coefficient friction and kinetic coefficient friction.
- 2 The coefficient friction depended on:
 - a Area of the contact surface.
 - b The shape of the two bodies.
 - c The nature of the two bodies.
 - d All the previous.
- 3 If μ_s, μ_k are the coefficients of static and kinetic friction respectively for two connected bodies , then :
 - a $\mu_s = \mu_k$
 - b $\mu_s < \mu_k$
 - c $\mu_s > \mu_k$
 - d no relation between them .

Answer the following :

- 4 A body of weight 13.5 kg.wt is placed on a rough horizontal plane , its coefficient of friction static between them is $\frac{2}{3}$, A horizontal force act on it its magnitude 7.5 kg.wt. show whether the body is about to move? Explain your answer.
- 5 A body of weight 45 kg.wt is placed on a rough horizontal plane, the coefficient of friction between it and the body equal $\frac{\sqrt{3}}{3}$. find:
 - a The magnitude of the smallest force horizontal sufficient to move the body on the plane.
 - b The magnitude and direction the resultant reaction.
- 6 A body of weight 26 newton is placed on a rough horizontal plane and the body become about to move when two horizontal forces of magnitudes 7, 8 newtons act on it and include an angle between them 60° . Find the static coefficient friction between the body and the plane.
- 7 A body of weight 10 kg.wt is placed on an inclined plane makes an angle of measure 30° with the horizontal, if the body is about to slide, find the force which acts on the line of the greatest slope to make the body about to move up the plane.
- 8 A body of weight 6 newtons is placed on a rough plane inclined the horizontal by an angle of cosine $\frac{4}{5}$ and the measure of the angle of friction between the body and the plane is 45° . Show that the body is kept in equilibrium , then find the magnitude of the force which acts on the body

For more Activities and Excersises, Visit www.sec3mathematics.com.eg

Moments

Unit

2



Introduction

Since the early history, people had depended upon the idea of levers to enable them to carry and transfer their objects from one place to another. The human locomotor system is similar to the idea on which levers are based on. Bones are the materialistic solid bodies acted by the muscular power joint to the bones to turn around a fixed point (the center). This necessitate to understand the rational effect of the force (moment of a force) in this unit, we are going to shed the light on the concept of the moment of a force about a point in a 2-D or 3-D coordinate system

Unit objectives

By the end of this unit and doing all the activities involved, the student should be able to:

- ⊕ Identify and find the moment of a force about a point in the space.
 - ⊕ Find the norm and direction of the moment of a force about a point.
 - ⊕ Find the moments of coplanar forces about a point lying in their plane.
 - ⊕ Identify the general theorem of moments
- “ if the system of forces acting on a rigid body has a resultant, then the algebraic sum of the moments of these forces about a certain point is equal to the moment of the resultant about this point “.
- ⊕ Solve various applications on the moments.

Key terms

- Moment
- Moment centre
- Moment axis
- Moment arm
- Rotation
- Resultant
- Moment component
- Anti clockwise
- Clockwise
- Algebraic measure of the moment
- Norm of the moment

Lessons of the unit

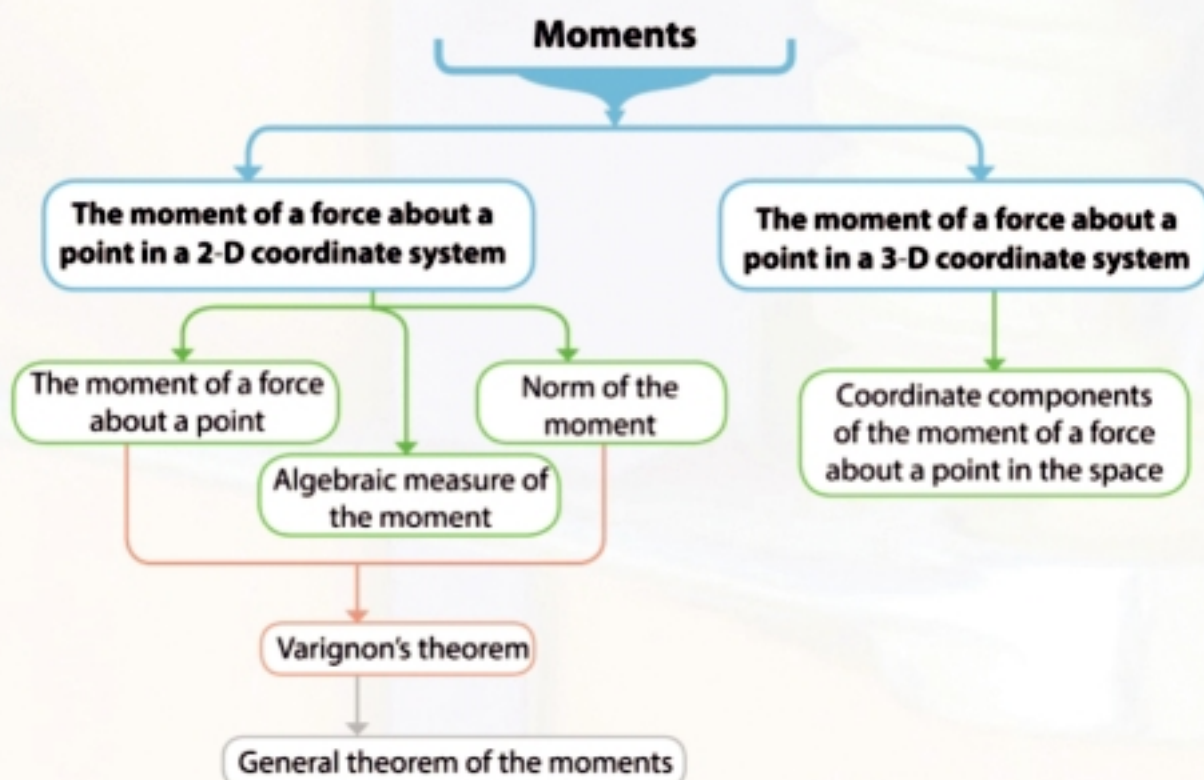
Lesson (2-1): The moment of a force about a point in a 2-D coordinate system

Lesson (2-2): The moment of a force about a point in a 3-D coordinate system

Materials

- Scientific calculator
- Computer graphics

Chart of the unit



Unit Two

2 - 1

The moment of a force about a point in 2D-coordinate system

You will learn

- The moment of a force about a point
- The moments of the planar forces about a point in their plane

Key terms

- Moment
- Moment centre
- Moment axis
- Moment arm

Materials

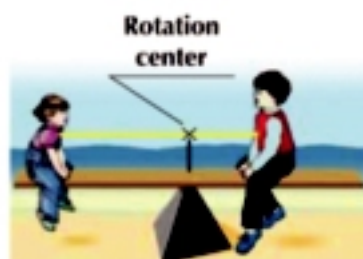
- Scientific calculator.

You have previously learned that the force is produced when a body acts on another body and this effect takes and my form (kinetic effect-formality effect). If a body moves from a place to another, the force acts on the body in a kinetic transitional way. If the body moves a rotational motion about a point, the force acts on the body in a kinetic rotational way. Here, we say that the force can rotate the body about a point. It is called the moment of a force about a point. This rotational effect of the force (moment) depends up on the magnitude of the force and how distant the action line of this force from this point is.



Think and discuss

- (1) The opposite figure shows two kids on a balanced swing in a horizontal position. which kid is (the heavier - the lighter) nearer to the rotation center?



What does the heavier kid do if he/she wants to rotate the swing where the lighter kid rises up?

- (2) The opposite figure shows a hand trying to tie a pipe. The most proper position to the force (F) to tie the pipe perfectly is (A, B, C).

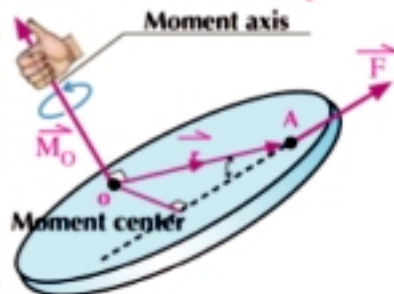


Learn

The moment of a force about a point in a 2D-coordinate system

The moment of a force \vec{F} about the point O is known as the ability of the force to rotate the body about point O. The rotational effect can be calculated by the relation $\vec{M}_O = \vec{r} \times \vec{F}$

where \vec{r} is the position vector of the of point A on the line of action the force about point O. The point O is called the moment center and the straight line passing through the point



and perpendicular to the plane containing the force (\vec{F}) and point (O) is called the moment axis. We notice that the moment of a force is a vector quantity. According to the rule of the right hand of the vector multiplication, the direction of the moment of a force about a point and perpendicular to the plane containing the force \vec{F} .

Critical thinking: is the moment of a force \vec{F} about point O based on the position of point A on the line of action of the force?

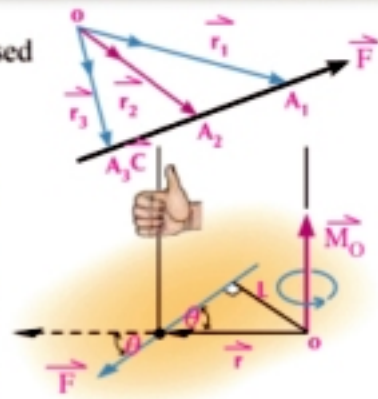
(1) The moment of a force about a point

From the definition of the vector multiplication of two vector, then:

$\vec{M}_O = (\|\vec{r}\| \|\vec{F}\| \sin \theta) \vec{C}$ where \vec{C} is a unit vector perpendicular to the plane of \vec{F} and \vec{r} where the rotation is from \vec{r} to \vec{F} in the direction of the vector \vec{C} and θ is the measure of the angle between \vec{r} and \vec{F}

let $\|\vec{F}\| = F$ and $\|\vec{r}\| \sin \theta = L$

where L is the length of the perpendicular segment on the action line of force \vec{F} (L is called the moment arm) then the moment of \vec{F} about point O is $\vec{M}_O = (FL) \vec{C}$ (1)



(2) Algebraic measure of the moment

If the force \vec{F} act on rotating about O in the anti clockwise direction, the algebraic measure of the moment vector is positive (the moment vector is in the direction of the vector \vec{C}). If the force \vec{F} acts on rotating about O in the clockwise direction,

the algebraic measure of the moment vector is negative (the moment vector is in the direction of the vector $-\vec{C}$)



(3) The norm of the moment the norm of the moment is $\|\vec{M}_O\| = FL$ (2)

(4) The moment of a force about a point on its line of action = zero

(5) The magnitude of the moment measuring unit:

the magnitude of the moment measuring unit = the magnitude of force measuring unit \times length measuring unit. Such as newton. meter, Dyne. cm , kg.wt. meter ...



the action line of the force passes through point O
 $\therefore M_O = \text{Zero}$

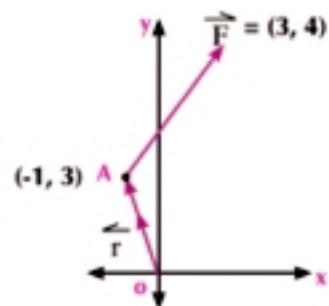
Example

1 If \hat{i} , \hat{j} and \hat{k} are a right system of the unit vectors and the force $\vec{F} = 3\hat{i} + 4\hat{j}$ actions at the point A (-1, 3) of a body find:

- a** the moment of the force \vec{F} about the origin point (0, 0)
- b** the length of the perpendicular segment from the point O and on the action line of the force \vec{F}

Solution

a $\vec{r} = \vec{OA} = \vec{A} - \vec{O}$
 $= (-1, 3) - (0, 0) = (-1, 3)$



$$\begin{aligned}\vec{M}_O &= \vec{r} \times \vec{F} \\ &= (-1, 3) \times (3, 4) = (-1 \times 4 - 3 \times 3) \hat{k} \\ &= -13 \hat{k}\end{aligned}$$

The norm of the moment = 13 moment unit and the algebraic measure of the moment vector = -13 moment unit

Interpreting the result: i.e. the force \vec{F} produces a rotation to the body about the point O and in the clockwise direction (the direction of the moment is in the direction of $-\hat{k}$)

- b** To find the length of the perpendicular drawn from O on the action line of the force F

$$\therefore \|\vec{M}_O\| = FL \quad \therefore L = \frac{\|\vec{M}_O\|}{F} = \frac{13}{\sqrt{3^2 + 4^2}} = \frac{13}{5} \text{ length unit.}$$

5 Try to solve

- 1** If \hat{i} , \hat{j} and \hat{k} are a right system of the unit vectors and the force $\vec{F} = \hat{i} - 2\hat{j}$ acts at the point A (2, 3) find:
- The moment of the force \vec{F} about point B (2, 1)
 - The length of the perpendicular segment from point A on the line of action of the force.

Critical thinking: what would it mean if the moment of the force about a point is vanished?



Learn

Principle of moments (Varignons theorem)

The moment of a force \vec{F} about a point equals the sum of the moments of the components of this force about the same point.

Let the force $\vec{F} = F_x \hat{i} + F_y \hat{j}$ acts at the point A whose position vector with respect to the point O is $\vec{r} = (x, y)$ then

$$\begin{aligned}\vec{M}_O &= \vec{r} \times \vec{F} \\ &= (x, y) \times (F_x, F_y)\end{aligned}$$

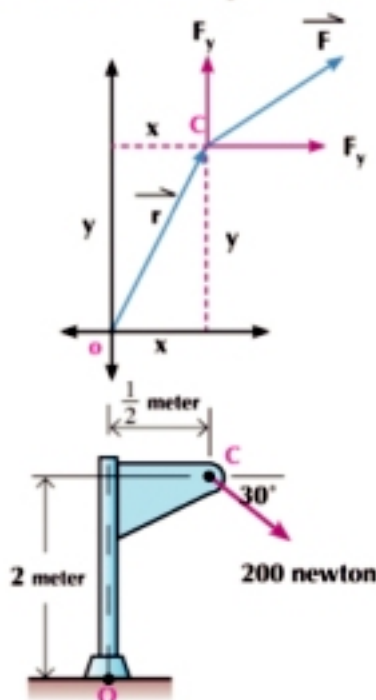
$$= (x F_y) \hat{k} + (-y F_x) \hat{k}$$

moment F_y about O + the moment of F_x about O



Example

- 2** In the figure opposite:
Find the algebraic measure of the moment of the force about the point O.



First Solution:

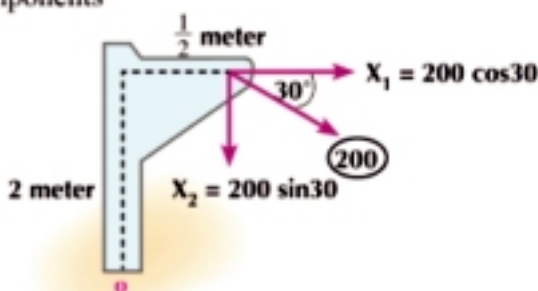
We factorize the force 200 newton into two components

$$F_1 = 200 \cos 30 = 100 \sqrt{3} \text{ newton}$$

$$F_2 = 200 \sin 30 = 100 \text{ newton}$$

In regard to Varignon's theorem

$$\begin{aligned} M_O &= -F_1 \times 2 - F_2 \times \frac{1}{2} \\ &= -100 \sqrt{3} \times 2 - 100 \times \frac{1}{2} \\ &= (-200 \sqrt{3} - 50) \text{ newton} \cdot \text{meter} \end{aligned}$$


Solution:

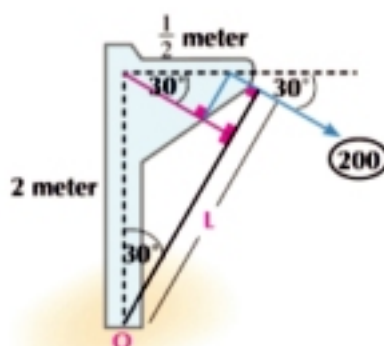
the length of the perpendicular segment from O on the line of action of the force = L

$$\text{where } L = 2 \cos 30 + \frac{1}{2} \sin 30 = (\sqrt{3} + \frac{1}{4}) \text{ meter}$$

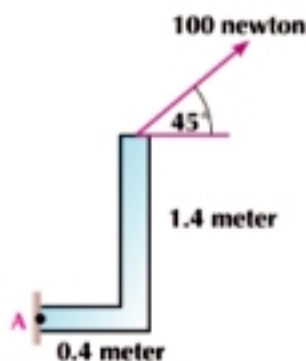
\therefore the force is working to rotate about O in the clockwise direction

\therefore the algebraic measure of the moment of a force is negative

$$\therefore M_O = -200 \times (\sqrt{3} + \frac{1}{4}) = (-200 \sqrt{3} - 50) \text{ newton} \cdot \text{meter}$$


Try to solve

- 2 In the figure opposite: calculate the algebraic measure for the moment of a force 100 newton about point A.


Theorem

The algebraic sum of the moments of a system of forces acting at a point about any point in space is equal to the moment of the resultant of these forces about the same point

Proof

Let $\vec{F}_1, \vec{F}_2, \dots, \vec{F}_n$ be a finite system of forces acting at point A and let that is point (O) be the point required to find the moments about it

$$\therefore \vec{r} = \vec{OA}$$

The sum of the moments of the forces about point O

$$= \vec{r} \times \vec{F}_1 + \vec{r} \times \vec{F}_2 + \dots + \vec{r} \times \vec{F}_n$$

$$= \vec{r} \times (\vec{F}_1 + \vec{F}_2 + \dots + \vec{F}_n)$$

$$= \vec{r} \times \vec{R}$$

= The moment of the resultant of these forces about the same point O



Example (The moment of a system of forces acting at a point)

- ③ The forces $\vec{F}_1 = \hat{i} + 2\hat{j}$, $\vec{F}_2 = (1, 3)$ and $\vec{F}_3 = 4\hat{i} + 4\hat{j}$ act at the point A (-2, 1). Find the sum of the moments of these forces about point B (0, 2), then find the moment of the resultant of these forces about B. What do you notice?

Solution

$$\vec{r} = \vec{BA} = \vec{A} - \vec{B} = (-2, -1)$$

$$\begin{aligned}\vec{M}_1 &= \vec{r} \times \vec{F}_1 \\ &= (-2, -1) \times (1, 2) \\ &= (-4 + 1)\hat{k} = -3\hat{k}\end{aligned}$$

$$\vec{M}_2 = \vec{r} \times \vec{F}_2 = (-2, -1) \times (1, 3) = (-6 + 1)\hat{k} = -5\hat{k}$$

$$\vec{M}_3 = \vec{r} \times \vec{F}_3 = (-2, -1) \times (4, 4) = (-8 + 4)\hat{k} = -4\hat{k}$$

\therefore The sum of the moment of the forces about point B

$$\begin{aligned}&= \vec{M}_1 + \vec{M}_2 + \vec{M}_3 \\ &= -3\hat{k} - 5\hat{k} - 4\hat{k} = -12\hat{k}\end{aligned}$$

Resultant of the force: $\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = (1, 2) + (1, 3) + (4, 4) = (6, 9)$

$$\begin{aligned}\therefore \text{Moment of the resultant} &= \vec{r} \times \vec{R} \\ &= (-2, -1) \times (6, 9) \\ &= (-18 + 6)\hat{k} = -12\hat{k}\end{aligned}$$

We notice that the sum of the moments of the forces about a point equals the moment of the resultant of the forces about the same point.

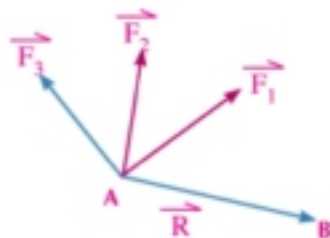
General theorem of moments

Theorem

The algebraic sum of the moments of forces about a point is equal to the moment of the resultant about this point.

Try to solve

- ③ The force $\vec{F}_1 = 3\hat{i} - \hat{j}$, $\vec{F}_2 = -2\hat{j} - 3\hat{j}$ act at point A (-1, 4). Find the sum of the moments of forces about point B (1, 1) hence find the moment of the resultant of these forces about point B.



Example

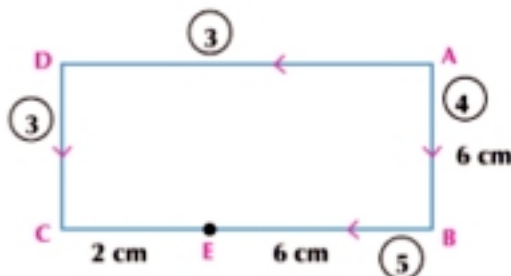
- 4 ABCD is a rectangle in which $AB = 6$ cm and $BC = 8$ cm forces of magnitudes 4, 5, 3 and 3 newtons act along the directions of \overrightarrow{AB} , \overrightarrow{BE} , \overrightarrow{DC} and \overrightarrow{AD} where $E \in \overline{BC}$, $BE = 6$ cm. Prove that the resultant of these forces passes through point E.

Solution

The sum of the algebraic measures of the moments of forces about point

$$E = -4 \times 6 + 3 \times 2 + 3 \times 6 = \text{zero}$$

According to the theorem of moments, the moment of the resultant about point E = zero
i.e. the resultant passes through point E



Try to solve

- 4 A B C D is a square of side length is 6 cm and $E \in \overline{BC}$ where $BE = 1$ cm, forces of magnitudes 1, 2, 3, 4, and F newtons act along \overrightarrow{AB} , \overrightarrow{BC} , \overrightarrow{CD} , \overrightarrow{DA} and \overrightarrow{AC} respectively. If the line of action of the resultant passes through point E, find the value of F.

Example

- 5 A force $\vec{F} = -2\hat{i} + 3\hat{j}$ acts at the point A (4, -3). Find the moment of \vec{F} about each of the points B(3, 1), C (1, 4) and D(-1, 2)

Solution

$$\vec{r}_1 = \overrightarrow{BA} = \vec{A} - \vec{B} = (1, -4)$$

$$\therefore \vec{M}_B = \vec{r}_1 \times \vec{F} = (1, -4) \times (-2, 3) = (3 - 8) \hat{k} = -5 \hat{k}$$

$$\vec{r}_2 = \overrightarrow{CA} = \vec{A} - \vec{C} = (3, -7)$$

$$\therefore \vec{M}_C = \vec{r}_2 \times \vec{F} = (3, -7) \times (-2, 3) = (9 - 14) \hat{k} = -5 \hat{k}$$

$$\vec{r}_3 = \overrightarrow{DA} = \vec{A} - \vec{D} = (5, -5)$$

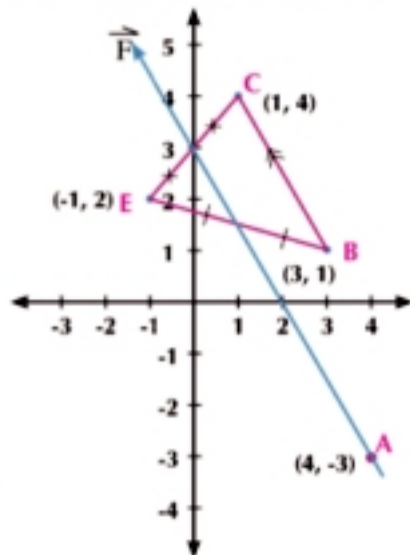
$$\therefore \vec{M}_D = \vec{r}_3 \times \vec{F} = (5, -5) \times (-2, 3) = (15 - 10) \hat{k} = 5 \hat{k}$$

From the previous example, we deduce that:

- If the moment of a force about point B = the moment of this point about point C, then the line of action of this force $\parallel \overline{BC}$
- If the moment of a force about point B = - the moment of this force about point D, then the line of action of this force bisects \overline{BD}

Try to solve

- 5 A force \vec{F} acts at point A (-3, 2). If the moment of \vec{F} about each of the two points B (3, 1) and C (-1, 4) equals $28 \hat{k}$, find \vec{F} .



Generalization of the previous conclusion

If a system of coplanar forces act on a body and A and B are two points at the same plane, then:

- (1) If the sum of the moments of forces about A = the sum of the moments of the forces about B then the line of action of the resultant // \overline{AB} .
- (2) If the sum of the moments of forces about A = - the sum of the moments of the forces about B, then the line of action of the resultants passes through the midpoint of \overline{AB}

Note: If the sum of the moments of the forces about a point- (say C) vanishes then either C lies on the line of action of the resultant or the resultant is the zero vector.

Example

- 6 The force $\vec{F}_1 = 2\hat{i} - \hat{j}$, $\vec{F}_2 = 5\hat{i} + 2\hat{j}$, $\vec{F}_3 = -3\hat{i} + 2\hat{j}$ act at point A(1, 1) prove using the moment that the line of action of the resultant is parallel to the straight line passing through the two points B(2, 1) and C(6, 4).

Solution

$$\therefore \vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 4\hat{i} + 3\hat{j}$$

$$\vec{r}_1 = \vec{BA} = A - B = (-1, 0)$$

$$\vec{M}_B = \vec{r}_1 \times \vec{F} = (-1, 0) \times (4, 3) = -3\text{ k}$$

$$\vec{r}_2 = \vec{CA} = A - C = (-5, -3)$$

$$\vec{M}_C = \vec{r}_2 \times \vec{F} = (-5, -3) \times (4, 3) = -3\text{ k}$$

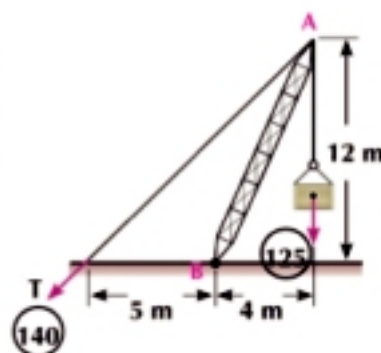
$$\therefore \vec{M}_B = \vec{M}_C, \vec{R} \neq \vec{0} \quad \therefore \text{line of action of } \vec{R} // \vec{BC}$$

Try to solve

- 6 The forces $\vec{F}_1 = \hat{i} + 2\hat{j}$, $\vec{F}_2 = 3\hat{i} - \hat{j}$ act at point A(-2, 3) prove using the moments that the line of action of the resultant bisects the straight segment drawn between the two points B(-1, 5) and C(1, 2).

Try to solve

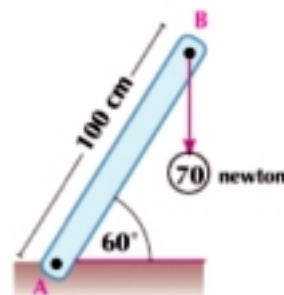
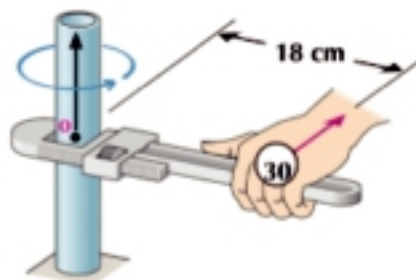
- 7 In the opposite figure: \overline{AB} represents a crane for lifting the goods. If the tension in the string is equal to 140 newtons, and the weight of the box is 125 newtons, find the sum of the two moments of the two forces about B.



Exercises 2 - 1

Complete the following

- 1 A force of a magnitude 50 newtons and is 8 cm it is a line of work from point A, then the norm of the moment about point A equals _____ newtons . cm
- 2 **In the opposite figure:** the norm of the moment of the force about the point (O) equals _____
- 3 A force of $4\hat{j}$ newton acts at a point whose position vector with respect to the origin point O equals $5\hat{i}$ meters, then the moment of the force about the origin point equals _____
- 4 If the moment of a force about a point is equals to zero, this means _____
- 5 If the moment of the force about point is constant, then the magnitude of the force is inversely proportional to _____
- 6 **The opposite figure:** shows a rod fixed by a hinge at A. If a vertical force of a magnitude 70 newton acts on the end B downward, then the norm of the moment of the force about A is equal to _____ newtons. meter



Choose the correct answer:

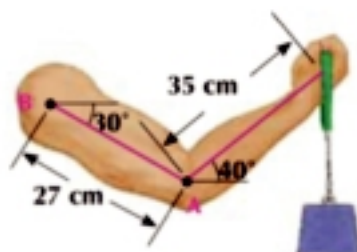
- 7 **The opposite figure** presents a door attached with a hinge at A. If a force \vec{F} acts on the door which of the following figures in which the force \vec{F} has the greatest moment about A?
 - a
 - b
 - c
 - d
- 8 A rod of length L. It can rotate easily about a point. at one of its ends. A force of a magnitude F acts on the other end and inclines on the rode with an angle measured θ if \vec{F} should be perpendicular to the rod, at which distance from the rotation center can F affect such that it has the same moment
 - a $L \sin \theta$
 - b $L \cos \theta$
 - c L
 - d $L \tan \theta$
- 9 If the moment of a force \vec{F} about point A is equal to its moment about point B then _____
 - a $\vec{F} \perp \overline{AB}$
 - b \vec{F} bisects \overline{AB}
 - c $\vec{F} \parallel \overline{AB}$
 - d \overline{AB} action line \vec{F} are skew



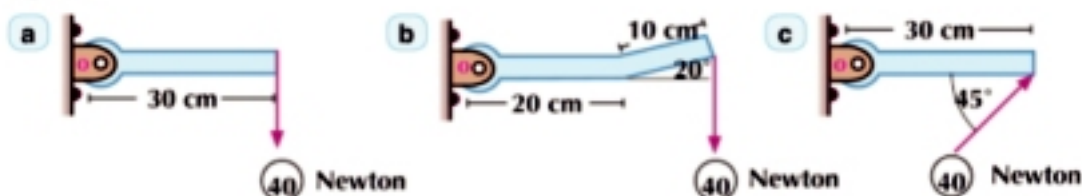
Answer the following questions

- 10 Two forces $\vec{F}_1 = M \hat{i} + 2 \hat{j}$ and $\vec{F}_2 = L \hat{i} - \hat{j}$ act at the two points $A_1(1, 1), A_2(-1, -2)$ respectively. Determine the value of the two constants M and L such that the sum of the two moments of those two forces about the origin point and about point $B(2, 3)$ vanishes.
- 11 The forces $\vec{F}_1 = 2 \hat{i} - \hat{j}$, $\vec{F}_2 = 5 \hat{i} + 2 \hat{j}$, $\vec{F}_3 = -3 \hat{i} + 2 \hat{j}$ act at point $A(1, 1)$. Prove using the moments that the line of action of the resultant is parallel to the straight line passing through the two points $(2, 1)$ and $(6, 4)$

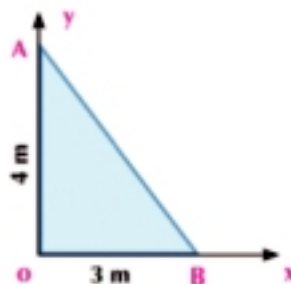
- 12 The opposite figure represents a person carrying a weight in his hand. If the norm of the moment of the weight about point A is equal to 80 newtons meter, find the moment of the weight about point B



- 13 Find the algebraic measure of the moment of the force about point O in each of the following figures



- 14 A force \vec{F} at the xy -plane acts on the triangle $A O B$. If the algebraic measure of the moment of \vec{F} about point O is equal to 84 newton . m , the algebraic measure of its moment about point A is equal to - 100 newton . m , and the algebraic measure of its moment at point B is equal to zero, determine \vec{F}



The moment of a force about a point in 3D- coordinate system

Unit Two 2 - 2

In the last lesson, you learned to find the moment of a force about a point in its plane. In this lesson, you are going to learn how to find the moment of a force about a point in the space.

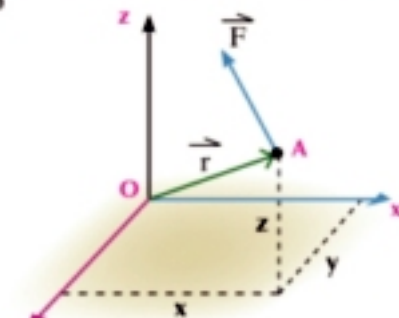


Learn

Moment of a force about a point in space

If $\vec{F} = (F_x, F_y, F_z)$ acts at point A (x, y, z) whose position vector with respect to point O $(0, 0, 0)$ is $\vec{r} = (x, y, z)$ then the moment of the force \vec{F} about point O is equal to

$$\begin{aligned} \vec{M}_O &= \vec{r} \times \vec{F} \\ &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix} \end{aligned}$$



Example

- 1 The force $\vec{F} = 2\hat{i} - \hat{j} + 3\hat{k}$ acts at point A $(-3, 1, 2)$, find the moment of the force \vec{F} about point B $(2, 2, -1)$ then calculate the length of the perpendicular segment from B to the line of action of the force



Solution

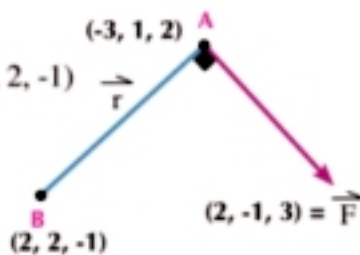
$$\vec{r} = \vec{BA} = \vec{A} - \vec{B} = (-3, 1, 2) - (2, 2, -1) = (-5, -1, 3)$$

$$\vec{M}_B = \vec{r} \times \vec{F}$$

$$\begin{aligned} &= (-5, -1, 3) \times (2, -1, 3) \\ &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -5 & -1 & 3 \\ 2 & -1 & 3 \end{vmatrix} = (-1 \times 3 - 3 \times -1)\hat{i} \\ &\quad - (-5 \times 3 - 3 \times 2)\hat{j} \\ &\quad + (-5 \times -1 - 2 \times -1)\hat{k} \end{aligned}$$

$$= 21\hat{i} + 7\hat{k} \text{ moment unit}$$

$$L = \frac{\|\vec{M}_B\|}{\|\vec{F}\|} = \frac{\sqrt{0^2 + 21^2 + 7^2}}{\sqrt{2^2 + (-1)^2 + 3^2}} = \sqrt{35} \text{ length unit}$$



You will learn

- Moment of a force about a point in the space
- Coordinate components of the moment of a force about a point in the space

Key terms

- Space
- Components
- Rotation
- Axis

Materials

- Scientific calculator

5 Try to solve

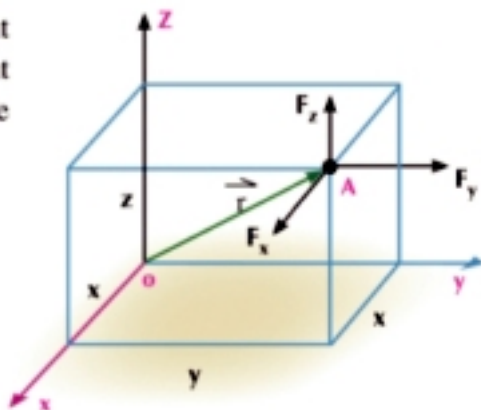
- 1 Find the moment of the force \vec{F} about the origin point where $\vec{F} = -2\hat{i} + 3\hat{j} + 5\hat{k}$ and acts at point A whose position vector with respect to the origin point is $\vec{r} = \hat{i} - \hat{j} + \hat{k}$ then find the length of the perpendicular segment drawn from the origin point to the line of action of the force \vec{F}

Coordinate components of the moment of a force about a point

Let the force $\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$ acts at point A whose position vector with respect to the origin point $\vec{r} = (x, y, z)$ then the moment of the force \vec{F} about the origin point O is equal to $\vec{r} \times \vec{F}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

$$= (yF_z - zF_y)\hat{i} + (zF_x - xF_z)\hat{j} + (xF_y - yF_x)\hat{k}$$



i.e. the moment of the force \vec{F} has three components each can be interpreted as follows:

the component of the moment in the direction of \hat{i} . It can be calculated by finding the moment of the components F_x , F_y and F_z about x-axis.

The component F_x does not have a rotational moment about x-axis since it is parallel to the axis whereas the component F_y produces the rotation about x-axis in the clockwise direction, then its moments is $-z \times F_y$.

The component F_z produces the rotation about x-axis in the anticlockwise direction, then its moment is $y \times F_z$ and the sum of the moments of the component about x-axis is equal to $y \times F_z - z \times F_y$

Similarly to the rest of the moment components in the direction of \hat{i} and \hat{k}

Example

- 2 If the force $\vec{F} = K\hat{i} + 4\hat{j} - \hat{k}$ acts at point A whose position vector with respect to the origin point is $\vec{r} = (1, 2, 2)$ and the component of the moment of the force \vec{F} about y-axis is equal to 7 moment unit, find the value of K then find the length of the perpendicular segment drawn from O on the line of action of \vec{F}

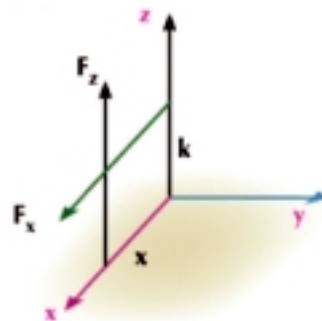
Solution

$$\vec{F} = (K, 4, -1) \longrightarrow F_x = K, F_y = 4, F_z = -1$$

$$\vec{r} = (1, 2, 2) \longrightarrow x = 1, y = 2, z = 2$$

the component of the moment of the force about y-axis = $zF_x - xF_z$

$$\therefore 2K - 1(-1) = 7$$



$$\therefore 2K + 1 = 7 \longrightarrow K = 3$$

$$\therefore \vec{M}_O = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 3 & 4 & -1 \end{vmatrix} = -10 \hat{i} + 7 \hat{j} - 2 \hat{k}$$

$$\therefore \text{the length of the perpendicular segment drawn from O on the line of action of the force} = \frac{\|\vec{M}_O\|}{\|\vec{F}\|}$$

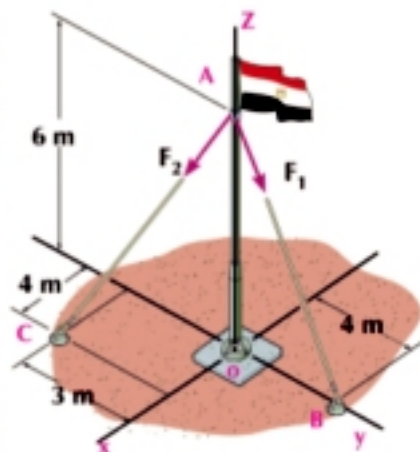
$$= \frac{\sqrt{(-10)^2 + (7)^2 + (-2)^2}}{\sqrt{3^2 + 4^2 + (-1)^2}} = \frac{3\sqrt{442}}{26} \text{ length unit}$$

Try to solve

- 2 If the force $\vec{F} = k \hat{i} + m \hat{j} - 2 \hat{k}$ acts at point A whose position vector with respect to the origin point is $\vec{r} = (3, 1, 1)$ if the two components of the moment of \vec{F} about x and y-axes are -1 and -8 respectively, find the value for each of k and m.

Example

- 3 The force $F_1 = 6\sqrt{13}$ newton and, $F_2 = \sqrt{61}$ newton in the direction of AB and AC as shown in the figure. Find:
- The sum of the moments of the forces about point O.
 - The moment of the resultants of the two forces about point O. What do you infer?



Solution

From the geometry of the figure, the coordinates of the points are:

$$A(0, 0, 6), B(0, 4, 0), C(4, -3, 0)$$

$$\vec{AB} = \vec{B} - \vec{A} = (0, 4, 0) - (0, 0, 6) = (0, 4, -6)$$

$$\vec{F}_1 = F_1 \left(\frac{\vec{AB}}{\|\vec{AB}\|} \right) = 6\sqrt{13} \left(\frac{(0, 4, -6)}{\sqrt{0^2 + 4^2 + (-6)^2}} \right) = 6\sqrt{13} \times \frac{(0, 4, -6)}{2\sqrt{13}}$$

$$\therefore \vec{F}_1 = (0, 12, -18)$$

$$\vec{AC} = \vec{C} - \vec{A} = (4, -3, 0) - (0, 0, 6) = (4, -3, -6)$$

$$\vec{F}_2 = F_2 \left(\frac{\vec{AC}}{\|\vec{AC}\|} \right) = \sqrt{61} \left(\frac{(4, -3, -6)}{\sqrt{4^2 + (-3)^2 + (-6)^2}} \right) = \sqrt{61} \times \frac{(4, -3, -6)}{\sqrt{61}}$$

$$\therefore \vec{F}_2 = (4, -3, -6)$$

$$\text{The moment of the force } \vec{F}_1 \text{ about point O} = \vec{OA} \times \vec{F}_1 = (0, 0, 6) \times (0, 12, -18)$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 6 \\ 0 & 12 & -18 \end{vmatrix} = -72 \hat{i}$$

The moment of the force \vec{F}_2 about point O = $\vec{OA} \times \vec{F}_2 = (0, 0, 6) \times (4, -3, -6)$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 6 \\ 4 & -3 & -6 \end{vmatrix} = 18 \hat{i} + 24 \hat{j}$$

a The sum of the moments of the forces about O

$$= -72 \hat{i} + 18 \hat{i} + 24 \hat{j} = -54 \hat{i} + 24 \hat{j} \quad (1)$$



the resultant of the two forces $\vec{F} = \vec{F}_1 + \vec{F}_2 = (0, 12, -18) + (4, -3, -6)$
 $= (4, 9, -24)$ and acts at point A

b The moment of the resultant about O = $\vec{OA} \times \vec{R} = (0, 0, 6) \times (4, 9, -24)$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 6 \\ 4 & 9 & -24 \end{vmatrix} = -54 \hat{i} + 24 \hat{j} \quad (2)$$

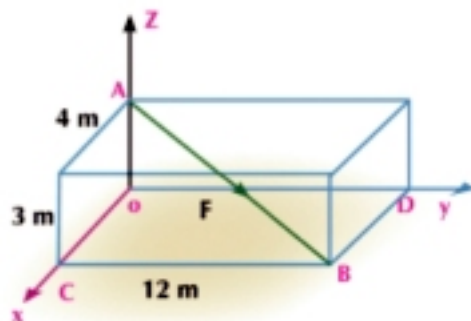
From 1, 2 we notice that

the sum of the moments of the forces about a point in the space is equal to the moment of the resultant of these forces about this point.

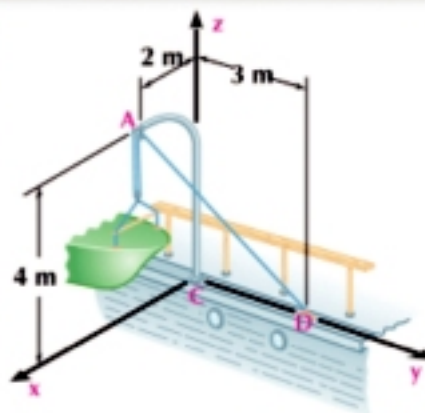
 **Exercises 2 - 2** 

- 1 If $\hat{i}, \hat{j}, \hat{k}$ a right system of the unit vectors and the force $\vec{F} = 2 \hat{i} + 3 \hat{j} - \hat{k}$ acts at point A(1, -1, 4), find:
 - a The moment of the force \vec{F} about the origin point O (0, 0, 0).
 - b The moment of the force \vec{F} about point B (2, -3, 1) then deduce the length of the perpendicular segment drawn from B to the line of action of the force.
- 2 If $\vec{F} = 2 \hat{i} + L \hat{j} - \hat{k}$ acts at point A(4, -2, 0) and the moment of \vec{F} about the origin point is equal to $2 \hat{i} + 4 \hat{j} + 16 \hat{k}$. Find the value of L.

- 3 In the opposite figure, a force of a magnitude 130 newton acts along the diagonal \vec{AB} in the cuboid whose dimensions are 3m, 4m and 12m as shown in the figure. Find the moment of the force \vec{F} about point D.



- 4 In the opposite figure, a rope is attached in point D passing over a smooth pulley at A and from the other end of the rope, a small boat suspends. If the tension magnitude of the rope \overline{DA} is equal to $10\sqrt{29}$ newton. Find the moment of the tension of the rope about point C.



- 5 A force \vec{F} acts at point $A(2, -1, 3)$ if the moment of \vec{F} about the origin point is equal to $21\hat{j} + 7\hat{k}$. Find \vec{F} where \vec{F} is parallel the x-axis
- 6 If the force $\vec{F} = 2\hat{i} + B\hat{j} + \hat{k}$ acts at point $A(-1, 3, -2)$ and the component of the moment of \vec{F} about x-axis is equal to -3 moment units. Find the value of B, hence find the length of the perpendicular segment drawn from the origin point to the line of action of the force.
- 7 ABCD is a right-angled trapezoid at B, $\overline{AD} \parallel \overline{BC}$, $AB = 8$ cm, $BC = 15$ cm, $AD = 9$ cm. \overline{DE} is drawn perpendicular to the plane of the trapezoid where $DE = 12$ cm. A force of a magnitude 75 newton acts along \overline{AE} . Find the norm of moment of the force about B.
- 8 If the moment force $\vec{F} = 2\hat{i} + 3\hat{j} - \hat{k}$ about the origin point O is equal to $\vec{M}_O = -5\hat{i} + 3\hat{j} - \hat{k}$ and if this force passes through a point whose y-coordinate is equal to 2. find the coordinates x and z for the point, hence find the length of the perpendicular segment drawn from the origin point to the line of action of the force.
- 9 A force $\vec{F} = 15\hat{i} - 25\hat{j} + 40\hat{k}$ acts at point $A(-3, -3, 2)$. Find the component of the moment of \vec{F} about y-axis.

UNIT SUMMARY

1 The moment of a force about a point : the moment of the force \vec{F} acting on a body about a point is known as the ability of the force \vec{F} to produce a rotation to the body about point O and the moment of the force \vec{F} is calculated by the relation $\vec{M}_O = \vec{r} \times \vec{F}$ where \vec{r} is the position vector of a point on the line of action of the force about point (O) and the direction of the moment is normal to the plane containing each of \vec{F} and \vec{r} .

2 The norm of the moment of a force about a point: if F represents the norm of the force \vec{F} and L represents the length of the perpendicular segment from point O to the line of action of the force, then the norm of the moment of \vec{F} about point O is calculated by the relation $\|\vec{M}_O\| = FL$.

3 Algebraic measure of the moment of a force about a point: if the force works to rotate the body about point O in the clockwise direction, then the algebraic measure of the moment vector is negative and if the force works to rotate the body about point O in anticlockwise direction, then the algebraic measure of the moment vector is positive.



4 The length of the perpendicular segment from point O to the line of action of the force \vec{F} is L where $L = \frac{\|\vec{M}_O\|}{\|\vec{F}\|}$

5 If the moment of the force about a point vanishes, then the line of action of this force passes through this point.

6 The principle of moments (varignon's theorem) the moment of the force \vec{F} about a point is equal to the sum of the moments of the components of this force about the same point

7 Theorem: The sum of the moments of a system of forces acting at a point about any point in space is equal to the moment of the resultant of these forces about the same point

8 If the sum of the moments of a system of forces about point A = the sum of the moments of these forces about point B, then the line of action of the resultant is parallel to \overline{AB}

9 If the sum of the moments of a system of forces about point A = - the sum of the moments of these forces about point B, then the line of action of the resultant bisects \overline{AB}

10 The moment of a force about a point in space $\vec{M}_O = \vec{r} \times \vec{F}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix} \text{ where } \vec{r} \text{ is the position vector of a point on the line of action of the force with respect to point O}$$

11 The components of the moment of a force in the direction of axes if $\vec{F} = (F_x, F_y, F_z)$ is a force acting at a point whose position vector about the origin point O is $\vec{r} = (x, y, z)$ then:

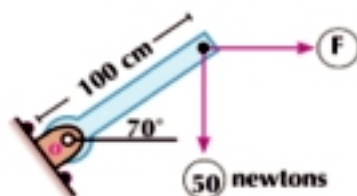
$(y F_z - z F_y)$ \longrightarrow the component of the moment of \vec{F} in the direction of x-axis

$(z F_x - x F_z)$ \longrightarrow the component of the moment of \vec{F} in the direction of y-axis

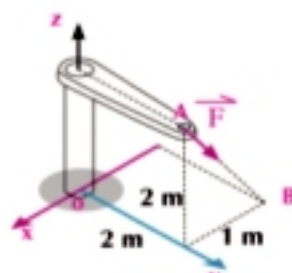
$(x F_y - y F_x)$ \longrightarrow the component of the moment of \vec{F} in the direction of z-axis

GENERAL EXERCISES

- 1 If the moment of the horizontal force F about point O is equal to the moment of the vertical force 50 newton about point O , find the value of F .

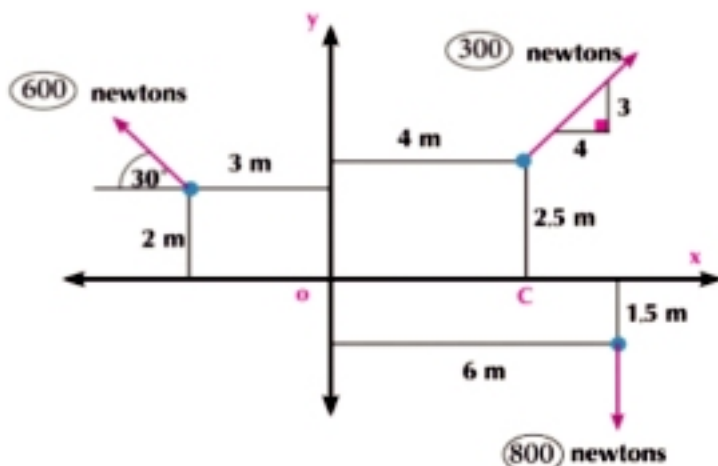


- 2 In the opposite figure calculate the moment of the force $F = 14\sqrt{5}$ newton about point O

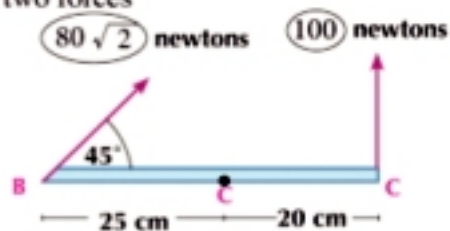


- 3 A force $\vec{F} = 7 \hat{j}$ acts at point $(-3, 0)$, find the moment of the force about point $(1, -2)$
- 4 A force $\vec{F}_1 = 2 \hat{i} + \hat{j}$ newton acts at a point whose position vector is $2 \hat{i} + 2 \hat{j}$ meter and another force $\vec{F}_2 = 5 \hat{i}$ newton acts at a point whose position vectors $-2 \hat{i} + \hat{j}$ meter. Find the sum of the moments of these forces about the origin point
- 5 If \hat{i} , \hat{j} and \hat{k} are a right system of the unit vectors, the force $\vec{F} = 3 \hat{i} + k \hat{j} + 4 \hat{k}$ acts at point $A(1, 0, -1)$ and the moment of the force \vec{F} about point $B(2, -1, 3)$ is equal to $12 \hat{i} - 8 \hat{j} - 5 \hat{k}$. Find the value of k .

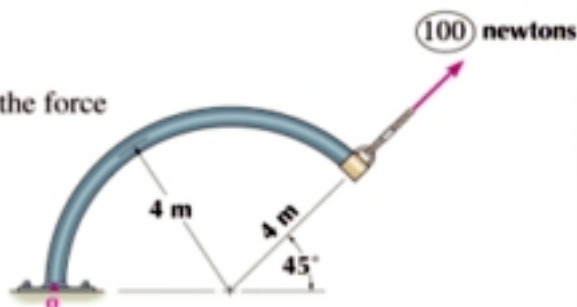
- 6 In the opposite figure, find the algebraic measure of the sum of the moments of the forces about point C .



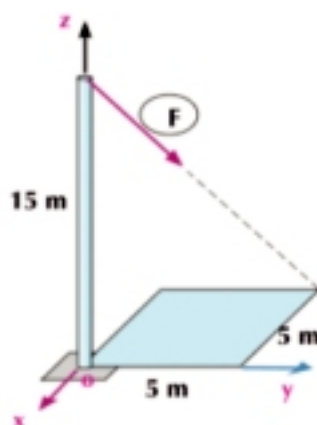
- 7 In the opposite figure, prove that the resultant of the two forces 100 newton and $80\sqrt{2}$ newton pass through point C



- 8 In the opposite figure Find the algebraic measure of the moment of the force 100 newton about point O



- 9 In the opposite figure Find the moment of the force $F = 15\sqrt{11}$ newtons about O



For more Activities and Exercises, Visit www.sec3mathematics.com.eg

Accumulative test

Choose the correct answer:

1. ABC is a right-angled triangle at B , $AC = 10$ cm and $m(\angle BCA) = \theta$ then $BC =$

a $10 \sin \theta$ b $10 \cos \theta$ c $10 \tan \theta$ d 5
2. The distance between the two points $(2, -1)$ and $(-1, 3)$ is equal to

a 4 b 5 c $\sqrt{5}$ d 2
3. The direction cosines of the vector $(-2, 2, 1)$ are:

a $2, -2, -1$ b $-\frac{2}{3}, \frac{2}{3}, \frac{1}{3}$ c $\frac{2}{3}, \frac{2}{3}, \frac{1}{3}$ d $\frac{-2}{\sqrt{5}}, \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}$
4. ABC is a triangle in which $AB = 8$ cm, $m(\angle ABC) = 70^\circ$ then the length of the perpendicular segment drawn from A on BC is equal to:

a $8 \cos 70$ b $8 \sin 70$ c $8 \tan 70$ d $\sqrt{8}$
5. If $\vec{A} = (2, -1)$, $\vec{B} = (3, 2)$ then $\vec{A} \cdot \vec{B} =$

a 7 b $\sqrt{65}$ c 4 d 8
6. If $\vec{A} = (-1, 2, 3)$, $\vec{B} = (2, 2, 1)$ then $\vec{A} \times \vec{B} =$

a $(4, 7, -6)$ b $(-4, -7, -6)$ c $(-4, 7, -6)$ d 5
7. If $\vec{F} = (2, -3, 4)$, acts at point $(1, 1, 1)$ then the component of the moment of \vec{F} about x-axis is equal to

a 7 b -2 c -5 d 2

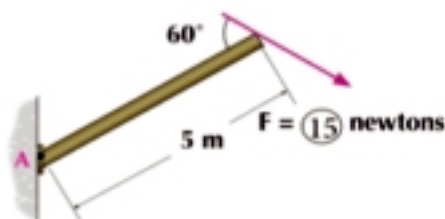
Answer the following questions :

8. $ABCD$ is a square of side length 10 cm. Forces of a magnitudes $3, 5, 8, 5\sqrt{2}$ kg.wt act along the directions $\vec{AB}, \vec{BC}, \vec{CD}$ and \vec{AC} respectively. Find the algebraic measure the sum of the moments of the forces:

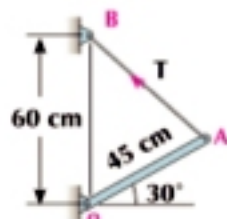
a about point A b about point B

c about the center of the square

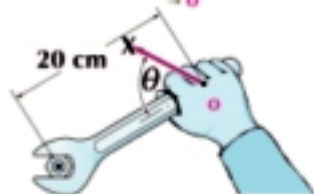
9. The opposite figure represents the effect of a force of 15 newton on an arm fixed in a hinge at A . Find the algebraic measure of the moment of the force about point A .



10. In the opposite figure the magnitude of the tension in the thread AB is 150 newton. Find the algebraic measure of the moment of the tension force about point O .

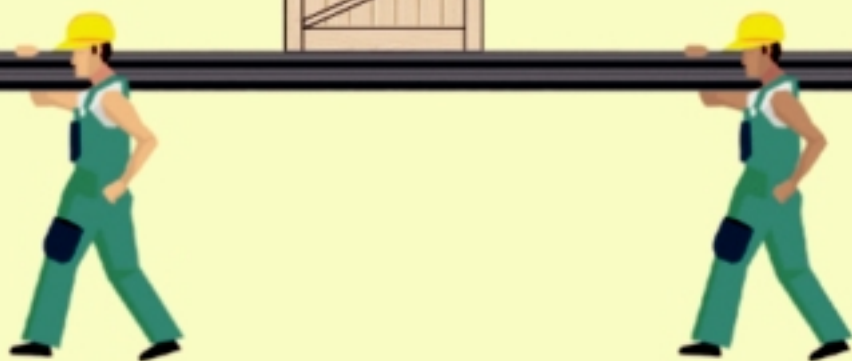


11. If the moment needed to rotate a nail is equal to 400 newton.cm, find the least value of the force F and the value of θ which satisfies the nail rotation.



Parallel coplanar forces

Unit 3



Introduction

In our previous study of the system of coplanar forces acting on a materialistic point, the lines of action of the resultant of these forces meet at one materialistic point. As a result, the line of action of the resultant of these forces passes through one point which is the common point of intersection to this system of forces. In this unit, we are going to learn a system of forces acting on a rigid body since the lines of action of these forces do not necessarily meet at a point. Our objective in this unit is to learn these forces whose lines of action are parallel to them and all lie in the same plane which is called "the parallel coplanar forces". You will learn the parallel coplanar forces when you are asked to find their resultant such that their direction, magnitude and point of action.

Unit objectives

By the end of this unit and by doing all the activities involved, the student should be able to:

- ⊕ Identify the parallel coplanar forces.
- ⊕ Identify the line of action of the resultant of two like and unlike forces.
- ⊕ Identify one of the two parallel forces if the other force and the resultant are known.
- ⊕ Find the moments of a system of parallel coplanar forces about a point
- ⊕ Find the resultant of a system of parallel coplanar forces.
- ⊕ Deduce that the sum of the moments of a system of parallel forces about a point is equal to the moment of the resultant about the same point.
- ⊕ Deduce that the sum of the moments of a system of parallel forces about a point is equal to zero if their resultant passes through such a point.
- ⊕ Deduce that the sum of the moments of a system of parallel forces about a point is equal to zero if the resultant of these forces vanishes.

Key Terms

- Parallel forces
- Resultant
- Magnitude
- Norm
- Point of action
- Reaction
- Weight
- Parallel
- Support
- Beam
- Tension
- Pulley
- Like forces
- Unlike forces

Unit Lessons

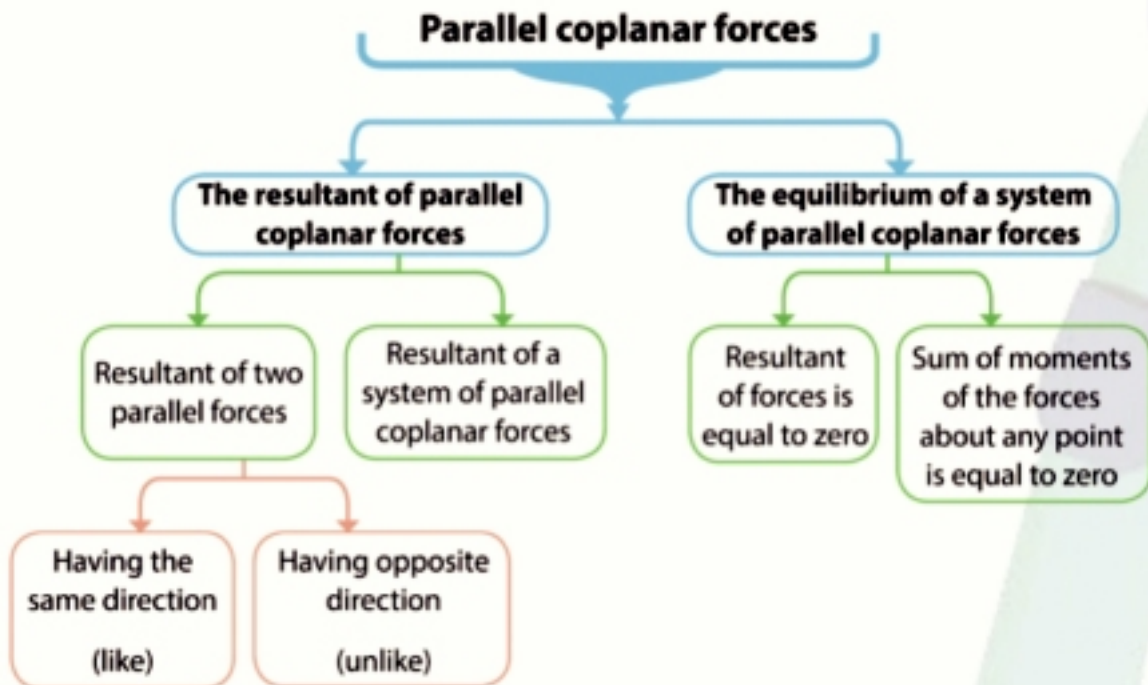
Lesson (3 -1): The resultant of parallel coplanar forces.

Lesson (2 -2): The equilibrium of a system of parallel coplanar forces.

Materials

- Scientific calculator
- Computer graphics

Unit planning guide



Unit Three

3 - 1

Resultant of a parallel coplanar forces



Cooperative work

You will learn

- The resultant of two like forces.
- The resultant of two unlike forces.
- The resultant of a system of parallel and coplanar forces.

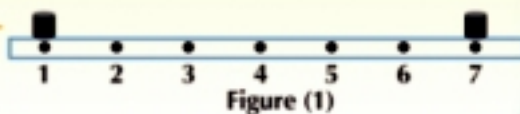
Key terms

- Parallel
- Resultant
- Magnitude
- Norm
- Point of action

Materials

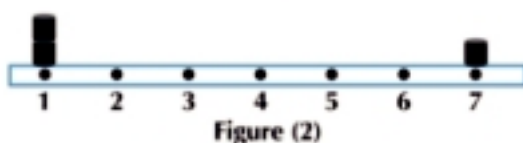
- Scientific calculator

Figure (1) shows a graduated wood ruler from 1 to 7 and two identical stones are placed at the ends of the ruler.



(1) Identify the position of a point on the ruler from which the ruler can be suspended in a horizontally equilibrium way.

(2) Does the position of the suspension point change if two weight are placed at one end as in figure (2)?



Identify the new suspension point if the position changes.

Resultant of two parallel forces having the same direction

(like forces)

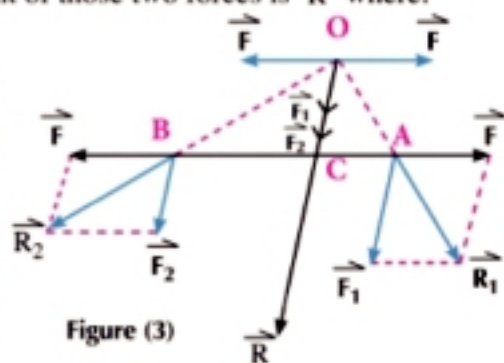
You have learned that the resultant of a system of coplanar forces $\vec{F}_1, \vec{F}_2, \dots, \vec{F}_n$ meet at a point is the force \vec{R} where $\vec{R} = \vec{F}_1 + \vec{F}_2 + \dots + \vec{F}_n$ and passes through the same point. In this lesson, you are going to find the resultant of a system of parallel coplanar forces.

You start to find the resultant of two parallel coplanar forces having the same direction (like forces).

Let \vec{F}_1 and \vec{F}_2 be two like forces forces, act at a rigid body at two points A and B, then the resultant of those two forces is \vec{R} where:

$$\vec{R} = \vec{F}_1 + \vec{F}_2.$$

To identify the position of the point of action of the resultant, let two forces of equal magnitude and opposite direction act at A and B and this does not change the action of the two forces \vec{F}_1 and \vec{F}_2 .



Let R_1 be the resultant of the two forces \vec{F}_1 and \vec{F} at A which represents a diagonal in the parallelogram and R_2 be the resultants of the two forces \vec{F} and \vec{F}_2 at B.

and let the two lines of action of the two resultants \vec{R}_1 and \vec{R}_2 intersect at O.

The force \vec{R}_1 can be replaced by their two original components \vec{F}_1 and \vec{F} and the force \vec{R}_2 can be also replaced by their two original components \vec{F}_2 and \vec{F} .

The force acting at point (O) are: \vec{F}_1 and \vec{F}_2 They act in the direction of \vec{OC} (parallel to the line of action of the two original forces) the two forces \vec{F} , \vec{F} which and act at two opposite directions where they can be removed without any change to the effect of the two forces \vec{F} and \vec{F} at point (O). The two forces \vec{F}_1 and \vec{F}_2 acting at point (O), act in the direction of OC and have the same effect of the two forces \vec{F}_1 and \vec{F}_2 acting at A and B, then their resultant is

$$\vec{R} = \vec{F}_1 + \vec{F}_2 \text{ and act in the direction of } \vec{OC}$$

since the forces \vec{F}_1 , \vec{F}_2 and \vec{R} are parallel, then

$$\frac{F}{F_1} = \frac{AC}{OC} \quad (1)$$

$$\text{and } \frac{F}{F_2} = \frac{BC}{OC} \quad (2)$$

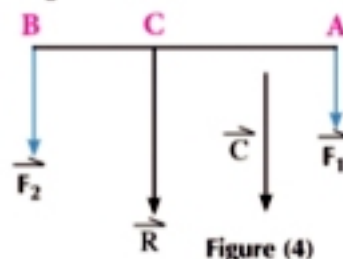
By dividing (2) by (1) then: $\frac{F}{F_2} \times \frac{F_1}{F} = \frac{BC}{OC} \times \frac{OC}{AC}$ i.e. $\frac{F_1}{F_2} = \frac{BC}{AC}$

$$\text{Thus: } F_1 \times AC = F_2 \times BC$$

In figure (4) by taking the unit vector \vec{C} in the direction of the two forces, then:

$$\vec{F}_1 = F_1 \vec{C} \quad , \quad \vec{F}_2 = F_2 \vec{C}$$

$\therefore \vec{R} = (F_1 + F_2) \vec{C}$ this means that the resultant is in the direction of the two forces and its magnitude is equal to the sum of the magnitude of the two forces i.e:



The resultant of two like forces is a force working in the same direction as the two forces, its magnitude is equal to the sum of the magnitudes of the two forces and its line of action divides the distance between the lines of action of the two forces from the inside in the inverse ratio to their magnitudes.

Example Identify the resultant of two like forces

- 1 Two like forces of magnitudes 5 and 7 newtons act at the two points A and B where $AB = 36$ cm find the resultant of the two forces

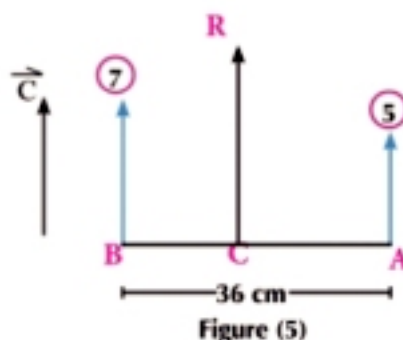
Solution

Let \vec{C} be the unit vector in the direction of the two forces

$$\therefore \vec{F}_1 = 5 \vec{C} \text{ and } \vec{F}_2 = 7 \vec{C}$$

the magnitude and direction of the resultant:

$$\vec{R} = \vec{F}_1 + \vec{F}_2 = 5 \vec{C} + 7 \vec{C} = 12 \vec{C}$$



Identifying the point of action of the resultant

Let the resultant acts at point $C \in \overline{AB}$ $\therefore \frac{AC}{CB} = \frac{7}{5}$ i.e. $\frac{AC}{36 - AC} = \frac{7}{5}$
 $\therefore 5 AC = 252 - 7 AC$ i.e. $AC = 21\text{cm}$

i.e. the magnitude of the resultant is equal to 12 newtons, it acts in the same direction of the two forces and it acts at a point which is 21 cm distant from A.

Try to solve

- 1 Two like forces of magnitudes 4 and 6 newtons act at the two points A and B where $AB = 25\text{cm}$. Find the resultant of the two forces

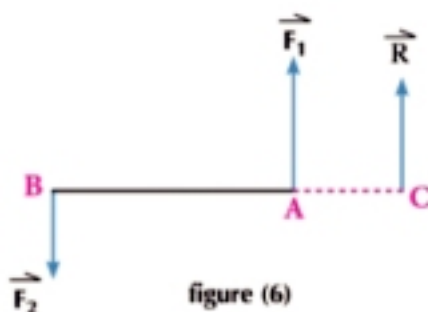
Critical thinking: Where does the point of action of the resultant lie if the two forces are equal.



Learn

Resultant of two unlike forces

Similarly in figure (6), if F_1 and F_2 are two parallel, unequal forces in the opposite directions act at the two points A and B in a rigid body and their resultant is \vec{R} then: $\vec{R} = \vec{F}_1 + \vec{F}_2$ and acts at point C which divides \overline{AB} externally in an inverse ratio of the magnitude of the two forces.



If $F_1 > F_2$ then $\frac{AC}{BC} = \frac{F_2}{F_1}$ i.e.: $F_1 \times AC = F_2 \times BC$

i.e. the resultant of two unequal and unlike forces is a force working in the direction of the force of the greater magnitude, its magnitude is equal to the difference between their magnitudes and its line of action divides the distance between the lines of action of the two forces externally from the side of the force of the greater magnitude in an inverse ratio to their magnitude.

Example Identifying the resultant of two unlike forces

- 2 Two unlike forces of magnitudes 40 and 100 newton and the distance between their two lines of action is 240 cm. Find their resultant.

Solution:

Let \vec{C} be a unit vector in the direction of the greater force

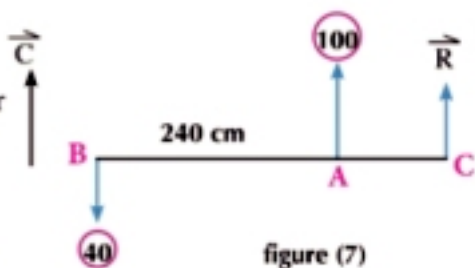
$$\therefore \vec{F}_1 = 100 \vec{C}, \quad \vec{F}_2 = -40 \vec{C}$$

the magnitude and direction of the resultant

$$\therefore \vec{R} = \vec{F}_1 + \vec{F}_2 = 100 \vec{C} - 40 \vec{C} = 60 \vec{C}$$

Identifying the point of action of the resultant let the resultant acts at point $C \in \overline{BA}$ where

$$\frac{CA}{CB} = \frac{40}{100}$$



$$\therefore \frac{CA}{240 + CA} = \frac{2}{5} \therefore 5 AC = 480 + 2 AC \therefore AC = 160 \text{ cm}$$

i.e the magnitude of the resultant is equal to 60 newtons, its direction is the same direction of the force 100 newtons, it acts at a point $\in \overrightarrow{BA}$ and $\notin \overrightarrow{AB}$ and it is 160 cm a part of A

5 Try to solve

- 2 Find the resultant of two unlike forces of magnitudes 7 and 12 newtons act at A and B where A and B = 20 cm

Critical thinking: What would you say about the resultant of two equal and unlike forces?

Theorem

The sum of the moments of a finite number of parallel coplanar forces about any point in its plane is equal to the moment of the resultant of these forces about the same point

Proof (is not required)

We start to prove this theorem in a special case when the system is made up of two forces only.

(1) If the two forces are like

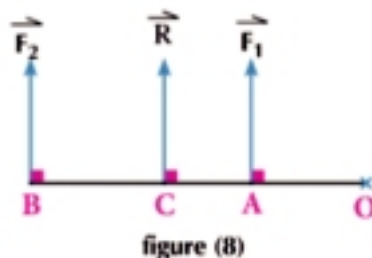
Let a point such as (O) lie in the plane of the two forces. From point O, draw a common perpendicular on the two lines of action of the two forces \vec{F}_1 and \vec{F}_2 to intersect them at the two point A and B respectively and to intersect the line of action of the resultant at point C.

Then, the algebraic sum of the moments of the forces about point O

$$\begin{aligned} &= -F_1 \times AO - F_2 \times OB = -F_1(OC - AC) - F_2(OC + CB) \\ &= -F_1 \times OC + F_1 \times AC - F_2 \times OC - F_2 \times CB \quad (1) \end{aligned}$$

but: $\frac{F_1}{F_2} = \frac{BC}{AC} = \text{i.e. } F_1 \times AC = F_2 \times BC$

$$\begin{aligned} \text{by substituting in (1)} \quad \therefore M_O &= -F_1 \times OC - F_2 \times OC \\ &= -(F_1 + F_2) \times OC \\ &= -R \times OC = \text{the moment of the resultant about point O} \end{aligned}$$



(2) If the two forces are unlike

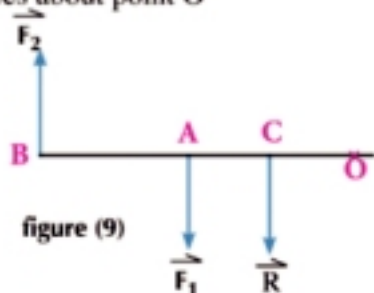
Let $F_1 > F_2$ then, then algebraic sum of the moments of the forces about point O

$$\begin{aligned} &= F_1 \times OA - F_2 \times OB \\ &= F_1(OC + AC) - F_2(OC + CB) \\ &= F_1 \times OC + F_1 \times CA - F_2 \times OC - F_2 \times CB \quad (2) \end{aligned}$$

but: $\frac{F_1}{F_2} = \frac{CB}{CA} = \text{i.e. } F_1 \times CA = F_2 \times CB$

by substituting in (2)

$$\begin{aligned} \therefore M_O &= F_1 \times OC - F_2 \times OC = (F_1 - F_2) \times OC \\ &= R \times OC = \text{the moment of the resultant about point O} \end{aligned}$$



(3) If the system is made up of any definite number of forces (more than two forces) whose resultant does not vanish, then the theorem can be proved by getting the resultant of any two forces whose resultant does not vanish and applying the theorem in pairs and so on until we get the resultant of the system.

Example Identifying one of two parallel forces if the other and the resultant are given

- (3)** Two parallel forces of magnitudes 20 and F newton act at the two points A and B. The magnitude of their resultants is 35 newtons and the distance between the lines of action of the known force and the resultant is equal to 15 cm, Find \vec{F} in each of the following two cases :
- The known force and the resultant are in the same direction.
 - The known force and the resultant are in the opposite directions.

Solution:

- a** Let \vec{C} be a unit vector in the direction of the resultant

$$\therefore \vec{R} = 35 \vec{C}, \quad \vec{F}_1 = 20 \vec{C}$$

$$\therefore \vec{R} = \vec{F}_1 + \vec{F}_2 \text{ i.e. } 35 \vec{C} = 20 \vec{C} + \vec{F}_2$$

$$\therefore \vec{F}_2 = 15 \vec{C}$$

i.e. the force \vec{F}_2 of a magnitude 15 newtons, and in the same direction of the known force and the resultant

\therefore The sum of the moments about point C is equal to the moment of the resultant about point C = zero

$$\therefore 20 \times 15 - 15 \times BC = \text{zero}$$

$$\therefore BC = 20 \text{ cm i.e. the force } F_2 \text{ acts at point B at a distance of 35 cm from A}$$

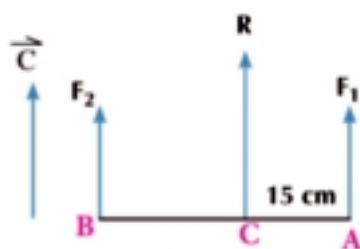


figure (10)

- b** Let \vec{C} be the unit vector in the direction of the resultant

$$\therefore \vec{R} = 35 \vec{C}, \quad \vec{F}_1 = -20 \vec{C}$$

$$\therefore \vec{R} = \vec{F}_1 + \vec{F}_2 \text{ i.e. } 35 \vec{C} = -20 \vec{C} + \vec{F}_2$$

$$\therefore \vec{F}_2 = 55 \vec{C}$$

i.e. the force \vec{F} of magnitude 55 newtons and its direction is in the same direction of the resultant force

\therefore the sum of the moments of forces about point C is equal to the moment of the resultant about C = zero

$$\therefore 20 \times 15 - 55 \times BC = \text{zero} \quad \text{i.e. } BC = \frac{60}{11} \text{ cm}$$

$$\text{i.e. force } F_2 \text{ acts at point B at a distance } \frac{105}{11} \text{ cm from A}$$

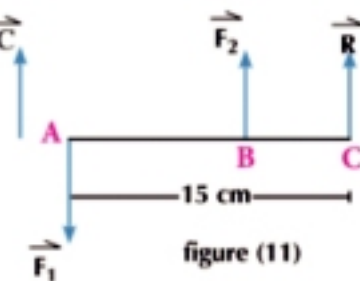


figure (11)

Try to solve

- (3)** Two parallel forces, the magnitude of their resultants is 350 newtons and the magnitude of one of the two forces is 500 newtons acting at a distance of 51 cm from the resultant. Find

the second force and the distance between the two lines of action of the two forces if the known force and the resultant have

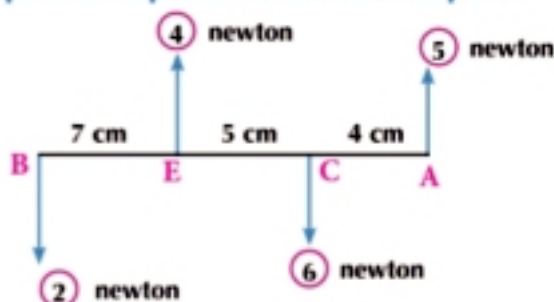
First: the same direction

Second: the opposite directions

Example The moments of a system of parallel coplanar forces about a point

- 4 The opposite figure represents a system of the parallel forces perpendicular to \overline{AB} . Find the algebraic measure of the sum of the moments of these forces about:

- a point A b point C



Solution

- a The force 5 newtons acts at point A, then its moment about A is equal to zero.

By considering the direction of revolving the forces about A (clockwise or anti clockwise direction), then the algebraic measure of the sum of the moments of forces about point A

$$= 6 \times 4 - 4 \times 9 + 2 \times 16 = 20 \text{ newton.cm}$$

- b The force 6 newtons acts at point C, then its moment about C is equal to zero.

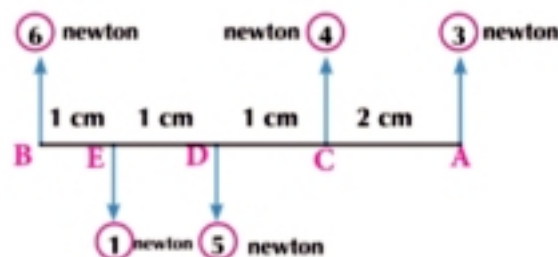
The algebraic measure of the sum of the moments of forces about point C

$$= 5 \times 4 - 4 \times 5 + 2 \times 12 = 24 \text{ newton.cm}$$

Try to solve

- 4 The opposite figure represents a system of parallel forces perpendicular to \overline{AB} . Find the algebraic measure of the sum of the moments of forces about

- a Point A b Midpoint of \overline{AB}



Example The resultant of a system of parallel coplanar forces

- 5 A, B, C, D and E are points lying on a straight line where:

$AB : BC : CD : DE = 2 : 3 : 4 : 7$. Five parallel forces having the same direction whose magnitudes are 30, 50, 20, 70 and 40 newtons act at the points A, B, C, D and E respectively. Find the resultants of these forces

Solution:

$$\text{Let } AB = 2x, BC = 3x$$

$$CD = 4x, DE = 7x$$

and let \vec{C} be the unit vector in the direction of the forces

$$\begin{aligned} \therefore \vec{R} &= \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 + \vec{F}_5 \\ &= 30 \vec{C} + 50 \vec{C} + 20 \vec{C} + 70 \vec{C} + 40 \vec{C} = 210 \vec{C} \text{ newtons} \end{aligned}$$

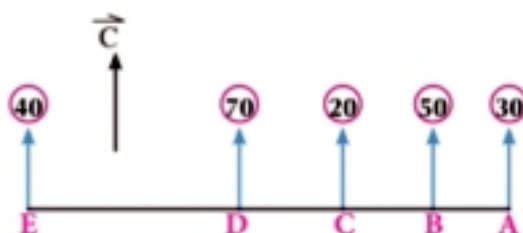


figure (12)

i.e the magnitude of the resultant is 210 newton in the same direction of the forces

to find the point of action of the resultant, let the resultant act at point $O \in \overline{AE}$

\therefore the sum of the moments of the forces about A is equal to the moment of the resultant about A

$$\therefore -50 \times 2x - 20 \times 5x - 70 \times 9x - 40 \times 16x = -210 \times AO$$

$$\therefore AO = \frac{1470x}{210} = 7x \text{ cm}$$

$\frac{AO}{AE} = \frac{7x}{16x} = \frac{7}{16}$ i.e the resultant acts at point (O) which divides \overline{AE} internally by a ratio

7 : 16 from the direction of A

5 Try to solve

- 5 If C, D and E $\in \overline{AB}$ such that $AC : CD : DE : EB = 1 : 3 : 5 : 7$. Parallel forces in the same directions and equal in the magnitudes act at points A, C, D, E and B. Prove that the resultant divides \overline{AB} by a ratio 3 : 5

Example the resultant of a system of parallel forces

- 6 In the opposite figure (fig.13) A, B, C, D and E are five points lying on a straight line. Two forces of magnitudes 20 and 30 newtons act vertically upwards at the two points B and D and two forces of magnitudes 40 and 60 newtons act vertically downwards at the points A and C. Find the magnitude, direction and the point of action of the resultant.

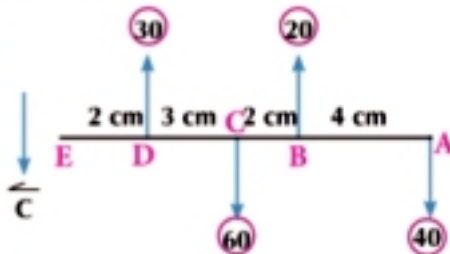


Figure (13)

Solution

Let \vec{C} the unit vector down as shown in figure (13)

$$\begin{aligned} \therefore \vec{R} &= \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 \\ &= -30 \vec{C} + 60 \vec{C} - 20 \vec{C} + 40 \vec{C} = 50 \vec{C} \end{aligned}$$

Let the resultant acts at a point on \overline{AE} and is distant x cm from A

\therefore the sum of the moments of forces about A = the moment of the resultant about A

$$-30 \times 9 + 60 \times 6 - 20 \times 4 = 50 \times X$$

$$\therefore X = 0,2$$

i.e the resultant acts at a point on \overline{AE} and at a distance of 0.2 cm from A

5 Try to solve

- 6 The opposite figure shows a light rod \overline{AB} . The parallel forces shown in the figure act on it. If the magnitude of the resultant is 300 newtons and it acts up at a point on the rod and is 4 meters from A. Find F and K

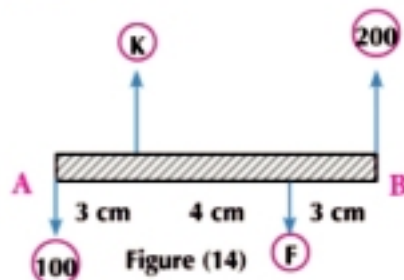


Figure (14)

Example The theoretical proof

- 7 \vec{F}_1 and \vec{F}_2 are two like forces which act at the two points A and B and their resultant is \vec{R} . If a force \vec{F}_2 moves parallel to itself in the direction of \vec{AB} a distance x cm, then prove that the resultant of the two forces moves in the direction of \vec{AB} a distance $(\frac{F_2}{F_1 + F_2})x$

Solution

In the first case:

Let the resultant act at point C

\therefore the moment of the resultant at A = the sum of the moments of the forces at A

$$\therefore R \times AC = F_2 \times AB \quad (1)$$

in the second case:

if force \vec{F}_2 parallel to itself moves in the direction of \vec{AB} a distance of x cm.

let the resultant act at C'

\therefore the moment of the resultant at A = the sum of the moments of forces at A

$$\therefore R \times AC' = F_2 \times AB' \quad (2)$$

By subtracting (1) from (2)

$$\therefore R (AC' - AC) = F_2 (AB' - AB) \quad \therefore R \times CC' = F_2 \times x$$

$$\therefore CC' = \frac{F_2}{R} \times x = (\frac{F_2}{F_1 + F_2})x$$

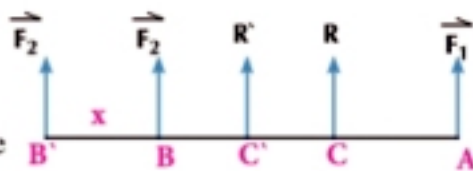


Figure (15)

Try to solve

- 7 Two like forces of magnitudes F and $2F$ act at the two points A and B. If the force $2F$ moves parallel to itself in the direction of \vec{AB} a distance x cm, prove that the resultant of the two forces moves in the same direction a distance $\frac{2}{3}x$

Example

- 8 Two forces $\vec{F}_1 = 2\hat{i} - 3\hat{j}$ and $\vec{F}_2 = 4\hat{i} - 6\hat{j}$ act at the two points A(1, 3) and B(4, 9) respectively. Find the resultant of the two forces and its point intersection of act line with \vec{AB} .

Solution

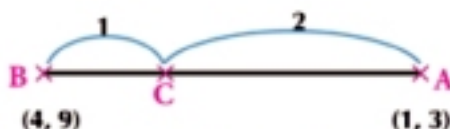
$$\vec{R} = \vec{F}_1 + \vec{F}_2 = 6\hat{i} - 9\hat{j}$$

we notice that $\vec{F}_2 = 2\vec{F}_1$ i.e the two forces are parallel and in the same direction.

Let the resultant act at point $C \in \vec{AB}$ where $\frac{AC}{CB} = \frac{2}{1}$

From the rule of the dividing a line segment internally point $C = (\frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}, \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2})$

$$\therefore C = (\frac{2 \times 4 + 1 \times 1}{2 + 1}, \frac{2 \times 9 + 1 \times 3}{2 + 1}) = (3, 7)$$



5 Try to solve

- 8 Two forces $\vec{F}_1 = 3\hat{i} - \hat{j}$ and $\vec{F}_2 = -9\hat{i} + 3\hat{j}$ act at the two points A(-1, 0) and B(1, 2) respectively. Find the resultant of the two forces and its point of intersection of line of action of \vec{AB} .


Exercises 3 - 1

Choose the correct answer :

- Two unlike forces of magnitudes 7 and 12 newtons, then their resultant is equal to:
 - 19 newton
 - 12 newton
 - 7 newton
 - 5 newton
- Two like forces of magnitudes 7 and 10 newtons act at the two points A and B where $AB = 51$ cm. If their resultant acts at point C then $AC =$
 - 30 cm
 - 27 cm
 - 21 cm
 - 12 cm
- Two like forces of magnitudes 5 and 7 newtons then their resultants is equal to
 - 12
 - 6
 - 2
 - 1

Answer the following questions:

In the exercises 4 - 6, two parallel forces \vec{F}_1 and \vec{F}_2 act at the two points A and B. If their resultant \vec{R} acts at point $C \in \overline{AB}$

- Find the magnitude and the direction of the resultant and the length of \overline{AC} in each of the following (the two forces are in the same direction)
 - $F_1 = 9$ newton, $F_2 = 17$ newton, $AB = 13$ cm
 - $F_1 = 23$ newton, $F_2 = 15$ newton, $AB = 57$ cm
 - $F_1 = 16$ newton, $F_2 = 10$ newton, $BC = 30$ cm
- If \vec{F}_1 and \vec{F}_2 are in the same direction, answer the following:
 - $F_1 = 8$ newton, $R = 13$ newton, $AC = 10$ cm find F_2 , AB
 - $F_2 = 6$ newton, $AC = 24$ cm, $AB = 56$ cm find F_1 , R
 - $F_1 = 6$ newton, $AC = 9$ cm, $CB = 8$ cm find F_2 , R
- If \vec{F}_1 and \vec{F}_2 are in opposite directions, answer the following:
 - $F_1 = 15$ newton, $R = 20$ newton, $AC = 70$ cm find F_2 , AB
 - $F_2 = 6$ newton, $AC = 24$ cm, $C \notin \overline{AB}$, $AB = 56$ cm find F_1 , R
 - $F_1 = 6$ newton, $AC = 9$ cm, $C \notin \overline{AB}$, $CB = 8$ cm find F_2 , R
- In each of the following, find the magnitude and the direction of the resultant and the distance of its point of action from point A

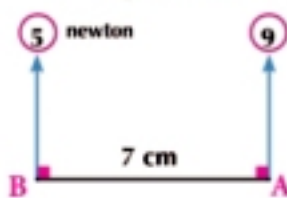


figure (16)



figure (17)

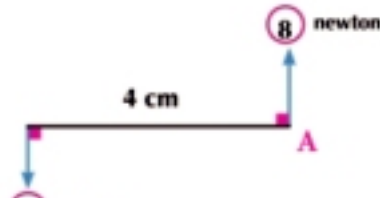
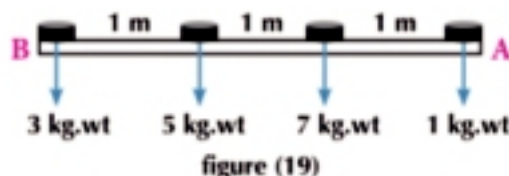


figure (18)

- 8 Two unlike forces of magnitudes 4 and 9 newtons act at the two points A and B where $AB = 15$ cm. Find their resultants.
- 9 If the resultant of the two parallel forces $7\vec{C}$ and $5\vec{C}$ newtons act at point $2\frac{1}{3}$ meters distant from the line of action of the smaller force, then find the distance between the two lines of action of the two forces.
- 10 Two parallel forces, the smaller one is 30 newtons and acts at the end A of a light rod \overline{AB} and the greater acts at the end B. If the magnitude of their resultant is 10 newtons and is 90 cm distant from end B. How long is the rod?
- 11 A, B, C, D and E are points on one straight line such that $AB = 4$ cm, $BC = 6$ cm, $CD = 8$ cm and $DE = 10$ cm. Five forces of magnitudes 60, 30, 50, 80 and 40 kg.wt act at the points A, C, D, B and E respectively and in a perpendicular direction to \overline{AE} such that the first three forces are in the same direction and the other two forces are in opposite directions. Identify the resultant of the system.
- 12 In figure (19) four weights of magnitudes 1, 7, 5 and 3 kg.wt have been placed on a light rod as in the figure. Identify the point of suspending the rod such that the rod remains horizontal.



- 13 Two like forces of magnitudes 5 and 8 newtons act at the two points A and B where $AB = 39$ cm. If another force of a magnitude F in the same direction is added to the first force, then the resultant will move 8 units. Find F.
- 14 A, B and C are three points on one straight line where $AB = 1$ meter, $AC = 3$ meter $B \in \overline{AC}$. The forces of magnitudes 2 and $\frac{1}{2}$ newtons act vertically downwards at the two points A and B respectively and a force of a magnitude 4 newtons acts vertically upwards at point C. Find the magnitude and the direction of the resultant and the distance of its point of action from point A.

You will learn

→ The equilibrium of a rigid body under the action of a system of parallel coplanar forces.

Key terms

- Reaction
- Weight
- Parallel
- Support
- Beam
- Tension
- Pulley
- Rotate

Materials

- Scientific calculator
- Mechanics lab

Think and discuss

- Does the rigid body get equilibrated if two parallel forces equal in magnitude, in opposite directions, and on the same line of action act on it?
- Does the rigid body get equilibrated if two parallel forces equal in magnitude, in opposite directions, but not on the same line of action act on it?
- Does the rigid body get equilibrated if a system of parallel coplanar forces such that the resultant of such forces vanishes acts on it?

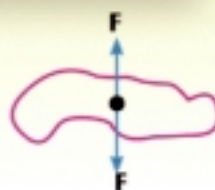


figure (20)



figure (21)

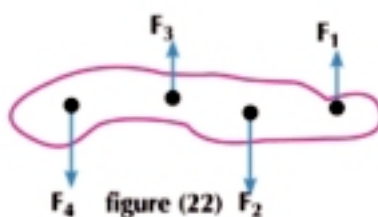


figure (22)



Activity

Equilibrium of a system of parallel coplanar forces

Aim of activity: This activity aims at verifying that if a rigid body is in an equilibrium under the action of a system of parallel coplanar forces whose number is more than three, then:

- 1 The sum of the algebraic measures of such forces is equal to zero.
- 2 The sum of the algebraic measures of the moments of such forces about any point in its plane is equal to zero.

Apparatus: A light graduated ruler - two capstan holders - two spring balances - weights - light strings.

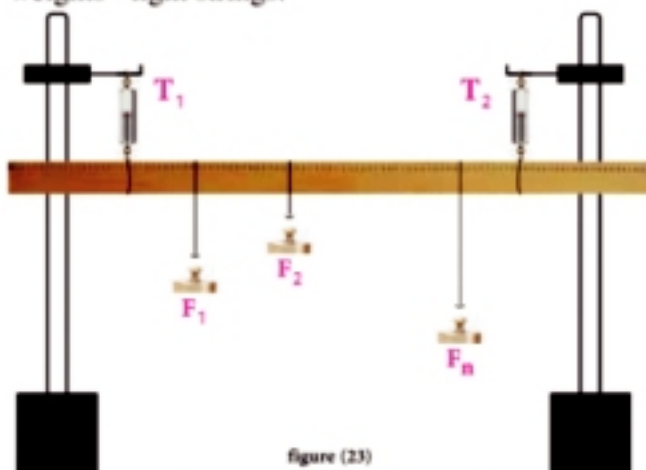


figure (23)

Doing the activity :

- 1 Fix the two spring balances each in a capstan holder, then suspend the ruler by two strings and adjust the apparatus such that the two balances and the two strings get vertical as in figure (23).
- 2 Suspend a number of proper weights in the ruler using the strings and adjust the positions and magnitudes of such weights until the ruler gets equilibrated horizontally. Let the magnitudes of the weight be F_1, F_2, \dots, F_n (these forces are directed vertically downwards).
- 3 Record the readings for each of the two balances to identify the two forces of tension. Let the two magnitudes of the two tensions be T_1 and T_2 (those two forces are directed vertically downwards). We find that :

$$T_1 + T_2 = F_1 + F_2 + \dots + F_n$$
- 4 Identify the distances of the weights from a point of the ruler points (let it be the midpoint of the ruler).
- 5 Identify the algebraic sum of the moments of all the forces: Let the magnitudes of the weights be $F_1, F_2, \dots, F_n, T_1$ and T_2 acting on the ruler about the chosen point to find that it is equal to zero.
- 6 Do the activity over several times changing the weights and their suspension points in each case. You get the following result :
 - 1- The sum of the algebraic measures of these forces = zero.
 - 2- The sum of the algebraic measures of the moments of these forces about any point in its plane = zero. In regard to the previous activity, we can formulate the next rule:

Rule

If a rigid body is in equilibrium under the action of system of parallel coplanar forces, then :

- 1- The sum of the algebraic measures of these forces (with respect to a parallel unit vector) is equal to zero (the resultant = zero).
- 2- The sum of the algebraic measures of the moments of these forces about any point in its plane is equal to zero.

**Example**

Equilibrium of a rigid body under the action of a system of parallel coplanar forces

- 1 The opposite figure shows a wood board of mass 30 kg for each meter of its length. If it rests horizontally on two supports A and B and carries a box of mass 240 kg, find the pressure exerted on each support.

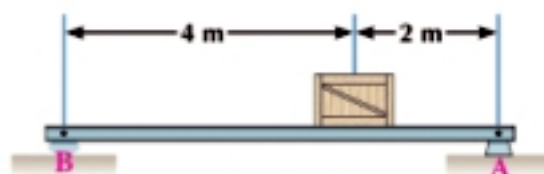


Figure (24)

Solution

Since the board is uniform, then its weight acts at its midpoint

Mass of the board = $30 \times 6 = 180$ kg

\therefore the weight of the board = 180 kg.wt

the reaction at each support is equal to the pressure exerted on it

The sum of the algebraic measures of the forces in the perpendicular direction to the board = 0

$$\therefore R_A + R_B = 240 + 180 \quad R_A + R_B = 420 \quad (1)$$

the sum of the algebraic measures of the moments of the forces about point B = zero

$$-180 \times 3 - 240 \times 4 + R_A \times 6 = \text{zero} \quad \text{i.e. } R_A = 250 \text{ kg.wt}$$

\therefore By substituting in (1) then $R_B = 170$ kg.wt

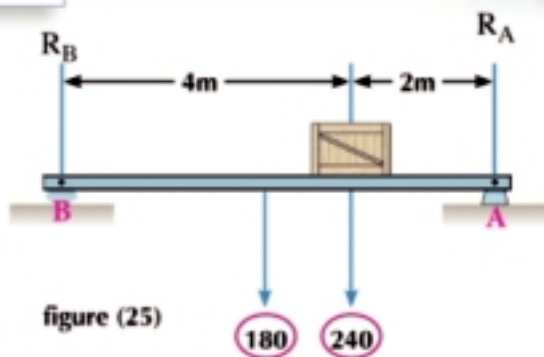


figure (25)

Critical thinking: What would happen to the reaction at each of A and B as the box gets closer from point A?

Try to solve

- Two men A and B carry a wood board of length 2 meters and weight 16 kg.wt acts at its midpoint and the board carries a box of weight 24 kg.wt as shown in figure (26). Find the pressure exerted on the shoulder of each man, then identify which point of the board the shoulder of man B should be to equilibrate the two pressures.

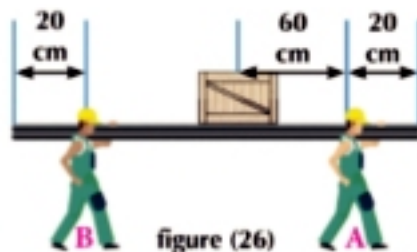


figure (26)

Example Equilibrium of a system of parallel coplanar forces

- AB is a uniform rod of length 90 cm and weight 60 newtons suspended horizontally by two vertical strings at its two ends A and B. Where should a weight of a magnitude 150 newton be suspended in order that the tension magnitude at A is twice its magnitude at B.

Solution

Let the weight of 150 newton suspended from a point distant x cm of A and the tension at B = T, and tension at A = 2T

\therefore the sum of algebraic measures of forces = zero

$$\therefore 2T + T - 150 - 60 = \text{zero, then } T = 70 \text{ newton}$$

\therefore The sum of algebraic measures of the moments of forces about A is equal to zero $\therefore 150 \times x + 60 \times 45 - T \times 90 = \text{zero}$

By substituting $T = 70$

$$\therefore 150x = 3600 \quad x = 24 \text{ cm}$$

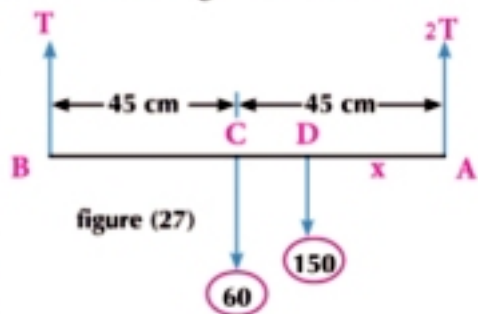


figure (27)

Try to solve

- AB A uniform wooden board of mass 10 kg and length 4 meters. If it rests horizontally on two supports; one of them at A and the other at a point distant 1 meter from B, then show at

which distance a 50 kg.wt child can stand on the board in order for the two reactions on the two supports get equal.

Example

- 3 \overline{AB} is a nonuniform wooden board of length 4 meters resting horizontally on two supports at C and D such that $AC = 1$ meter and $BD = 1\frac{1}{2}$ meter. If the maximum distance a 780 newton man can move on the board from A to B without getting the board imbalanced is 3 meters and the maximum distance the same man can move from B to A is $3\frac{1}{2}$ meters. Find the weight of the board and its point of action.

Solution

Let the weight of the board be equal to (w) newton and act at a point distant x meter from the end A.

First case:

When the man travels the maximum distance 3 meters from A to B the board is about to rotate about D.

i.e the reaction of the support at C vanishes.

\therefore The sum of moments of forces about D = zero

$$780 \times \frac{1}{2} - w(2\frac{1}{2} - x) = \text{zero}$$

$$\therefore w(2\frac{1}{2} - x) = 390 \quad (1)$$

Second case:

When the man travels the maximum distance $3\frac{1}{2}$ meter from B to A the board is about to rotate about C. i.e the reaction of the support at D = zero

\therefore The sum of moments of forces about C = zero

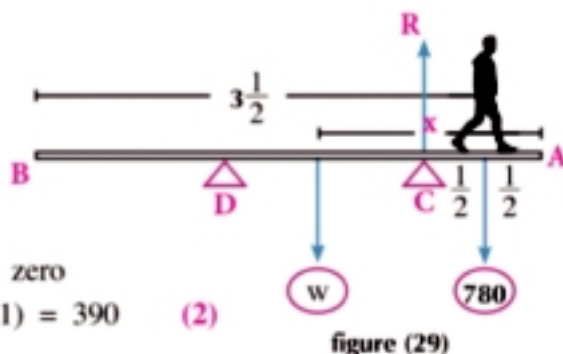
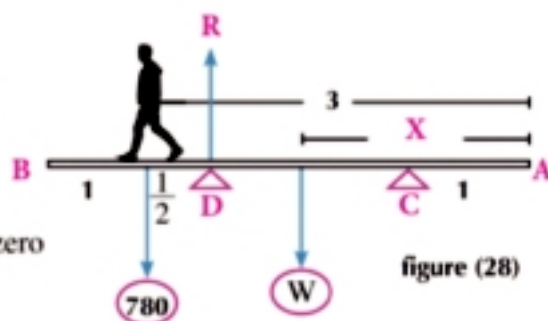
$$\therefore w(x - 1) - 780 \times \frac{1}{2} = 0 \quad \therefore w(x - 1) = 390 \quad (2)$$

from (1), (2)

$$\therefore x - 1 = 2\frac{1}{2} - x \text{ then } x = 1.75 \text{ meters}$$

By substituting in (2), we find that $w = 520$ newtons

i.e the weight of the board is equal to 520 newtons and acts at a point distant 1.75 meters from the end A.



Try to solve

- 3 \overline{AB} is a rod of length 90 cm and weight 50 newtons acting at its midpoint rests horizontally on two supports. One of them at end A and the other is distant 30 cm from B carries a 20-newton weight at a point distant 15 cm from B. Find the value of pressure exerted on each support and find the magnitude of the weight which should be suspended from end B such that the rod is about to rotate. What is the value of the pressure exerted on C hence?



Exercises 3 - 2



In each of the following figures, a light rod is equilibrium horizontally. Find the magnitude (norm) each of the forces F and K

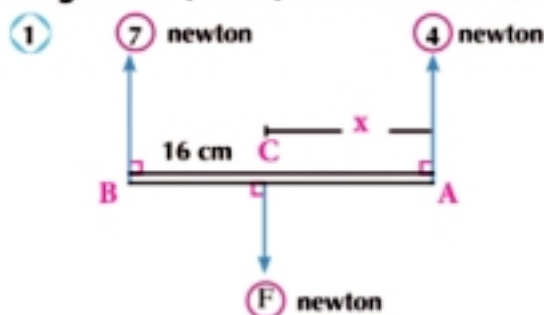


figure (34)

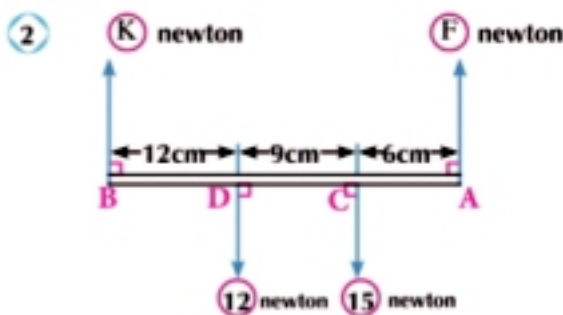


figure (35)

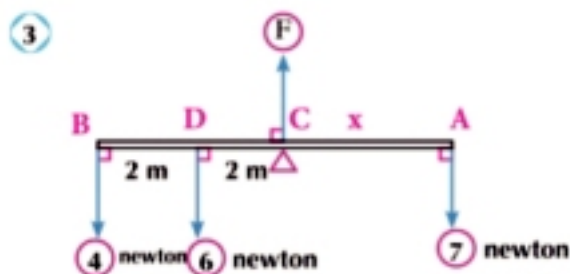


figure (36)

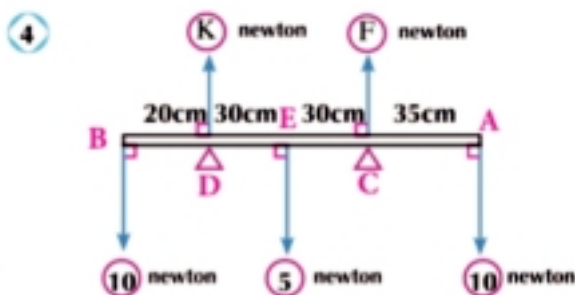


figure (37)

Answer the following :

- 5 A uniform rod of length 2 meters and mass 75 kg rests horizontally on two supports at its ends. A weight of magnitude 15 kg.wt is suspended from a point on the rod distant 50 cm from one end. Find the reaction at each support.
- 6 A uniform rod of length 3 meters and mass 4 kg carries two bodies of masses 5 kg and 1.5 kg at its two ends. Find the position of the suspension point on the rod in order for the rod to equilibrate horizontally.
- 7 AB is a nonuniform rod of length 120 cm. If a weight of a magnitude 1 newton is fixed at its end B and a weight of a magnitude 16 newtons is suspended from end A. If the rod gets equilibrated in this case at a point distant 30 cm from A and if the weight existed at A is decreased to get 8 newtons, then the rod gets equilibrated at a point distant 40 cm from A. Find the weight of the rod and the distance from its point of action to A.

- 8 Figure (38) shows a motorcycle of mass 200 kg and its weight acts at the vertical line passing through the midpoint between the two wheels, and the mass of the motorcyclist is 84 kg and its weight acts the vertical line distant 1 meter behind the front wheel. Find the reaction of the ground on each of the two wheels for each of the following cases :



figure (38)

- a The motorcycle is without a motorcyclist.
 b The motorcycle and the motorcyclist.
- 9 \overline{AB} is a rod of length 60 cm and weight 400 gm.wt acts at its midpoint on a wedge distant 20 cm from A to keep the rod horizontally in an equilibrium state by a light vertical string attached by its end B. Find:
- a The magnitude of the string tension and the reaction of the wedge.
 b The magnitude of the weight required to be suspended from A to make the tension in the rod about to vanish.
- 10 AB is a uniform rod of length 60cm and weight 10 gm.wt acting at its midpoint. It is suspended horizontally by vertical strings. One of them is attached at point A and the other is attached at point C where $AC = x$ cm. A weight of a magnitude 12 gm.wt is suspended at point D where $AD = 25$ cm. If the maximum tension each string can stand is 15 gm.wt, find the values in which x lies and find the maximum and minimum values of the tension in each of the two strings.
- 11 Based ruler light \overline{AB} measured by cm rests horizontally on two supports at C, D where $C \in \overline{AD}$, $2AC = 2BD = CD$, suspended weight of a magnitude (w) newton from the point (m) on the ruler on the verge of a coup if the suspended from A weight of magnitude 10 newton Or if suspended from B weight of magnitude 6 newton . Find the magnitude (w) and prove that $\frac{AM}{MB} = \frac{9}{7}$.
- 12 Two men A and B carry a body of mass 90 kg suspended from a strong light iron rod. If the distance between the two men is 60 cm and the suspension point of the body is distant 20 cm from A. What is the magnitude that each man can carry of this weight? If man B cannot carry more than 50 kg.wt, identify the maximum distance from A the weight can be suspended at until man B can keep carrying the rod.
- 13 The equilibrium parallel coplanar forces \vec{F}_1 , \vec{F}_2 , \vec{F}_3 and \vec{F}_4 act at point A = (2, -1), B (-4, -3), C = (3, 5), D = (-1, 0) respectively, if $\vec{F}_1 = 3\hat{i} + 4\hat{j}$, $\|\vec{F}_2\| = 20$ newtons in the same direction of \vec{F}_1 . Find each of \vec{F}_3 and \vec{F}_4 if they act at the opposite direction of \vec{F}_1 .

UNIT SUMMARY

1 The resultant of two parallel forces having the same direction (like forces):

$\vec{F}_1 = F_1 \vec{C}$, $\vec{F}_2 = F_2 \vec{C}$ act at A and B then

the resultant $\vec{R} = \vec{F}_1 + \vec{F}_2$ and acts at point $C \in \overline{AB}$ such that : $\frac{AC}{CB} = \frac{F_2}{F_1}$

2 The resultant of two parallel forces having opposite directions (unlike forces):

$\vec{F}_1 = F_1 \vec{C}$, $\vec{F}_2 = -F_2 \vec{C}$ ($F_1 > F_2$) act at A and B then :

$\vec{R} = \vec{F}_1 + \vec{F}_2 = (F_1 - F_2) \vec{C}$ and acts at point $C \in \overline{BA}$ such that : $\frac{AC}{CB} = \frac{F_2}{F_1}$

3 The moments of the parallel coplanar forces :

Theorem (the sum on the moments of any finite number of parallel coplanar force about any point in its lane is equal to the moment of the resultant of these forces about the same point).

4 The resultant of a system of parallel forces :

If the forces \vec{F}_1 , \vec{F}_2 , ..., \vec{F}_n are parallel and acting at point A_1 , A_2 , ... A_n then

their resultant is \vec{R} where $\vec{R} = \vec{F}_1 + \vec{F}_2 + \dots + \vec{F}_n$ and acts at point C where

$$A_1C = \frac{\| \text{sum of moments of forces about } A_1 \|}{\| \vec{R} \|}$$

5 The equilibrium of a system of the parallel coplanar forces:

Rule

If a rigid body is in equilibrium under the action of system of parallel coplanar forces, then :

- 1- The sum of the algebraic measures of these forces (with respect to a parallel unit vector) is equal to zero (the resultant = zero).
- 2- The sum of the algebraic measures of the moments of these forces about any point in its plane is equal to zero.

For more activities and exercises, visit www.sec3mathematics.com.eg

GENERAL EXERCISES

Complete :

- 1 Two unlike forces of magnitudes 10 and 15 newtons act at A and B respectively where $AB = 35$ cm then the resultant acts at point C where $AC = \dots\dots\dots$
- 2 The sum of the moments of a system of parallel coplanar forces about a point is equal to
- 3 Two like forces of magnitudes F and $2F$ act at the two points A and B respectively where $AB=39$ cm, then the resultant acts at point C where $AC = \dots\dots\dots$

Answer the following :

- 4 Two parallel forces, the magnitudes of their resultant is 250 newtons and the magnitude of one of those two forces is 150 newton act on a distance 40 cm from the resultant. Find the magnitude of the second force and the distance between the two lines of action of the two forces if the known force and the resultant act:

First: in the same direction.

Second: in opposite directions.

- 5 A, B, C and D are four points on one straight line such that $AB = 32$ cm, $BC = 40$ cm and $CD = 8$ cm. Two parallel forces of magnitudes 8 and 10 newton act at A and C respectively and two forces of magnitude 7 and 3 newton act at B and D in opposite direction to the forces at A and C. Find the resultant of this system and the distance of its point of action from A.
- 6 The weights 2, 3, 4 and 5 w kg have been placed on a light rod such that they are distant 2, 3, 4 and 5 cm from one of its end respectively. Find the distance of the suspension point of the rod from this end such that the rod equilibrates horizontally.
- 7 AB is a rod of length 100 cm and weight 10 newtons acting at its midpoint rests horizontally on two supports one of them at A and the other at a point distant 25 cm from B. Find the weight that should be suspended in end B of the rod so that the value of the reaction of the support near from end B is equal to 6 times the value of the reaction of the support at A. Find the reaction of each support in this case.
- 8 AB is a nonuniform rod of length 80 cm and weight 20 kg.wt rests vertically on two supports at C and D such that $AC = BD = 10$ cm. A weight of magnitude 40 kg.wt is suspended from A and the rod becomes to rotate about C. Find the distance from the point of action of the rod weight to A, then find the maximum weight that can be suspended from B and lifting the suspended weight from A without getting imbalanced .
- 9 ABCD is a nonuniform rod that rests horizontally on two smooth supports at B and C such that $AB = 6$ cm and $CD = 7$ cm and the point of action of the rod weight divides it in ratio 2:3 from its end A. It's found that if a weight of a magnitude 120 gm.wt is suspended from the end A or a weight of 180 gm.wt is suspended from the end D, the rod is about to rotate. Find the weight of the rod and the distance between the two supports.
- 10 AB is a uniform rod of length 120 and weight 60 newtons acting at its midpoint rests horizontally on a support at its end B and it's kept in equilibrium by a vertical string fixed at a point (on the rod of 40 cm from A) and carries a weight of a magnitude 20 newtons at a point distant 20 cm from A. Identify the value of the tension in the string and the pressure exerted on the support. What is the magnitude of the weight that should be suspended in end A so that the rod is about to get separated from the support? What is the value of tension in the string then?

General Equilibrium

Unit 4



Unit Introduction

The equilibrium is a branch of mechanics since it intersects in studying the conditions which a set of forces should satisfy such that if these forces act at a point or a rigid body, the point or the body keep in static. Long ago, people were interested in the topic of equilibrium before other branches of mechanics. The beginning of the applications of such a science was many years B.C when ancient Babylonians and Egyptians had used the principles of equilibrium and simple machine rules to lift weights extremely high to construct their temples and pyramids. According to the information available, it seems that the beginning of the topic of the equilibrium had been before Archimedes in the fourth century B.C.

In the seventeenth century, this science was stated of its current content as the theoretical formulation of mechanics fully completed at the age of Newton.

The mechanics-equilibrium researches had the greatest interest within the Islamic Arab civilization where some books were translated from Greek language to Arabic language. One of these books was physics. After studying the translated books well, new amendments were used, others were enlarged and fundamental additions were done to contribute in improving this science. Ibn al-Haytham, Abu Sahl al quhi, Al-Byroni, Ibn Sina, Al-khayam and others are to be honored in this field. They had written books about the centers of weights, pullies and about the equilibrium of fluids.

In this unit, you are going to identify the conditions of the equilibrium of a system of forces and solve some life applications.

Unit objectives

By the end of this unit and by doing all the activities involved, the student should be able to:

- Determine the general conditions of the equilibrium in a plane.
- Determine the general conditions of the equilibrium of a body under the action of a set of coplanar forces.
- Solve various applications on the equilibrium of a ladder or a rod on rough horizontal ground and smooth vertical wall.
- Solve life applications on the equilibrium of a ladder or a rod on rough horizontal ground and a rough vertical wall.

Key terms

- Friction
- General Equilibrium
- Vertical Reaction
- Resultant
- Algebraic Component
- Horizontal Component
- Vertical Component
- Equilibrium of Original Body
- Triangle of force

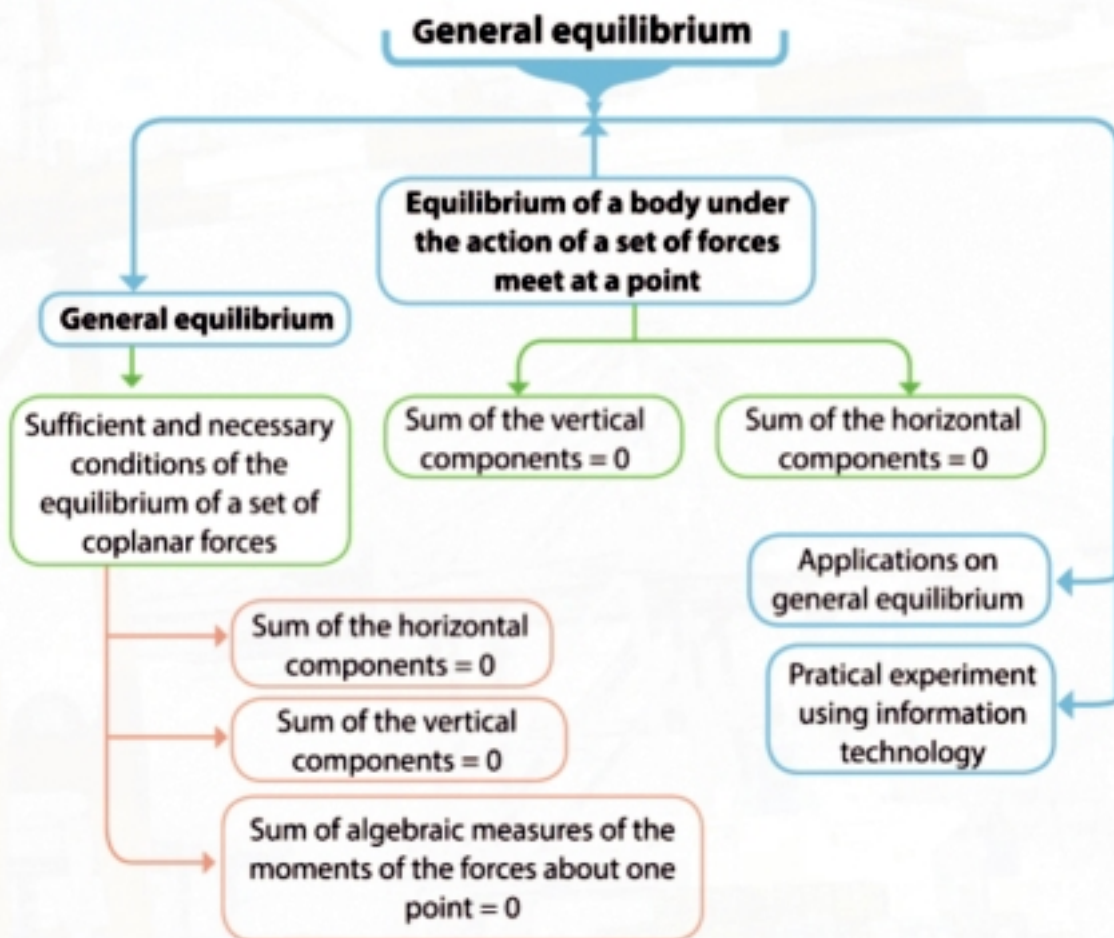
Unit Lessons

Lesson (4 -1): General equilibrium

Materials

Scientific calculator

Unit planning guide



Equilibrium of rigid body

You will learn

- ↻ The vanishing of the moments of a set of forces about any point.
- ↻ The sufficient and necessary conditions of the equilibrium of a set of coplanar forces .

Key terms

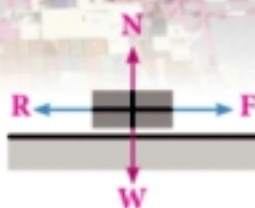
- ↻ General Equilibrium
- ↻ Vertical reaction
- ↻ Horizontal component
- ↻ Vertical component
- ↻ Equilibrium of original body
- ↻ Triangle of force

Materials

- ↻ Scientific Calculator

You have previously learned that if two meeting forces or more act at a point in a rigid body and its position does not alter, it is said that the body is in equilibrium. You have also previously learned the equilibrium of a rigid body under the action of two forces, then under the action of three coplanar forces. You have also identified the rule of the triangle of forces and lami's rule

, then you have learned the equilibrium of a rigid body under the action of a system of parallel forces meeting at a point. The opposite figure shows an example for the equilibrium in two dimensions in order to get the body static under the action of the four forces shown in the figure. The sum of each of the horizontal components and vertical components ought to be equal to zero. By applying the condition of the equilibrium stating that the sum of each of the horizontal components and vertical component= zero. $N - W = 0$ and $F - R = 0$ by considering the positive directions are the right and upwards.



Think and discuss

Can you find the magnitude of the tension forces acting on this body in the opposite figure?

The weight of the body acts in the vertical direction downwards and its magnitude is 400 newtons. According to the definition of the tension, the two directions of the other two forces should be along the two ropes such that they are come out from the body. Let the tension in the two ropes be T_1 and T_2 . We draw the graphical diagram as in figure (2). How can you write down the components of T_1 and T_2 ? We factorize each of T_1 and T_2 into two components in two orthogonal directions, then we write the equation of the equilibrium.

$$T_1 \cos 55^\circ = T_2 \cos 35^\circ$$

$$\therefore 0.57T_1 = 0.82T_2 \dots\dots\dots (1) ,$$

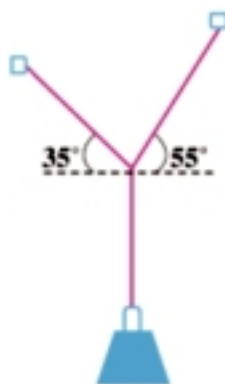
$$T_1 \sin 55^\circ + T_2 \sin 35^\circ = 400$$

$$\therefore 0.82T_1 + 0.57T_2 = 400 \dots\dots (2)$$

By solving the two equations, we find that:

$$T_1 \simeq 328.81 \text{ newtons} , T_2 \simeq 228.52 \text{ newtons}$$

Can you check your results?



400 newton

Fig (1)

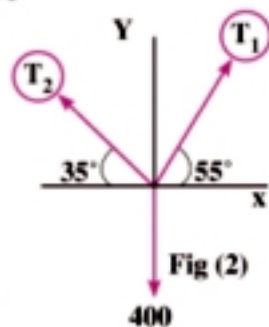


Fig (2)

The law of finding the resultant of the two forces F_1 and F_2 which include an angle α can be used as follows: $R = \sqrt{F_1^2 + F_2^2 + 2 F_1 F_2 \cos \alpha}$

So that: $R = \sqrt{(328.81)^2 + (228.52)^2} \approx 400$ newton and this matches with the weight of the body

what would you expect if the set of the previous forces are not meeting in one point?

In this case, the previous condition stating that the sum of each of the horizontal components and vertical components is equal to zero is not sufficient and it is possible that the body may move even if this condition is satisfied. This because there is another condition which should be satisfied until the body gets in a case of static equilibrium. In figure (3) we find that the ruler is in equilibrium under the action of its weight vertically downward (W) and the vertical reaction (R); where $R = W$. If the ruler is pushed by two forces equal in magnitude and in opposite two directions ($-F$ and F), the ruler is no longer static and starts to rotate although the first condition is satisfied. As a result, it is necessary to search for another condition related to the rotation in order to keep the body in a static state.

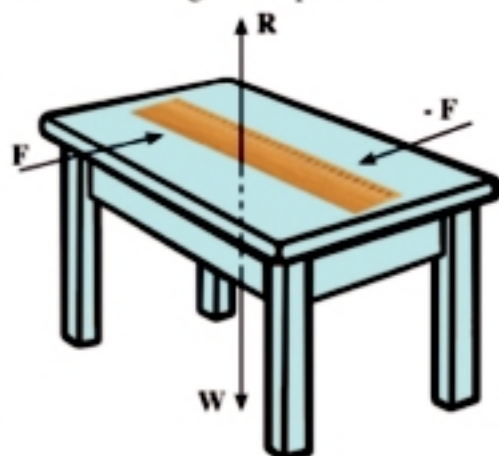


Fig (3)

Vanishing of the moments of a set of forces about any point

Definition:

The moments of rotation acting on a body in clockwise direction are in equilibrium with the moments of rotation in anticlockwise direction until the body gets in a static state.

Thus:

the body under the action of a set of coplanar forces is in a static equilibrium state if the following two conditions are satisfied:

(1) The resultant vector of the forces to the set ($\vec{R} = \vec{0}$) vanishes

(2) The moments of forces about a point ($\vec{M} = \vec{0}$) vanishes

These condition are sufficient and necessary for the equilibrium of a set of coplanar forces.

Figure (4): shows a set of the orthogonal unit vectors ($\hat{i}, \hat{j}, \hat{k}$) where \hat{i} and \hat{j} lie on the plane of the forces and in turn \hat{k} is perpendicular to this plane.

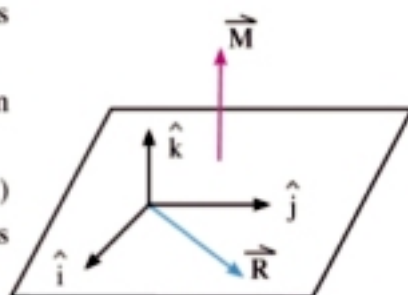


Fig (4)

Thus, the resultant vector \vec{R} can be factorized in the two directions of \hat{i} and \hat{j} , where as the moment vector \vec{M} is parallel to the unit vector \hat{k}

then: $\vec{R} = x \hat{i} + y \hat{j}$ and $\vec{M} = M \hat{k}$

where: x = the sum of the algebraic components of the forces of the system in the directions of \hat{i} ,

y = the sum of the algebraic component for the forces of the system in the direction of \hat{j} ,

M = the sum of the algebraic measures of the moments for the forces of the system in the direction of \hat{k} .

From that, we find that if $x = 0$, $y = 0$ and $M = 0$ then: $\vec{R} = \vec{0}$, $\vec{M} = \vec{0}$

Since we do not determine the two directions of \hat{i} and \hat{j} in the plane, we can deduce the following formulation:

The sufficient and necessary conditions for the equilibrium of a body under the action of a set of the coplanar forces

For a set of coplanar forces to be in equilibrium, it is necessary and sufficient that the following conditions are to be satisfied:

- (1) The sum of the algebraic components of the forces in two orthogonal direction lying in their plane vanishes.
- (2) The sum of the algebraic measures of the moments of the forces about one point in its plane vanishes.

These conditions can be expressed mathematically as follows:

$$x = \text{zero}, \quad y = \text{zero}, \quad M = \text{zero}$$

Example Equilibrium of a horizontal rod fixed in a hinge

- Figure (5) represents a uniform rod of length 120 cm and weight 4 newtons attached to a hinge against a vertical wall. A weight of 3 newtons has been suspended in the rod and its free end is attached by a rope to a point on the wall. If the rod is in a static equilibrium horizontally, find the magnitude of tension in the rope and the magnitude and direction of the reaction of the hinge.

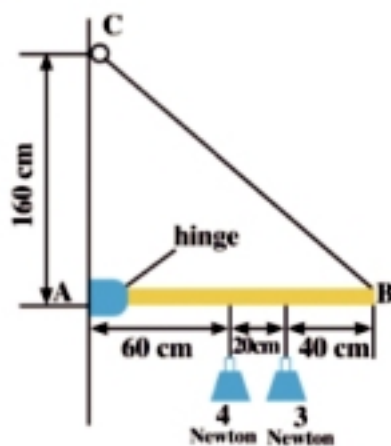


Fig (5)

Solution

Figure (6) represents the graphical diagram of the example. The rod is in equilibrium under the action of four forces which are:

The weight of the rod 4 newtons vertically downwards, the weight 3 newton vertically downward.

The force of tension T in the rope acts in the direction of \vec{BC} , and makes its line of action an angle of measure θ to the horizontal.

The reaction at the hinge which is represented by the two orthogonal components x_1 and y_1 as shown in the figure.

By writing down the conditions of the equilibrium:

$$x = 0, \quad y = 0, \quad M = 0$$

$$\therefore x_1 = T \cos \theta$$

$$\therefore x_1 = \frac{3}{5} T \dots\dots\dots (1)$$

$$y_1 + T \sin \theta = 3 + 4$$

$$\therefore y_1 + \frac{4}{5} T = 7 \dots\dots\dots (2)$$

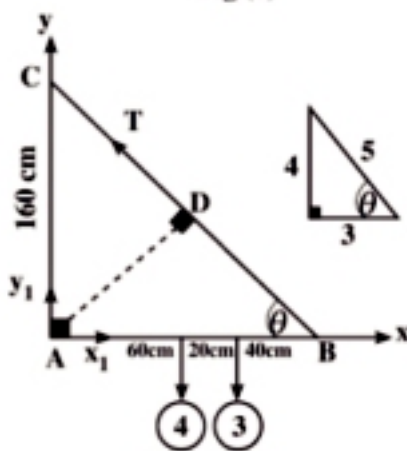


Fig (6)

$$\begin{aligned} \therefore M = 0 & \qquad \therefore T \times \frac{160 \times 120}{200} - 4 \times 60 - 3 \times 80 = 0 \\ \therefore 96 T = 480 & \qquad \therefore T = 5 \text{ newton} \end{aligned}$$

By substituting in (1) and (2)

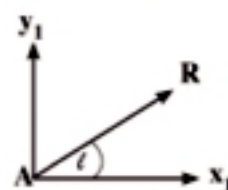
$$\begin{aligned} \therefore x_1 &= \frac{3}{5} \times 5 = 3 \text{ newton} , \\ y_1 + \frac{4}{5} \times 5 &= 7 \qquad \therefore y_1 = 3 \text{ newton} \end{aligned}$$

Thus, the magnitude and direction of the reaction of the hinge can be determined. let R be the magnitude of this force and L is the measure of angle of inclination of its line of action \overrightarrow{AX}

$$\therefore R = \sqrt{x_1^2 + y_1^2} = \sqrt{9 + 9} = 3\sqrt{2} \text{ newton}$$

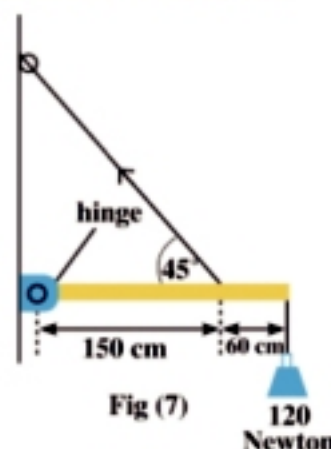
$$\tan L = \frac{y_1}{x_1} = \frac{3}{3} = 1 \quad \therefore L = 45^\circ$$

i.e. : the magnitude of the force of the reaction of the hinge = $3\sqrt{2}$ newtons and makes an angle of measure 45° with \overrightarrow{AX} .



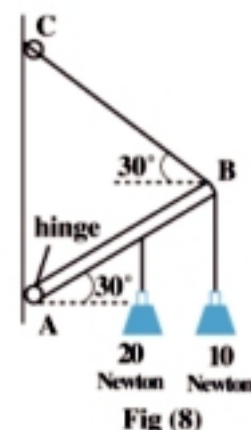
Try to solve

- 1 Figure: (7) represents a rod of a negligible weight and of length 210 cm is hinged at a vertical wall by a hinge. A weight of 120 newtons is suspended in the rod. If the rod is in horizontally static equilibrium, find the magnitude of tension in the rope and the magnitude and direction of the reaction of the hinge, where $\sin \theta = \frac{4}{5}$



Example Equilibrium of an inclined rod fixed at a hinge

- 2 Figure (8) represents a uniform rod AB of weight 20 newtons inclined at 30° to the horizontal. A weight of 10 newtons is suspended at its end B and by a rope BC inclined at 30° to the horizontal, If the rod is in static equilibrium, find the magnitude of tension in the rope and the magnitude and direction of the reaction of the hinge.



Solution

Figure (9) represents a graphical diagram to the example. (notice that the triangle ABC is equilateral).

The rod is in equilibrium under the action of four forces which are:

Its weight is 20 newtons and acts vertically downwards at the midpoint of the rod.

The weight is 10 newtons and acts vertically downwards at the end of the rod.

The tension T in the rope and acts in the direction of \overrightarrow{BC} .

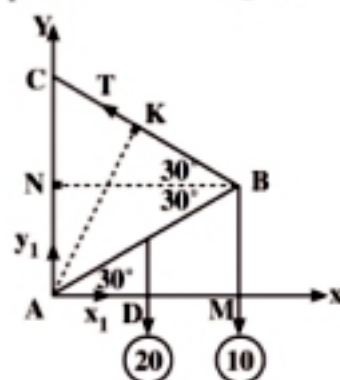
The reaction of the hinge whose two component x_1 and y_1 are orthogonal.

By applying the conditions of the equilibrium which are:

$$x = 0, y = 0, M_a = 0 \quad (\text{let the rod length} = 2\ell)$$

$$\therefore M = 0 \quad \therefore T \times AK - 20 \times AD - 10 \times AM = 0$$

$$\therefore T \times \sqrt{3} \ell = 20 \times \frac{\sqrt{3}}{2} \ell + 10 \times \sqrt{3} \ell \quad \therefore T = 20 \text{ newton}$$



$$x_1 = T \cos 30^\circ = 20 \times \frac{\sqrt{3}}{2} = 10\sqrt{3} \text{ newton}$$

$$y_1 + T \sin 30^\circ = 10 + 20 \quad \therefore y_1 = 30 - 20 \times \frac{1}{2} = 20 \text{ newton}$$

$$\therefore R = \sqrt{x_1^2 + y_1^2} \quad \therefore R = \sqrt{300 + 400} = 10\sqrt{7} \text{ newton}$$

$$\therefore \tan L = \frac{y_1}{x_1} = \frac{2}{\sqrt{3}} = 1.1547$$

$\therefore L = 49^\circ 6'$ (where L is the angle of inclination of the reaction of the hinge with the horizontal)

Try to solve

- 2 A uniform rod AB of length 60 cm and weight 8 newtons is hinged at its end A to a hinge fixed at a vertical wall. A weight of 6 newtons is suspended at a point in the rod distant 40 cm from the end A. The rod is being kept in a horizontal position by a light string attached at one of its two ends with the end B of the rod while the other end of the string is fixed at a point on the wall distant 80 cm vertically upwards from A. Find the tension in the string and the reaction of the hinge .

Example Equilibrium of a ladder on two orthogonal planes one of them is rough

- 3 A uniform ladder of weight 20 kg.wt rests in a horizontal ground with one of its ends and the other end on a smooth vertical wall and the ladder is being kept in a vertical plane and inclined at 60° to the horizontal. If it is known that the coefficient of friction between the ladder and the ground is equal to $\frac{1}{2\sqrt{3}}$. Prove that the maximum distance a girl of weight 60 kg.wt can ascend the ladder is equal to half the length of the ladder.



Solution

The ladder is in equilibrium under the action of the forces:

The weight of the ladder 20 kg.wt and acts vertically downwards at its midpoint.

The weight of the girl 60 kg.wt and acts vertically downwards at a distance x from the ladder base.

The reaction of the rough ground at end A whose two components the vertical R_1 and the horizontal μR_1 .

The reaction of the smooth wall R_2 and its orthogonal on the wall

By applying the conditions of the equilibrium which are :

$$x = 0, y = 0, M_A = 0 \quad \text{let the ladder length} = l,$$

and the maximum distance the girl can ascend = x then the rod is about to move

$$\therefore R_1 = 20 + 60 = 80 \text{ kg.wt ,}$$

$$R_2 = \frac{1}{2\sqrt{3}} R_1$$

$$\therefore R_2 = \frac{1}{2\sqrt{3}} \times 80 = \frac{40}{\sqrt{3}} \text{ kg.wt (1)}$$

$$\therefore M_A = 0$$

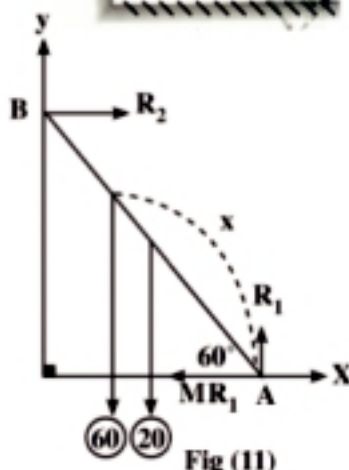


Fig (11)

$$\therefore 20 \times \frac{\ell}{2} \cos 60^\circ + 60 \times x \cos 60^\circ - R_2 \times \ell \sin 60^\circ = 0 \dots (2)$$

from (1), (2)

$$\therefore 5\ell + 30x - \frac{\sqrt{3}}{2}\ell \times \frac{40}{\sqrt{3}} = 0 \quad \therefore 30x - 15\ell = 0 \quad \therefore x = \frac{15}{30}\ell = \frac{1}{2}\ell$$

\therefore The maximum distance the girl can ascend is equal to half the length of the ladder.

P Try to solve

- 3 A uniform ladder AB of weight 30 kg.wt and length 4 meters rests with its end A on a smooth horizontal plane and its other end B against a smooth vertical wall. The ladder is being kept in a vertical plane and inclined at 45° by a horizontal rope joining the end A with a point of the horizontal plane lying vertically under B exactly. If a man of weight 80 kg.wt ascends the ladder, prove that the tension of the rope increases whenever the man ascends. If the rope does not stand tension more than 67 kg.wt, find the length of the maximum distance the man can ascend without cutting the rope.

Example Equilibrium of a rod on two rough orthogonal planes

- 4 A uniform rod rests with its upper end on a vertical wall, such that the coefficient of friction between the wall and rod is equal to $\frac{1}{2}$, and with its lower end on a horizontal plane such that the coefficient of friction between the horizontal plane and the rod is equal to $\frac{3}{4}$. Find the tangent of the angle of inclination of the rod to the horizontal when it is about to slide.

Solution

The rod is in equilibrium under the action of the forces:

The weight of the rod (w) and acts vertically downwards.

The reaction of end A on the horizontal plane and its two orthogonal components R_1 and μR_1 .

The reaction of end B on the vertical wall and its two orthogonal components R_2 and $\mu' R_2$.

Let the rod length be ℓ , and inclined at an angle of measure θ to the horizontal. By applying the conditions of equilibrium which are:

$$x = 0 \quad , \quad y = 0 \quad , \quad M_A = 0$$

$$\therefore R_2 = \mu' R_1 \therefore R_2 = \frac{3}{4} R_1 \quad , \quad R_1 + \mu' R_2 = W \quad \therefore R_1 + \frac{1}{2} R_2 = W$$

$$\therefore R_2 = \frac{3}{4} (W - \frac{1}{2} R_2) \quad \therefore R_2 = \frac{3}{4} W - \frac{3}{8} R_2$$

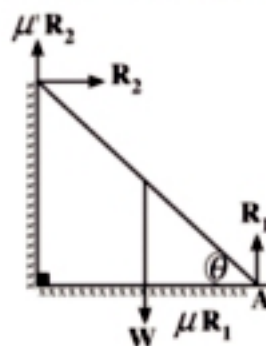
$$\therefore \frac{11}{8} R_2 = \frac{3}{4} W \quad \therefore R_2 = \frac{6}{11} W \quad \dots (1)$$

$$\therefore M_A = 0 \therefore W \times \frac{\ell}{2} \cos \theta - R_2 \times \ell \sin \theta - \frac{1}{2} R_2 \times \ell \cos \theta = 0 \text{ By dividing both sides by } \ell \cos \theta \text{ then multiplying } \times 2$$

$$\therefore W - 2R_2 \tan \theta - R_2 = 0 \quad \therefore R_2 (2 \tan \theta + 1) = W \quad \dots (2)$$

from (1), (2)

$$\therefore \frac{6}{11} W \times (2 \tan \theta + 1) = W \quad \therefore 2 \tan \theta + 1 = \frac{11}{6} \quad \therefore \tan \theta = \frac{5}{12}$$



Try to solve

- 4 A uniform rod AB of weight 40 newtons rests with its end A on a vertical wall such the coefficient of friction between the wall and the rod is equal to $\frac{1}{2}$ and with its end B on a horizontal ground such that the coefficient of friction between the wall and the rod is equal to $\frac{1}{3}$. If the minimum horizontal force making the end B of the rod about to move towards the wall is equal to 60 newtons. Find, in the position of the equilibrium, the measure of the angle of inclination of the rod to the horizontal known that the rod is being kept in a horizontal plane.

Example Equilibrium of a bar on a rough horizontal plane and a smooth wedge

- 5 A uniform bar AB of weight 5 kg.wt and length 30 cm rests with its end A on rough horizontal ground and at one of its point C on a smooth wedge which is 12.5 cm up on the ground surface. If the bar is about to slip when it inclines at 30° to the horizontal ground and lies in a vertical plane, find:

First : the reaction of the wedge.

Second : the coefficient of friction between the end A and ground.

Solution

We notice that $AC = 25$ cm

The bar is in equilibrium under the action of the forces:

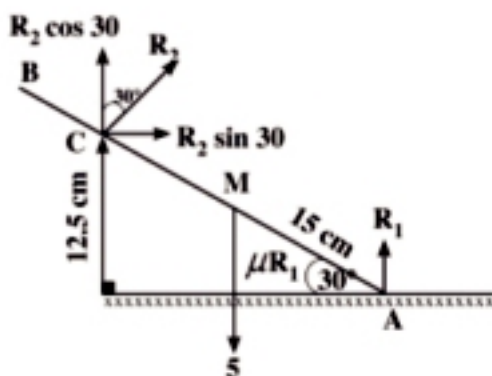
The weight of the bar 5 kg.wt and acts vertically downwards.

The reaction at end A of the ground and its two orthogonal components R_1 and μR_1 .

The reaction of the wedge at the bar R_2 , and it is perpendicular to the bar at the point of tangency C.

By applying the conditions of the equilibrium which are : $x = 0$, $y = 0$, $M_A = 0$

$$\therefore M_A = 0 \quad \therefore 5 \times 15 \cos 30^\circ - R_2 \times 25 = 0$$



$$\therefore R_2 = \frac{3\sqrt{3}}{2} \dots\dots\dots (1)$$

From the two equations of the equilibrium : $x = 0$, $y = 0$

$$\therefore R_2 \sin 30^\circ - \mu R_1 = 0$$

$$\therefore R_2 = 2 \mu R_1 \quad \text{By substituting from (1)}$$

$$\therefore 2 \mu R_1 = \frac{3\sqrt{3}}{2}$$

$$\therefore \mu R_1 = \frac{3\sqrt{3}}{4} \dots\dots\dots (2)$$

$$R_1 + R_2 \cos 30^\circ = 5$$

$$\therefore R_1 + \frac{\sqrt{3}}{2} R_2 = 5 \quad \text{By substituting from (1)}$$

$$\therefore R_1 + \frac{\sqrt{3}}{2} \times \frac{3\sqrt{3}}{2} = 5$$

$$\therefore R_1 = 5 - \frac{9}{4} = \frac{11}{4} \text{ kg.wt.}$$

By substituting the value of R_1 in equation (2) to find the value μ .

$$\therefore \mu \times \frac{11}{4} = \frac{3\sqrt{3}}{4}$$

$$\therefore \mu = \frac{3\sqrt{3}}{11}$$

P Try to solve

- 5 A uniform rod AB of weight 20 newtons and length 60 cm rests with its end A at a rough horizontal plane and at one of its points C at a smooth wedge which is 25 cm upon the horizontal plane. If the rod is about to slip as it is inclined at 30° to the horizontal, find the reaction of the wedge and the coefficient of friction between the rod and the plane known that the rod lies in a vertical plane.

**Exercises 4 - 1****First : Put (✓) or (x):**

- For a set of coplanar forces which do not meet at a point to be in equilibrium, it is necessary and sufficient that the vector sum of the forces vanishes.
- For a set of coplanar forces acting on a body to be in equilibrium, it is necessary and sufficient that the sum of the algebraic components of the forces in each two orthogonal directions lying on their plane vanishes.
- If the sum of the algebraic components of the forces of a set vanishes and its moment about one point in its plane vanishes, then this set is in equilibrium.
- A ladder is in equilibrium if it rests with one of its ends on a smooth horizontal ground and its other end on a rough vertical wall.

Second : Complete:

- The sufficient and necessary conditions for a set of forces to be in equilibrium are _____
- If a rod rests with one of its ends on a smooth wedge, then the reaction of the wedge at the rod is _____
- If a body of weight 6 newtons is rested upon a rough horizontal place and the coefficient of friction between the plane and the body is $\frac{1}{3}$, then the magnitude of the horizontal force making the body about to move is equal to _____

Third : Answer the following questions

- A uniform rod AB of weight 4 newtons and length 120 cm, one of its end is attached at a hinge fixed at its end B and the hinge is fixed at a vertical wall. A weight of 6 newtons is suspended at a point on the rod distant 20 cm from A. Then the rod is kept in a horizontal position by a fine rope BC whose end C is fixed at a point on the wall lying vertically above A exactly and distant 90 cm from A. Find the tension in the rope and the direction of the reaction at the hinge.
- A uniform bar of weight 4 kg.wt is hinged at its end A to a vertical wall and carries a weight of 2 kg.wt at B. The bar is kept in a position inclined at 30° to the horizontal upwards by a rope equal to it in length and one of its ends is attached to the end B of the bar, and its other end to point C of the wall lying vertically above A and at a distance equal to the bar length. Find the tension in the rope and the reaction of the hinge.
- A uniform rod of weight (W) rests on a vertical wall with its upper end; the coefficient of friction between the wall and rod is equal to $\frac{1}{2}$ and with its lower end on a horizontal plane; the coefficient of friction between the plane and rod is equal to $\frac{3}{4}$. Find the tangent of the angle which the rod makes with the horizontal when it is about to slip.

UNIT SUMMARY

If a rigid body is in equilibrium under the action of two forces, then the point of action of any of the two forces can be traveled into another point on its line of action without acting at the equilibrium of the body.

If three coplanar forces meeting at a point, are in equilibrium and a triangle whose sides are parallel to the lines of action of these force is drawn, the triangle sides lengths are proportional to the magnitudes of the corresponding forces.

If a body under the action, of three coplanar forces but not parallel is in equilibrium, then the lines of action of these forces meet at one point.

The conditions of the equilibrium of a body under the action of a system of coplanar forces meeting at a point:

- The algebraic sum of the components of the forces in the direction of $\overrightarrow{OX} = \text{zero}$
- The algebraic sum of the components of the forces in the direction of $\overrightarrow{OY} = \text{zero}$

The vanishing of the moment of a set of forces about any point : the moments of rotation acting on a body in the clockwise direction are in equilibrium to the moments of rotation in the anticlockwise direction so that the body is in an equilibrium state.

The sufficient and necessary condition for a set of coplanar forces to be in equilibrium: for a set of coplanar forces to be in equilibrium, it is necessary and sufficient to satisfy the next conditions:

- The sum of the algebraic components of the forces in the two orthogonal direction lying in their plane vanishes.
- The sum of the algebraic measures of the moment of these forces about one point in their plane vanishes.
- These conditions can be expressed mathematically as follows: $x = 0$, $y = 0$, $M = 0$

For more activities and exercises, visit www.sec3mathematics.com.eg

GENERAL EXERCISES

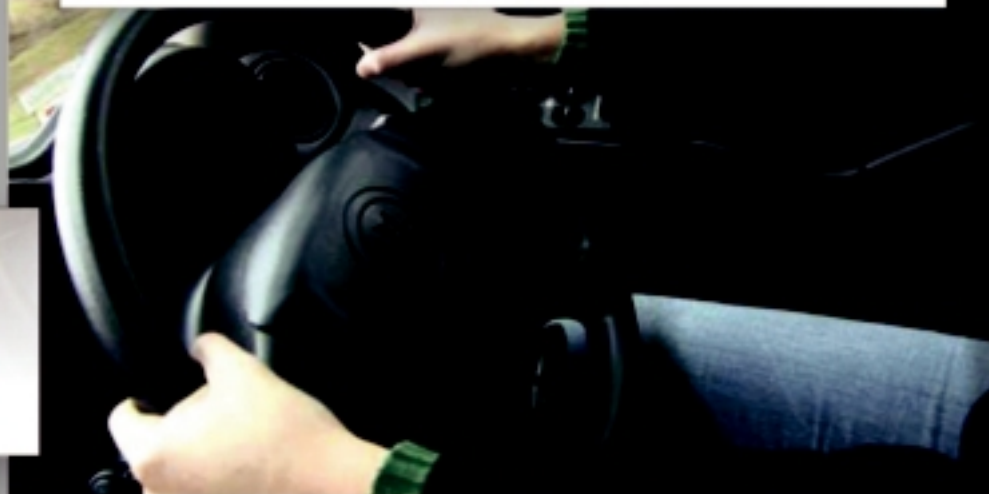
- 1 A non uniform rod AB of length 140 cm rests on horizontal ground with its end B and on a vertical wall with its end A. If the two coefficient of friction between the rod and both of the ground and the wall are equal to $\frac{1}{2}$ and $\frac{1}{3}$ respectively and if the rod is about to slip as the measure of its angle of inclination at 45° to the horizontal, find the distance between the weight center of the rod and the end B.
- 2 A uniform rod AB of length 260 cm and weight 43 newtons rests in a vertical wall with its end A and on horizontal ground with its end B on a rough horizontal ground. If the two coefficients of friction between the rod and both the wall and ground are equal to $\frac{1}{4}$ and $\frac{1}{2}$ respectively. If the end B is at distant 100 cm from the wall, find the horizontal force acting at the end B to make the rod be about to move towards the wall.
- 3 A uniform ladder of weight 40 kg.wt rests on a smooth vertical wall with an end and on rough horizontal ground with the other end such that it lies in a vertical plane perpendicular to the wall and inclinds at 45° to the horizontal. A young man of weight equal to the ladder weight ascends and the ladder is about to slip when the young man ascends $\frac{3}{4}$ of the ladder length. Find the coefficient of friction between the ground and ladder. If the young man wants to ascend the ladder to the top, find the minimum horizontal force acts on the lower end of the ladder so that the young man can ascend the ladder to the top.
- 4 A uniform rod AB of weight 15 kg.wt rests on a rough horizontal ground with its end A and on smooth vertical wall with end B such that the rod lies in a vertical plane perpendicular to the wall and inclines at 45° , to the horizontal. A horizontal force F acts at point C of the rod such that AC is equal to $\frac{1}{4}$ of the rod length, then the end A becomes about to move towards wall. If the coefficient of friction between the rod and the ground is equal to $\frac{1}{2}$, find the force F and the reaction of the wall.
- 5 A uniform ladder rests on a rough vertical wall with an end and on a rough horizontal ground with the other end and the coefficient of friction between the ladder and both the wall and ground is equal to $\frac{1}{2}$. If the ladder is in equilibrium in a vertical plane perpendicular to the wall and inclines at an angle of tangent $\frac{4}{3}$. Prove that a man of weight equal twice the ladder weight cannot ascend more than $\frac{1}{2}$ of the ladder length without getting imbalanced.
- 6 A uniform rod of weight (W) rests on a rough vertical wall with an end and on rough horizontal ground with the other end and the coefficient of friction between the rod and wall is equal to $\frac{1}{4}$ and the coefficient of friction between the rod and ground is equal to $\frac{1}{3}$. If the rod is in equilibrium at a vertical plane perpendicular to the wall, find the tangent of the angle of inclination of the rod to the vertical as the rod is about to slip.
- 7 A uniform ladder is in equilibrium in a vertical plane on a vertical wall and horizontal ground. If the measure of the angle of friction between the ladder and both the wall and ground is L, prove that the measure of the angle of inclination of the ladder to the vertical as the ladder is about to slid is $\theta = 2L$.
- 8 A uniform rod AB of weight rests in 10 kg.wt rests in rough horizontal ground with end A and on a smooth vertical wall with end B such that the rod is at a vertical plane perpendicular to the wall and inclines at 45° to the horizontal ground if the coefficient of friction between the rod and the ground is equal to $\frac{3}{4}$, find the minimum horizontal force acts at end A of the rod to make it be about to slide away from the wall and the reaction of the wall.

Accumulative test

- 1 Three coplanar forces of magnitudes 4, 5 and 6 act at a materialistic point. What is the measure of the angle between the last two forces if this set of forces is in equilibrium.
- 2 If a pendulum sphere of weight 600 kg.wt is translated until the string makes an angle of measure 30° with the vertical under the action of a force on the sphere in a perpendicular direction to the string, find the force and tension in the string.
- 3 A weight of 26 newtons is suspended in two strings of lengths 25 cm and 60 cm. The other two ends of the two strings are fixed at two points of a horizontal line. If the distance between the other two ends is 65 cm, find the tension in each of the two strings.
- 4 A body of weight (w) is suspended by two strings inclining at θ° and 30° to the vertical to get the body equilibrated as the tension is 12 newtons in the first string and 9 newtons in the second string . Find the value of the weight (w) and the measure of angle θ .
- 5 A uniform solid sphere of weight 30 gm.wt rests on two planes with its surface. If the sphere is in equilibrium between two smooth planes one of them is vertical and the other inclinds at 60° to the vertical , find the magnitude of two forces of the pressure exerted on each of the two planes.
- 6 A uniform rod of length 100 cm and weight 20 newton (acting at its midpoint) is suspended at its two ends by two light strings whose two ends are fixed at a point in a room ciling. If the two strings are perpendicular and the length of one of them is 60 cm, find the tension in each of the two strings when the rod is freely suspended and in equilibrium .
- 7 A uniform rod AB (its weight act at its midpoint)attached in a vertical wall with its end A by a hinge and the rod is attracted from the point B by a horizontal force F kg.wt until the rod is in equilibrium in a position it makes an angle of measure 30° to the vertical. Find F and the reaction at the hinge.
- 8 A uniform rod in a vertical plane rests on a smooth vertical wall with its upper end and on a horizontal plane with its lower end the coefficient of friction between the horizontal plane and the rod is equal to $\frac{1}{4}$. Find the tangent of the angle which the rod makes to the horizontal as the rod is about to slip.
- 9 A uniform ladder of weight 14 kg.wt rests on rough horizontal ground with end A and on a rough vertical wall with end B. The coefficient of friction between the ladder and the ground is $\frac{3}{7}$ and the coefficient of friction between the ladder and the wall is $\frac{1}{3}$. If the ladder is in equilibrium at a vertical plane perpendicular to the wall when it inclinds at 45° to the horizontal, find the minimum horizontal force acts at the end A of the ladder to make it be about to move towards the wall .
- 10 A uniform ladder of length 10 meters and weight 20 kg.wt rests on rough horizontal ground with its end A. The coefficient of friction between the ground and the ladder is $\frac{1}{4}$. The ladder also rests in a smooth vertical wall with its end B. Prove that the ladder cannot be in equilibrium when the end B is distant 8 meters from the ground surface.
- 11 A uniform ladder AB of weight 9 kg.wt rests on rough horizontal ground with its end A and on a rough vertical wall with its end B. If the two coefficient of friction at A and B are $\frac{5}{6}$ and $\frac{1}{2}$ respectively, then the end A is pulled by a horizontal force \vec{F} making the ladder be about to slip away from the wall and the ladder makes an angle of measure 45° to the horizontal. Find the force \vec{F} (the ladder is in a vertical plane perpendicular to the wall).

Couples

Unit 5



Unit introduction

In the previous unit, you learned to obtain two parallel forces in opposite direction by replacing them into two forces meeting at a point (concurrent). You noticed that it can be possible as long as the two forces are equal. But if the two parallel forces are equal in the magnitude, they cannot be replaced by two non-parallel forces. We always obtain two parallel forces equal in magnitude and of opposite direction. Hence, such two forces cannot be obtained at the same time.

In regard to what is mentioned above, we believe that the system made up of two parallel forces of equal magnitudes and of opposite direction has a new name in statistics which is called "couple" in this unit, we are going to demonstrate the concept and definition of a couple and, calculate its moment, the equilibrium of a rigid body under the action of two coplanar couples, and the moment of the resultant couple. This unit ends up with learning the sum of any finite number of couples.

By the end of this unit and by doing all the activities involved, the student should be able to:

- ⊕ Identify the concept of a couple.
- ⊕ Find the moment of a couple.
- ⊕ Deduce that the moment of a couple is a constant vector.
- ⊕ Identify equivalent couples and equilibrium of two couples.
- ⊕ Identify the concept of the equilibrium of a body under the action of two coplanar couples.
- ⊕ Find the resultant of several couples
- ⊕ Prove that a system of forces is equivalent to a couple (the resultant = zero and the moments about any point \neq zero) or the sum of moments of the forces about three non-collinear points = a constant \neq zero.
- ⊕ Prove that a system of forces is equivalent to a couple using the definition.
- ⊕ Identify the theorem stating " If many coplanar forces act on a rigid body and are completely represented by the sides of a polygon. taken the same cyclic order. then the system is equivalent to a couple.
- ⊕ Solve various applications on the couples.

Key Terms

- ☛ Couple
- ☛ Line of action
- ☛ Equilibrium
- ☛ Rigid body
- ☛ Equivalence

Unit Lessons

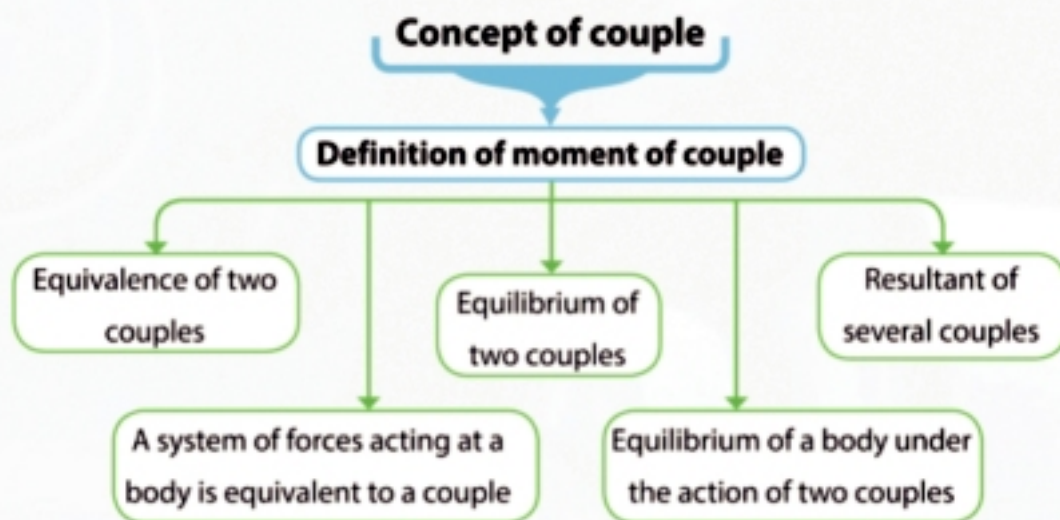
Lesson (5-1): Couples

Lesson (5-2): Resultant couple

Materials

Scientific calculator

Unit planing guide



You will learn

- ↪ Couple and moment of couple
- ↪ Equivalence of two couples
- ↪ Equilibrium of a body under the action of two or more couples

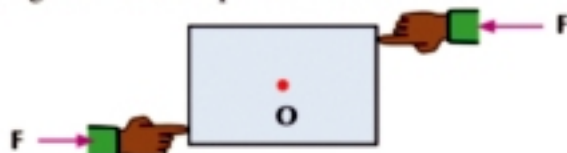
Key terms

- ↪ Couple
- ↪ Line of action
- ↪ Equilibrium
- ↪ Rigid body
- ↪ Equivalence

Materials

- ↪ Scientific calculator
- ↪ Mechanical Lab

Introduction: Some people may believe that if the resultant of the forces acting on a body is equal to zero, then this body is kept at rest (static). If you look at the opposite figure, you will find two forces of equal magnitudes and opposite direction (their resultant is equal to zero). You see that this body will move a rotational motion about (O). The speed of rotation depends upon several things which the students can discover through the next cooperative work:



Cooperative work

The opposite figure represents a graduated ruler two parallel forces in opposite direction of magnitude 15 newtons of each act at its ends. With your classmates. Find the sum of the moments of the two forces about each of the points A, B, C, D, E, F and G and place your results in the next table:



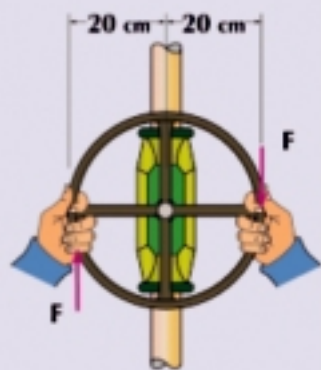
The points	A	B	C	D	E	F	G
Sum of moments of two forces							

What do you notice from the results?

Learn Couple

Definition

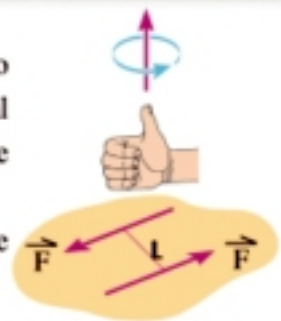
Couple: a system of two forces of equal magnitudes and opposite directions and acting in different lines of action.



Moment of a couple

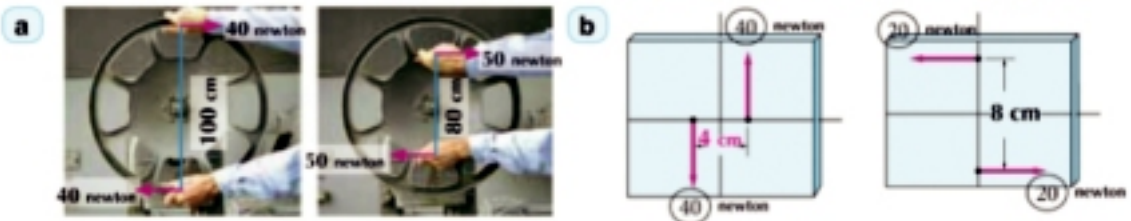
The moment of a couple is known as the sum of the moments of the two forces of the couple about a point in space, and its magnitude is equal to the product of the magnitude of one of the two forces by the distance between them and is denoted by $M = \|\vec{M}\|$

$\therefore \|\vec{M}\| = F \times L$ where $F = \|\vec{F}\|$ and L is called the arm of the couple



Example

1 Find the algebraic measure of the moment of the couple in each of the following figures:

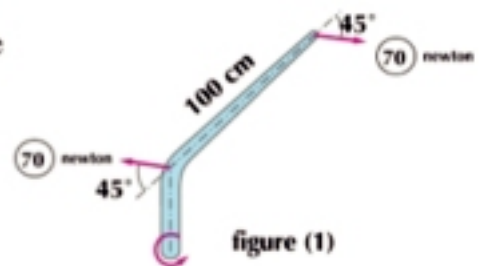


Solution

- a The algebraic measure of both the couples in figure (A) is equal to - 4000 newton. cm
- b The algebraic measure of both the couples in figure (B) is equal 160 newton. cm. Notice the increase of the distance between the two forces, the decrease of the magnitudes of the two forces, and the constancy of the magnitude of the moment.

Try to solve

1 Find the algebraic measure of the moment of the couple in the following figure:



Theorem

The moment of a couple is a constant vector independent of the point about which we take the moments of the two forces.

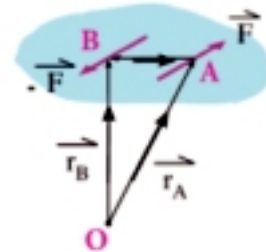
The proof (is not required)

Let the two forces \vec{F} and $-\vec{F}$ act at the two points A and B respectively and let point O be a general point in space.

We find the sum of the moments of the forces about point O

$$\vec{M} = \vec{OA} \times \vec{F} + \vec{OB} \times -\vec{F} = (\vec{OA} - \vec{OB}) \times \vec{F}$$

$$\therefore \vec{OA} - \vec{OB} = \vec{BA} \quad \therefore \vec{M} = \vec{BA} \times \vec{F}$$



The last form of the moment shows that the moment of the couple independent of the position of point O about which we take the moments of the two forces.

Example

- ② If the two forces $\vec{F}_1 = 2\hat{i} + b\hat{j}$ and $\vec{F}_2 = a\hat{i} - 5\hat{j}$ form a couple and act at the two points A (-1, 3) and B (2, 2) respectively, find the value of a and b, and the moment of the couple.

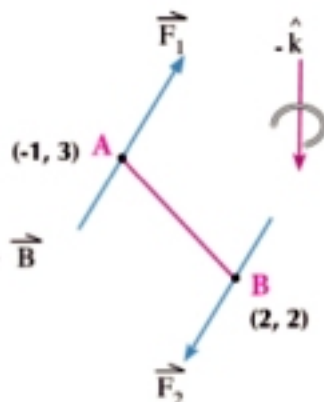
Solution

∴ The two forces form a couple

$$\therefore \vec{F}_1 = -\vec{F}_2$$

$$\therefore a = -2, \quad b = 5$$

$$\begin{aligned} \text{The moment of the couple} &= \text{moment of } \vec{F}_1 \text{ about B} \\ &= \vec{BA} \times \vec{F}_1 \quad \text{where } \vec{BA} = \vec{A} - \vec{B} \\ &= (-3, 1) \times (2, 5) \\ &= (-15 - 2)\hat{k} = -17\hat{k} \end{aligned}$$



Try to solve

- ① If \vec{F}_1 and \vec{F}_2 are two forces of a couple where $\vec{F}_1 = -3\hat{i} + 2\hat{j}$ acting at point A (1, 1), \vec{F}_2 acting at point B (-1, 2), find \vec{F}_2 , moment of the couple, and the length of the perpendicular drawn from A on the line of action of \vec{F}_2 .

Equilibrium of a rigid body under the action of two or more coplanar couples

Definition

A rigid body is said to be in equilibrium under the action of two coplanar couples if the sum of their moments is the zero vector

If \vec{M}_1 and \vec{M}_2 are the two moments of two couples, then the condition of the equilibrium of a body under the action of the two couples is $\vec{M}_1 + \vec{M}_2 = \vec{0}$

In general: If the body is acted on by several coplanar couples of moments $\vec{M}_1, \vec{M}_2, \dots, \vec{M}_n$, then the condition of the equilibrium of the body under the action of such couples is $\vec{M}_1 + \vec{M}_2 + \dots + \vec{M}_n = \vec{0}$

Corollary

A rigid body is said to be in equilibrium under the action of two or more coplanar couples if the sum of the algebraic measures of the moments of the couple vanishes

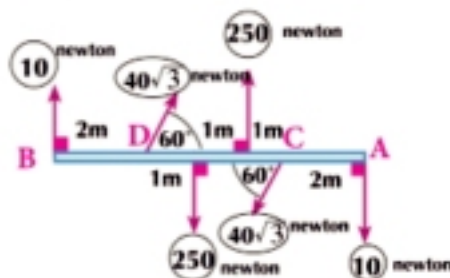
Example

- ③ The forces shown in the figure act at the light rod AB. prove that the rod is in equilibrium.

Solution

The two forces 10, 10 from a couple.

The algebraic measure of its moment is $M_1 = -10 \times 7 = -70$ newton. meter



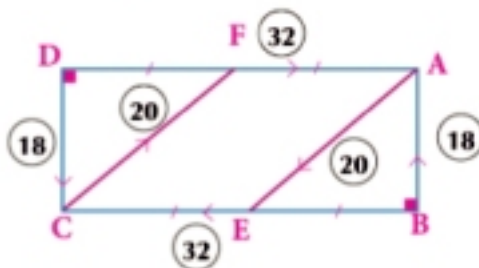
The two forces $40\sqrt{3}$ and $40\sqrt{3}$ form a couple. The algebraic measure of its moment $M_2 = -40\sqrt{3} \times 3 \sin 60 = -180$ newton. meter

the two forces 250 and 250 form the couple of the algebraic measure of its moment $M_3 = 250 \times 1 = 250$ newton. meter

$\therefore M_1 + M_2 + M_3 = -70 - 180 + 250 = \text{zero} \quad \therefore$ the rod is in equilibrium.

P Try to solve

- 2 In the opposite figure: ABCD is a rectangle. E and F are the midpoints of \overline{BC} and \overline{AD} respectively, $AB = 6\text{cm}$ and $BC = 16\text{cm}$. If the forces acting in newton and their magnitudes and directions are shown in the figure, prove that the system is in equilibrium.



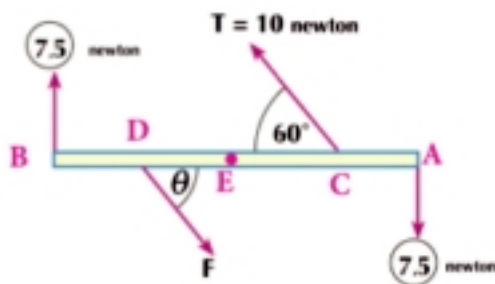
Example

- 4 AB is a rod of negligible weight is horizontally suspended by a pin at its midpoint, two forces each of a magnitudes 7.5 newtons act at its two ends that at one end is vertically upwards and at the other end it is vertically down wards. It is also pulled by a string in a direction making an angle of measure 60° with the rod from point C on it. Find the magnitude and direction and the point of action of the force which if acts with the previous forces on the rod will keep it in equilibrium in a horizontal position, given that the tension in the string is of a magnitude 10 newtons and the rod length is 30 cm.

Solution

The two forces 7.5 and 7.5 newtons at A and B form a couple. The algebraic measure of its moment is $M_1 = -7.5 \times 30 = -225$ newtons. cm
Since the rod is to be in equilibrium, the force of tension in the string and the other force should form a couple. The algebraic measure of its moment is 225 newtons. cm

\therefore the other force $F = T = 10$ newton, $\theta = 60^\circ$ and $10 \times CD \sin 60 = 225$
 $\therefore CD = 15\sqrt{3}$ cm i.e. point D is distant $15\sqrt{3}$ cm from point C on the rod.



P Try to solve

- 3 ABCDEF is a regular hexagon, The forces 3, 9, F_1 , 3, 9, F_2 gm.wt act along the directions \overrightarrow{AB} , \overrightarrow{BC} , \overrightarrow{DC} , \overrightarrow{DE} , \overrightarrow{EF} and \overrightarrow{AF} respectively. Find the value for each of F_1 and F_2 so that the system is in equilibrium.

Example

- 5 ABCD is a fine lamina in the form of a square whose side length is 60 cm and of weight 200 gm.wt acts at the point at which the diagonals meet. The lamina is suspended by a pin from a small hole near the vertex A such that its plane is vertical and a couple acts on its plane of a magnitude $3000\sqrt{2}$ gm.wt. Find, in the position of equilibrium, the measure of the angle of inclination of \overline{AC} to the vertical.

Solution

In the position of equilibrium, the lamina is under the action of two forces which are; the weight of the lamina and the reaction of the pin at A, in addition to the external couple.

Let the external couple act in the anticlockwise direction (as in the figure). Since the couple is in equilibrium, only with a similar couple, then the reaction at point A and the weight form a couple whose algebraic measure of its moment is

$$M_2 = -200 \times AM \sin \theta$$

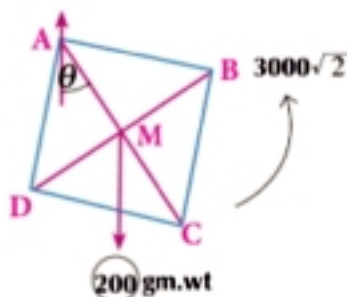
$$M_1 + M_2 = \text{zero}$$

$$\text{where } \sin \theta = \frac{1}{2}$$

$$\text{where } AM = 30\sqrt{2}$$

$$3000\sqrt{2} - 200 \times 30\sqrt{2} \sin \theta = \text{zero}$$

$$\theta = 30^\circ \text{ or } 150^\circ$$



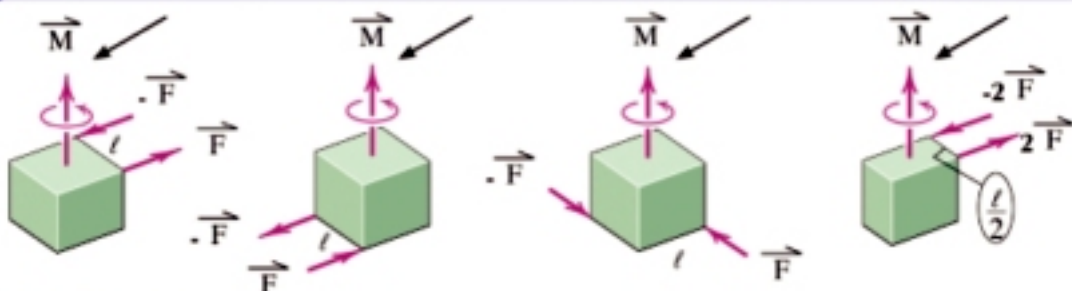
Try to solve

- 4 A rod of length 40 cm and weight 2.4 kg.wt act at its midpoint. It can easily rotate at a vertical plane about a fixed hinge at its end. A couple of moment 24 kg.wt.cm and of orthogonal direction to the vertical plane at which the rod can rotate in acts on the rod. Identify the magnitude and the direction of the reaction of the hinge and the angle of inclination of the rod to the vertical in the equilibrium position.

Equivalent of two couples

Definition

Two couples in the same plane are equivalent if the algebraic measures of their moments are equal



Example

- 6 ABCD is a rectangle in which, $AB = 12$ cm and $BC = 16$ cm. Two equal forces each of a magnitude 96 newtons in the directions of \overline{AB} and \overline{CD} act on it. Find the magnitudes of

two equal forces acting at A and C in a direction parallel to \overleftrightarrow{BD} such that the couple formed from the first two forces is equivalent to the couple formed from the second two forces.

Solution

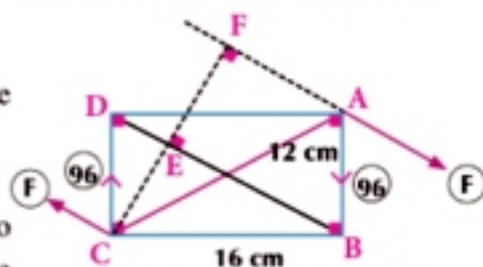
The two forces 96 and 96 newtons form a couple whose algebraic measure of its moment is
 $M_1 = -96 \times 16 = -1536$ newtons. cm
 since the two couples are equivalent, then the two forces at A and C act on the rotation in the clockwise direction (as in the figure).

$$\therefore M_2 = -F \times CF$$

$$\therefore M_2 = -F \times 19.2$$

$$\therefore \text{two couples are equivalent} \quad \therefore M_1 = M_2$$

$$\therefore -F \times 19.2 = -1536 \quad \therefore F = 80 \text{ newton}$$



from Euclidean theorem

$$CE = \frac{AB \times AD}{BD}$$

$$CE = \frac{16 \times 12}{20} = 9.6 \text{ cm}$$

$$CF = 2 \text{ CE}$$

$$CF = 19.2 \text{ cm}$$

Try to solve

- 5 AB is a light rod of length 50 cm, two forces each of a magnitude 30 newtons act at A and B in two opposite directions. Two other forces, each of a magnitude 100 newtons, act in two opposite directions at points C and D of the rod where $CD = 30$ cm such that they form a couple equivalent to the couple formed by the first two forces. Find the measure of the angle of inclination of the other two forces on the rod.

Exercises 5 - 1

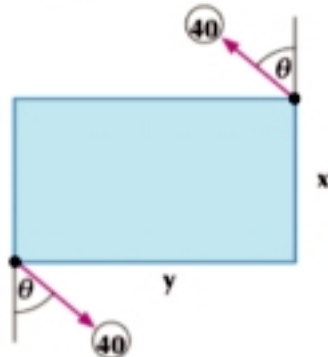
Choose the correct answer:

- The couple is:
 - Two parallel forces of equal magnitudes and in the same direction.
 - Two perpendicular forces of equal magnitudes.
 - Two parallel forces of equal magnitudes and on one line of action.
 - Two parallel forces of equal magnitudes, in opposite directions and are not on one line of action.
- Which of the following conditions does not change the effect of the couple on the body?
 - Translating the couple into a new position in its plane.
 - Translating the couple into another plane parallel to its plane
 - Rotating the couple in its same plane.
 - All what previously mentioned.
- The two forces acting on the steering wheel of a car and producing the rotation of the steering wheel form:
 - Friction.
 - Couple.
 - Perpendicular force on the steering wheel.
 - Non-zero resultant.

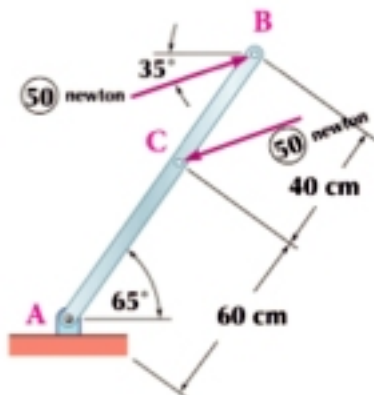
- 4 To form a couple of two forces, the two forces should be :
- a Equal in magnitudes.
 - b Opposite in directions.
 - c Not on one line of action.
 - d All what previously mentioned.
- 5 If M_1 and M_2 are the algebraic measures of the moments of two couples and $M_1 + M_2 = \text{zero}$ then:
- a The two couples are equivalent
 - b The two couples are not in equilibrium
 - c The two couples are equilibrium
 - d The two couples are equivalent to a force
- 6 The product of the magnitude of a force of the two forces of a couple and the arm of the couple is called:
- a The resultant of the couple.
 - b The moment of the couple.
 - c The moment of one force of the two forces of the couple.
 - d Nothing of the previous.
- 7 If $\vec{F}_1 = 3\hat{i} - b\hat{j}$ and $\vec{F}_2 = a\hat{i} - 5\hat{j}$ form a couple, then (a, b) =
- a (3, -4)
 - b (3, 5)
 - c (-3, 5)
 - d (-3, -5)
- 8 If the norm of the moment of a couple is 350 newtons. m and the magnitude of one of its two forces is 70 newtons, then the arm length of the couple is equal to:
- a 50 meter
 - b 5 meters
 - c 5 cm.
 - d 24500 cm.

Answer the following questions :

- 9 The opposite figure shows two forces each of a magnitude 40 newtons act on the two edges of a lamina in the form of a triangle of dimensions X and Y cm. Find the moments of the couple of the two forces in each of the following cases:
- a $x = 3\text{cm}$, $y = 4\text{cm}$, $\theta = \text{zero}$
 - b $x = y = 6\text{cm}$, $\theta = \frac{\pi}{4}$
 - c $x = 0$, $y = 5\text{cm}$, $\theta = 30^\circ$
 - d $x = 6\text{cm}$, $y = 0$, $\theta = 60^\circ$
 - e $x = 5\text{cm}$, $y = 12\text{cm}$, $\tan \theta = \frac{5}{12}$

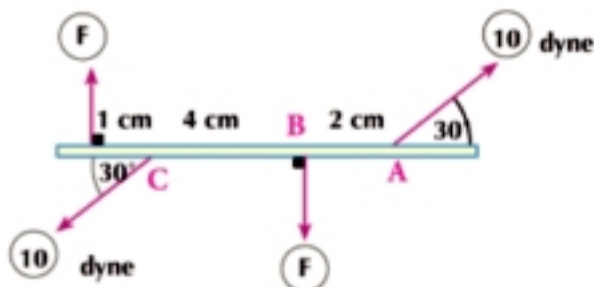


- 10 The opposite figure shows two forces each of a magnitude 50 newtons, act on a lever AB find the algebraic measure of the moment of the couple in two methods:
- a Using the perpendicular distance between two forces.
 - b By finding the sum of the moments of the two forces about point A



- 11 Two forces $(3\hat{i} - 5\hat{j})$ and $(-3\hat{i} + 5\hat{j})$ newtons act at the two points A and B respectively whose position vectors are, $(6\hat{i} + \hat{j})$ and $(4\hat{i} + \hat{j})$ meters. Prove that the system is equivalent to a couple and find its moment.
- 12 Two forces $(a\hat{i} + b\hat{j})$ and $(5\hat{i} - 2\hat{j})$ newtons act at the two points C and D respectively where $C(-2, 1)$ and $D(3, 1)$. If the two forces form a couple, find the value for each a and b, then find the moment of the couple and the perpendicular distance between the two lines of action of the forces.

- 13 The opposite figure represents an equilibrium rod under the action of four forces. Find the value of F.



- 14 ABCD is a rectangle in which $AB = 8\text{ cm}$, $BC = 6\text{ cm}$, x, y, z and L are midpoints of \overline{AB} , \overline{BC} , \overline{CD} and \overline{DA} respectively, the forces of magnitudes F, F, F, F, 6 and 6 newtons act in the direction of \overrightarrow{AX} , \overrightarrow{CZ} , \overrightarrow{YX} , \overrightarrow{LZ} , \overrightarrow{CY} and \overrightarrow{AL} respectively. If the rectangle is in equilibrium, find the value of F.
- 15 AB is a rod of length 60 cm and weight 18 newtons, acts at its midpoint. The rod can rotate easily about a fixed horizontal pin passing through a small hole in the rod at point C which is distant 15 cm from A. If the rod rests with its end B on a smooth horizontal table and the end A is pulled horizontally by a string until the reaction of the table is equal to the weight of the rod, find the tension in the string and the reaction of the pin known that the rod is in equilibrium as it inclines at 60° to the horizontal.
- 16 ABCD is a fine lamina in the form of a rectangle in which $AB = 18\text{ cm}$, $BC = 24\text{ cm}$ and its weight equals 20 newtons acting at the intersection point of the two diagonals. The lamina is suspended by a pin in a small hole near the vertex D such that its plane is vertical. If the lamina is acted by a couple whose moment is 150 newtons and in a perpendicular direction of the plane of the lamina, find the angle of inclination of \overline{DB} to the vertical in the equilibrium position.
- 17 ABCD is a square of side length 10 cm, Two forces each of magnitude 60 newtons act in the directions of \overrightarrow{BA} and \overrightarrow{DC} . Find two forces equal in the magnitude, acting at A and C, parallel to \overrightarrow{BD} and forming a couple equivalent to the couple formed by the first two forces.



Think and discuss

You will learn

- ▷ The sum of coplanar couples (resultant couple)
- ▷ The condition of a system of coplanar forces equivalent to a couple

- 1) What is the action occurring on a body if this body is under the action of a couple?
- 2) Does the body under the action of a couple move in a linear motion or circular motion?
- 3) If the resultant of a system of concurrent coplanar forces is equal to zero; can these forces represent a couple?
- 4) If the resultant of a system of non-concurrent forces is equal to zero, can these forces represent a couple?



Learn

The system of the coplanar forces equivalent to a couple

A system of coplanar forces $\vec{F}_1, \vec{F}_2, \dots, \vec{F}_n$ is said to be equivalent to a couple if the following two conditions are satisfied together :

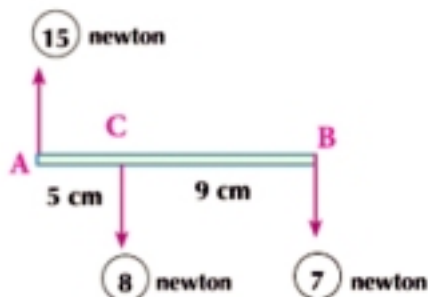
- 1) Vanishing the resultant of the forces (or the sum of the algebraic components of the forces in any direction = zero)
- 2) The sum of the moments of the forces about any point does not vanish.

Notice: satisfying one of the two conditions only is not enough to prove that the system is equivalent to a couple. If the resultant of the concurrent forces, vanishes, then the system of the forces is in equilibrium but is not equivalent to a couple.



Example

- 1 AB is a light rod acted on by the forces shown in the figure. Prove that the system of forces is equivalent to a couple and find the algebraic measure of its moment.



Solution

Let \vec{C} be the unit vector in the direction of the force 15 newtons

$$\therefore \vec{R} = 15\vec{C} - 8\vec{C} - 7\vec{C} = \vec{0}$$

i.e. the resultant vanishes

\therefore Either the system is in equilibrium or it is equivalent to a couple.

Key terms

- ▷ Resultant couple
- ▷ Equivalent

Materials

- ▷ Scientific calculator.

As a result, we find the sum of the moments of the forces about a point (say A)

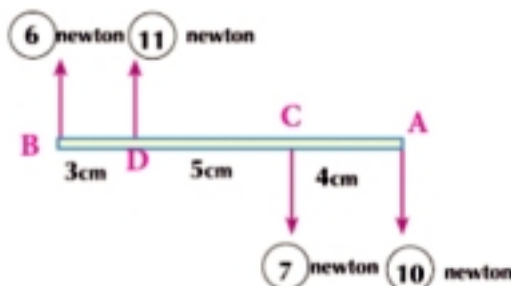
$$M_A = -8 \times 5 - 7 \times 14 = -138$$

\therefore the system is equivalent to a couple and the algebraic measure of its moment is equal to -138 newtons.cm

Critical thinking: Find the sum of the moments of the forces about each of B, C. What do you notice?

Try to solve

- 1 In the opposite figure, prove that the system is equivalent to a couple and find the algebraic measure to its moment.



Rule

If three coplanar forces act on a rigid body and are completely represented by the sides of a triangle, taken the same way round, then this system is equivalent to a couple, the magnitude of its moment is equal to twice the area of the triangle multiplied by the magnitude of the force represented the unit length.

Proof (is not required)

The directed straight segments \overrightarrow{AB} , \overrightarrow{BC} and \overrightarrow{CA} represent the three forces completely.

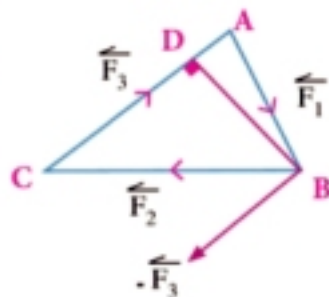
i.e. in the magnitude, direction and line of action, let m represent the magnitude of the force of the unit length

$$\text{i.e. } m = \frac{F_1}{AB} = \frac{F_2}{BC} = \frac{F_3}{AC}$$

$$\therefore \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$$

$$\overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} = \vec{0}$$

$$\therefore \overrightarrow{F_1} + \overrightarrow{F_2} = -\overrightarrow{F_3}$$



i.e. the resultant of the two forces $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ is the force $(-\overrightarrow{F_3})$ acting at point B - thus the system is equivalent to the two forces $\overrightarrow{F_3}$ acting at C and $(-\overrightarrow{F_3})$ acting at point B, it is equivalent to a couple.

To identify the magnitude of the moment of such a couple, draw a perpendicular from B to \overline{AC} to meet it at point D.

$$\text{The magnitude of the moment of the couple} = \|\overrightarrow{F_3}\| \times BD$$

$$\text{But } \|\overrightarrow{F_3}\| = AC \times m$$

$$\text{The magnitude of the norm of the couple} = AC \times m \times BD$$

$$= (AC \times BD) \times m = m \times \text{twice the area of triangle ABC}$$

Example

- 2 ABC is a triangle, in which $AB = BC = 17$ cm and $AC = 16$ cm. Forces of magnitudes 340, 340 and 320 newtons act at \overrightarrow{AB} , \overrightarrow{BC} and \overrightarrow{CA} respectively. Prove that the system is equivalent to a couple and find the magnitude of its moment.

Solution

$$\text{Since } \frac{340}{17} = \frac{340}{17} = \frac{320}{16} = 20$$

\therefore The magnitude of the force representing the unit length is equal to 20 newton and since the forces are taken in one cyclic order (taken the same way round)

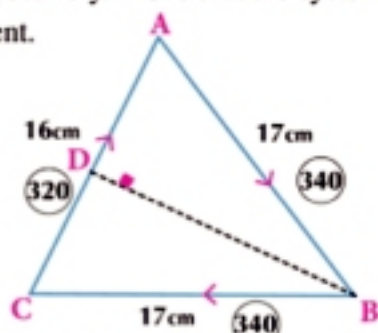
\therefore the system is equivalent to a couple

the magnitude of the moment of the couple = twice the area of $\triangle ABC$ \times the magnitude of the force representing the unit couple

To find the area of \triangle we draw $\overline{BD} \perp \overline{AC}$ to bisect it

$$\therefore BD = \sqrt{17^2 - 8^2} = 15 \text{ cm}$$

$$\therefore \text{the magnitude of the moment of the couple} = 2 \times \frac{1}{2} \times 16 \times 15 \times 20 = 4800 \text{ newtons. cm}$$



Try to solve

- 2 ABC is a right-angled triangle at B in which $AB = 30$ cm and $BC = 40$ cm. Forces of magnitudes 6, 8 and 10 newtons act at \overrightarrow{AB} , \overrightarrow{BC} and \overrightarrow{CA} respectively. Prove that the system is equivalent to a couple and find the magnitude of its moment.

Generalization: If a system of coplanar forces act on a rigid body and are completely represented by the sides of a closed polygon taken the same cyclic order, then this system is equivalent to a couple. The magnitude of its moment is equal to the product of twice the surface area of the polygon by the magnitude of the force representing the unit length.

Example

- 3 ABCD is a quadrilateral in which $AB = AD = 20$ cm, $BC = CD = 10\sqrt{7}$ cm, $m(\angle A) = 120^\circ$. forces represented by the directed straight segments \overrightarrow{AB} , \overrightarrow{BC} , \overrightarrow{CD} and \overrightarrow{DA} act on. If the system ends to a couple the magnitude of its moment is $180\sqrt{3}$ newtons. cm in the direction of ABCD. Find the magnitude of the forces acting on the sides of the figure.

Solution

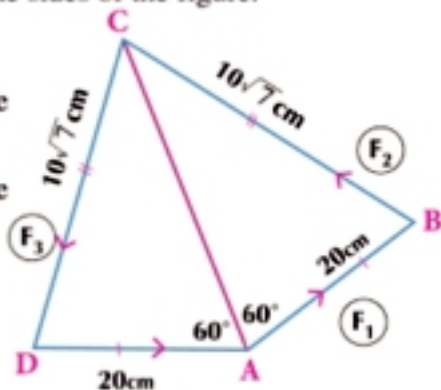
\therefore The forces act on the sides of the polygon and take the same way round

\therefore the magnitude of the moment = twice the area of the figure $\times m$

$$\text{Twice the area of the figure} \times m = 180\sqrt{3} \quad (1)$$

from the geometry of the figure $\triangle ABC \cong \triangle ADC$

from the cosine rule in $\triangle ABC$



$$\begin{aligned}(BC)^2 &= (AB)^2 + (AC)^2 - 2 AB \times AC \times \cos(\widehat{BAC}) \\ \therefore (10\sqrt{7})^2 &= 20^2 + (AC)^2 - 2 \times 20 \times AC \times \cos 60 \\ \therefore 700 &= 400 + (AC)^2 - 20 AC \\ \therefore (AC)^2 - 20 AC - 300 &= \text{zero} \\ \therefore (AC + 10)(AC - 30) &= 0 \quad \text{then } AC = 30\end{aligned}$$

$$\begin{aligned}\text{Area of figure ABCD} &= 2 \times \text{area } \triangle ABC \\ &= 2 \times \frac{1}{2} \times AB \times AC \times \sin 60 = 20 \times 30 \times \sin 60 = 300\sqrt{3} \text{ cm}^2\end{aligned}$$

By substitution in (1)

$$\therefore 2 \times 300\sqrt{3} \times m = 180\sqrt{3} \text{ then } m = \frac{3}{10}$$

$$\therefore \frac{F_1}{AB} = \frac{F_2}{BC} = \frac{F_3}{CD} = \frac{F_4}{DA} = m$$

$$\therefore \frac{F_1}{20} = \frac{F_2}{10\sqrt{7}} = \frac{F_3}{10\sqrt{7}} = \frac{F_4}{20} = \frac{3}{10}$$

then $F_1 = 6$ newtons , $F_2 = 3\sqrt{7}$ newtons , $F_3 = 3\sqrt{7}$ newtons , $F_4 = 6$ newtons

Try to solve

- 3 ABCD is a trapezium in which $\overline{AD} \parallel \overline{BC}$, $\overline{AB} \perp \overline{BC}$, $AB = 6$ cm, $BC = 9$ cm and $AD = 3$ cm. The forces \vec{F}_1 , \vec{F}_2 , \vec{F}_3 and \vec{F}_4 completely represented by the directed straight segments act at \overline{DA} , \overline{CD} , \overline{BC} and \overline{AB} respectively. If the system is equivalent to a couple the magnitude of its moment is 360 newtons.cm in the direction of ABCD. Find the magnitude for each of \vec{F}_1 , \vec{F}_2 , \vec{F}_3 and \vec{F}_4 .

Rule

If the sum of the algebraic measures of the moments of a system of coplanar forces with respect to three non collinear points in its plane is equal to a non zero constant, then the system is equivalent to a couple. The algebraic measure of its moment is equal to the value of the constant.

Proof (is not required)

For any such system of forces reduces to either a single force \vec{F} or a single couple or it is in equilibrium.

It is clear that the forces are not in equilibrium since the sum of the algebraic measures of the moments of the forces about a point does not vanish. Let the system be equivalent to one force of a magnitude F , the three points be A , B and C and their distances to the line of action of the force be L_1 , L_2 and L_3 respectively.

$$\therefore F \times L_1 = F \times L_2 = F \times L_3 = \text{a constant magnitude}$$

By dividing by F where $F \neq \text{zero}$ $\therefore L_1 = L_2 = L_3$

i.e. points A , B and C lie on one straight line parallel to the line of action of F and this does not match with the hypothesis.

\therefore The system forces is not equivalent to a force

\therefore The system is equivalent to a couple. The algebraic measure of its moment is equal to the value of the constant.



Example

- 4 ABCD is a trapezium in which $\overline{AD} \parallel \overline{BC}$, $m(\angle B) = 90^\circ$, $AB = 12$ cm, $BC = 18$ cm and $AD = 9$ cm. Forces of magnitudes 200, 600, 500, 1200 and $300\sqrt{13}$ kg.wt act at \overline{BA} , \overline{BC} , \overline{CD} , \overline{DA} and \overline{AC} respectively. Prove that the system is equivalent to a couple and find its moment.

Solution

Calculate the sum of the algebraic measures of the moments of the forces with respect to three non-collinear points. Let them be A, B and C.

$$M_A = -600 \times 12 - 500 \times AO$$

$$\text{where } AO = 9 \sin \theta = 9 \times \frac{12}{15} = 7.2$$

$$\therefore M_A = -600 \times 12 - 500 \times 7.2 = -10800 \text{ kg.wt.cm}$$

$$M_B = -1200 \times 12 - 500 \times BL + 300\sqrt{13} \times BE$$

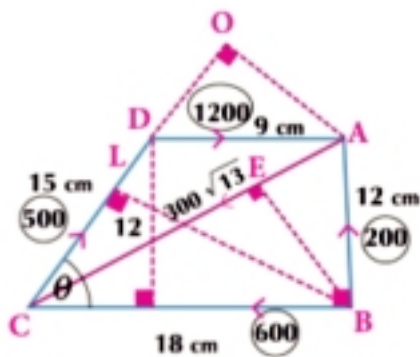
$$\text{where } BL = 18 \sin \theta = 18 \times \frac{12}{15} = 14.4$$

$$, BE = \frac{12 \times 18}{6\sqrt{13}} = \frac{36}{\sqrt{13}}$$

$$\therefore M_B = -1200 \times 12 - 500 \times 14.4 + 300\sqrt{13} \times \frac{36}{\sqrt{13}} = -10800 \text{ kg.wt.cm}$$

$$\therefore M_C = 200 \times 18 - 1200 \times 12 = -10800 \text{ kg.wt.cm}$$

\therefore The system is equivalent to a couple acting on rotation in the clockwise direction. The magnitude of its moment is 10800 kg.wt.cm



Try to solve

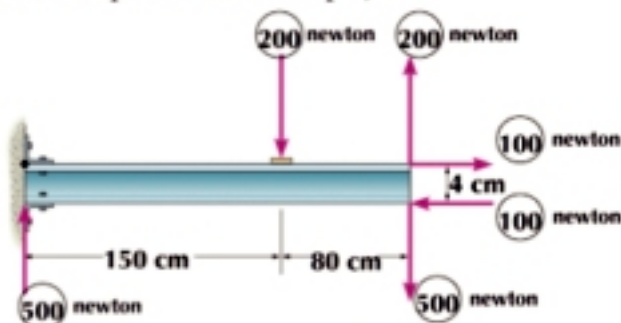
- 4 ABCD is a square of side length 10 cm, $E \in \overline{CB}$, $F \in \overline{CD}$, such that $CE = CF = 30$ cm. Forces of magnitudes 40, 10, 20, 30 and $20\sqrt{2}$ kg.wt.cm act at \overline{AB} , \overline{BC} , \overline{CD} , \overline{DA} and \overline{EF} respectively. Prove that the system is equivalent to a couple and find its moment.

Resultant couple

The sum of two coplanar couples is known as the couple whose moment is equal to the sum of the two moments of those two couples $\vec{M} = \vec{M}_1 + \vec{M}_2$ and the sum of the two coplanar couples is called the resultant couple (the system is equivalent to a couple).

Example

- 5 In the opposite figure find the algebraic measure of the resultant couple.



Solution

The two forces 200 and 200 newtons form a couple the algebraic measure of its moment

$$M_1 = 200 \times 0.8 = 160 \text{ newtons .meter}$$

The two forces 500 and 500 newtons form a couple the algebraic measure of its moment

$$\vec{M}_2 = -500 \times 2.3 = -1150 \text{ newtons. meter}$$

The two forces 100 and 100 newtons form a couple the algebraic measure of its moment

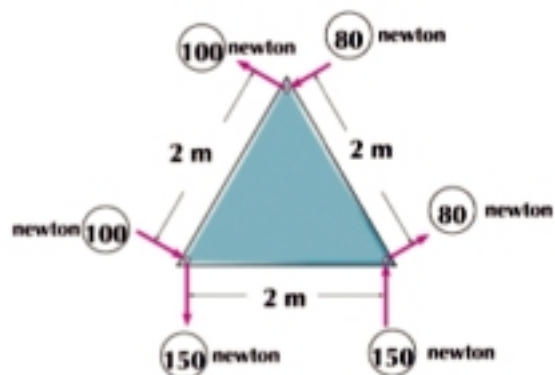
$$M_3 = -100 \times 0.04 = -4 \text{ newtons. meter}$$

The resultant couple = $M_1 + M_2 + M_3$

$$= 160 + (-1150) + (-4) = -994 \text{ newtons. meter}$$

Try to solve

- 5 The opposite figure represents a uniform lamina in the form of an equilateral triangle. If a forces act on the lamina as shown in the figure, find the algebraic measure of the moment of the resultant couple.



Example

- 6 ABCD is a square of side length 10 cm. Two forces, each of a magnitude 40 kg.wt act at \vec{AD} and \vec{CB} and two forces each of a magnitude 70 kg.wt at \vec{AB} , \vec{CD} . Find the algebraic measure of the moment of the resultant couple.

Solution

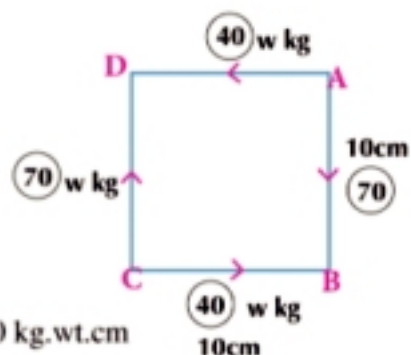
The two forces 40 and 40 form a couple the algebraic measure of its moment

$$M_1 = 40 \times 10 = 400 \text{ kg.wt.cm}$$

the two forces 70 and 70 form a couple the algebraic measure of its moment

$$M_2 = -70 \times 10 = -700 \text{ kg.wt.cm}$$

the resultant couple = $M_1 + M_2 = 400 + (-700) = -300 \text{ kg.wt.cm}$



Try to solve

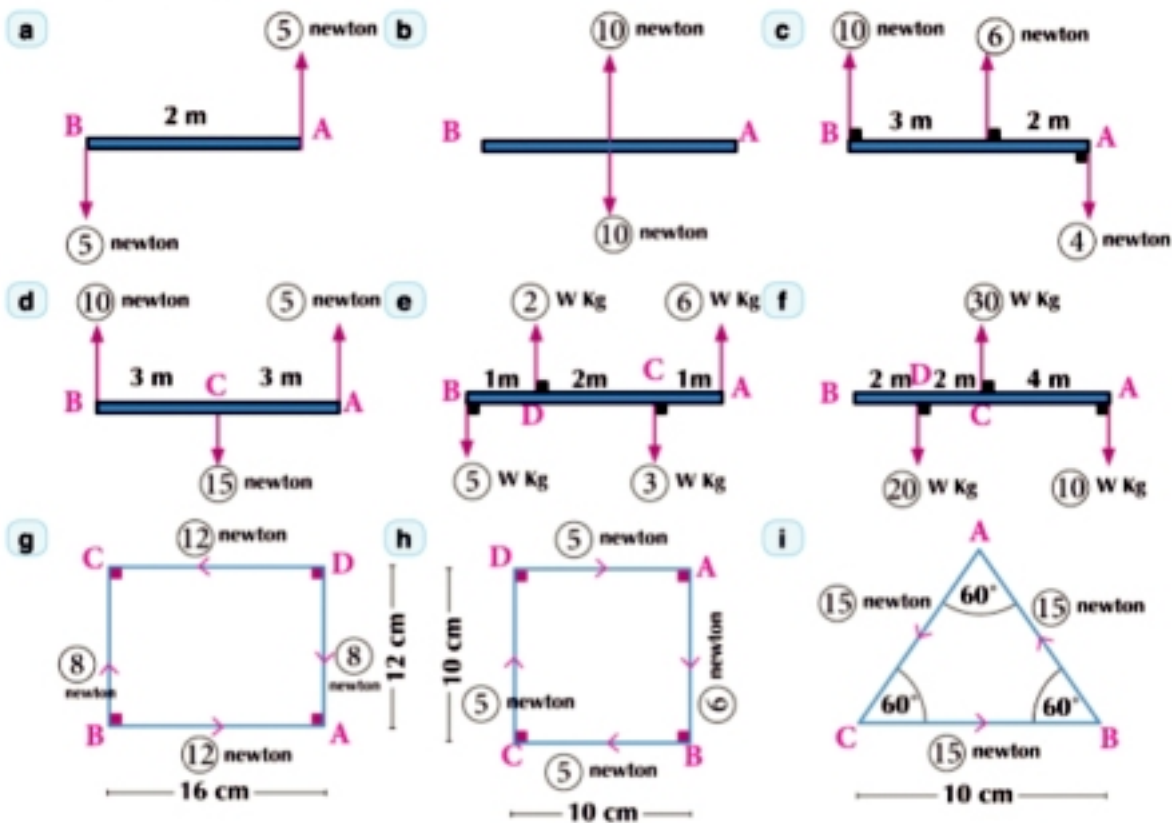
- 6 ABCD is a rectangle in which $AB = 60 \text{ cm}$, $BC = 160 \text{ cm}$, X and Y midpoints of \vec{BC} and \vec{AD} respectively. The forces of magnitudes 200, 200, 400, 400, F and F newtons act in the direction \vec{AB} , \vec{CD} , \vec{CB} , \vec{AD} , \vec{XA} and \vec{YC} , respectively. If the algebraic measure of the moment of the resultant couple is equal to 6400 newton.cm, find the value of: F.



Exercises 5 - 2

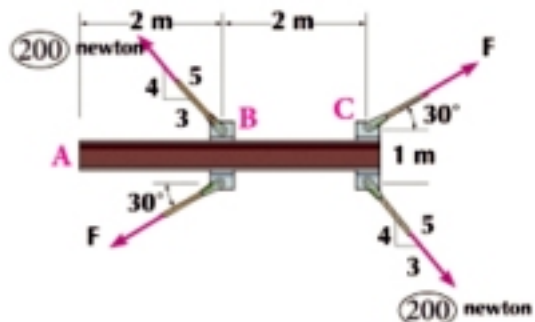


- 1 Show which of the following system of forces is equivalent to a couple and find the algebraic measure of its moment:

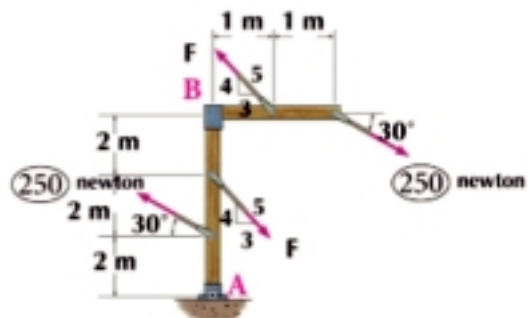


- 2 ABCD is a square of side length 3 meters. The forces of magnitudes 5, 2, 5 and 2 newtons act in the directions of \overrightarrow{BA} , \overrightarrow{BC} , \overrightarrow{DC} and \overrightarrow{DA} , respectively. Show that the system is equivalent to a couple and find the magnitude of its moment.
- 3 ABCD is a rectangle in which $AB = 6\text{ cm}$, $BC = 8\text{ cm}$ forces of magnitudes 7 kg.wt act at \overrightarrow{AB} , \overrightarrow{BC} , \overrightarrow{CD} and \overrightarrow{DA} respectively prove that the system is equivalent to a couple and find the magnitude of its moment.
- 4 ABCD is a rectangle in which $AB = 30\text{ cm}$, $BC = 40\text{ cm}$ forces of magnitudes 15, 30, 15 and 30 gm.wt act at \overrightarrow{BA} , \overrightarrow{BC} , \overrightarrow{DC} and \overrightarrow{DA} respectively. Prove that this system is equivalent to a couple and find its moment, then find the two forces acting at A and C perpendicular to \overline{AC} such that the system is in equilibrium.
- 5 ABCD is a rhombus of side length 10 cm, $m(\angle BAC) = 120^\circ$. Forces of magnitudes 20, 15, 20 and 15 kg.wt act at \overrightarrow{AB} , \overrightarrow{BC} , \overrightarrow{CD} and \overrightarrow{DA} respectively. Prove that the system is equivalent to a couple and find the magnitude of its moment. Find the two forces acting at B and D perpendicular to \overline{BD} such that the system is in equilibrium.

- 6 The opposite figure represents an arch. The shown force in the figure acts on it. If the algebraic measures of the moment of the resultant couple is equal to $200 - 200\sqrt{3}$ newtons. meter, find F.



- 7 ABCD is an isosceles trapezium in which $\overline{AD} \parallel \overline{BC}$, $AD = 9$ cm $AB = DC = 15$ cm and $BC = 33$ cm. The forces of magnitudes 45, 99, 45 and 27 act in the directions of \overline{AB} , \overline{BC} , \overline{CD} , \overline{DA} respectively. Prove that the system is equivalent to a couple and find the magnitude of its moment.
- 8 ABCDEF is a regular hexagon of side length 15 cm. Forces of magnitudes 40, 50, 30, 40, 50 and 30 newtons act at \overline{AB} , \overline{CB} , \overline{CD} , \overline{DE} , \overline{OE} and \overline{FA} respectively. Find the moment of the resultant couple.
- 9 ABCDE is a regular pentagon of side length 15 cm. Forces of magnitudes $10w$ kg act at \overline{AB} , \overline{BC} , \overline{CD} , \overline{DE} and \overline{EA} respectively. Prove that the system is equivalent to a couple and find the magnitude of its moment.
- 10 ABC is a triangle in which $AB = BC = 6$ cm, $m(\angle ABC) = 120^\circ$. Forces of magnitudes 18, 18 and $18\sqrt{3}$ act at \overline{AB} , \overline{BC} and \overline{CA} respectively prove that the system is equivalent to a couple and find the magnitude of its moment.
- 11 ABCD is a square of side length 60 cm. Forces of magnitudes 10, 20, 80 and 50 newton act at \overline{AB} , \overline{BC} , \overline{CD} and \overline{DA} respectively and two forces of magnitude $50\sqrt{2}$ and $20\sqrt{2}$ newtons act in \overline{AM} and \overline{DB} respectively. Prove that the system is equivalent to a couple the magnitude of its moment is 4800 newtons. cm
- 12 In the figure opposite, find F which makes the algebraic measure of the moment of the resultant couple equal to $150 - 500\sqrt{3}$



UNIT SUMMARY

Definition of couple: it is a system of forces formed from two forces of equal magnitudes and opposite directions and acting in different lines of action.

The moment of a couple: it is known as the sum of the moments of two forces of the couple about a point in space. Its magnitude is equal to the product of the magnitude of one of the two forces by the distance between them.

Theorem: the moment of a couple is a constant vector independent of the point about which we take the moment of the two forces.

Equilibrium of two couples: the two couples are said to be in equilibrium if the sum of their two moments is the zero vector.

Equilibrium of a body under the action of several couples if several coplanar couples which the direction of their moments are $\vec{M}_1, \vec{M}_2, \dots, \vec{M}_n$ act a rigid body, then the condition of equilibrium of the body under the action of these couples is $\vec{M}_1 + \vec{M}_2 + \dots + \vec{M}_n = \vec{O}$

Equivalent couples: two coplanar couples in the same plane are equivalent if the two algebraic measures of the two moments of their vectors are equal.

The system of the coplanar forces equivalent to a couple: A system of coplanar forces $\vec{F}_1, \vec{F}_2, \dots, \vec{F}_n$ is said to be equivalent to a couple if the following two conditions are satisfied together:
1 - The resultant of forces is equal to the zero vector ($\vec{F}_1 + \vec{F}_2 + \dots + \vec{F}_n = \vec{O}$)
2 - The sum of moments of the forces about any point in space does not vanish.

Rule 1: If three coplanar forces act on a rigid body and are completely represented by the sides of a triangle, taken the same way round, then this system is in equivalent to couple the magnitude of its moment is equal to twice the area of the triangle by the magnitude of the force representing the unit length.

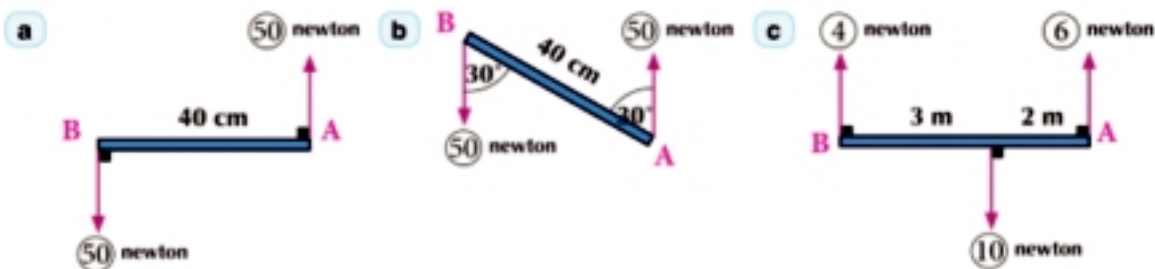
Generalization: if a system of coplanar forces act on a rigid body and are completely represented by the sides of a closed polygon taken the same way round, then this system is equivalent to a couple the magnitude of its moment is equal to the product of twice the surface area of the polygon by the magnitude of the force representing the unit length.

Rule 2: If the sum of the algebraic measures of the moments of a system of coplanar forces with respect to three points in its plane and are not lying in the same straight line is equal to a constant magnitude unequal to zero, then the system is equivalent to a couple the algebraic measure of its moment is equal to to a constant magnitude.

Resultant couple: The sum of two coplanar couples is known as the couple whose moment is equal to the sum of the two moments of those two couples $\vec{M} = \vec{M}_1 + \vec{M}_2$ and the sum of the two coplanar couples is called the resultant couple (the system is equivalent to a couple):

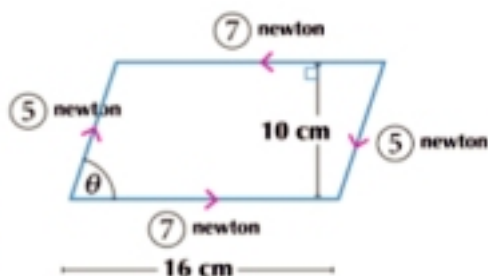
GENERAL EXERCISES

- 1 Find the algebraic measure of the moment of the resultant couple in each of the following figures:



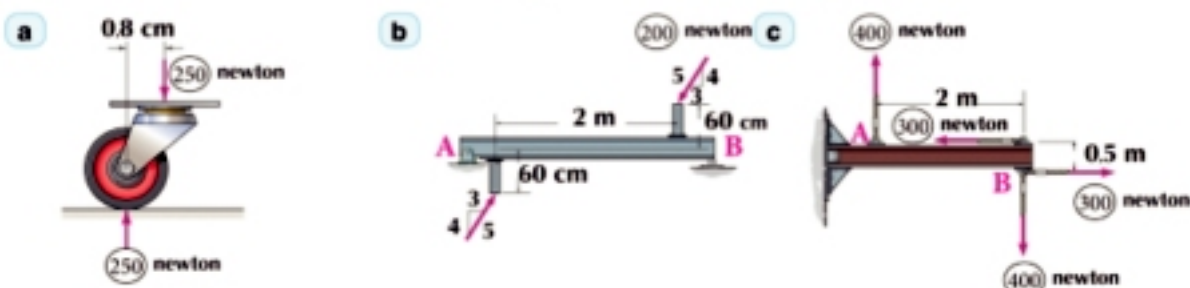
- 2 The opposite figure shows a lamina in the form of a parallelogram acted on by two couples. Find:

- The algebraic measure of the moment of the couple formed by the two forces 7 and 7.
- The algebraic measure of the moment of the couple formed by the two forces 5 and 5 newtons when $\theta = 60^\circ$.
- What is the value of θ if the algebraic measure of the moment of the resultant couple is equal to 30 newtons. cm?
- What is the value of θ if the lamina is in equilibrium?



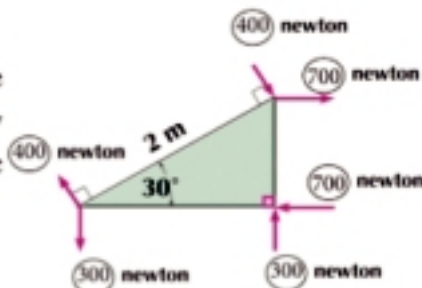
- 3 A uniform rod AB of length 20 cm can rotate in a vertical plane about a fixed horizontal pin passing through a small hole in the rod at point $C \in \overline{AB}$ where $AC = 5$ cm. If the rod is in equilibrium in a horizontal position under the action of two forces each of a magnitude 50 newtons act at its two ends A and B in opposite directions and make an angle of measure 30° to the rod, find the weight of the rod and the reaction in the pin.
- 4 $ABCD$ is a rectangle in which $AB = 30$ cm, $BC = 40$ cm forces of magnitudes 1, 2, 4, 6 and 5 kg.wt act at \overrightarrow{AB} , \overrightarrow{BC} , \overrightarrow{CD} , \overrightarrow{DA} and \overrightarrow{AC} respectively. Prove that the system is equivalent to a couple and find the magnitude of its moment.

- 5 Determine the moment of the couple acting in each of the following figures:



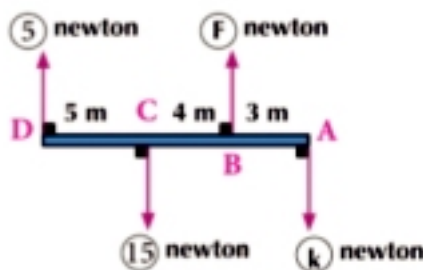
GENERAL EXERCISES

- 6 In The opposite figure shows a lamina in the form of a right-angled triangle. It is acted up on by the forces shown in the figure, find the algebraic measure of the moment of the resultant couple.



- 7 ABCD is a square of side length 20 cm, forces of magnitudes 3, 5, 3, 5 w kg act at \overrightarrow{BA} , \overrightarrow{BC} , \overrightarrow{DC} , \overrightarrow{DA} respectively and two forces each of a magnitude $4\sqrt{2}$ kg.wt act at the two points A and C in the direction of \overrightarrow{BD} and \overrightarrow{DB} respectively. Find the magnitude of the resultant couple equivalent to the system.
- 8 Two forces $\vec{F}_1 = a\hat{i} - 3\hat{j}$ and $\vec{F}_2 = -5\hat{i} + b\hat{j}$ act at two points C(2, -1) and D(0, -2) respectively and form a couple. Find the value for each of a and b, then find the moment of the couple and the perpendicular length between the two forces.
- 9 The force $\vec{F}_1 = 6\hat{j}$ acts at the origin point and the force $\vec{F}_2 = -6\hat{j}$ at point (2, 0). Show that the sum of the moments of the forces about any point (x, y) does not depend upon x and y.
- 10 The forces $\vec{F}_1 = 2\hat{i} - 4\hat{j}$, $\vec{F}_2 = \hat{i} - 3\hat{j}$, $\vec{F}_3 = -3\hat{i} + 7\hat{j}$ act at point A (-1, 1), B (-2, 3), C (0, 1) respectively. Prove that the system of forces is equivalent to a couple and find the magnitude of its moment

- 11 The opposite figure shows a system of forces acting on rod AD forms the couple of the algebraic measure of its moments is equal to -75 newton. m. Find the value for each of F and k.



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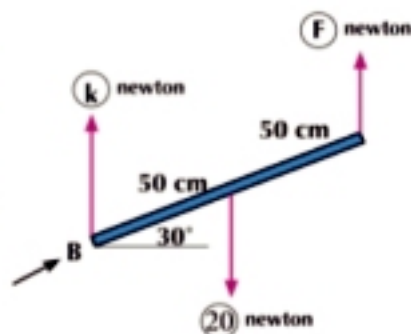
ACCUMULATIVE TEST

Choose the correct answer:

- If the two forces each of a magnitude 4 and 8 newtons act at a point and the measure of the angle between them is 120° , then the magnitude of their resultant is:
 - 12
 - 4
 - $4\sqrt{3}$
 - $8\sqrt{3}$
- If two forces parallel and in the same direction each of a magnitude 5 and 7 newtons act at two points A and B, then the magnitude of their resultant is equal to:
 - 12
 - 2
 - $\sqrt{74}$
 - $\sqrt{24}$
- If the force $\vec{F} = 2\hat{i} - 3\hat{j}$ acts at point A (1, -2) then the moment of \vec{F} about point B (-1, 4) is equal to:
 - $-6\hat{k}$
 - $6\hat{k}$
 - $22\hat{k}$
 - $-22\hat{k}$
- If the force $\vec{F} = \hat{i} + 2\hat{j} - 3\hat{k}$ acts at point A (2, -1, 3) then the moment of \vec{F} about origin point is equal to:
 - $-3\hat{i} + 9\hat{j} + 5\hat{k}$
 - $-\hat{i} - 2\hat{j} + \hat{k}$
 - $3\hat{i} - 9\hat{j} - 5\hat{k}$
 - $2\hat{i} - 5\hat{j} + \hat{k}$
- If three coplanar forces equal in magnitudes and concurrent at a point are in equilibrium, then the measure of the angle between the two forces is equal to:
 - 30°
 - 60°
 - 90°
 - 120°

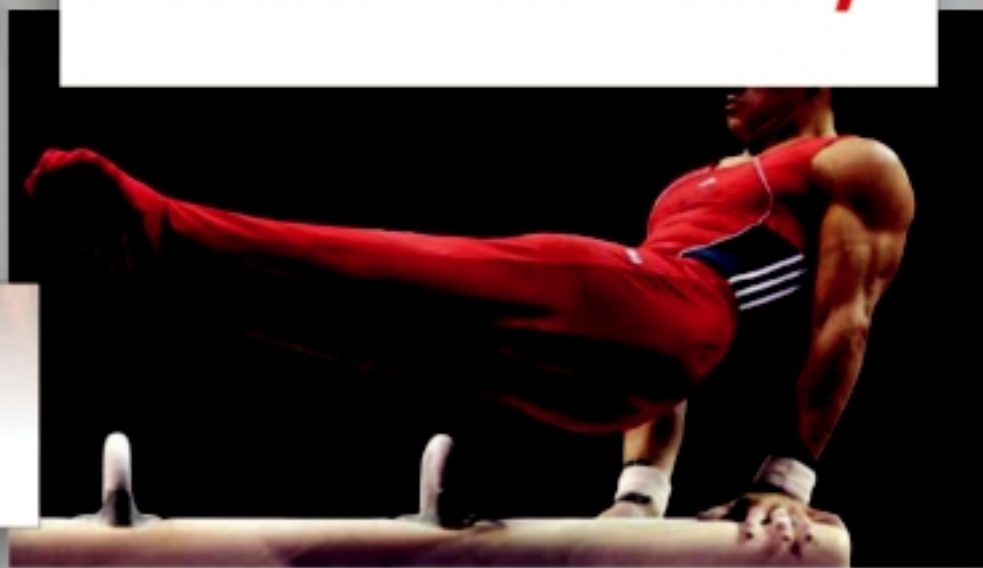
Answer the following questions:

- The opposite figure shows a uniform rod AB in an equilibrium state under the action of the shown forces. Find F and K.
- If $\vec{F} = L\hat{i} + M\hat{j}$ acts at point A (3, -2) and if the moment of the force \vec{F} about the origin point is equal to $\vec{0}$ about point B (-1, 2) is equal to $-8\hat{k}$, find the value of each of L and m.
- If the force $\vec{F} = \hat{i} + b\hat{j} + c\hat{k}$ acts at point A (-2, 1, 2) and the two components of the moment of \vec{F} about Yz axes are 2, -3 respectively, find the value for each of b and c.
- ABC is an isosceles triangle in which $m(\angle B) = 120^\circ$, $AC = 12\sqrt{3}$ cm forces of magnitudes 6, 7, $8\sqrt{3}$ newtons act at \overline{AC} , \overline{CB} and \overline{AB} respectively. Find the sum of the moment of forces about the midpoint of \overline{BC} .
- A, B, C are three points lying in one straight line where $AB = 6$ cm, $BC = 4$ cm, $C \in \overline{AB}$. Two parallel forces act at the two points A and B of the magnitude of their resultant is equal to 24 newtons and act at point C. Find the magnitude for each of the two forces.
- The parallel forces $\vec{F}_1, \vec{F}_2, \vec{F}_3, \vec{F}_4$ act at point A(1, 1), B(-2, 1), C(3, 3) and D(-2, 0) respectively. If the forces are in equilibrium and $\vec{F}_1 = \hat{i} + 2\hat{j}$ and $\|\vec{F}_2\| = 2\sqrt{5}$ newtons in the opposite direction of \vec{F}_1 , find each of \vec{F}_2, \vec{F}_3 and \vec{F}_4



Center of Gravity

Unit 6



Unit Introduction

However the bodies differ (made up of a great number of particles) in shape and the outer appearance even they are of the same weight; they are acted by the force of the gravity which always acts vertically down wards. (in the direction of Earth's center). Scientists have found that the resultant of the forces acting on all the parts which the body is made up of is equal to the weight of the body. Furthermore, they have found that the resultant of such forces acting on a body centralize at a point and then the center of gravity of the rigid body is the constant point of that body through which the line of action of the resultant of the gravitational forces to the points of the mentioned body at any of its positions in space passes. It is worth to mention that the center of gravity is a geometric point lying outside the body in case of being in the ring shape and its parts centralize in this point. The center of gravity of some regular bodies have been identified quite easily since this point is the position of its geometrical center (such as uniform geometrical laminas, discs, spheres and so on ...) For irregular bodies (such as the human body), the method to identify their center of gravity is conducted throughout different scientific methods. It is worth to mention that the center of gravity of the human body has been identified using some computer programs which supply the computer with information about the weight of the human body and the weight of each part of the whole parts and the dimensions of their centers of gravity in the orthogonal coordinate systems.

Of the studies concerned in such a topic is the study of (Brown and Ficher) which has identified the height of the center of gravity of the human body to be 54.8% of its height measured from under the foot. (Crosky) has also mentioned that the center of gravity for men is higher than of women. The study of the center of gravity is quite essential in regard to the sports motion in the field of biomechanics of sports. We are going to learn all about that through the activities included in this unit.

In this unit, you will learn the center of gravity of a system of particles and find its components in orthogonal coordinate systems with identifying the center of gravity of the rigid body and the complex laminas. In this unit, you are also going to learn some applications on the center of gravity in different daily life fields.

Unit objectives

By the end of this unit and by doing all the activities involved , the student should be able to:

- Identify the center of gravity of a rigid body.
- Identify the relation among the weight of a body, the center of gravity, Equilibrium and the gravity.
- Identify the center of gravity of a system of particles.
- Identify the position vector of the center of gravity for a rigid body about the origin point.
- Deduce the components of the center of gravity in an orthogonal Cartesian coordinate system.
- Deduce the center of gravity of a rigid body suspended freely.
- Deduce the center of gravity of two physical points of distance L between them.
- Deduce the center of gravity of a uniform fine rod.
- Deduce the center of gravity of a uniform fine lamina in the form of a parallel gram.
- Deduce the center of gravity of a uniform fine lamina in the form of a triangle.
- Identify the negative mass method to calculate the center of gravity of a body after eliminating a part of it.
- Identify the center of gravity of some bodies which have the properties of symmetry.

Key terms

- Negative Mass
- Symmetry
- Center of Gravity
- Physical point
- two-dimensional system
- Three-dimensional system
- free suspension

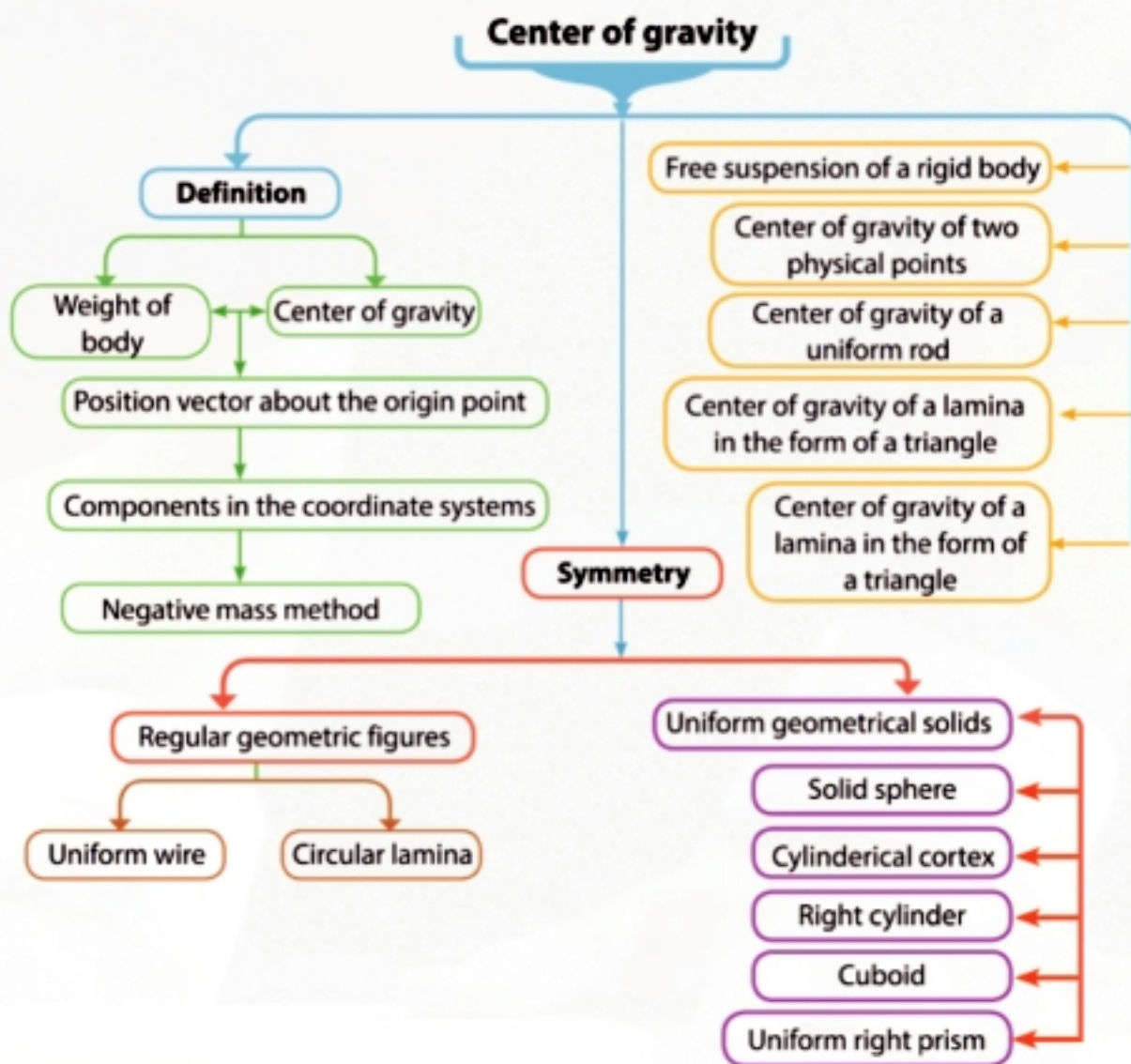
Unit lessons

- (6 -1): The center of gravity
- (6 -2): The negative mass method

Materials

Scientific calculator.

Unit planning guide





Preface

You will learn

- ↻ The center of gravity of a rigid body.
- ↻ The weight of the body, the center of its gravity and gravitation.
- ↻ The center of gravity of two physical points.
- ↻ The position vector of the center of gravity of a rigid body.
- ↻ The center of gravity in the orthogonal coordinate system.
- ↻ The free suspension of the rigid body.
- ↻ The center of gravity of a uniform fine rod.
- ↻ The center of gravity of a uniform lamina in the form of a parallelogram.
- ↻ The center of gravity of a uniform lamina in the form of a triangle.

Key terms

- ↻ Center of Gravity
- ↻ Particle
- ↻ Physical point
- ↻ two-dimensional system
- ↻ Three-dimensional system
- ↻ free suspension

Materials

- ↻ Scientific calculator

Through your previous study to the motion or static of the bodies, you have noticed that we have not given the volumes or dimensions of such bodies much care. It has been sufficient to represent such bodies by a point, small circle or a rectangle supposing that the force acting on the body passes through a single point lying on its midpoint.



Figure (1)

The center of gravity of a rigid body:

If we look at the huge bodies which move in a transitional way only, we will find that each point in these bodies moves the same way exactly. In turn, considering the body is equivalent to a single point is possible in this case.

But if we have a huge body moves randomly (as transition or rotation), then each point in this body will move in a different way from other points.

For example, when a bat is pushed in air, then its motion upward and downward motion is more complicated than the motion of a metallic sphere. This be cause the existence of the revelational motion of the bat during its transitional motion. In other words, each point of the bat has a different motion than other different points. In figure (2), you will find that there is a certain point on the bat that moves on the pathway which we know about the launched body such as the motion of the small metallic sphere as it is launched in air.

It is clear that this point moves as if the mass of the bat centralizes at this point and the weight of the bat acts on this point. This certain point is called the center of gravity which seems as if the whole body is assembled at it. This means that the center of the gravity is a hypothesis point expressing the resultant of the weights of the elements of the rigid body and it is also the equilibrium point. We can also say that it is the point at which the weight of the body is equally distributed about it in all directions.



Figure (2)

The weight, center of gravity and gravitation of body:

If we generally consider the body (gas, liquid, solid) a system of the physical points, then the gravitation acts on the body by the force of its weight and it is at each point of these points. If we consider that the Earth is a homogenous sphere, then the line of action of the weight of each point acts at the straight line connecting between this point and the Earth's center. As the bodies which we meet in our daily life and belonging to our study are extremely tiny comparatively to the Earth. Since they are extremely distant from the Earth's center, then the lines of action of the weights of the physical points forming a body are parallel. As a result, they can be formed in a singular force which equals in regard to the magnitude the sum of the weights of these points and acts vertically down wards towards the Earth.

The gravitational force acts on all the parts of the body, but when the moments are taken, the gravitational force (weight of body) acts at a single point in the body called the center of gravity of the body.

The center of gravity of a system of particles

If we consider A_1, A_2, A_3, \dots a system of particles forming a rigid body and W_1, W_2, W_3, \dots are the weight of these particles respectively and act vertically downwards as in figure (2).

- The resultant of the two parallel force W_1, W_2 acting at A_1, A_2 respectively and passing through point G_1 is $(W_1 + W_2)$ so : $W_1 \times A_1 G_1 = W_2 \times A_2 G_1$ disregarding the position of the body to the ground since the distance between A_1 and A_2 is constant because the body is rigid and in turn G_1 keeps constant.

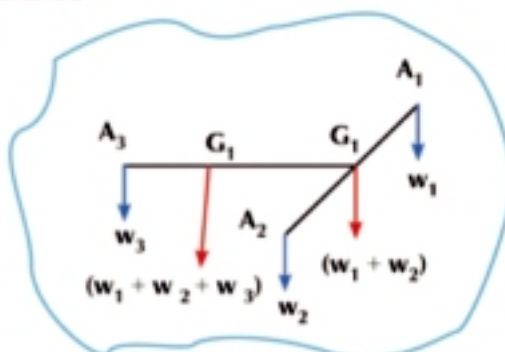


Figure (2)

- The resultant of the two parallel forces $(W_1 + W_2), W_3$ is $(W_1 + W_2 + W_3)$ and let its point of action be point G_2 then : $W_3 \times A_3 G_2 = (W_1 + W_2) \times G_1 G_2$ and the distance $A_3 G_1$ remain constant. So G_2 is a constant point what ever the position of the particles are at points A_1, A_2, A_3 .
- By repeating the previous steps with respect to all other particles forming the body, we finally get the weight of the body. We find that it is equal to the sum of all the weights of the particles and always passes through a point of a constant position.

Definition

The center of gravity of a rigid solid body is a constant point. with the body, through which the line of action of the resultant of the weights of the particles which the body is made up of passes. Its position does not change with respect to the body whatever the position of the body changes with respect to the ground.

Note: The center of gravity of a rigid body changes by the changes of its shape since the dimensions of the particles forming this body change.

per coherent body weight of a single center

The body of a uniform density:

It is the body whose the mass of its unit length, areas or volumes taken from any part of it is constant.

The center of gravity of two physical points (particles)

If the masses of the two particles are m_1 and m_2 in two positions x_1 and x_2 on the x -axis respectively with respect to an observer standing at the origin point O as in figure (3), then the center of gravity of those two particles with respect to the observer is determined by the relation:

$$x_m = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

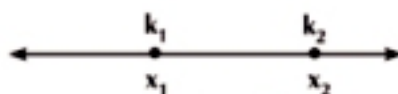


Figure (3)

Example The center of gravity of two physical points

- ① Two physical particles of weights 2 newtons, 4 newtons, and the distance between them is 3 meters. Find the center of gravity with respect to the particle 2 newtons.

Solution

Let the line connecting the two bodies lie on x -axis and consider the origin point lies at the body 2 newtons then: $x_1 = 0$, $x_2 = 3$, $w_1 = 2$, $w_2 = 4$

Use the relation : $x_G = \frac{w_1 x_1 + w_2 x_2}{w_1 + w_2}$

the : $x_G = \frac{2 \times 0 + 4 \times 3}{2 + 4} = \frac{12}{6} = 2$

i.e : the center of gravity of the two physical particles is distant 2 meters from position the body 2 newtons.

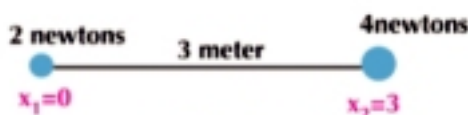


figure (5)

Try to solve

- ① Two physical particles of weights 3 newtons, 5 newtons and the distance between them is 8 meters. Find the center of gravity about the position particle 3 newtons.

The position vector of the center of gravity of a rigid body about the origin point

If $W_1, W_2, W_3, \dots, W_n$ are the weights of the particles forming the rigid body and $\vec{r}_1, \vec{r}_2, \vec{r}_3, \dots, \vec{r}_n$ are the position vectors of these particles about the origin point, then the position vector \vec{r} to the center of gravity of the rigid body about the origin point is determined by the relation:

$$\vec{r}_G = \frac{W_1 \vec{r}_1 + W_2 \vec{r}_2 + W_3 \vec{r}_3 + \dots + W_n \vec{r}_n}{W_1 + W_2 + W_3 + \dots + W_n} \dots \dots \dots (1)$$

$$W_1 = m_1 g$$

$$W_2 = m_2 g$$

$$W_n = m_n g$$

By writing each of weights $w_1, w_2, w_3, \dots, w_n$ as the product of the corresponding mass by the magnitude of the gravitational acceleration, and dividing each of the numerator and denominator by g , we get the relation.

$$\vec{r}_G = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + m_3 \vec{r}_3 + \dots + m_n \vec{r}_n}{m_1 + m_2 + m_3 + \dots + m_n} \dots\dots\dots (2)$$

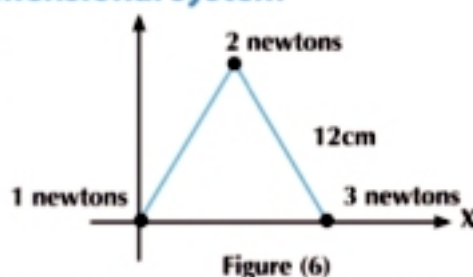
The previous vector relations can be written in terms of the components in the direction of the orthogonal coordinate axes \vec{ox} and \vec{oy} to get the following:

$$x_G = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \dots + m_n x_n}{m_1 + m_2 + m_3 + \dots + m_n}$$

$$y_G = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3 + \dots + m_n y_n}{m_1 + m_2 + m_3 + \dots + m_n}$$

Example The center of gravity of a 2-dimensional system

- 2 In figure (6): Find the center of gravity of three weights of magnitudes 1, 2 and 3 newtons placed at the vertices of an equilateral triangle of side length 12 cm.



Solution

The data of the problem can be placed in a table as follows, considering the axes are orthogonal as in figure (5):

Weight	1 newtons	2 newtons	3 newtons
x	0	6	12
y	0	$6\sqrt{3}$	0

$$\therefore x_G = \frac{w_1 x_1 + w_2 x_2 + w_3 x_3}{w_1 + w_2 + w_3}$$

$$\therefore x_G = \frac{1 \times 0 + 2 \times 6 + 3 \times 12}{1 + 2 + 3} = 8 \text{ cm}$$

$$\therefore y_G = \frac{w_1 y_1 + w_2 y_2 + w_3 y_3}{w_1 + w_2 + w_3}$$

$$\therefore y_G = \frac{1 \times 0 + 2 \times 6\sqrt{3} + 3 \times 0}{1 + 2 + 3} = 2\sqrt{3} \text{ cm}$$

The position of the center of gravity of the system is $G(8, 2\sqrt{3})$

Critical thinking: Does the center of gravity change by changing the positions of orthogonal axes? Explain.

Try to solve

2. A $\triangle ABC$ is an equilateral triangle of side length 4. The points D , E and F are midpoints of \overline{BC} , \overline{CA} and \overline{AB} respectively. The weights 5, 1, 3, 2, 4 and 6 kg.wt are placed at points A , B , C , D , E and F respectively. Find the center of gravity of the system from B .

Important notes: The free suspension of a rigid body

If a rigid body is suspended from a point A freely, then its center of gravity lies C on the vertical line passing through the suspension point since the body, in this case, is in equilibrium under the action of the two shown forces in figure (6) which are:

- (1) The tension in the string, (2) The weight of the body and works vertically downwards

Thus, those two force are to be of equal magnitude, opposite directions and having the same line of action. Hence the center of gravity of the body G must lie along the vertical line connecting point A

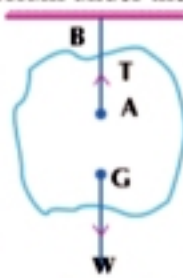


Figure (6)

The center of gravity of uniform rods and laminas

- 1- The center of gravity of a uniform rod is at its midpoint.
- 2- The center of gravity of a uniform fine lamina bounded by a parallelogram is at its geometrical center (intersection point of the diagonals)
- 3- The center of gravity of a uniform lamina bounded by a triangle is the intersection point of the medians of this triangle (centroid point).
- 4- The center of gravity of a uniform lamina in the shape of a circle is its center.

Example The center of gravity of a uniform rod

3. A uniform rod \overline{AC} of length $2L$ is bent at its midpoint B , then suspended from end A freely. If \overline{BC} is horizontal as the rod is in equilibrium, prove that $\cos(\angle ABC) = \frac{1}{3}$.

Solution

Let the weight of the rod \overline{AB} be equal to (w) and act at its midpoint M_2 and the weight of the rod \overline{BC} be equal to (w) and act at its midpoint M_1

Note that : $AB = BC$ and $m(\angle ABC) = \theta$

\therefore The rod is in equilibrium when it is suspended from point A and \overline{BC} is horizontal.

\therefore The center of gravity located on the line \overleftrightarrow{AD} .

$\therefore W \times M_1 D = W \times N D \quad \therefore M_1 D = ND \dots \dots (1)$

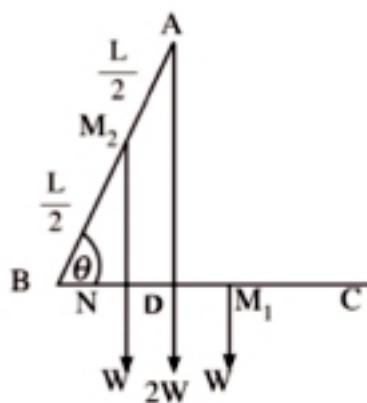


Figure (7)

From geometry of the figure:

$$\therefore \overline{AD} \parallel \overline{M_2N}, m_2 \text{ is midpoint } \overline{AB} \quad \therefore N \text{ is midpoint } \overline{BD}$$

$$\therefore DN = BN \dots\dots\dots (2) \quad \text{from (1) and (2)}$$

$$\therefore m_1 D = ND = BN = \frac{1}{3} \times \frac{L}{2} = \frac{L}{6}$$

$$\therefore \cos \theta = \frac{NB}{BM_2} = \frac{L}{6} \div \frac{L}{2} = \frac{L}{6} \times \frac{2}{L} = \frac{1}{3}$$

Try to solve

- 3 A thin wire of uniform thickness and density in the form of a trapezium ABCD in which $AB = 15 \text{ cm}$, $BC = 12 \text{ cm}$, $CD = 10 \text{ cm}$, $m(\angle ABC) = m(\angle BCD) = 90^\circ$. Find the distance between the center of gravity of the wire and the two sides \overline{AB} and \overline{BC} .

Example

- 4 A uniform fine lamina in the form of a parallel gram ABCD in which $AB = 40 \text{ cm}$, $BC = 20 \text{ cm}$ and $m(\angle BCA) = 90^\circ$. If the lamina is suspended at point (E) on \overline{CD} to be in equilibrium when \overline{CD} is horizontal, find the length of \overline{CE} .

Solution

\therefore The vertical line passing through the suspension point must pass through the center of gravity of the lamina.

$$\therefore \overline{EM} \text{ is vertical}$$

$$\therefore \overline{CD} \text{ is horizontal } \therefore m(\angle MEC) = 90^\circ.$$

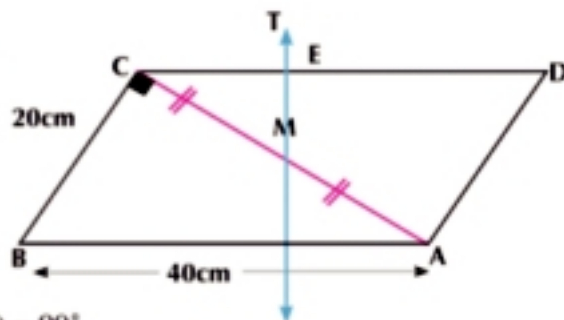


Figure (8)

From the triangle ABC, we find that $m(\angle ACB) = 90^\circ$,
 $BC = \frac{1}{2} AB$

$$\therefore m(\angle CAB) = 30^\circ, \quad AC = 20\sqrt{3} \text{ cm}$$

$$\therefore MC = 10\sqrt{3} \text{ cm}, \quad \therefore m(\angle MCE) = 30^\circ$$

$$\therefore CE = MC \cos 30^\circ \quad \therefore CE = 10\sqrt{3} \times \left(\frac{\sqrt{3}}{2}\right) = 15 \text{ cm}.$$

Try to solve

- 4 A uniform squared lamina of weight (w) is suspended freely from the vertex A and a weight of $\frac{1}{4} w$ is fixed at vertex B. Prove that the tangent of the angle of inclination of the diagonal \overline{AC} to the vertical in the equilibrium position is equal to $\frac{1}{5}$.

Critical thinking:

Prove that the center of gravity of a uniform lamina in the form of a triangle is coincident with the weight of three equal masses placed at the vertices of the triangle.

Example Finding the center of gravity of a triangular lamina

- 5 A uniform fine lamina of mass 3 kg in the form of a figure triangle ABC in which $AB = AC$, $BC =$ the height of the triangle = 6 cm. the masses 3, 2, 3 and 4 kg are fixed at points A, B, C and D respectively, such that D is the midpoint of \overline{AB} . Determine the center of gravity of the system and prove that the center of gravity is distant 4 cm from C. If the lamina is suspended from C freely, find in the equilibrium position the measure of the angle of inclination for each of \overline{BC} and \overline{AC} to the vertical.

Solution

Consider the two orthogonal directions are \overrightarrow{CX} and \overrightarrow{CY} , then point C is the origin point.

Distribute the mass of lamina 3 kg at the vertices A, B and C into 3 equal masses each of 1 kg, then the fixed masses at A, B and C are 4, 3, 4 and 4 kg as shown in the figure.

The data of the problem can be placed in the table as follows:

	at A	at B	at C	at D
Mass	4	3	4	4
X	3	6	0	4.5
Y	6	0	0	3

$$\therefore x_G = \frac{4 \times 3 + 3 \times 6 + 4 \times 0 + 4 \times 4.5}{4 + 3 + 4 + 4} = 3.2 \text{ cm},$$

$$\therefore y_G = \frac{4 \times 6 + 0 + 0 + 4 \times 3}{4 + 3 + 4 + 4} = 2.4 \text{ cm},$$

$$\therefore G = (3.2, 2.4)$$

\therefore Let the distance between the center of gravity G and C be GC

$$\therefore GC = \sqrt{(3.2)^2 + (2.4)^2} = 4 \text{ cm}$$

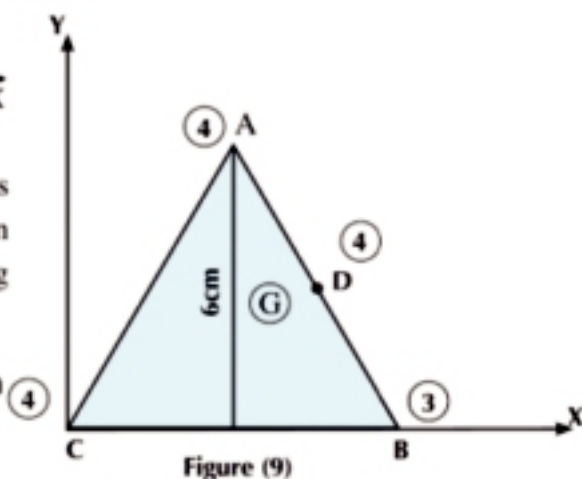


Figure (9)

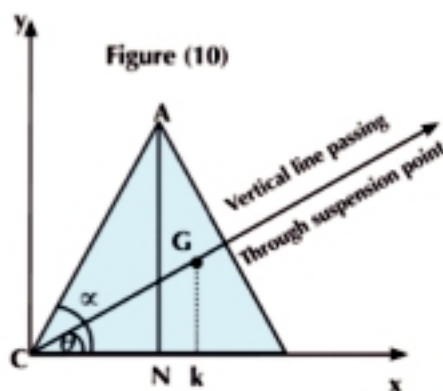


Figure (10)

Finding the measure of the angle of inclination \overline{CB} to the vertical

Draw \overline{CG} to be the vertical line passing through the suspension point (C) and let θ be the measure of the angle of inclination of \overline{CB} to \overline{CG} , and draw $\overline{GK} \perp \overline{CB}$.

$$\therefore \tan \theta = \frac{GK}{KC} = \frac{2.4}{3.2} = 0.75 \qquad \therefore \theta = 36^\circ 52' 12''$$

Finding the measure of the angle of inclination of \overline{CA} to the vertical:

Calculate the measure of angle ACB Let it be α where $\tan \alpha = \frac{AN}{NC} = \frac{6}{3} = 2$

$$\therefore \alpha = 63^\circ 26' 6''$$

$$\begin{aligned} \therefore \text{The measure of the angle of inclination of } \overline{CA} \text{ to the vertical } CG &= \alpha - \theta \\ &= 63^\circ 26' 6'' - 36^\circ 52' 12'' = 26^\circ 33' 54'' \end{aligned}$$

Try to solve

- 5 In figure (11) A fine lamina of mass 300 gm in the form of an equilateral triangle ABC of side length 12 cm, A mass of 100 gm is stuck to the lamina at the trisection point \overline{AB} from A. Determine the center of gravity of the system about the two orthogonal axes \overrightarrow{AX} and \overrightarrow{AY} .

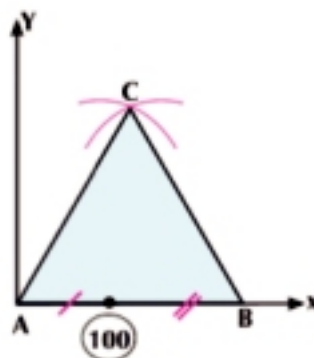


Figure (11)

Example Finding the center of gravity of a squared lamina

- 6 A uniform fine lamina in the form of a square ABCD of side length L in which E and F are the midpoints of \overline{AB} and \overline{AD} respectively. If the triangle AEF is bent about the side \overline{EF} such that point A is coincident with the square M, determine the center of gravity of the lamina in its new position.

Solution

Consider the mass of the lamina is (4m) and F' is the midpoint of the side \overline{BC} in the new position.

Let the lamina be formed of three parts as follows:

- The triangular lamina (formed from two layers) FEM whose mass (m) is equal to one fourth of the mass of the lamina and let its center of gravity be G_1
- The squared lamina EBF'M whose mass is (m) and let its center of gravity be G_2
- The rectangular lamina FFCD whose mass is (2m) and let its center of gravity be G_3 .

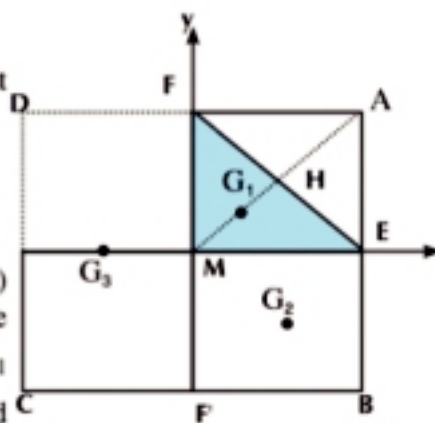


Figure (12)

Then, the lamina in its new position is equivalent to a system formed of three masses, the mass of (m) at G_1 and another mass equal to it at G_2 and the mass of (2m) at G_3 as shown in figure (12).

Center of Gravity

Consider \overline{MX} , \overline{MY} are two orthogonal directions such that the first axis passes through point E and the second axis passes through point F, as shown in the same figure, and consider point h is the midpoint of \overline{EF} then:

$$\begin{aligned} AM &= \frac{1}{2} \sqrt{AE^2 + EM^2} \\ &= \frac{1}{2} \sqrt{\frac{L^2}{4} + \frac{L^2}{4}} = \frac{1}{2} \sqrt{\frac{L^2}{2}} = \frac{1}{2\sqrt{2}} L \end{aligned}$$

$$\therefore MG_1 = \frac{2}{3} MH = \frac{2}{3} \times \frac{1}{2\sqrt{2}} L = \frac{1}{3\sqrt{2}} L$$

$$\therefore G_1 = \left(\frac{1}{3\sqrt{2}} L \cos 45^\circ, \frac{1}{3\sqrt{2}} L \sin 45^\circ \right) = \left(\frac{L}{6}, \frac{L}{6} \right)$$

The center of gravity G_2 for the squared lamina EBFM lies in the center of the square i.e. $G_2 = \left(\frac{L}{4}, \frac{L}{4} \right)$

The center of gravity G_3 for the rectangular lamina FFCD lies in its center also i.e. $G_3 = \left(-\frac{L}{4}, 0 \right)$

To find the center of gravity of the lamina in its new position, use the following :

$$\therefore x_G = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \dots + m_n x_n}{m_1 + m_2 + m_3 + \dots + m_n}$$

$$\therefore x_G = \frac{m \times \frac{L}{6} + m \times \frac{L}{4} + 2m \times \left(-\frac{L}{4}\right)}{m + m + 2m} = -\frac{1}{48} L$$

$$\therefore y_G = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3 + \dots + m_n y_n}{m_1 + m_2 + m_3 + \dots + m_n}$$

$$\therefore y_G = \frac{m \times \frac{L}{6} + m \times \left(-\frac{L}{4}\right) + 2m \times \text{zero}}{m + m + 2m} = -\frac{1}{48} L$$

Solution

In figure (13) and through out data of the table, the following components are determined:

	G_1	G_2	G_3	G_4
Mass	m	m	m	m
X	$\frac{L}{6}$	$\frac{L}{4}$	$-\frac{L}{4}$	$-\frac{L}{4}$
Y	$\frac{L}{6}$	$-\frac{L}{4}$	$-\frac{L}{4}$	$\frac{L}{4}$

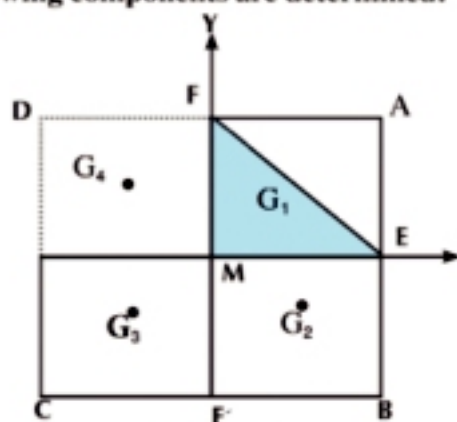


Figure (13)

$$x_G = m \times \frac{\frac{L}{6} + \frac{L}{4} - \frac{L}{4} - \frac{L}{4}}{4m} = -\frac{1}{48}L$$

$$y_G = m \times \frac{\frac{L}{6} - \frac{L}{4} - \frac{L}{4} + \frac{L}{4}}{4m} = -\frac{1}{48}L$$

P Try to solve

- 6 A fine lamina of a uniform density in the form of a rectangle ABCD in which AB = 6 cm, BC = 10 cm and E ∈ AD such that AE = 6 cm, if the triangle ABE is bent about the side BE until AB is coincident with BC completely, find the position of the center of gravity of the lamina after bending it with respect to \overline{CB} , \overline{CD} .



Exercises 6 - 1

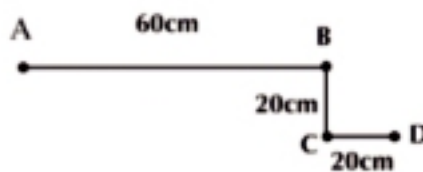
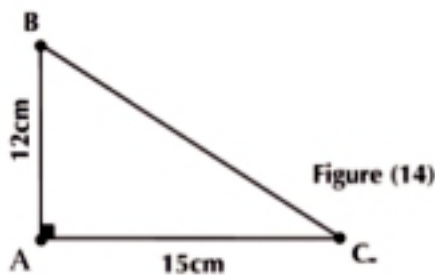


First: put (✓) or (X) for each phrase of the following:

- 1 The center of gravity of a rigid body is constant and it is not necessary to lie on a particle of the particles of this body.
- 2 If a non-uniform lamina bounded by a triangle is suspended from one of its vertices freely, then the vertical line passing through the suspension point passes through the intersection point of the medians of the triangle.
- 3 If three equal masses are placed at the midpoints of an equilateral triangle, then its center of gravity lies on the intersection point of the medians of the triangle.
- 4 The center of gravity of a uniform fine lamina bounded by a triangle is coincident with the center of gravity of three equal masses placed at the vertices of this triangle.
- 5 The center of gravity of a uniform fine lamina bounded by a parallelogram lies at the intersection point of its diagonals.
- 6 If four equal masses are placed at the vertices of an isosceles trapezium, then the center of gravity of the system acts at the intersection point of its diagonals.
- 7 If a rigid body is suspended freely, then the vertical straight line passing through the center of gravity of this body passes through the suspension point.
- 8 The center of gravity of two physical points separated by a constant distance lies on the straight segment drawn between them and divides its length by a ratio equal to the ratio between their masses.
- 9 If a lamina of uniform thickness and density and bounded by an equilateral triangle is suspended freely from one of its vertices, then the side opposite to this vertex is horizontal.
- 10 If four equal masses are placed at the vertices of a parallelogram, then the center of gravity of the system acts at the intersection point of the diagonals of the parallelogram.

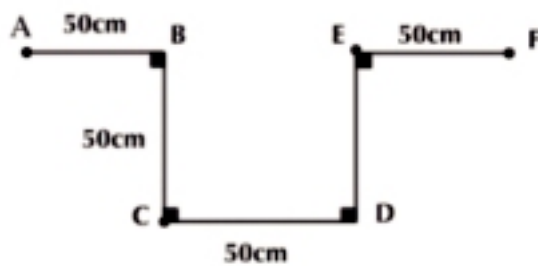
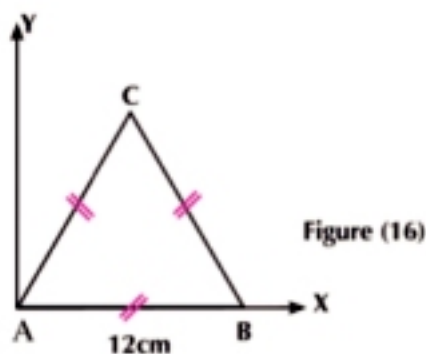
Second: Answer the following questions

- 11 Find the center of gravity of two physical particles of weights 4 newtons and, 6 newtons and the distance between them is 5 meters.
- 12 Where does the center of gravity of a system, made up of three masses distributed as follows: $m_1 = 1$ kg at the position $p_1 (0, 0)$, $m_2 = 1$ kg at the position $p_2 (3, 0)$, $m_3 = 2$ kg at the position $p_3 (3, 4)$, lie.
- 13 Find the center of gravity of the following distribution:
 $w_1 = 3$ newtons at $(4, -1)$, $w_2 = 5$ newtons at $(0, 3)$ and
 $w_3 = 4$ newtons $(-2, 3)$ where the forces are like.
- 14 Determine the center of gravity for each of the following systems according to the given data in the table:



Mass	20 kg	40 kg	30 kg
Position	at A	at B	at C

Mass	m	m	m	m
Position	at A	at C	at B	at D



Mass	4 kg	5 kg	3 kg
Position	at A	at B	at C

Mass	8 kg.wt	3 kg.wt	2 kg.wt	2 kg.wt
Position	at A	at C	at E	at F

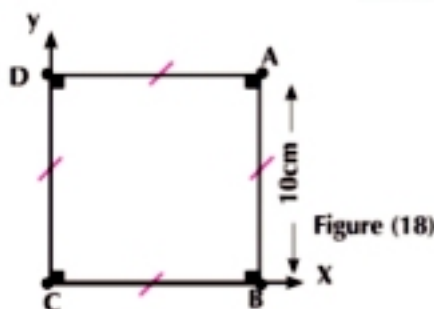


Figure (18)

Mass	20 gm	30 gm	10 gm	40 gm
Position	at A	at B	at C	at D

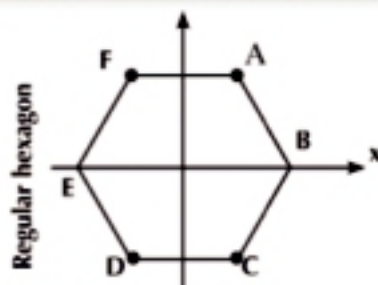


Figure (19)

Mass	10 gm	15 gm	5 gm	20 gm
Position	at A	at C	at D	at F

Third: Answer the following questions:

- 15 AB is a uniform rod of length 90 cm and mass 5 kg. C and D are its two trisection points from end A. Masses of magnitudes 1, 2, 3 and 4 kg are placed at A, B, C and D respectively. Determine the distance from the center of gravity of the system to end A.
- 16 AB is a non-uniform rod of length 30 cm and weight 500 gm.wt. Two weights of magnitudes 100 and 200 gm.wt are fixed at the two ends A and B of the rod respectively to make the center of gravity of the system at the midpoint of the rod. Determine the center of gravity of the rod about end A.
- 17 ABC is a lamina in the form of an equilateral triangle of mass 3kg and M is its center of gravity. Masses of magnitudes 2, 2 and 11 kg are placed at the vertices A, B and C respectively. Prove that the center of gravity of the system lies at the midpoint of \overline{MC} .
- 18 A squared lamina of a uniform density of weight 40 gm.wt is freely suspended from the vertex A and a weight of a magnitude 10 gm.wt is fixed at vertex B. Find the measure of the angle of inclination of the diagonal \overline{AC} to the vertical in the equilibrium position.
- 19 Figure (24): AB is a fine wire of a uniform density is bent at B and C. Find the distance from the center of gravity from each of \overline{AB} and \overline{CB} , then find in the equilibrium position, the measure of the angle of inclination of \overline{AB} to the vertical if the wire is freely suspended from A.
- 20 ABCD is a square of side length L, drawn on \overline{BC} an isosceles triangle BCE such that the vertex E lies outside the square. Find the center of gravity of the lamina of uniform thickness and density bounded by the resulted figure given that the square side length is twice the triangle height length.
- 21 A lamina of a uniform density is made up of two parts, the rectangle : ABCD in which AB = 12 cm, BC = 16 cm and an isosceles triangle CED in which DE = CE = 10 cm and the vertex E lies outside the rectangle. Determine the center of gravity of the lamina.
- 22 ABCD is a lamina of a uniform thickness and density in the form of a rectangle in which AB = 12 cm, BC = 16 cm and point E is the intersection point of its diagonals \overline{AC} and \overline{BD} , the triangle AED is separated and fixed above the triangle BEC. Find the center of gravity of the lamina in this case. If the lamina is freely suspended from point C, find the tangent of the angle of \overline{CB} to the vertical.

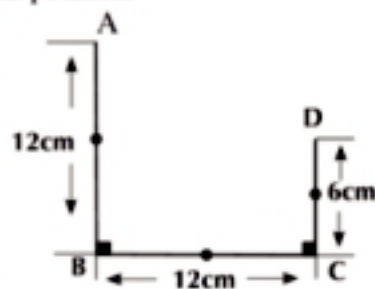


Figure (24)

You negative

Key terms

- Negative mass
- Symmetry

Materials

- Scientific calculator
- Computer graphics.

Preface

You have previously learned that the center of gravity of a rigid body is a constant point in the body through which the line of action of the resultant of the weights of the particles which this body is made up of passes. We have also found the components of the center of gravity in a 2 - dimensional system in the orthogonal coordinate system. Then you learned that the center of gravity of a rigid body suspended freely lies on the vertical straight line passing through its suspension point.

In this lesson, you are going to learn the negative mass method to calculate the center of gravity of a body after removing a part away. Besides, you will identify the center of gravity of some symmetrical bodies.

The negative mass method

Considering a body of mass m and center of gravity G . If the left side is (cut off) removed as in (figure 20) G_1 is the center of gravity of the removed part and G_2 is the center of gravity of the remaining right part. If \vec{r}_1 , \vec{r}_2 and \vec{r}_G are the position vectors G_1 , G_2 and G respectively about the origin point (O) considering that the mass of the origin body (m) and the removed part (considering its mass is negative) is $(-m_1)$ then the mass of the remainder ($m - m_1$) so \vec{r}_G is given by the relation:

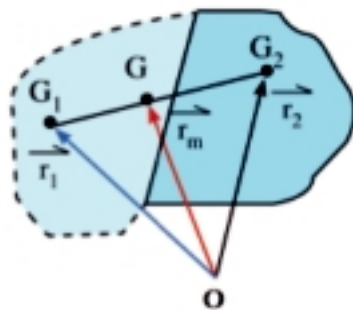


Figure (20)

$$\vec{r}_G = \frac{m_1 \vec{r}_1 + (m - m_1) \vec{r}_2}{m} \text{ by multiplying both by } m, \text{ then:}$$

$$m \vec{r}_G = m_1 \vec{r}_1 + (m - m_1) \vec{r}_2$$

$$\text{i.e. : } (m - m_1) \vec{r}_2 = m \vec{r}_G - m_1 \vec{r}_1$$

$$\text{i.e. : } \vec{r}_2 = \frac{m \vec{r}_G - m_1 \vec{r}_1}{m - m_1}$$

By substituting \vec{r}_G and \vec{r}_1 in terms of their algebraic components in the direction of the orthogonal axes \vec{OX} and \vec{OY} , we get the coordinates of the remaining part which are:

$$x_2 = \frac{m x - m_1 x_1}{m - m_1}, y_2 = \frac{m y - m_1 y_1}{m - m_1}$$

Where (x, y) is the center of gravity of the origin body whose mass is m and (x_1, y_1) is the center of gravity of the removed (cut off) part whose mass is m_1 . This rule means that when you want to find the center of gravity of the remaining part, it is considered as if it is made up of two bodies which are:

- (1) The origin body and the mass is m
- (2) The cut off body considering its mass is $(-m_1)$ and it was named after that- the negative mass method.

Example

- 1 Four equal masses each of a magnitude 100 gm are placed at the vertices of the square ABCD.

First: Determine the center of gravity of the system about \vec{AB} and \vec{AD} .

Second : If the mass placed at vertex C is removed, determine the center of gravity of the remaining system.

Solution

First : Let the square side length = L cm, point E is the center of the square ABCD and A is the origin point as in figure (21).

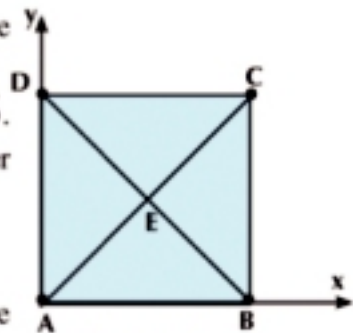


Figure (21)

	at A	at B	at C	at D
Mass	100	100	100	100
x	0	L	L	0
y	0	0	L	L

$$\therefore x_G = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + m_4 x_4}{m_1 + m_2 + m_3 + m_4}$$

$$\therefore x_G = \frac{100 \times 0 + 100 \times L + 100 \times L + 100 \times 0}{100 + 100 + 100 + 100} = \frac{L}{2} \text{ cm}$$

$$\therefore y_G = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3 + m_4 y_4}{m_1 + m_2 + m_3 + m_4}$$

$$\therefore y_G = \frac{100 \times 0 + 100 \times 0 + 100 \times L + 100 \times L}{100 + 100 + 100 + 100} = \frac{L}{2} \text{ cm}$$

The coordinate of the center of gravity of the system is $(\frac{L}{2}, \frac{L}{2})$ i.e at the center of the square at point E

Second: After removing the existed mass at c i.e 100 gm, then:

The center of gravity of the origin system (where the mass $m = 400$ gm) is point E = $(x, y) = (\frac{L}{2}, \frac{L}{2})$

The center of gravity of the removed mass $m_1 = 100$ gm at the vertex C is (L, L)

Center of Gravity

The center of gravity of the remaining part, let $G_2 = (x_2, y_2)$ identified from:

$$x_2 = \frac{m x - m_1 x_1}{m - m_1} = \frac{400 \times \frac{L}{2} - 100 \times L}{400 - 100} = \frac{L}{3} \text{ cm}$$

$$y_2 = \frac{m y - m_1 y_1}{m - m_1} = \frac{400 \times \frac{L}{2} - 100 \times L}{400 - 100} = \frac{L}{3} \text{ cm}$$

\therefore The center of gravity of the remaining system is $(\frac{L}{3}, \frac{L}{3})$

Try to solve

- 1 Can you solve example (1) using the different ways you learned in the last lesson? Explain and write down such ways if existed.

Example

- 2 Figure (22) : A B C is an equilateral triangle of side length 20 cm, D is the intersection point of its medians and, E is the midpoint of BC, masses of magnitudes 10, 20, 30, 30 and 50 are fixed at points A, B, C, D and E respectively. Identify the centre of gravity of the system. Where does the center of gravity of the remaining system about vertex C lie if the mass fixed at B is removed?

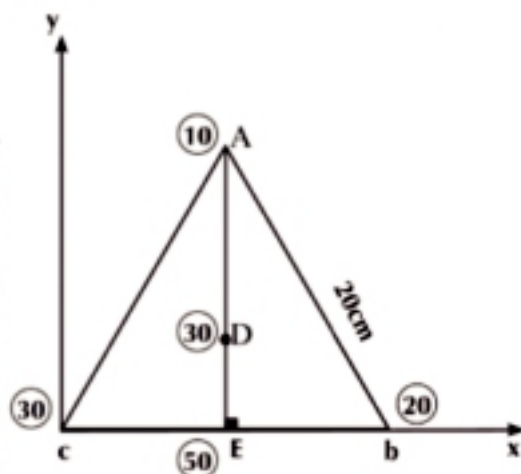


Figure (22)

Solution

Identifying the center of gravity of a system

Take \overrightarrow{CX} and \overrightarrow{CY} two orthogonal directions considering that C is the origin point.

$$AE = 20 \sin 60^\circ = 10\sqrt{3}, \quad DE = \frac{10\sqrt{3}}{3}$$

By forming the following coordinate table:

	at C	at E	at B	at A	at D
Mass	30	50	20	10	30
x	0	10	20	10	10
y	0	0	0	$10\sqrt{3}$	$\frac{10\sqrt{3}}{3}$

$$\therefore x_G = \frac{50 \times 10 + 20 \times 20 + 10 \times 10 + 30 \times 10}{30 + 50 + 20 + 10 + 30} = \frac{65}{7}$$

$$y_G = \frac{10 \times 10\sqrt{3} + 30 \times \frac{10\sqrt{3}}{3}}{30 + 50 + 20 + 10 + 30} = \frac{10\sqrt{3}}{7}$$

i.e the two coordinates of the center of gravity are $(\frac{65}{7}, \frac{10\sqrt{3}}{7})$ from point C.

When removing the mass existed at B:

$$\therefore x_2 = \frac{m_{xG} - m_1 x_1}{m - m_1} = \frac{140 \times \frac{65}{7} - 20 \times 20}{140 - 20} = \frac{15}{2}$$

$$y_2 = \frac{m_{yG} - m_1 y_1}{m - m_1} = \frac{140 \times \frac{10\sqrt{3}}{7} - 20 \times 0}{140 - 20} = \frac{5\sqrt{3}}{3}$$

\therefore The new center of gravity after cutting of the mass 20 at B is $(\frac{15}{2}, \frac{5\sqrt{3}}{3})$

P Try to solve

- 2 Five equal masses are placed at the vertices A, B, C, D and E of the square ABCD where E is the intersection point of its diagonals and the side length of the square is 12 cm. Identify the center of gravity of the system. If the mass placed at B is removed, identify the center of gravity of the remaining system about \overline{AB} and \overline{AD} .

The center of gravity of some symmetric bodies

The symmetry of a fine lamina of a uniform density by considering \overline{AB} The axis of symmetry of the uniform lamina, then it divides the lamina into two completely symmetrical parts in regard to the shape and in turn to the mass as in figure (23).

Consider that G_1, G_2 are the two centers of gravity of those two parts, then it is clear that the axis of symmetry intersects the line segment $\overline{G_1G_2}$ on the orthogonality from its midpoint.

Since the center of gravity of the lamina is the same center of gravity of two equal masses placed at G_1 and G_2 , then it lies at the midpoint of $\overline{G_1G_2}$ i.e on the axis of symmetry we can deduce the following:

If a geometrical axis of symmetry of a fine lamina of a uniform density exists, then its center of gravity lies on this axis.

Some geometrical solids of uniform density

The symmetry of the geometrical solids is typically similar to the symmetry of the geometrical shapes by replacing the axis of symmetry with the plane of symmetry. Figure(24) shows that.

Thus, we can deduce that:

If a geometrical plane of symmetry of a solid of a uniform density is existed, then its center of gravity lies in this plane.

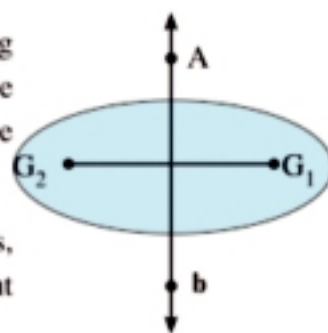


Figure (23)

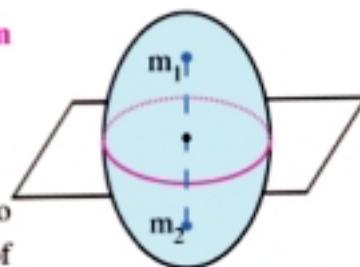


Figure (24)

Center of Gravity

From the previous symmetry of the uniform geometrical shape and the uniform geometrical solids, we can identify some special cases for the center of gravity as follows:

- 1 - The center of gravity of a wire of a uniform density in the form of a circle lies in the center of the circle.
- 2 - The center of gravity of a lamina of a uniform density in the form of a circle lies in the center of the circle.
- 3 - The center of gravity of a spherical crust (cortex) of a uniform density lies in the center of the sphere.
- 4 - The center of gravity of a solid sphere of a uniform density lies in the center of the sphere.
- 5 - The center of gravity of a solid of a uniform density in the form of a cuboid lies in its geometrical center.
- 6 - The center of gravity of a right circular cylindrical crust of a uniform density lies at the midpoint of its axis.
- 7 - The center of gravity of a right solid circular cylinder of a uniform density lies at the midpoint of its axis.
- 8 - The center of gravity of a uniform right prism lies at the midpoint of the axis parallel to its lateral edges and passing through the two centers of gravity of its two bases considering them as two fine laminas of uniform densities.

Example

- 3 A uniform fine board of an area 200 cm^2 . A circular hole of an area 40 cm^2 , is punctured (bored). If the distance between the center of the hole and the center of the board is 4 cm , determine the center of gravity of the remaining part of the board.

Solution

	Mass	Distance between center of gravity and E
Board	m	0
Hole	$-\frac{1}{5} m$	4
Remaining part	$\frac{4}{5} m$	x

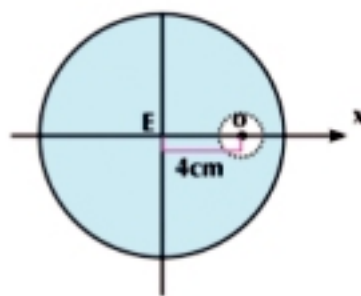


Figure (25)

Let the mass of the board be m

$$\therefore \text{the area of the board} = 200 \text{ cm}^2,$$

$$\text{the surface area of the hole} = 40 \text{ cm}^2.$$

$$\therefore \text{The mass of the hole} = -\frac{1}{5} m.$$

$$\therefore x_2 = \frac{m x_G - m_1 x_1}{m - m_1} = \frac{m \times 0 - \frac{1}{5} m \times 4}{m - \frac{1}{5} m} = \frac{-4 m}{5 m - m} = -1$$

\therefore The center of gravity of the remaining part is distant 1 cm from E in the direction \overrightarrow{OE} .

P Try to solve

- 3 A fine lamina of a uniform thickness and density in the form of a circular disc whose center is the origin point and the radius length is 6 unit length. Two circular discs, the center of one of them is $(-1, -3)$ and radius length is a unit length and the center of the other disc is $(1, 2)$ and radius length is 3 unit length are cut off. Find the center of gravity of the remaining part of the origin disc.

Example

- 4 A uniform fine lamina in the form of a rectangle ABCD in which $AB = 30$ cm, $BC = 80$ cm. The triangle ABE where E is the midpoint of \overline{AD} , is cut off, then the remaining part is freely suspended from the vertex C. Identify the measure of the angle of inclination of the side \overline{CB} to the vertical in the equilibrium position, and if the mass of the lamina is m , what is the mass which should be placed at the vertex D until \overline{BC} inclines at 45° to the vertical in the equilibrium position?

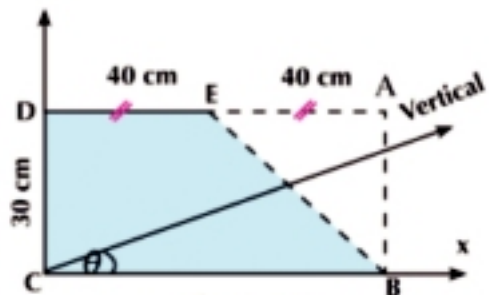


Figure (26)

Solution

First : Finding the measure of the angle of inclination of side \overline{CB} to the vertical

$$\frac{\text{Area of triangle ABE}}{\text{Area of square ABCD}} = \frac{\frac{1}{2} \times 30 \times 40}{30 \times 80} = \frac{1}{4}$$

$$\therefore \text{Mass of rectangle ABCD} = m \text{ and mass } \triangle ABE = -\frac{1}{4} m$$

Construct the following coordinate table:

	Rectangle	Triangle
Mass	m	$-\frac{1}{4}m$
x	40	$\frac{80 + 80 + 40}{3} = \frac{200}{3}$
y	15	$\frac{0 + 30 + 30}{3} = 20$

$$\therefore x_2 = \frac{m \times 40 - \frac{1}{4}m \times \frac{200}{3}}{m - \frac{1}{4}m} = \frac{280}{9}$$

$$y_2 = \frac{m \times 15 - \frac{m}{4} \times 20}{m - \frac{1}{4}m} = \frac{40}{3}$$

$$\tan \theta = \frac{y}{x} = \frac{40}{3} \div \frac{280}{9} = \frac{3}{7}$$

$$\therefore \theta = 23^\circ 12'$$

Second : When a weight of a magnitude W is suspended at D until \overline{BC} inclines at 45° to the vertical to be in an equilibrium position.

$$\tan \theta = \frac{y}{x} \qquad \therefore \tan 45^\circ = \frac{y}{x}$$

$$\therefore 1 = \frac{y}{x} \qquad \therefore y = x$$

	Rectangle	Triangle	Weight at O
Mass	m	$-\frac{1}{4}m$	w
x	40	$\frac{200}{3}$	0
y	15	20	30

$$\therefore x_3 = y_3$$

$$\therefore \frac{40m - \frac{200}{12}m}{\frac{3}{4}m + W} = \frac{15m - 5m + 30W}{\frac{3}{4}m + W}$$

$$\therefore \frac{40}{3}m = 30W \qquad \therefore W = \frac{4}{9}m.$$

P Try to solve

- 4 A uniform fine lamina in the form of a rectangle $ABCD$ in which $AB = 6$ cm and $BC = 8$ cm, A squared piece of side length 4cm, is cut off. Find the distance between the center of gravity of the remaining part and both of \overline{CD} and \overline{CB} . If the remaining part is freely suspended from the vertex C , find the tangent of the angle of inclination of \overline{CB} to the vertical in the equilibrium position.



Exercises 6 - 2

**First: Complete the following:**

- 1 The constant point through which the line of action of the resultant of the weights of the particles from which the body is made up of whatever the position of the body change with respect to the ground is called _____
- 2 The center of gravity of the rigid body suspended freely lies on the vertical straight line passing through _____
- 3 The center of gravity of a fine rod of a uniform density lies at _____
- 4 The center of gravity of a uniform lamina bounded by a parallelogram lies at _____
- 5 The center of gravity of a uniform lamina bounded by a triangle lies at _____
- 6 If a geometrical axis of symmetry of a fine lamina of a uniform density is existed, then its center of gravity lies on _____
- 7 If a geometrical plane of symmetry of a solid of a uniform density is existed, then its center of gravity lies at _____
- 8 The center of gravity of a lamina of a uniform density bounded by a circle lies at _____
- 9 The center of gravity of a solid sphere of a uniform density lies at _____
- 10 The center of gravity of a solid of a uniform density in the form of a cuboid lies at _____
- 11 The center of gravity of a right circular cylindrical crust of a uniform density lies at point ...

Second: Answer the following questions:

- 12 Four equal masses are placed at the vertices A, B, C and D for a square of side length 80 cm. Then a fifth equal mass is added at its center. Identify the center of gravity of the system. If the mass existed at vertex A is removed, identify the center of gravity of the system using the negative mass method.
- 13 A uniform fine lamina in the form of a circular disc of radius length 30 cm. A part in the form of a circular disc of radius length 10 cm and its center is distant 20 cm from the center of the lamina is cut off. Find the center of gravity of the remaining part.

Center of Gravity

- 14 ABC is an equilateral triangle of side length 12 cm and G is its center of gravity. The triangle GBC is cut off. Identify the center of gravity of the remaining part.
- 15 A uniform fine lamina in the form of an isosceles triangle ABC in which $AB = AC$ and \overline{AD} is the height of the triangle of length 45 cm. A straight line is constructed parallel to the base \overline{BC} and passes through the center of gravity of the lamina to intersect \overline{AB} and \overline{AC} at E and F respectively. Prove that the center of gravity of the quadrilateral EBCF lies on \overline{AD} and is distant 7 cm from point D.
- 16 A uniform wire of length 100 cm is bent in the form of five sides of a regular hexagon ABCDEF. Identify the distance between the center of gravity of the wire and the center of the hexagon. If the wire is freely suspended from the end A, identify the measure of the angle of inclination of \overline{AB} to the vertical in the equilibrium position.
- 17 A uniform fine lamina bounded by rectangle ABCD where $AB = 30$ cm, $BC = 60$ cm. E is the midpoint of \overline{AD} and N is the midpoint of \overline{DC} . If the triangle EDN is separated from the lamina and the remaining part is freely suspended from point B, find the tangent of the angle which \overline{BC} makes with the vertical in the equilibrium position.
- 18 A fine lamina of a uniform density in the form of square ABCD of side length 36 cm, its two diagonals intersect at M, \overline{DM} is bisected at point E and the triangle EAD is separated. Identify the center of gravity of the remaining part of the lamina. If the lamina is freely suspended from point A until it gets in equilibrium in a vertical plane, find the inclination of \overline{AB} to the vertical.
- 19 A uniform lamina in the form of a square ABCD of side length 8 cm, If a circular disc of radius length 2 cm and distant 3 cm from each of \overline{AB} , \overline{BC} , Identify the distance between the center of gravity of the remaining part and both of \overline{DC} , \overline{AD} .
- 20 A uniform fine lamina bounded by a square ABCD of side length 40 cm. A circular hole of area 100 cm^2 and its center is at a point on the diagonal \overline{BD} and divides it internally in a ratio of 1 : 4 from B, is board, then suspended freely from vertex A. Identify the measure of the angle of inclination of side \overline{AB} to the vertical in the equilibrium position.

UNIT SUMMARY

Unit summary

- 1) **The center of gravity of a rigid body** is a constant point in the body through which the line of action of the resultant of the weights of the particles from which the body is made up of whatever the position of the body changes with respect to the ground.

2) **Notes about the center of gravity:**

The center of gravity of the rigid body changes by the change in its shape due to the change of the dimensions of the particles which the body is made up of.

The body of a uniform density is the body which the mass of the unit length, areas or volumes taken from any part of it is constant.

3) **The position vector of the center of gravity of a rigid body about the origin point:**

If m_1, m_2, m_3, \dots are the masses of the particles which the rigid body is made up of and, $\vec{r}_1, \vec{r}_2, \vec{r}_3, \dots, \vec{r}_n$ are the position vectors of these particles about the origin point, then the position vector \vec{r}_G of the center of gravity of the rigid body about the origin point is identified by the relation:

$$\vec{r}_G = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + m_3 \vec{r}_3 + \dots + m_n \vec{r}_n}{m_1 + m_2 + m_3 + \dots + m_n}$$

It is expressed in terms of the components of the center of gravity in the orthogonal Cartesian coordinate system as follows:

$$X_G = \frac{m_1 X_1 + m_2 X_2 + m_3 X_3 + \dots + m_n X_n}{m_1 + m_2 + m_3 + \dots + m_n}$$

$$Y_G = \frac{m_1 Y_1 + m_2 Y_2 + m_3 Y_3 + \dots + m_n Y_n}{m_1 + m_2 + m_3 + \dots + m_n}$$

- 4) **The free suspension of the rigid body** : The center of gravity of the rigid body suspended freely lies on the vertical straight line passing through the suspension point.
- 5) **The center of gravity of a uniform fine rod** : the center of gravity of a fine rod of a uniform density lies at its midpoint.
- 6) **The center of gravity of a uniform fine lamina in the form of a parallelogram**: The center of gravity of a uniform lamina bounded by a parallelogram lies at its geometrical center C (intersection point of diagonals).
- 7) **The center of gravity of a uniform fine lamina in the form of a triangle**: The center of gravity of a uniform lamina bounded by a triangle lies at the intersection point of the medians of the triangle.
- 8) **The negative mass method**: Considering the mass of the origin body is (m) and the part cut off (considering its mass is negative) is ($-m_1$), then the mass of the remaining part is ($m - m_1$) thus \vec{r}_2 is given by the relation:

UNIT SUMMARY

$$\vec{r}_2 = \frac{m \vec{r}_G - m_1 \vec{r}_1}{m - m_1}$$

The previous vector relation can be written in terms of the components in the direction of the orthogonal coordinates:

\vec{OX} , \vec{OY} , we obtain the follows:

$$x_2 = \frac{mx_G - m_1 x_1}{m - m_1}, y_2 = \frac{m y_G - m_1 y_1}{m - m_1}$$

- 9) **The symmetry of a fine geometrical lamina of a uniform density:** If a geometrical axis of symmetry for a fine lamina of a uniform density exists, then its center of gravity lies on the line of this axis.
- 10) **The symmetry of a geometrical solid of a uniform density:** If a geometrical plane of symmetry for a solid of a uniform density is existed then its center of gravity lies in this plane.
- 11) **Some special cases of the center of gravity :**
- The center of gravity of a wire of a uniform density in the form of a circle lies in the center of the circle.
 - The center of gravity of a lamina of a uniform density in the form of a circle lies in the center of the circle.
 - The center of gravity of a spherical crust (cortex) of a uniform density lies in the center of the sphere.
 - The center of gravity of a solid sphere of a uniform density lies in the center of the sphere.
 - The center of gravity of a solid of a uniform density in the form of a cuboid lies in its geometrical center.
 - The center of gravity of a right circular cylindrical crust of a uniform density lies at the midpoint of its axis.
 - The center of gravity of a right solid circular cylinder of a a uniform density lies at the midpoint of its axis.
 - The center of gravity of a uniform right prism lies at the midpoint of the axis parallel to its lateral edges and passing through the two centers of gravity of its two bases considering them as two fine laminas of uniform densities.

GENERAL EXERCISES

General Exercises

First : Choose the correct answer:

- ① The center of gravity of three equal masses each of 2 kg placed at the vertices of a right- angled triangle whose length of the two legs of the right angle are 3 cm and 4 cm is:
- a $(1, \frac{4}{3})$ b $(2, \frac{3}{2})$
 c $(\frac{4}{3}, 1)$ d $(\frac{3}{3}, 2)$
- ② The center of gravity of two physical particles separated by a constant distance lies on the line segment connecting them and divides its length in ratios:
- a direct. b inverse. c random. d constant.
- ③ The center of gravity of the next system : $m_1 = 1$ kg at $(2, 3)$, $m_2 = 2$ kg at $(-2, 1)$ $m_3 = 3$ kg at $(0, 1)$ is:
- a $(-\frac{1}{3}, \frac{4}{3})$ b $(\frac{7}{6}, \frac{4}{3})$ c $(-\frac{1}{3}, \frac{2}{3})$ d $(0, 1)$
- ④ The center of gravity of a system made up of two masses of 6 and 9 kg, wt distant 10 meters from each other is distant meters from the first mass:
- a 3 mater b 4 mater c 5 mater d 6 mater
- ⑤ The center of gravity of a uniform fine lamina in the form of an equilateral triangle of side length 12 cm is distant..... from one of the vertices of the triangle:
- a $2\sqrt{3}$ cm b $4\sqrt{3}$ cm c 6 cm d $6\sqrt{3}$ cm
- ⑥ If uniform fine lamina in the form of an equilateral triangle is suspended by a string from a point on one of its edges dividing it in ratio 1 : 2, then the angle of inclination of this edge to the vertical is equal to :
- a 22.5° b 30° c 45° d 60°
- ⑦ In the opposite figure, ABCD is a wire of length 32 cm in which $AB = 2 BC = 2 CD = 16$ cm , then the distance between the center of gravity of the wire and both \overrightarrow{BC} and \overrightarrow{BA} respectively is:
- a $(3, 3)$ b $(4, 4)$ c $(3, 5)$ d $(4, 8)$

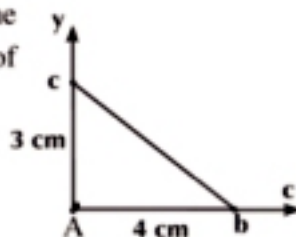


Figure (35)



Figure (36)

Second: Answer the following question :

- ⑧ ABC is a right angled triangle at B , in which $AB = 3$ cm and $BC = 4$ cm . Three equal masses each of a magnitude m are placed at B , the midpoint of \overline{AB} and midpoint of \overline{AC} find the center of gravity of these three masses.

- 9 A fine lamina of a uniform density bounded by the right-angled triangle ABC at B in which $AB = BC = 9$ cm. If the triangle ABM where M is the center of gravity of the lamina is cut off and the remaining part is freely suspended from point B, Find the tangent of the angle of inclination of \overline{BC} to the vertical in the equilibrium position.
- 10 AB is a uniform rod of length 24 cm and mass 2 kg. A mass of a magnitude 2 kg is attached at point A and another mass of a magnitude 3 kg is attached at point C lying on the rod and is distant 8 cm from point B. Find the distance between the center of gravity of the system and point B.
- 11 ABCD is a square of side length 4 cm. The masses 6, 4, 3 and 2 gm are attached at A, B, C and D respectively. Another mass of magnitude 10 gm is attached at the midpoint of \overline{AB} . Identify the distance between the center of gravity of the system and both \overline{CD} and \overline{CB} .
- 12 A fine wire of a uniform thickness and density is bent in the form of a right-angled triangle ABC at B in which $AB = 3$ cm and $BC = 4$ cm. Find the distance between the center of gravity of the wire and both \overline{BA} and \overline{BC} , then find the distance between the center of gravity of the wire and point B.
- 13 A fine wire of uniform thickness and density and length 40 cm. If the wire is bent in the form of a trapezium ABCD in which $AB = 16$ cm, $CD = 8$ cm, $DA = 6$ cm, $m(\angle DAB) = m(\angle CDA) = 90^\circ$. Find the distance between the center of gravity of the wire and the two sides \overline{AD} and \overline{AB} . If the wire is freely suspended from A, Find the tangent of the angle which \overline{AB} makes with the vertical in the equilibrium position.
- 14 A fine lamina of uniform thickness and density in the form of a trapezium ABCD in which $m(\angle A) = m(\angle D) = 90^\circ$, $CD = 40$ cm, $AD = 60$ cm and $AB = 120$ cm. Identify the distance between the center of gravity of the lamina and both \overline{AD} , \overline{AB} .
- 15 A wire of uniform thickness and density, length 120 cm and mass 600 gm is bent in the form of a right-angled triangle at B where $AB = 30$ cm, If a mass of a magnitude m kg is attached at vertex A, then the wire is freely suspended from vertex B to be in equilibrium when \overline{AC} is horizontal, find m .
- 16 A fine lamina of uniform thickness and density in the form of a circular disc whose center is the origin point and radius length 24 cm, Two circular discs the center of one of them is $(-2, -12)$ and radius length is 4 cm where the center of the other disc is $(6, 10)$ and the radius length is 12 cm are cut off. Find the center of gravity of the remaining part of the disc.
- 17 ABCD is a uniform fine lamina in the form of a rectangle in which $AB = 40$ cm, $BC = 60$ cm, and its diagonals intersect at M. The triangle BCM is cut off and the remaining part is freely suspended from vertex C. Identify the tangent of the angle of inclination of \overline{CB} to the vertical in the equilibrium position.
- 18 ABCD is a uniform fine lamina in the form of a square of side length 48 cm and mass 40 gm. The two points L and M are the midpoints of \overline{AB} and \overline{AD} respectively. The triangle ALM

is cut off and a mass equivalent to the mass of the removed triangle is attached at each of C and D. Another mass of twice the mass of the removed triangle is attached at B. If the system is freely suspended from point C, Find the tangent of the angle of inclination of \overline{BC} to the vertical in the equilibrium position.

- 19 ABC is a fine lamina of uniform thickness and density in the form of a right - angled triangle at B where $AB = 12$ cm, $BC = 20$ cm and X , Y and Z are the midpoints of \overline{AB} , \overline{BC} and \overline{CA} respectively. The triangle CYZ is cut off and coincided with the triangle Y B X. If the system is freely suspended from point B. Find the tangent of the angle of inclination of \overline{BC} to the vertical in the equilibrium position.
- 20 A fine lamina of uniform thickness and density in the form of an isosceles triangle ABC where $AB = AC = 26$ cm, $BC = 20$ cm. $\overline{AD} \perp \overline{BC}$ is drawn to intersect \overline{BC} at D , If E is the midpoint of \overline{AD} and the triangle EBC is cut off, find the distance between of the center of gravity of the remaining part and point E.
- 21 A fine lamina of uniform thickness and density in the form of a square ABCD of side length 48 cm and M is the intersection point of its diagonals. The triangle CMD is cut off, then stuck on the triangle CMB such that \overline{MD} is coincident to \overline{MB} . Find the distance between the center of gravity of the lamina and both \overline{BA} and \overline{BC} .
- 22 A fine lamina of uniform thickness and density in the form of a rectangle ABCD whose center is M where $AB = 16$ cm and $BC = 20$ cm. The two points E and F are taken on \overline{AB} where $AE = BF = 3$ cm , If the triangle MEF is cut off, Find the distance between the center of gravity of the remaining part and both \overline{CD} and \overline{AD} . If this part is freely suspended from D , Find the tangent of the angle which \overline{DA} makes to the vertical in the equilibrium position.
- 23 Masses of magnitudes 10 , 20 , 10 , 30 , 10 and 40 are attached at vertices A , B , C , D , E and F respectively for a uniform hexagon of side length 60 cm. Find the distance between the center of gravity of this system and the center of the hexagon.
- 24 A fine lamina of a uniform density in the form of a rectangle ABCD in which $AB = 25$ cm and $BC = 16$ cm. Let point $E \in \overline{BC}$ and point $F \in \overline{BA}$ such that $BE = 10$ cm The triangle BEF is cut off and the lamina rests in a vertical plane such that its edge \overline{CE} is coincident with a smooth horizontal table, then the lamina is about to rotate about (E) . Find the length of \overline{BF} .

For more activities and exercises, visit www.sec3mathematics.com.eg

Rectilinear Motion

Unit

1



Introduction

In this unit, you are going to learn the rectilinear motion of a moving particle, analyzing this motion, and studying the vectors of the position displacement, velocity, and the acceleration of the motion of a particle. The motion will be identified at any moment during the motion of the particle in a rectilinear motion whether this motion is uniform or uniform using the methods of differential and integral to deduce the elements of study. The rectilinear motion will be analyzed graphically through the curves of the motion and using it to solve different problems. The study will not only be for the particle in motion but other different bodies such as cars, trains, and planes will be taken into consideration.

Unit objectives

By the end of this unit and by doing all the activities involved, the student should be able to:

- ❖ Use the related time to express the velocity if the displacement is a function of time ($v = \frac{ds}{dt}$)
 - ❖ If each of s , v , and a are functions in time, then:
 - ▶ $v = \frac{ds}{dt} \Leftrightarrow \int v dt = \int ds \quad \therefore s = \int v dt$
- ❖ Use the related time to express the acceleration if the velocity is a function of time ($a = \frac{dv}{dt}$)
 - ❖ $a = \frac{dv}{dt} \Leftrightarrow \int a dt = \int dv \quad \therefore v = \int a dt$
- ❖ Express the acceleration as a function of the displacement if the velocity is a function of the displacement ($a = v \frac{dv}{ds}$)
 - ❖ If a is a function of displacement then:
 - ▶ $a = v \frac{dv}{ds} \Leftrightarrow \int a ds = \int v dv$

Key terms

- Rectilinear Motion
- Position
- Displacement
- Distance
- Average Velocity
- Average Speed
- Velocity
- Speed
- Average Acceleration
- Acceleration

Lessons of the unit

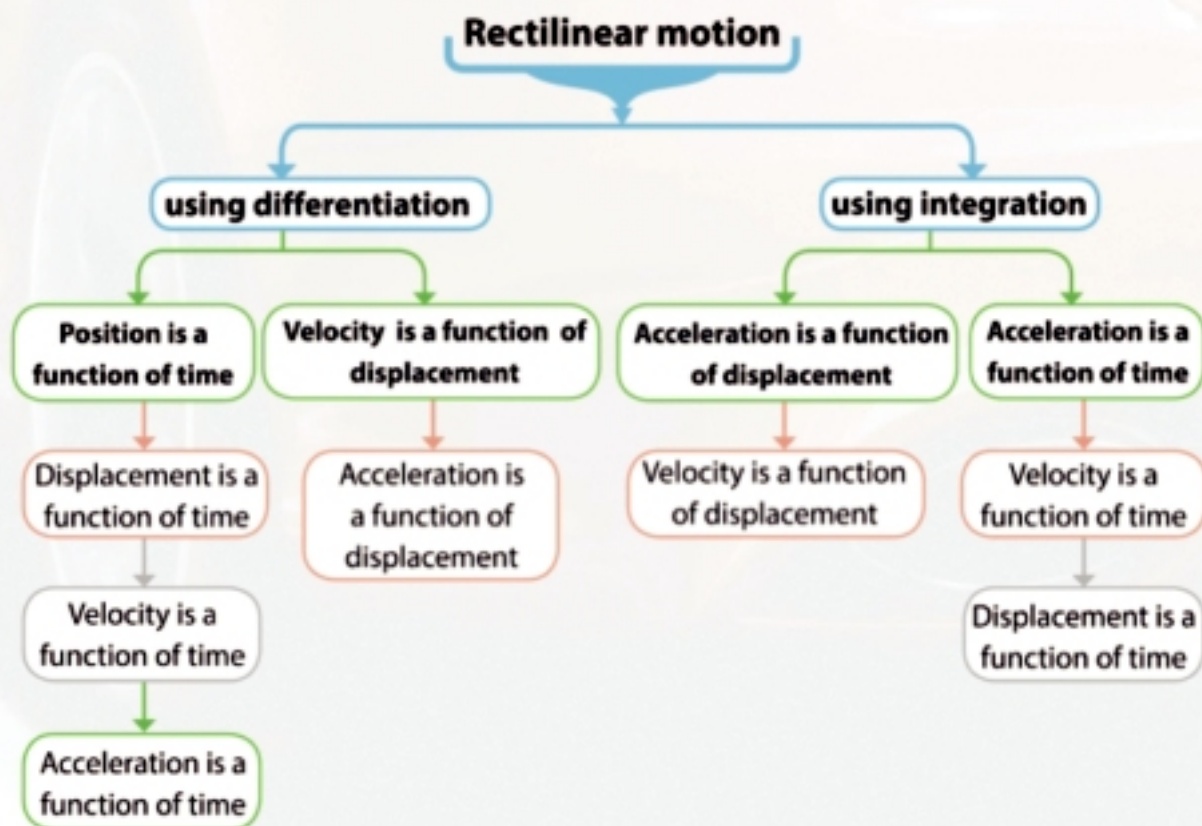
(1 -1): Differentiation of the vector functions

(1 -2): Integration of the vector function.

Materials

- Scientific calculator
- Computer graphics

Unit planning guide





Think and discuss

You will learn

→ If \vec{s} is a function of time then:

$$\vec{v} = \frac{d\vec{s}}{dt}$$

→ If \vec{v} is a function of time, then:

$$\vec{a} = \frac{d\vec{v}}{dt}$$

→ If \vec{v} is a function of displacement \vec{s} then

$$\vec{a} = \vec{v} \frac{d\vec{v}}{ds}$$

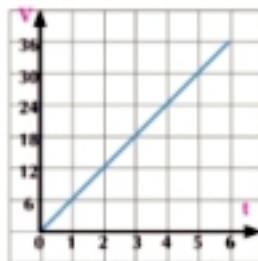
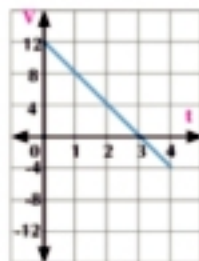
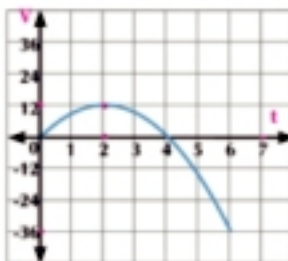
Key terms

- Rectilinear motion
- Position of the particle
- Displacement
- Distance
- Speed
- Velocity
- Average velocity
- Instantaneous velocity
- Average acceleration
- Acceleration

Materials

- Scientific calculator.
- Computer graphics

Each of the following graphical figures represents the velocity-time graph of a particle moving in a straight line.



After you study these curves,

- 1 Can you identify the velocity V of the particle at the starting of the motion, after 2 seconds and after 4 seconds from the beginning of the motion?
- 2 How can you calculate the displacement of the body at $t = 2$ and $t = 4$?
- 3 Can you identify the acceleration of motion of the moving particle?



Learn

1- Rectilinear motion

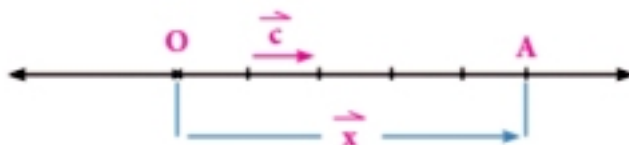
If a particle moves in a straight line, it is said that it moves in a linear motion.

2- Position of the particle

When a particle moves in a linear motion, at this moment it takes up a certain position on the straight line. To identify the position of \vec{x} for a moving particle at any moment t , we choose a constant point "O" on the straight line as an origin point and determine the positive direction along the line.

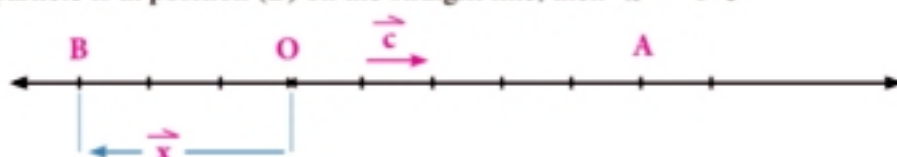
For example:

When the particle is at position (A) on the straight line, then $\vec{x} = 5\vec{c}$



where \vec{c} is a unit vector in the direction \vec{OA} ,

But if the particle is at position (B) on the straight line, then $\vec{x} = -3\vec{c}$



Note that the position of the particle is a vector quantity and it can be expressed as a function of time t .

i.e. $\vec{x}(t)$ and the magnitude of \vec{x} in the international system is measured in metre.

3- Displacement

The displacement \vec{s} of a particle is known as the change of its position.



If a particle moves from position A to position A' on a straight line **then:**

Displacement $\vec{s} = \Delta \vec{x}$ where $\Delta \vec{x} = \vec{x}' - \vec{x}_0$, and in this case $\Delta \vec{x}$ is positive since the particle's final position A' is on the right of the particle's initial position A. If the final position of the particle is on the left of the initial position of the particle, then $\Delta \vec{x}$ will be negative.

so generally $\vec{s}(t) = \vec{x}(t) - \vec{x}(0)$

- ⊙ The displacement \vec{s} of a particle is a vector quantity and it can be expressed as a function of time t i.e. $\vec{s}(t)$, and the displacement \vec{s} is distinguished from the distance traveled by the particle. Specifically, the distance is a standard positive quantity and represents the length of the whole pathway traveled by the particle.
- ⊙ The symbols x and s can be used to express the algebraic measure of the position vector \vec{x} and the displacement \vec{s} respectively.
- ⊙ If the position of the body at the beginning of measuring the time is at the origin point, then $\vec{x}_0 = \vec{0}$ and $\vec{s} = \vec{x}$

4- Velocity

If $\vec{s} = \Delta \vec{x}$ is the displacement of the particle during a period of time Δt , then the average velocity vector \vec{V}_a is equal to the quotient of the displacement over time

$$\text{i.e. } \vec{V}_a = \frac{\Delta \vec{x}}{\Delta t} = \frac{\vec{x}(t + \Delta t) - \vec{x}(t)}{\Delta t}$$

and the instantaneous velocity vector \vec{V} at any moment t is defined by the relation:

$$\vec{V} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{x}}{\Delta t} = \lim_{\Delta t \rightarrow 0} \frac{\vec{x}(t + \Delta t) - \vec{x}(t)}{\Delta t}$$

From the definition of the derivative: we can deduce that $\vec{V} = \frac{d\vec{x}}{dt}$ (the slope of the tangent

to the curve of position -time)

since \vec{x} is a constant vector **Alinalt**, then: the velocity vector is equal to the rate of change of displacement with respect to the time $\vec{v} = \frac{d\vec{s}}{dt}$ (the slope of the tangent to the displacement-time graph) and the magnitude of the velocity vector is calculated in the unit m/sec in the international system of units.

⇒ The symbol V can be used to express the algebraic measure of the velocity vector \vec{v} .

Example

- 1 A stone is projected vertically upwards and its height x after t second from the projection is given by the relation $x = 49t - 4.9t^2$ where x is in meters.
- Find the maximum height the projected body can reach.
 - Find the algebraic measure of the velocity vector when the stone is 78.4 meters high, then find its norm velocity.
 - Graph both the position-time graph and the velocity-time graph and use them to analyze the motion.

Solution

In the coordinate system of the motion in a straight line, consider x measures the height (position) at the projection point.

v is positive in case of moving upwards.

$$\therefore x(t) = 49t - 4.9t^2 \quad \therefore v(t) = \frac{dx}{dt} \quad \therefore v(t) = 49 - 9.8t$$

- a The stone reaches the maximum height when $V = 0$
 $\therefore 49 - 9.8t = 0 \quad \therefore t = 5 \text{ sec}$
 \therefore the maximum height $x(5) = 49 \times 5 - 4.9 \times 5^2 = 122.5$ meters

- b The stone is 78.4 meters high when $x = 78.4$
 $\therefore 49t - 4.9t^2 = 78.4 \quad \therefore 4.9t^2 - 49t + 78.4 = 0$

By dividing both sides of the equation by 4.9 we find that: $t^2 - 10t + 16 = 0$

$$\therefore (t - 2)(t - 8) = 0 \quad \therefore t = 2 \text{ sec or } t = 8 \text{ sec}$$

$$\therefore v(2) = 49 - 9.8 \times 2 = 29.4 \text{ meters / sec} \quad \therefore v(8) = 49 - 9.8 \times 8 = -29.4 \text{ meters / sec}$$

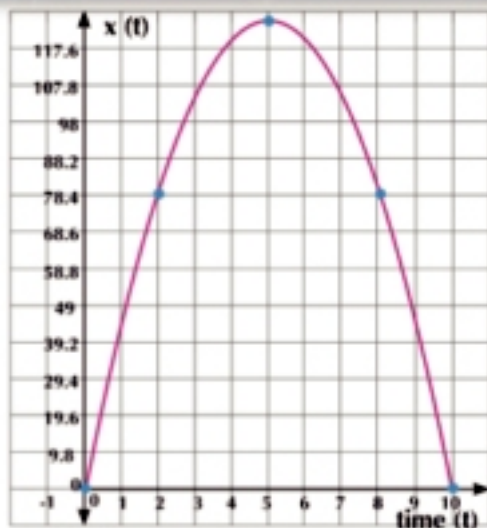
i.e.: the stone is 78.4 meters once ascending after 2 sec and once descending after 8 sec

The algebraic measure of the velocity vector is either 29.4 or -29.4

$$\therefore \text{the norm velocity of the stone in the two cases} = |\pm 29.4| = 29.4 \text{ meters/sec}$$

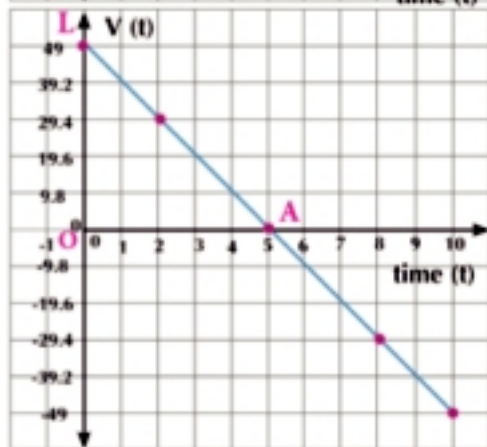
b From the position-time graph, we find that:

- The stone reaches the maximum height 122.5 meters when $t = 5$ sec (the curve vertex point).
- The stone gets back to the projection point once again when $t = 10$ sec (point B (10, 0))
- The ascending stage took 5 seconds, and the descending stage took another 5 seconds.
- The stone was at 78.4 meters high when $t = 2$ sec, and $t = 8$ sec



From the velocity-time graph, we find that:

1- The starting(initial) velocity of the stone was 49 meters/sec and it gets to decrease during the time interval $]0, 5[$ until the stone got instantaneously static when $t = 5$ when the stone reaches its maximum height, then its velocity increases in the opposite direction in the time interval $]5, 10[$ until it turns back to the projection point when $t = 10$ sec at the same projection velocity 49 meters/sec.



2- The maximum height of the stone can be calculated by the velocity-time graph in two methods:

- The maximum height = area $\triangle OAL = \frac{1}{2} OA \times OL = \frac{1}{2} \times 5 \times 49 = 122.5$ square meters
- By calculating the integration. This method will be discussed in details later.

Critical thinking: How could you calculate from the velocity-time graph in Example (1) the distance traveled by the stone until the stone turns back to the throwing point and also its displacement during this time?

5 Try to solve

- 1 A particle moves in a straight line such that its position \vec{x} at any time t is given by the relation $\vec{x}(t) = (t^2 - 4t + 3) \vec{c}$ where x is measured in meter, t in sec and \vec{c} is the unit vector in the direction of the motion of the body.
 - a Find the displacement of the particle during the first three seconds.
 - b Find the average velocity vector of the particle when $t \in [0, 2]$
 - c Find the velocity vector of the particle when $t = 4$
 - d Through the velocity-time graph and the position-time graph, analyze the motion of the particle and show: when does the particle change the direction of its motion?

5- Acceleration

If $\Delta \vec{v}$ expresses the change of the velocity vector during time interval Δt then the average acceleration \vec{a}_a is given by the relation

$$\vec{a}_a = \frac{\Delta \vec{v}}{\Delta t} \quad \text{i.e. } \vec{a}_a = \frac{\vec{v}(t + \Delta t) - \vec{v}(t)}{\Delta t}$$

The instantaneous acceleration \vec{a} (at any time) t is identified by

$$\text{the relation } \vec{a} = \lim_{\Delta t \rightarrow 0} \frac{\vec{v}(t + \Delta t) - \vec{v}(t)}{\Delta t}$$

From the definition of the derivative, we can deduce that: $\vec{a} = \frac{d\vec{v}}{dt}$

i.e. the acceleration is the rate of change of the velocity vector with respect to the time (the slope of the tangent to the velocity-time graph)

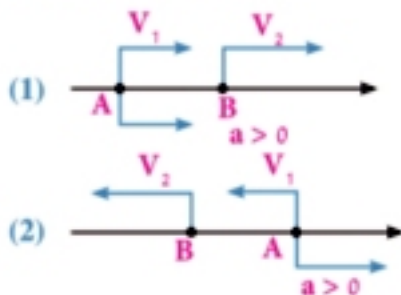
the magnitude of the acceleration vector is calculated by the unit $\text{m} / \text{sec} / \text{sec}$ (m/sec^2) in the international system of units

From the previous, we find that: if $\vec{x}(t)$ the position of the particle is a function of time t , then the velocity vector $\vec{v} = \frac{d\vec{x}}{dt}$ and we can deduce that the acceleration $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{x}}{dt^2}$

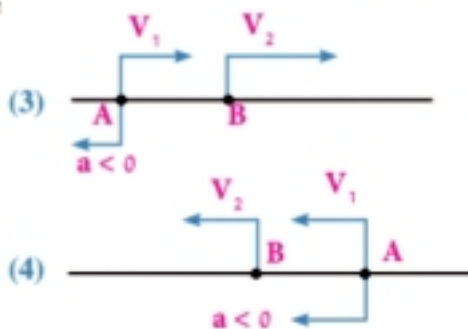
Attention: when we refer to the algebraic measures for each of the position, velocity vector and acceleration, we use the symbols x , v , a respectively.

The algebraic measure of the velocity vector and acceleration:

- 1-** If $a > 0$ then V increases. This means that the particle speeds up in the positive direction figure (1) or the particle speeds down in the negative direction figure (2) and in the two cases $\Delta V > 0$



- 2-** If $a < 0$ then V decreases. This means that the particle speed down in the positive direction figure (3) or the particle speed up in the negative direction figure (4).



- 3-** In each of the cases (2) and (3), it is said that the body moves in retardation while in each of the cases (1) and (4) it is said that the body moves in (acceleration).

i.e.: the body moves in an acceleration motion if \vec{v} and \vec{a} have the same direction ($V a > \text{zero}$) and moves retardation (deceleration) if \vec{v} and \vec{a} are in opposite directions ($V a < \text{zero}$)

Example

- 2 If the algebraic measure of the displacement of a particle moving in a straight line is given by the relation $s = t^3 - 6t^2 + 9t$ where s is measured in meter and t in second
- Find the acceleration of the particle when the velocity vanishes.
 - Find the norm velocity of the particle when the acceleration vanishes.
 - Find the distance covered by the particle during the time interval from $t = 0$ to $t = 2$.

Solution

$$\therefore s = t^3 - 6t^2 + 9t \quad \therefore v = \frac{ds}{dt} = 3t^2 - 12t + 9$$

$$\therefore a = \frac{dv}{dt} = 6t - 12$$

- a The velocity of the particle vanishes when $3t^2 - 12t + 9 = 0$

$$\therefore t^2 - 4t + 3 = 0$$

$$(t - 1)(t - 3) = 0 \quad \text{when } t = 1, t = 3$$

$$a(1) = 6(1) - 12 = -6 \text{ m/sec}^2$$

$$a(3) = 6(3) - 12 = 6 \text{ m/sec}^2$$

- b The acceleration of the particle vanishes when $6t - 12 = 0$

$$\therefore t = 2$$

$$\text{norm velocity} = |v(2)| = |3 \times 4 - 12 \times 2 + 9| = 3 \text{ m/sec}$$

- c From the study of the velocity-time graph of the motion of the particle or by studying the sign $V(t)$, we find that the particle moves in the positive direction in the interval $0 \leq t < 1$ then it changes the direction of its motion to move in the opposite direction in the interval $1 < t < 3$.

\therefore the distance traveled from $t = 0$ to $t = 2$ during the first and second seconds

$$= |s(1) - s(0)| + |s(2) - s(1)|$$

$$= |4 - 0| + |2 - 4| = 6 \text{ meters}$$



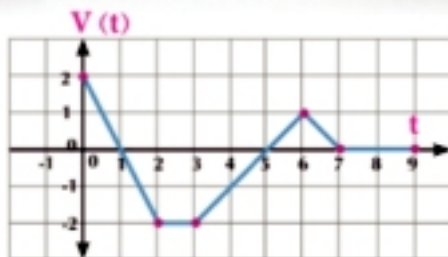
Critical thinking: Use the figure above to show the acceleration intervals and slowdown intervals to the motion of the particle

Try to solve

- 2 If the velocity \vec{v} of a particle is given as a function of time t by the relation $\vec{v}(t) = -(t^2 - 6t + 5) \vec{c}$ where \vec{c} is a unit vector in the direction of the motion of the particle.
- At what time does the particle change the direction of its motion?
 - At what time does the velocity of the particle increase? decrease?
 - Find the acceleration of the motion of the particle when its velocity vanishes.

Critical thinking:

The opposite figure shows the velocity $V = v(t)$ of a particle moving in a straight line.



- What time does the particle move forward? Backward?
- What time is the acceleration of motion be positive? Negative? What time does the acceleration of motion vanish?
- What time is the velocity of the particle be maximum?
- What time does the particle stop (rest) for a period more than one second?

Deducing the acceleration when the velocity is a function of position:

If $V = f(x)$, $x = g(t)$

By using the chain rule, we can deduce that: $\frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt}$

i.e.: $a = v \cdot \frac{dv}{dx}$

It is another form for the acceleration which can be used when the velocity vector \vec{V} is a function of position \vec{x}

Example

- A particle moves in a straight line such that the algebraic measure of its velocity vector \vec{V} is given by the relation $V = \frac{1}{20} (400 - x^2)$ where x expresses the algebraic measure of the position \vec{x} . Find the algebraic measure of the acceleration of motion \vec{a} when $x = 15$

Solution

$$\therefore v = \frac{1}{20} (400 - x^2)$$

$$\therefore \frac{dv}{dx} = \frac{-1}{10} x$$

$$\therefore a = v \frac{dv}{dx}$$

$$a = \frac{-1}{200} x (400 - x^2)$$


when $x = 15$ unit length.

$$\therefore a = \frac{-1}{200} \times 15 (400 - 225)$$

$$a = -\frac{105}{8} \text{ Acceleration unit}$$

Try to solve

- A particle moves in a straight line such that the relation between v and x is given in the form $v = \frac{5}{4+x}$ where v is measured in m/sec, x is measured in meter. Find the acceleration of motion when $x = 2$ meter.

 **Example**

- 4 A particle moves in a straight line such that the relation between V and x is given in the form $v^2 = 5(9 - x^2)$. Find the acceleration of motion when the velocity vanishes, knowing that the velocity is measured in m/sec and x is measured in meter.

 **Solution**

$$\therefore v^2 = 5(9 - x^2)$$

$$\therefore 2a = -10x$$

The velocity vanishes when $v = 0$

$$a(3) = -5 \times 3 = -15 \text{ m/sec}^2$$

$$\therefore 2V \frac{dv}{dx} = -10x$$

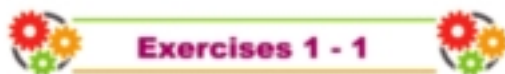
$$\therefore a = -5x$$

$$\text{i.e. } 5(9 - x^2) = 0 \quad \therefore x = \pm 3$$

$$a(-3) = -5 \times -3 = 15 \text{ m/sec}^2$$

 **Try to solve**

- 4 A particle moves in a straight line such that the algebraic measure of its velocity vector V is given in a relation with the algebraic measure of its position x in the form $v^2 = \frac{1}{8(4 - x^2)}$. Find a in terms of x where a is the algebraic measure of the acceleration of motion, then find the minimum velocity of the moving particle.

 **Exercises 1 - 1**
Choose the correct answer:

- 1 When a particle moves in a straight line in a constant velocity, then the magnitude of its acceleration _____
- a** increases **b** decreases **c** is constant and is not equal to zero
d zero
- 2 The change in the position vector of a particle moving in a straight line is called the _____
- a** displacement **b** distance **c** velocity vector
d acceleration vector
- 3 A particle moves in a straight line such that $V = 3e^{t^2}$ then its starting (initial) velocity is equal to _____
- a** 3 **b** e **c** $3e^2$ **d** e^2
- 4 A particle moves in a straight line and the equation of its motion $x = \tan t$ then the acceleration of motion a is equal to _____
- a** $\sec^2 t$ **b** $2 \sec t$ **c** $2v x$ **d** vx
- 5 A particle moves in a straight line and the equation of its motion $x = 2 + \ln(t + 1)$ then _____
- a** Its velocity and the acceleration of motion always decrease.
b Its velocity and the acceleration of motion always increase.
c The velocity decreases and the acceleration of motion increases.
d The velocity increases and the acceleration of motion decreases.

6 Choose the appropriate chart in front of each sentence of the following sentences :

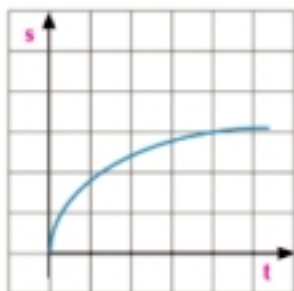


figure a

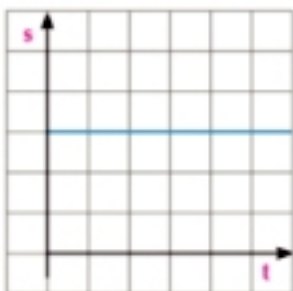


figure b

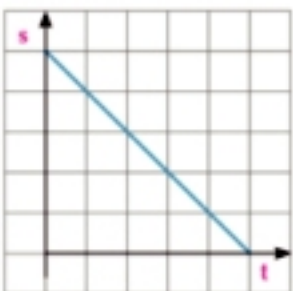


figure c

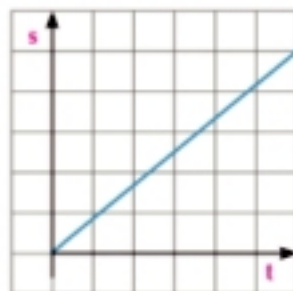


figure d

- (1) the body stalled (2) Body back wards (3) The body is moving forward at a constant at speed
(4) speed of the particle decreases

7 Choose the appropriate chart in front of each sentence of the following sentences:

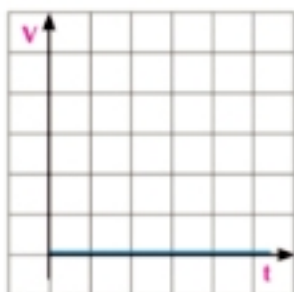


figure a

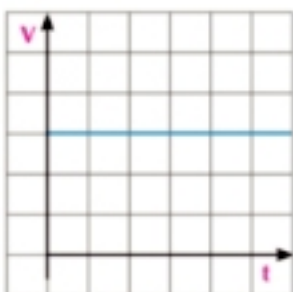


figure b

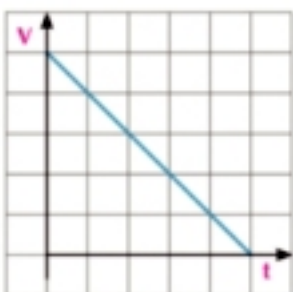


figure c

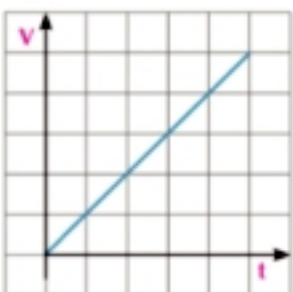


figure d

- (1) The particle move declaration (2) The body is moving constant speed (3) the body stalled
(4) the particle move at accelerated

8 In each of the graphs drawn (position-time graph), identify the sign of the algebraic measure of the velocity vector, then whether the particle moves in acceleration or deceleration.

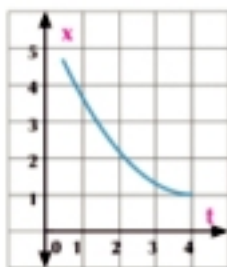


figure (1)

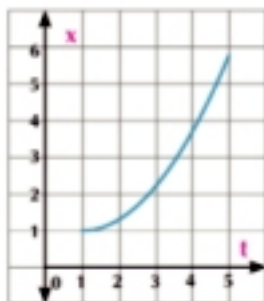


figure (2)

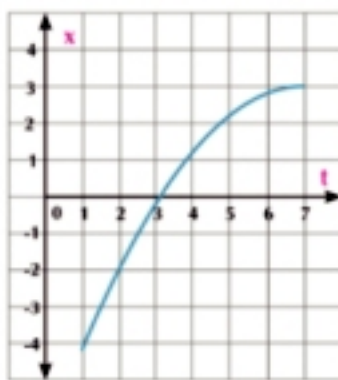


figure (3)

- 9 In each of the graphs drawn (**velocity-time graph**), identify the sign of the acceleration and tell whether the particle moves in acceleration or deceleration.



figure (1)

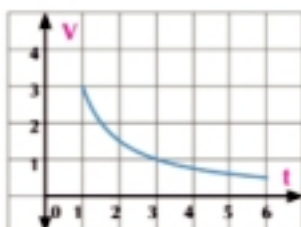


figure (2)

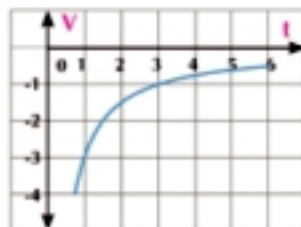


figure (3)

- 10 You have three graphs (1), (2) and (3), each of them represents the position-time graph. The graphs (4), (5) and (6) each represents the velocity-time graph. Match each graph from group A with its corresponding graph from group B.

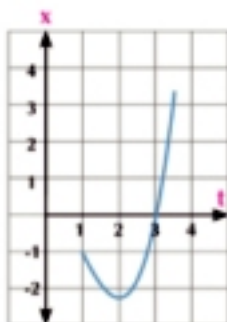


figure (1)

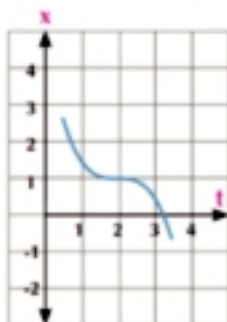


figure (2)



figure (3)

Group A

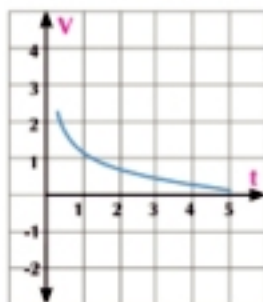


figure (4)

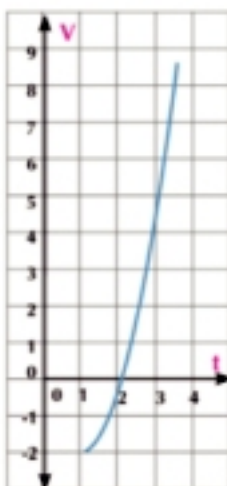


figure (5)

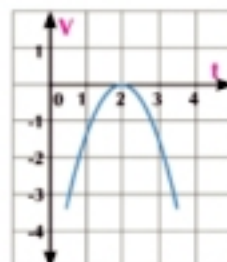


figure (6)

Group B

- 11 If $V = 3x$, find a in terms of x , then find a when $x = 2$
- 12 A particle moves in a straight line such that the algebraic measure of the velocity V is given in a relation with the algebraic measure of the position x in the form $V = x + \frac{1}{x}$. Find the acceleration of motion when $x = 2$ where x is measured in meter and, V is measured in m/sec.
- 13 A particle moves in a straight line such that the algebraic measure of its velocity is given in a relation with the algebraic measure of the position x in the form $V = \frac{1}{x^2}$. Find a in terms of x , then find a when $x = \frac{1}{2}$.
- 14 A particle moves in a straight line such that the algebraic measure of the velocity V is given in a relation with the algebraic measure of the position x in the form $V^2 = 16 - 9 \cos x$. Find the maximum velocity of the particle and the acceleration of motion then.
- 15 A particle moves in a straight line such that the equation of its motion is given in the form $x(t) = 3 \cos t + 4 \sin t$ where x is measured in meter and t in second.
- Find the algebraic measure of the displacement \vec{s} when $t = \frac{\pi}{2}$, $t = \pi$
 - Find the algebraic measure of the velocity vector \vec{v} when $t = 0$, $t = \frac{\pi}{2}$, $t = \pi$
 - Find the maximum displacement of the particle.
- 16 A particle moves in a straight line according to the relation $x = a \sin k t$ such that x express the algebraic measure of the position, t time, $a, k \in \mathbb{R}$.
- Find the relation between V and x such that V is the algebraic measure of the velocity vector.
 - Find V when $x = \frac{a}{2}$.
 - Find the time taken until $x = \frac{-a}{2}$ and find the acceleration of motion then.

Integration of vector functions

Unit One

1 - 2



Co-operative work

The graph drawn represents the velocity-time graph and the acceleration-time graph in one graphical figure. Show that the area under the acceleration-time graph during any time interval $[0, t]$ is equal to the velocity of the particle at time t .

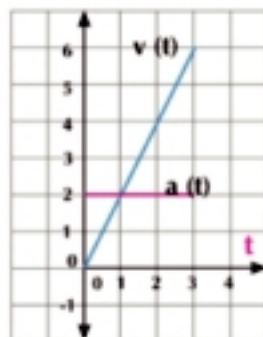


figure (1)

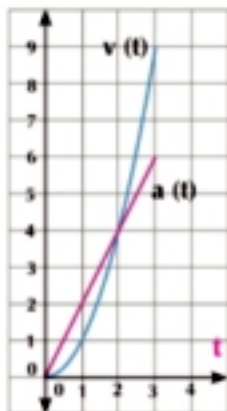


figure (2)

The graphs drawn can be used on any other graphic program to draw other graphs to prove the required.



Learn

1- Definite integration:

$$\int_a^b f'(x) \cdot dx = f(b) - f(a)$$

i.e: $\int_1^4 (x^2 + 2x - 1) dx$

$$= \left[\frac{x^3}{3} + \frac{2x^2}{2} - x \right]_1^4$$

$$= \left[\frac{x^3}{3} + x^2 - x \right]_1^4 = \left[\frac{64}{3} + 16 - 4 \right] - \left[\frac{1}{3} + 1 - 1 \right] = 33$$

The definite integration will be learned in the integral.

2- Deducing the velocity and displacement:

(i) If $a = \frac{dv}{dt}$ then $\int a dt = \int dv$
 $\therefore v = \int a dt$ (1-1)

Remember



The symbols a , v and x are used to refer to the algebraic measures for each of the acceleration, velocity vector and the position respectively.

You will learn

\rightarrow If each of a and V are functions of time, then:

$$\rightarrow x = \int V dt$$

$$V = \int a dt$$

\rightarrow If the acceleration is a function of position, then

$$\int a dx = \int V dV$$

Key terms

\rightarrow Integration

Materials

\rightarrow Scientific calculator.

\rightarrow Computer graphics.

To identify a single acceleration of motion to be congruent to the given acceleration $a(t)$, the initial conditions for each of the initial velocity V_0 and the starting (initial) position x_0 should be stated

the indefinite integration can be replaced by the definite by integration with the proper limits of integration, then we obtain.

$$v_0 \int^v dv = \int_0^t a dt$$

$$\therefore v - v_0 = \int_0^t a dt \quad (1-2)$$

= the area under the acceleration-time graph

If the acceleration a is constant, then: $V - V_0 = a \int_0^t dt$

$$\therefore v = v_0 + at \quad (1-3)$$

Notice that:

The equation (1-3) can be used only when the acceleration is constant but if the acceleration is a function of time, we use the equation (1-1) or (1-2) in regard to the data of the problem.

(ii) if $V = \frac{dx}{dt}$ then: $\int V dt = \int dx$

$$\therefore x = \int V dt \quad (2-1)$$

use the definite integration and the proper limits of integration to find that:

$$x_0 \int^x dx = \int_0^t v dt$$

$$\therefore x - x_0 = \int_0^t v dt \quad (\text{Notice that } x - x_0 = s) \quad (2-2)$$

The displacement = the area under the velocity-time graph

If the acceleration is constant, velocity (V) can be substituted from the equation (1-3) to find that:

$$\therefore x - x_0 = \int_0^t (v_0 + at) dt \quad \therefore x - x_0 = v_0 t + \frac{1}{2} at^2$$

$$\therefore x = x_0 + v_0 t + \frac{1}{2} at^2 \text{ by using in case constancy of acceleration}$$

(iii) If $a = v \frac{dv}{dx}$ then: $\int a dx = \int v dv \quad (3-1)$

Use the definite integration with the proper limits of integration to find that:

$$v_0 \int^v v dv = \int_{x_0}^x a dx$$

$$\frac{1}{2} (v^2 - v_0^2) = \int_{x_0}^x a dx \quad (3-2)$$

= The area under the acceleration-displacement graph

At the constancy of the acceleration a , then: $v^2 - v_0^2 = 2a \int_{x_0}^x dx$

$$\therefore v^2 = v_0^2 + 2a(x - x_0) \quad (\text{notice that } x - x_0 = s) \quad (3-3)$$

$$\therefore v^2 = v_0^2 + 2as$$

Example

- 1 A particle starts to move in a straight line from the origin point with initial velocity of a magnitude 8m/sec and the acceleration of motion after t second is given by the relation $(3t - 2)$, find each of the velocity of the particle and its displacement after 2 sec from the starting of motion.

Solution

$$\begin{aligned} \therefore a &= 3t - 2 & \therefore v &= \int (3t - 2) dt & \therefore v &= \frac{3}{2}t^2 - 2t + c \\ \therefore V_0 &= 8\text{m/sec} & \therefore v &= 8\text{m/sec when } t = 0 & \therefore c &= 8 \\ \therefore V &= \frac{3}{2}v^2 - 2t + 8 & \therefore v(2) &= \frac{3}{2} \times 4 - 2 \times 2 + 8 = 10 \text{ m/sec} \\ \therefore V &= \frac{3}{2}v^2 - 2t + 8 & \therefore x &= \int (\frac{3}{2}v^2 - 2t + 8) dt \\ \therefore x &= \frac{1}{2}v^3 - v^2 + 8v + c' \\ \therefore x &= 0 \text{ when } t = 0 & \therefore c' &= 0 & \therefore x &= \frac{1}{2}v^3 - v^2 + 8t \\ s(2) &= x(2) - x(0) = \frac{1}{2}(2)^3 - (2)^2 + 8(2) = 16 \text{ meters} \end{aligned}$$

Another solution:

$$\begin{aligned} \therefore a &= 3t - 2 & \therefore \frac{dv}{dt} &= 3t - 2 \\ \therefore \int_8^v dv &= \int_0^t (3t - 2) dt & \therefore v - 8 &= \frac{3}{2}t^2 - 2t \\ \therefore v &= \frac{3}{2}t^2 - 2t + 8 & \therefore v(2) &= \frac{3}{2} \times 4 - 2 \times 2 + 8 = 10 \text{ m/sec} \\ \therefore s &= \int_0^t v dt & \therefore s(2) &= \int_0^2 (\frac{3}{2}t^2 - 2t + 8) dt \\ \therefore s(2) &= [\frac{1}{2}t^3 - t^2 + 8t]_0^2 = \frac{1}{2}(2)^3 - (2)^2 + 8(2) = 16 \text{ meters} \end{aligned}$$

Try to solve

- 1 A particle moves in a straight line starting from rest and distant 8 meters from a constant point on the straight line. If $a = 6t - 4$ where a is measured in m/sec^2 , find the relation between the velocity and the time and between the displacement and the time.

Example

- 2 A particle moves in a straight line such that the algebraic measure of its velocity is given as a function of time by the relation $v = 6t^2 - 24$ where V is measured in (m/sec) . Find when the velocity of the particle reaches 72 m/sec, then find the magnitude of the acceleration of the particle when its velocity reaches 30 m/sec and the displacement of the particle during the interval $t \in [1, 4]$.

Solution

$$\therefore V = 6t^2 - 24$$

- 1- When the velocity of the particle reaches 72 m/sec

$$\therefore 6t^2 - 24 = 72 \quad \therefore 6t^2 = 96 \quad t^2 = 16 \quad t = 4 \text{ sec}$$

- 2- When the velocity of the particle reaches 30 m/sec

$$\therefore 6t^2 - 24 = 30 \quad \therefore 6t^2 = 54 \quad t^2 = 9 \quad t = 3 \text{ sec}$$

$$\therefore a = \frac{dv}{dt} \quad \therefore a = 12t \quad a(3) = 36 \text{ m/sec}^2$$

- 3- The magnitude of the displacement during the interval $t \in [1, 4]$

$$s = \int_1^4 v \, dt \quad s = \int_1^4 (6t^2 - 24) \, dt = [2t^3 - 24t]_1^4$$

$$s = (2 \times 64 - 24 \times 4) - (2 - 24) = 54 \text{ m}$$

Try to solve

- 2 A car starts moving from rest in a straight line from a constant point on the line and the algebraic measure of its velocity vector after time t is given by the relation $v = 3t^2 + 2t$ where v is measured in m/sec and t in seconds. Find each of the acceleration of motion and the displacement of the car when $t = 2$

Example

- 3 A car starts moving from rest in a straight line from a constant point on the line and the algebraic measure of its velocity vector after time t is given by the relation $v = 3t^2 - 6t$ where v is measured in m/sec and t in second. Find each of the average velocity vector and the average velocity during the time interval $0 \leq t \leq 3.5$

Solution

$$\therefore v = 3t^2 - 6t \quad \therefore v = 3t(t - 2)$$



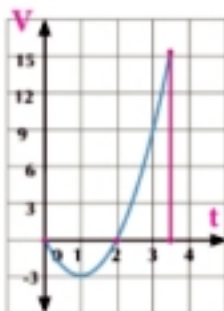
We find that the car changes the direction of its motion after 2 seconds and the process of investigating the sign of $V(t)$ of the velocity-time graph shows that

$$\therefore s = \int_0^{3.5} v \, dt = \int_0^{3.5} (3t^2 - 6t) \, dt$$

$$\therefore s = [t^3 - 3t^2]_0^{3.5} = (3.5)^3 - 3(3.5)^2 = \frac{49}{8}$$

$$\therefore \text{the average velocity vector } \vec{V}_a = \frac{\frac{49}{8} \vec{c}}{3.5 - 0} = 1.75 \vec{c}$$

since \vec{c} is a unit vector in the direction of motion and the algebraic measure of the average velocity vector is equal to 1.75 m/sec



The distance traveled during the time interval $t \in [0, 3.5]$

$$= \int_0^2 v \, dt + \int_2^{3.5} v \, dt = |[t^3 - 3t^2]_0^2| + |[t^3 - 3t^2]_2^{3.5}| = 4 + \frac{49}{8} + 4 = \frac{113}{8} \text{ meters}$$

$$\therefore \text{The average velocity} = \frac{\frac{113}{8}}{3.5 - 0} = \frac{113}{28} \approx 4.04 \text{ m/sec}$$

 **Solution**

- a** $\because a = 40 e^{-x}$ $\therefore \int a \, dx = \int v \, dv$
 $\therefore 40 \int_0^x e^{-x} \, dx = \int_8^v v \, dv$ $\therefore -80(e^{-x} - 1) = v^2 - 64$
 $\therefore -40[e^{-x}]_0^x = \frac{1}{2}[v^2]_8^v$ $\therefore v^2 = 144 - 80e^{-x}$
 $\therefore v^2 = 144 - 80e^{-x}$
- b** When $v = 10$ m/sec we find that: $80e^{-x} = 44$
 $\therefore e^x = \frac{20}{11}$ $\therefore x = \ln \frac{20}{11}$ meters
- c** $\because v^2 = 144 - \frac{80}{e^x}$ $\therefore e^x > 0$ for all the values of x
 $\therefore \frac{80}{e^x} \rightarrow 0$ when $x \rightarrow \infty$ \therefore the maximum velocity = 12 m/sec

 **Try to solve**

- 5** A particle moves in a straight line with initial velocity of a magnitude 2m/sec from a constant point on the straight line such that $a = e^x$. Find v^2 in terms of x , then find v when $x = 4$ meters and x when $V = 20$ m/sec.


Exercises 1 - 2

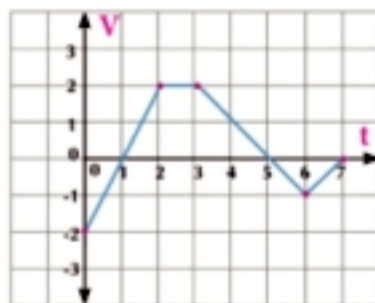

In all problems, let the particle move in a straight line, x , V and a be the algebraic measures for each of the position, velocity vector and acceleration respectively.

Choose the correct answer:

- 1** If $v = 3t^2 - 2t$, $x = 1$ when $t = 0$ then:
a $x = 6t - 2$ **b** $x = 3t^2 - 2t + 1$ **c** $x = t^3 - t^2 + 1$ **d** $x = t^2 - t - 1$
- 2** If $v = 1 + \sin t$, $x = -2$ when $t = 0$ then:
a $x = t + \cos t$ **b** $x = t - \cos t$ **c** $x = t - \cos t + 2$ **d** $x = t - \cos t - 2$
- 3** If $v = 3t - 2$, then s in the interval $[0, 2]$ is
a 1 unit length **b** 2 unit length **c** 3 unit length **d** 4 unit length
- 4** If $v = 3t^2 - 2t$, then the distance covered within the interval $[0, 2]$
a $\frac{4}{27}$ unit length **b** 4 unit length **c** $\frac{112}{27}$ unit length **d** $\frac{116}{27}$ unit length
- 5** If $v = t^3 - 3t^2 + 2t$, then the distance covered within the time interval $[0, 3]$
a $\frac{1}{4}$ unit length **b** $\frac{1}{2}$ unit length **c** $\frac{9}{4}$ unit length **d** $\frac{11}{4}$ unit length

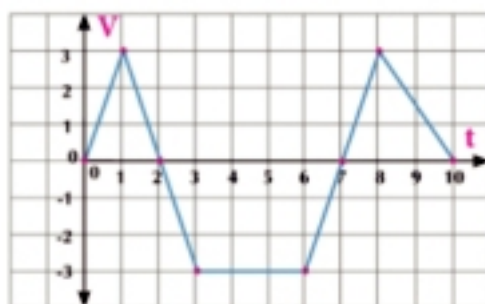
- 6 If $a = 3$, $v_x = -1$, then s within the time interval $[0, 2]$.
- a $\frac{1}{6}$ unit length b 4 unit length c $\frac{25}{6}$ unit length d $\frac{13}{3}$ unit length
- 7 If $a = 3$, $V_x = -1$ then the distance covered within the time interval $[0, 2]$.
- a $\frac{1}{6}$ unit length b 4 unit length c $\frac{25}{6}$ unit length d $\frac{13}{3}$ unit length

- 8 From the velocity-time graph in the opposite figure, the magnitude of displacement is equal to _____



- a 3 unit length
b 5 unit length
c 7 unit length
d 8 unit length

- 9 From the velocity-time graph in the opposite figure, then the distance traveled = _____



- a 4.5 unit length
b 10.5 unit length
c 13.5 unit length
d 19.5 unit length

- 10 A particle is thrown vertically upwards with initial velocity of a magnitude 5.6 m/sec from a point 24.5 above the earth's surface. Find each of V and x in terms of t , then find the maximum height the particle can reach.
- 11 A particle moves in a straight line with initial velocity 2 m/sec from a constant point such that $a = 2t - 6$ where a is measured in m/sec^2 . Find each of V and x in terms of t and x when $V = 18$ m/sec.
- 12 A particle moves in a straight line from a constant point starting at rest such that $a = 8 - 2t^2$ where a is measured in m/sec^2 . Find the maximum velocity of the particle and the distance traveled maximum speed up.
- 13 A particle moves in a straight line from a constant point on the straight line starting at rest such that $a = \frac{3}{8}x^2$ where a is measured in m/sec^2 and x in meter. Find the velocity of the particle when $x = 2$ meters, then find its position when $V = 4$ m/sec.
- 14 A particle moves in a straight line with initial velocity 3 m/sec from a constant point such $a = 6x + 4$ where a is measured in m/sec^2 and x in meter. Find v^2 in terms of x , then find the velocity of the particle when $x = 2$ and x when $v^2 = 87$.

GENERAL EXERCISES

Choose the correct answer

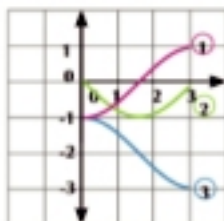
- ① If $x = t^2 - 3t + 2$ then the particle changes its motion when:
a $t = 1, t = 2$ **b** $t = 1$ **c** $t = 1.5$ **d** $t = 2$
- ② If $x = 6t - t^2$, then the distance traveled within the time interval $0 \leq t \leq 6$ is
a zero **b** 9 **c** 18 **d** 36
- ③ If $V(t) = 9.8t + 5$ where $x(0) = 10$, then $x(10)$
a zero **b** 530 **c** 540 **d** 550
- ④ If $V(t) = \frac{2}{\pi} \cos\left(\frac{2t}{\pi}\right)$, and $x(\pi^2) = 1$ then $x(t)$
a $\frac{2}{\pi} \sin\left(\frac{2t}{\pi}\right) + 1$ **b** $\frac{2}{\pi} \sin\left(\frac{2t}{\pi}\right) - 1$
c $\sin\left(\frac{2t}{\pi}\right) + 1$ **d** $\sin\left(\frac{2t}{\pi}\right) - 1$
- ⑤ If $a(t) = -4 \sin 2t$, and $v(0) = 2$, $x(0) = -3$ then $x(\pi)$
a -3 **b** 0 **c** 2 **d** 3

- ⑥ The graph drawn in the opposite figure represents the position of a particle and its velocity vector and the acceleration of motion. Which of the following choices represents the position time graph, the velocity-time graph and the acceleration-time graph respectively.



- a** 3, 2, 1
b 1, 3, 2
c 3, 1, 2
d 1, 2, 3

- ⑦ The graph drawn in the opposite figure represents the position of a particle and its velocity vector and the acceleration of motion. Which of the following choices represents the position time graph, the velocity-time graph and the acceleration-time graph respectively.



- a** 3, 2, 1 **b** 1, 2, 3
c 2, 3, 1 **d** 3, 1, 2

- ⑧ A particle moves in a straight line according to the relation $x = 4 \cos t$ where x is measured in cm and t in second, find:
a V when $t = \frac{\pi}{2}$ **b a when $t = \pi$**
- ⑨ A particle moves in a straight line from a constant point on the straight line according to the relation $V = \sin t - \cos t$, find $x\left(\frac{\pi}{2}\right)$

- 10 A particle moves in a straight line such that the algebraic measure of its displacement s is given as a function of time t by the relation $s = t^3 - 6t^2 + 9t$ where s is measured in meter and t in second.
- Find the acceleration of motion at the moments the velocity vanishes.
 - Find the velocity of the particle when $a = 0$
 - When does the velocity of the particle increase? decrease?
 - Find the distance covered during the first five seconds.
- 11 A particle moves in a straight line according to the relation $a(t) = -2$ with initial velocity of a magnitude 3 m/sec from a constant point on the straight line. Find each of the displacement and the distance traveled within the time interval $[1, 4]$.
- 12 A particle moves in a straight line according to the relation $s = t^3 - 3t^2$ where s is measured in meter and t in second. Find:
- The acceleration of motion when the velocity vanishes
 - The average velocity and the average velocity vector within the time interval $[0, 5]$.
- 13 A particle moves in a straight line with initial speed of a magnitude -2 m/sec from a position distant 3 meters in the positive direction from a constant point on the straight line such that $a = 2t + 1$. Find x at the moment the velocity vanishes.
- 14 A and B are two points on a singular straight line. A particle moves at rest starting from point A in the direction of \overrightarrow{AB} such that the algebraic measure of its velocity is given by the relation $v = 0.4t + 0.9t^2$ where v is measured in m/sec and t in second. After two seconds from the motion of the first particle another particle moves starting from point B in the direction of \overrightarrow{BA} from rest with a constant acceleration of a magnitude 0.2 m/sec^2 . The two particles meet after 5 seconds of the motion of the first particle. Find the distance between A and B.



For more activities and exercises, visit www.sec3mathematics.com.eg

Newton's Laws of Motion

Unit 2



Unit Introduction

Thanks to the discovery of the universal gravitation law by the British scientist, Isaac Newton (1642-1727), who is considered one of the icons of the scientific revolution in the field of modern mechanics, The German scientist, Johann Kepler (1571 - 1630), has stated some mathematical rules which control the motion of the planets around the sun. In addition to the work of the Islamic Scientists which had been translated during the previous centuries. The Italian scientist, Galileo Galilei (1564 - 1642), had established the mechanics. He had conducted several experiments on the fallen or thrown bodies and the bodies moving horizontally. Through his experiments, he had discovered a lot of the important properties of the motion of the bodies. Thanks to him it was discovered that the bodies moving on horizontal surfaces without resistance keep moving in a uniform velocity. It is thought that Galileo had discovered the first and second laws of Newton's laws of motion. Isaac Newton had collected his overall researches in a book named (Bernspaa) which means the mathematical principles of the natural philosophy. This book is considered one of the most important scientific books that appeared in the modern age. Newton had formulated his three laws. The Newton's law of universal gravitation had clarified the concept that the force can be occurred under the action of a distance. Bodies attract each other even if they are not contacting. For example, the Earth attracts the bodies by a force called (the weight force). With respect to the mass, we notice that its static definition does not allow us to identify the mass of objects or bodies but to only compare the masses through the resistance of their weights. The mass can get a dynamic definition through studying the motion of bodies in this unit. You are going to learn the mass, momentum and Newton's laws of motion with application on these laws and you will also learn the motion on a rough or smooth plane and the motion of the simple pulleys..

By the end of this unit and by doing all the activities involved, the student should be able to:

- ⊕ Identify the concept of momentum and its measuring units- the change of momentum.
- ⊕ Identify Newton's laws (first, second, third).
- ⊕ Identify the relation between the force and acceleration:
 - If the force F is a function of time t i.e. $F = f(t)$ then:
$$F = m \frac{dV}{dt} \text{ i.e. } \int F dt = m \int dV$$
 - If the force F is a function of displacement S i.e. $F = f(S)$ then:
$$F = mV \frac{dV}{dS} \text{ i.e. } \int F dS = m \int V dV$$
- ⊕ Apply Newton's laws of motion in daily life situations such as: a body is placed in a lift moving in a uniform acceleration (motion of the bodies connected by strings):
 - ⊕ Motion of the simple pulley.
 - ⊕ Motion of a body in a smooth plane (horizontal - inclined).
 - ⊕ Identify the motion on a rough plane (horizontal - inclined)
 - ⊕ Motion of a system of two bodies connected by a string passing over a smooth pulley.
 - ⊕ Motion of a system of two bodies connected by a string one of them is hanging free and the other moves on a rough horizontal plane.

- ⊕ Motion of a system of two bodies connected by a string. One moves on a smooth inclined plane and the other moves vertically.
- ⊕ Motion of a system of two bodies connected by

a string. One of them moves on a rough inclined plane and the other moves vertically.

Key Terms

- ⊕ Momentum
- ⊕ Linear momentum
- ⊕ Mass
- ⊕ Velocity
- ⊕ Change of momentum
- ⊕ Newton's first law
- ⊕ Inertia
- ⊕ Inertia principle
- ⊕ Force
- ⊕ Newton's second law
- ⊕ Equation of motion
- ⊕ Weight
- ⊕ Newton's third law
- ⊕ Pressure
- ⊕ Reaction
- ⊕ Lift motion
- ⊕ Spring scale
- ⊕ Pressure scale
- ⊕ Balance
- ⊕ Inclined plane
- ⊕ Smooth plane
- ⊕ Rough plane
- ⊕ Kinetic friction
- ⊕ Static friction
- ⊕ Pulley

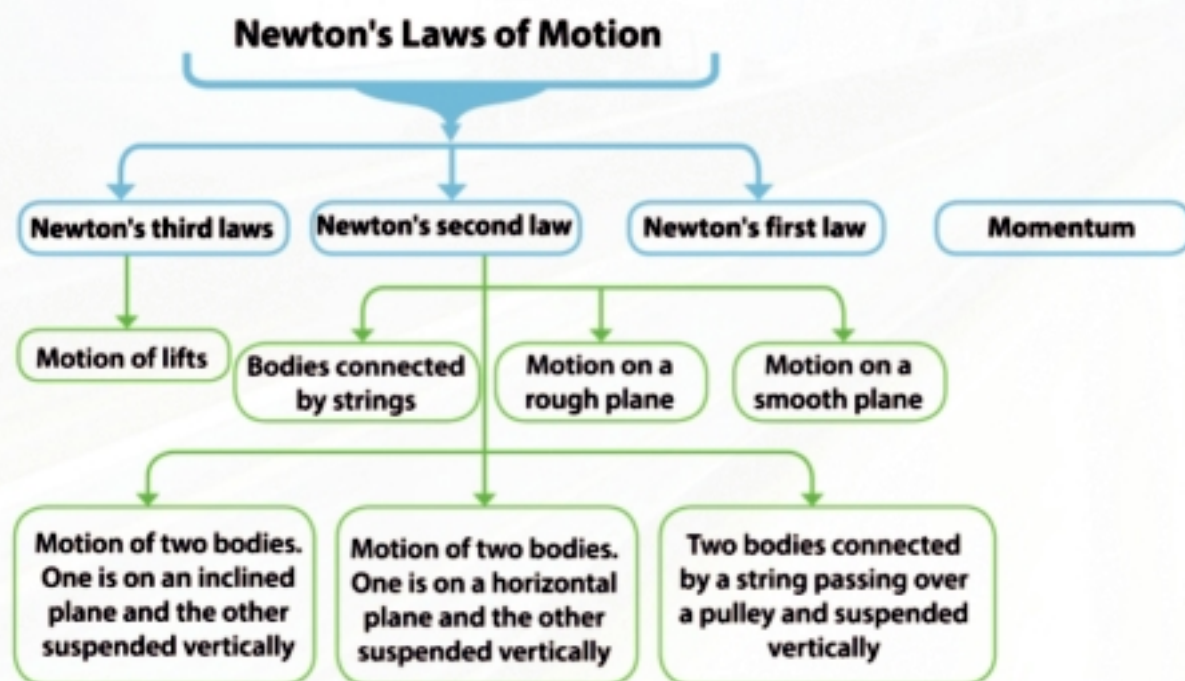
Unit Lessons

- (2 - 1): Momentum
- (2 - 2): Newton's first law
- (2 - 3): Newton's second law
- (2 - 4): Newton's third laws
- (2 - 5): Motion of a body on a smooth inclined plane
- (2 - 6): Motion of a body on a rough inclined plane
- (2 - 7): Simple pulleys

Materials

- Scientific calculator -
- Computer graphics

Unit Planing Guide



You will learn

- ▷ The concept of momentum
- ▷ The measuring units of the momentum
- ▷ The change of momentum.

Key terms

- ▷ Momentum
- ▷ Linear momentum
- ▷ Mass
- ▷ Velocity
- ▷ Change of momentum

Materials

- ▷ Scientific calculator.



Think and Discuss

- 1 What is the action of a big stone placed on the roof of a house? What is the action of shooting a bullet from a barrel of a gun on this roof?
- 2 What is the action of placing a grain of sand on your palm? What is the action of this grain of sand if it moves in a storm in the direction of a car moving fast towards the storm?

From the examples, notice that:

- 1 - Shooting a bullet, however its mass is so small, on the roof of the house will lead to the bullet being imbedded in the roof for a distance since the velocity of the bullet is much faster than the velocity of the stone.
- 2 - The grain of sand, however its mass is very small, can scratch off the windshield glass since it acquired a momentum with respect to the car and the momentum vector of the grain of sand has become extremely strong due to the increase of its relative velocity vector.



Learn

Momentum

The momentum of a moving body is a vector quantity of the same direction of the velocity of this body and its magnitude at a moment is estimated by the product of the mass of this body by its velocity at this moment. The momentum vector is denoted by the symbol \vec{H} .

$$\vec{H} = m \vec{v}$$

In case of the rectilinear motion, each of \vec{H} and \vec{v} are parallel to the straight line on which the motion occurs. Each of \vec{H} and \vec{v} can be expressed in terms of the algebraic measure for each of them:

$$H = m V$$

Since H and V are the two algebraic measures for the momentum vector and velocity respectively.

Measuring units of momentum

The magnitude unit of momentum = mass unit \times velocity unit

In the international system of units, the magnitude of momentum is measured in kg. m/sec
 i.e.: $H \text{ (kg. m / sec)} = m(\text{kg}) \times V(\text{m/sec})$.

Notice that: At the constancy of the mass of the body, H is proportional to V and the relation between them is linear. As a result the momentum in this case is called the linear momentum.

Example Definition of momentum

- 1 Calculate the momentum of a bike whose mass is 35 kg and moves in a uniform speed of a magnitude 12 m/sec towards East.

Solution

$$\therefore H = mV \quad \therefore H = 35 \times 12 = 420 \text{ kg. m/sec}$$

The momentum of the bike = 420 kg. m/sec towards East.



Figure (1)

Try to solve

- 1 Calculate the momentum of a train whose mass is 40 tons moving towards the North with a uniform speed of a magnitude 72 km/h.
- 2 Calculate the momentum of a car whose mass is 800 kg moving towards the Southwest with a uniform speed of a magnitude 126km/h.

Example Using the vectors

- 2 A car of mass 2 tons moves in a straight line such that $\vec{X} = (3t^2 - 4t + 1) \vec{C}$ where \vec{C} is the unit vector in the direction of the motion of the car. If x is measured in meter, find the magnitude of the momentum of the car when it starts to move, then after 3 seconds of its motion.

Solution

$$\therefore \vec{X} = (3t^2 - 4t + 1) \vec{C}$$

$$\therefore \vec{V} = \frac{dx}{dt} = (6t - 4) \vec{C}$$

- (1) At the beginning of the motion $t = 0$, $\vec{V} = -4 \vec{C}$

$$\therefore \vec{H} = m \vec{V}$$

$$\therefore \vec{H} = 2000 (-4 \vec{C}) = -8000 \vec{C}$$

the magnitude of momentum = 8000 kg. m/sec

- (2) when $t = 3$ sec, then $\vec{V} = (6 \times 3 - 4) \vec{C} = 14 \vec{C}$

$$\therefore \vec{H} = m \vec{V} \quad \therefore \vec{H} = 2000 (14 \vec{C}) = 28000 \vec{C}$$

the magnitude of momentum = 28000 kg. m/sec.



Figure (2)

Try to solve

- 3 A car of mass 1200 kg moves in a straight line such that $S = t^3 - 12 t^2$ where S is measured in meter, Find the momentum of the car after 4 sec from the beginning of the motion.

The change of momentum

If the two velocity vectors of a moving body at two successive moments t_1 and t_2 respectively are \vec{v}_1 and \vec{v}_2 then the change of momentum of the body is determined by the relation:

$$\Delta \vec{H} = m \Delta \vec{v}$$

where m is the mass of the moving body, $\Delta \vec{v}$ is the change occurring in the value of its velocity

\therefore The change of momentum of a body $\Delta H = m (\vec{v}_2 - \vec{v}_1)$

If $\vec{a}(t)$ is the acceleration of the moving body, then:

$$\Delta H = m \int_{t_1}^{t_2} a \, dt$$

Example The change of momentum

- 3 A rubber ball of mass 200 gm is let to fall on a horizontal surface from a height of 90 cm to rebound up to a height of 40 cm. Calculate using kg.m/sec unit the magnitude of the change of momentum of the ball as a result of impact.

Solution

Let \vec{c} be the unit vector directed vertically upwards.

Studying the motion of the ball in case of falling.

$$\therefore v^2 = V^2 + 2gS$$

$$\therefore V_1^2 = 0 + 2 \times 980 \times 90$$

$$v_1 = 420 \text{ cm / sec}$$

$$\therefore \vec{v}_1 = 420 \vec{c}$$

Studying the motion of the ball in case of rebounding.

$$\therefore v^2 = V^2 + 2gS$$

$$\therefore 0 = V_2^2 - 2 \times 980 \times 40$$

$$v_2 = 280 \text{ cm / sec}$$

$$\therefore \vec{v}_2 = -280 \vec{c}$$

The change of momentum $\Delta \vec{H} = m(\vec{v}_2 - \vec{v}_1)$

$$= \frac{200}{1000} (-2.8 - 4.2) \vec{c} = -1.4 \vec{c}$$

\therefore the magnitude of the change of momentum = 1.4 kg. m/sec

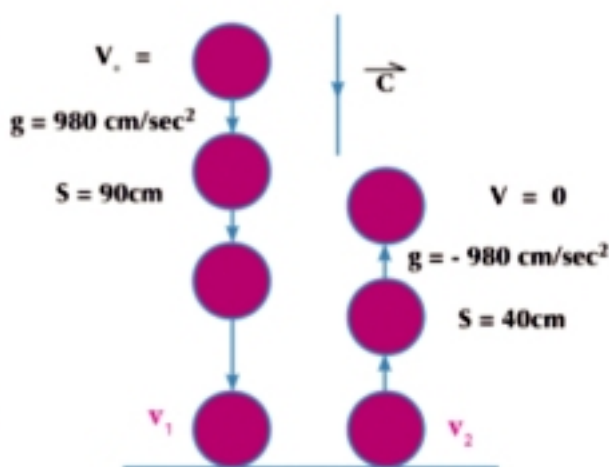


Figure (3)

Try to solve

- 4 A stone of mass 800 gm is let to fall from rest for two seconds then it impings with a surface of a pond and sinks down in a uniform velocity to travel 12 meters in three seconds. Find the momentum of the stone as a result of impact against water surface.

Example Using integration

- 4 A body moves in a straight line such that the acceleration of its motion a is given as a function of time t by the relation $a = 2t - 6$ where a is measured in m/sec^2 , unit and time t in second. Calculate the momentum of the body in the time interval $3 \leq t \leq 5$ if the mass of the body is 8 kg.

Solution

$$\therefore \Delta H = m \int_3^5 a \, dt$$

$$\begin{aligned} \therefore \Delta H &= 8 \int_3^5 (2t - 6) \, dt = 8 [t^2 - 6t]_3^5 \\ &= 8 [(25 - 30) - (9 - 18)] = 32 \text{ kg. m/sec} \end{aligned}$$

$$\therefore \Delta \vec{H} = 32 \vec{C} \text{ where } \vec{C} \text{ is the unit vector in the direction of the motion of the body.}$$

Try to solve

- 5 A car of mass 1.5 tons, moves in a straight line such that $a(t)$ is given by the relation $a = 12t - t^2$ where a is measured in m/sec^2 , unit and time t in sec, find:
- The change of momentum of the car during the first six seconds.
 - The change of momentum of the car during the time interval $[2, 14]$

Critical thinking:

In the billiards game, when the white ball strikes one of the other balls, we find that the motion of the two balls changes and the motion of the white ball slows down and its direction may change and in turn, its momentum slows down while the other ball starts to move and its momentum speeds up. **Explain.**



Figure (4)



Exercises 2 - 1



Choose the correct answer:

- 1 The momentum of a bullet whose mass is 100 gm moving at velocity 240m/sec is:
- 24×10^{-3} gm. m/sec
 - 24kg. m/sec
 - 24×10^4 gm. m/sec
 - 24×10^3 kg. m/sec

- 2 The momentum of a car whose mass is 2 tons moving in a straight line at velocity 54 km/h is:
- a 108 tons. m/sec b 30 ton. km/h
 c 30000 kg. m/sec d 108000 kg. m/sec
- 3 A body of a mass 500 gm is let to fall from a height of 4.9 meters on the ground surface, then its momentum as it comes the ground is:
- a 2.45 kg. m/sec b 4.9kg. m/sec
 c 2450 kg. m/sec d 4900 kg. m/sec
- 4 A rocket of mass 4 tons including the fuel is launched at velocity 200 m/sec, and it throws out the fuel at a constant rate of a magnitude 100 kg per second. If the momentum is constant, then the velocity of the rocket after 10 seconds in km/h unit is:
- a $\frac{800}{3}$ b 600
 c 800 d 960
- 5 A missile of mass 1 kg is launched at velocity 720 km/h towards a tank of mass 50 tons moving towards the mortar at velocity 20 m/sec, then:
- (1) The magnitude of the momentum of the missile with respect to the tank is:
- a 200 kg. m/sec b 220 kg. m/sec
 c 10^7 kg. m/sec d 1.1×10^7 kg. m/sec
- (2) The magnitude of momentum of the tank with respect to the missile is:
- a 200 kg. m/sec b 220 kg. m/sec
 c 10^7 kg. m/sec d 1.1×10^7 kg. m/sec

Answer the following questions:

- 6 A ball of mass 200 gm moves horizontally at a uniform velocity of magnitude 40 m/sec to collide with a vertical wall. If the magnitude of the change of momentum as a result of impact is 12 kg.m/sec, calculate the velocity of rebounding the ball.
- 7 A body of mass 90 gm is let to fall and after three seconds of falling, the body collides with a viscous liquid surface to imbed in it with a uniform velocity to travel 2.2 meters in half a second. Calculate the change of momentum due to the impact.
- 8 A rubber body of mass 100 kg moves horizontally with velocity 120 cm/sec. When it collides with a vertical wall and rebounds in a perpendicular direction on the wall, it loses two thirds of the magnitude of its velocity. Calculate the change of momentum of the rubber body due to the impact.
- 9 From a point under the ceiling of a room of a distance 240 cm, a ball of mass 40 gm is thrown with velocity 980 cm/second vertically upwards to collide with the ceiling and in turn, its momentum changes by a magnitude 0.4 km.m/sec. Find the velocity of rebounding the ball.

- 10 A rubber ball of mass $\frac{1}{2}$ kg is let to fall from a height of 8.1 meters on horizontal ground, then it vertically rebounds upwards to a height of 3.6 meters after it impings with the ground. Calculate the change of the momentum of the ball due to the impact with the ground.
- 11 A train wagon of a mass 15 tons moves horizontally with velocity of a magnitude 40 m/sec to collide with a barrier at the end of the railroad, then it rebounds back with velocity 30 m/sec. Calculate the change of its momentum.
- 12 A body of mass 1 kg is thrown vertically upwards with velocity 58.8 m/s. Calculate the change of its momentum in the following time intervals:
- a [2 , 5] b [4 , 8] c [7 , 11]
- 13 A body whose mass at any time t in second is equal to $\frac{1}{5}(t + 5)$ kg, moves in a straight line and its displacement at any time t is given in the form $\vec{s} = \frac{1}{2}(t^2 - 4t + 3)\vec{c}$ where \vec{c} is the unit vector in the direction of the motion of the body and the magnitude of \vec{s} is given in meter.
- a Find the momentum of the body at any time t .
- b Find the momentum of the body during the time interval [2 , 5]
- 14 A body of mass 12 kg moves in a straight line such that \vec{s} is given as a function of time t by the relation $\vec{s} = t(6 - t)\vec{c}$ where \vec{c} is the unit vector in the direction of the motion. If the magnitude of \vec{s} is in meter unit and t in second, find the change of momentum of the body in the following time intervals:
- a [1, 2] b [2,5] c [4, 6]
- 15 A body moves in a straight line with a uniform acceleration $a = -3\text{m/sec}^2$ and with initial velocity 5m/sec. If the mass of the body is 18 kg, find the magnitude of the change of the momentum in the following time intervals:
- a [0 , 3] b [1,2]
- 16 A body of mass 48 gm, moves in a straight line such that $a = (3t - 12)\text{m/sec}^2$. Calculate the change of momentum motion in the following time intervals:
- a [1, 3] b [3, 5]

You will learn

- Newton's first law
- The inertia principle

Key terms

- Newton's first law
- Inertia
- Inertia principle
- Force

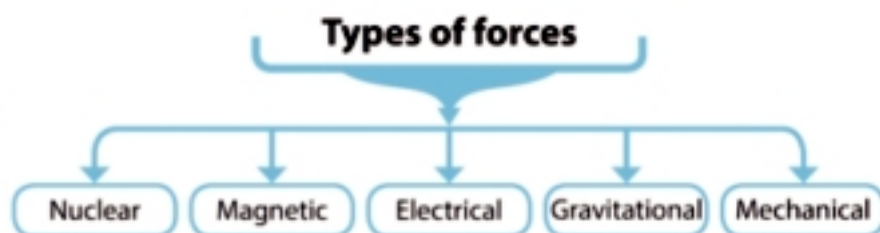
Materials

- Scientific calculator
- Computer graphics

Introduction:

In our daily life, we deal with several types of different forces which may act on the moving bodies to change their velocity. For example, when a person pushes or pulls a car. Forces can also act on the bodies at rest to preserve them in their state of rest for example, the book placed on the office or the picture hanged up on a wall. The action of force may be contact such as pulling the spring or pushing a box or may be acted -at - a distance such as the repulsion or attraction of two poles of a magnet. The particle at rest is said to be in an equilibrium state when the resultant of the forces acting on it is equal to zero.

There are several types of forces existing in nature. They are either mechanical, gravitational, electrical, magnetic or nuclear forces. You are going to study the first and second types only in this unit.



To study the mechanical forces, we start to study Newton's laws of motion

Newton's First Law

Within this law, Isaac Newton stated what happens to a body when the vanish of the forces acting on it.

Every body preserves in its state of rest or of moving uniformly unless acted upon by an unbalanced external force by an external effect

From Newton's first law, we notice that:

- (1) The body at rest preserves at rest unless acted upon by a force to move it and the body in a uniform motion preserves at motion unless acted upon by a force changes its motion.
- (2) In the formulation of the law, the expression "force" means the resultant of all the forces acting upon the body. The force is measured by the newton unit as in the international system of units Newton.
- (3) The law considers the two states of rest and uniform motion in a straight line in an equivalent position since both states represent the "natural state" of the body when the resultant of the forces acting on

the body is equal to zero.

- (4) The law shows that the body which is at rest or moving uniformly in a straight line (i.e. when the body is in its normal state) cannot change its state and that's why Newton's first law is named "law of inertia".

Inertia

From Newton's first law, we can deduce that the bodies naturally endeavor to preserve their states of rest or of moving uniformly in a straight line and this reluctance or resistance to the change is known as inertia.

Inertia principle:

Every body, as much as in its state, endeavors to preserve its present state, whether it be of rest or of moving uniformly forward in a straight line.



Activity

The relation between mass and inertia

- The following activity shows that the mass is the measure to the quantity of the inertia.
- Bring balls such that one of them is a golf ball of weight about 500 gm.wt and the other is a bowling ball of weight about 5 kg.wt.
- Which ball needs a greater force to move?
- Undoubtedly, the bowling ball requires a force greater to start moving than that of the golf ball.
- This is because the bowling ball endeavors to preserve its present state (rest position). i.e. its inertia is greater due to its great mass which is approximated ten times of the mass of the golf ball.



Figure (5)

Force

Newton's first law includes a definition of the force as it is the effect which changes or tries to change the state of the body whether of rest or of moving uniformly forward in a straight line.



Example

(Body at rest)

- 1 The opposite Figure shows a body at rest and a system of forces acting on it. Find F_1 and F_2 .

Solution

- ∵ The body is at the rest state
 ∴ the vertical forces are in equilibrium
 $\therefore F_2 + 40 = 70$
 $\therefore F_2 = 30$ newton
 ∵ the horizontal forces are in equilibrium
 $\therefore F_1 = 20 + F_2$
 $\therefore F_1 = 50$ newton

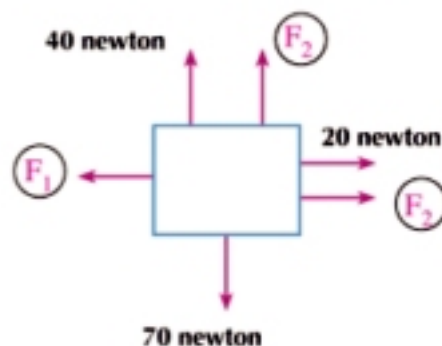
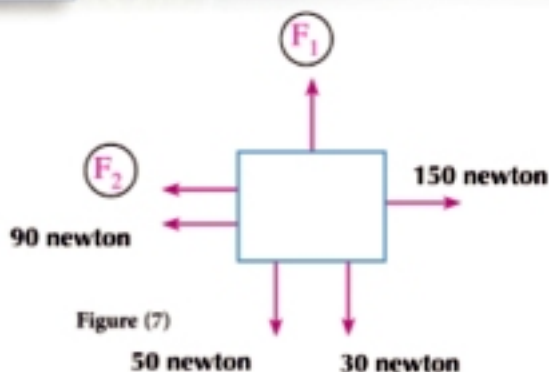


Figure (6)

Try to solve

- 1 The opposite Figure shows a body at rest and a system of forces acting on it. Find F_1 and F_2 .

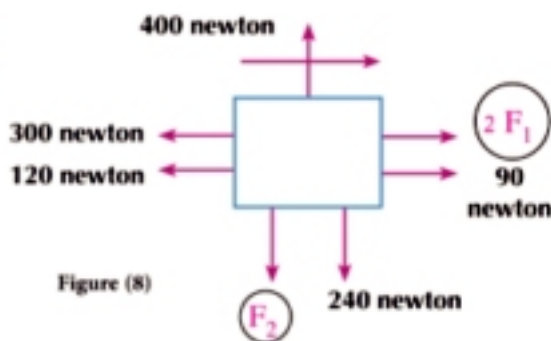


Example (Body at rest)

- 2 The opposite Figure shows a body moving horizontally in the direction shown with a uniform velocity of magnitude 8m/sec. Find F_1 and F_2 .

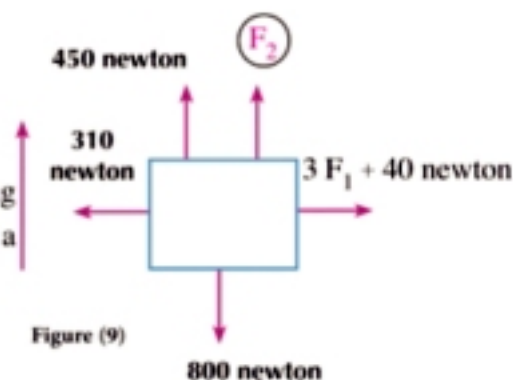
Solution

∴ The body is in a uniform motion state
 ∴ The horizontal forces are in equilibrium
 $\therefore 2F_1 + 90 = 300 + 120$
 $\therefore F_1 = 165$ newton
 ∴ The vertical forces are in equilibrium
 $\therefore 240 + F_2 = 400$
 $\therefore F_2 = 160$ newton



Try to solve

- 2 The opposite Figure shows a body moving vertically upwards with a uniform velocity and a system of forces acting on it. Find F_1 and F_2 .



Example

- 3 A train of mass 200 tons moves under the action of a resistance proportional to the square of its velocity. If this resistance is 9.6 kg.wt per ton of the mass of the train when the velocity of the train is 72 km/h, find the maximum velocity for the train if the engine of the train can entrain (pull) it with a uniform force of magnitude 4.32 ton.wt.

Solution

Let the resistance = r_1 when the velocity of the train is v_1 .
 The resistance = r_2 when the velocity of the train is v_2 .
 ∴ The resistance is proportional to the square of its velocity.

Given data

$r_1 = 9.6 \times 200$
 $= 1920$ sec kg
 $v_1 = 72$ km / h



Figure (10)

$$\therefore \frac{r_1}{r_2} = \frac{V_1^2}{V_2^2}$$

The train reaches the maximum velocity when the resistance is completely equal to the force of pulling the train.

If v_2 is the maximum velocity for the train, then $r_2 = 4.32 \text{ tons.wt} \quad \therefore r_2 = 4320 \text{ kg.wt}$

$$\therefore \frac{r_1}{r_2} = \frac{V_1^2}{V_2^2} \quad \frac{1920}{4320} = \frac{72 \times 72}{V_2^2} \quad \therefore v_2 = 108 \text{ km/h.}$$

P Try to solve

- 3 A train of mass 240 tons is pulled by an engine with a uniform force of magnitude 12 ton.wt. If the resistance of the train motion is proportional to the square of the train's velocity and the resistance is 8 kg.wt per ton of the moving mass when the train's velocity is 45 km/h, calculate the maximum velocity of the train.

Example

- 4 A parachutist lands vertically as the parachute is open. If the total weight of the parachutist and his equipments is 90 kg.wt and the air resistance is proportional to the square of its velocity and the landing maximum velocity of the parachutist is 15 km/h, find the air resistance of the parachutist when his velocity is 10 km/h.

Solution

The parachutist reaches the landing maximum velocity when the resistance is equal to the weight of the parachutist and his equipments.

If r_1 is the air resistance when the velocity of the parachutist is v_1

$$\therefore r_1 = 90 \text{ kg.wt} \quad \text{when } v_1 = 15 \text{ km/h}$$

If r_2 is the air resistance when the velocity of the parachutist is v_2

$$\therefore r_2 = ? \quad \text{when } v_2 = 10 \text{ km/h}$$

\therefore The resistance is proportional to the square of the velocity

$$\therefore \frac{r_1}{r_2} = \frac{V_1^2}{V_2^2} \quad \therefore \frac{90}{r_2} = \frac{15 \times 15}{10 \times 10}, \quad r_2 = 40 \text{ kg.wt}$$



Figure (11)

P Try to solve

- 4 A man tied in a parachute lands vertically. If the air resistance is directly proportional to the square of his velocity and if the air resistance is equal to $\frac{4}{9}$ of the total weight of the man and his equipments when his velocity is 12 km/h, find the landing maximum velocity of the man.

Example

- 5 A body moves in a straight line under the action of three forces $\vec{F}_1 = 4\hat{i} + 3\hat{k}$, $\vec{F}_2 = -\hat{i} + 4\hat{j} - 15\hat{k}$ and \vec{F}_3 such that its displacement vector \vec{S} is given as a function of time by the relation $\vec{S} = 2t\hat{i} - t\hat{j} + \hat{k}$ find the magnitude of \vec{F}_3 .

Solution

$$\therefore \vec{s} = 2t \hat{i} - t \hat{j} + \hat{k} \quad \therefore \vec{v} = \frac{d\vec{s}}{dt} = 2 \hat{i} - \hat{j}, \quad \vec{a} = \frac{d\vec{v}}{dt} = \vec{0}$$

\therefore The body moves

$$\therefore \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{0}$$

$$\therefore \vec{F}_3 = -\vec{F}_1 - \vec{F}_2$$

$$\vec{F}_3 = (-4, 0, -3) + (1, -4, 15) \quad \vec{F}_3 = -3 \hat{i} - 4 \hat{j} + 12 \hat{k}$$

$$F_3 = \|\vec{F}_3\| = \sqrt{9 + 16 + 144} = 13 \text{ force unit.}$$

Try to solve

- 5 A body moves with a uniform velocity under the action of a system of forces \vec{F}_1 , \vec{F}_2 and \vec{F}_3 where $\vec{F}_1 = a \hat{i} - 5 \hat{j} + 7 \hat{k}$, $\vec{F}_2 = -3 \hat{i} + 4 \hat{j}$, $\vec{F}_3 = 2 \hat{i} + 4 \hat{j} + c \hat{k}$, find each of a, b and c.


Exercises 2 - 2

Choose the correct answer:

- 1 A car of mass 4 tons moves on a horizontal road with a uniform velocity. If the force of the engine is 120 kg.wt, then the resistance of motion per ton of the mass is:
 a 4 sec tons b 30 kg.wt c 120 kg.wt d 480 kg.wt
- 2 A body moves in a straight line with a uniform velocity under the action of the two forces $\vec{F}_1 = 2a \hat{i} - 3 \hat{j} + 4 \hat{k}$ and $\vec{F}_2 = 6 \hat{i} + b \hat{j} - c \hat{k}$ then $a + b + c$ equals:
 a 4 b 3 c -3 d -4
- 3 If a body of mass 20 kg.wt slips with a uniform velocity on an inclined plane to the horizontal with angle of measure 30° , then the resistance of the plane in kg.wt equals:
 a zero b 10 c $10\sqrt{3}$ d 20
- 4 A body moves with a uniform velocity under the action of three forces \vec{F}_1 , \vec{F}_2 , \vec{F}_3 where $\vec{F}_1 = 5 \hat{i} + 7 \hat{j} + 35 \hat{k}$, $\vec{F}_2 = 5 \hat{j} + 49 \hat{k}$, then the magnitude of \vec{F}_3 is:
 a 49 force unit. b 54 force unit. c 85 force unit. d 103 force unit.
- 5 A parachutist lands vertically downwards and the air resistance to its motion is proportional to the square of his velocity and v_1 is his velocity when the air resistance is equivalent to $\frac{9}{25}$ of his weight and, v_2 is the landing maximum velocity to the parachutist. Calculate $v_1 : v_2$
 a 9: 25 b 25: 9 c 3: 5 d 5: 3

Answer the following questions:

- 6 In each of the following situations, the body is at rest under the action of a system of forces:

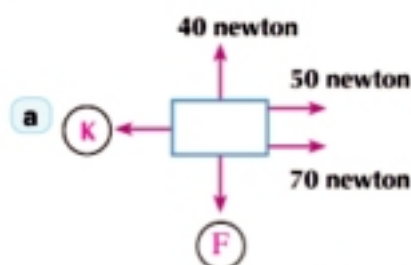


Figure (13)

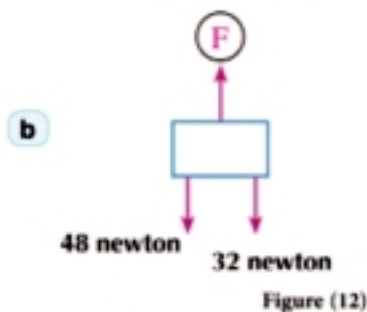


Figure (12)

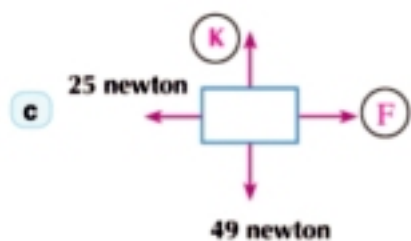


Figure (15)

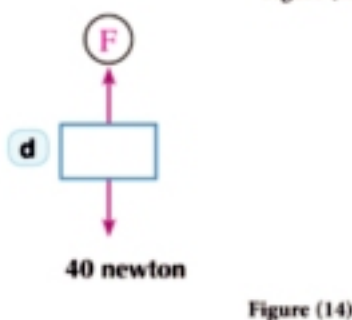


Figure (14)

Find the magnitude of the ungiven force in each case:

- 7 In each of the following situations, the body moves with a uniform velocity V under the action of a system of forces.

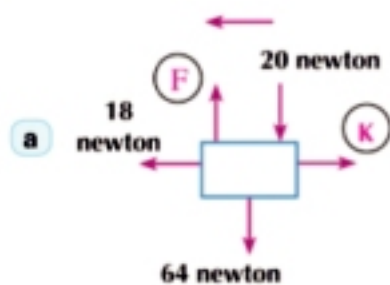


Figure (17)

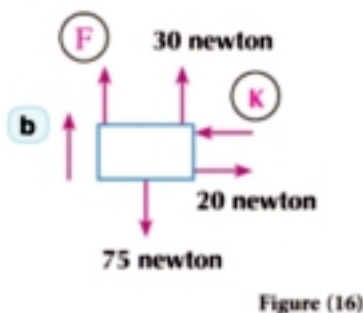


Figure (16)

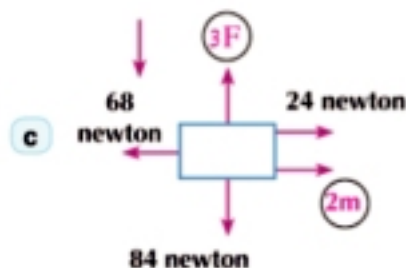


Figure (19)

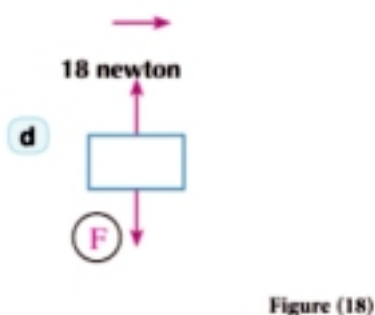


Figure (18)

Find the magnitude of the non-given force in each case:

- 8 A car of mass 8 tons moves with a uniform velocity under the action of a constant resistance of a magnitude 6 kg.wt per each ton of its mass. What is the force of the car's engine?
- 9 A train of mass 240 tons moves with a uniform velocity and the force of the train's engine is 4 w tons. Find the magnitude of the resistance per ton of the train's mass?
- 10 A car of mass 3 tons moves under the action of a resistance proportional to the car's velocity. If this resistance is 8 kg.wt per each ton of the car's mass when its velocity is 36 km/h, find the maximum velocity of the car if the engine pulling force is 120 kg.wt.
- 11 A train of mass 200 tons moves under the action of a resistance proportional to the square of its velocity. If this resistance is 8 kg.wt per ton of the train's mass when its velocity is 70km/h. Find the maximum velocity of the train if the engine pulls with a constant force of a magnitude 6.4 tons.wt
- 12 A train of mass 300 tons is pulled by an engine with a constant force of a magnitude 810 kg.wt under the action of a resistance proportional to the square of its velocity. If the maximum velocity of the train is equal to 30m/sec, find the rate of the resistance per ton of the train's mass when its velocity is 90 km/h.
- 13 The total weight of a parachutist and his equipments is 80 kg.wt and the air resistance to his motion is proportional to the square of his velocity. If this resistance is equal to 45 kg.wt when the velocity of the parachutist is 4.5 km/h, find the maximum velocity which the parachutist can acquire during his landing.
- 14 The total weight of a soldier and his equipments is 90 kg.wt and the air resistance to his motion is proportional to the square of his velocity. If the landing maximum velocity of the solvdier is 12 km.h, find the air resistance when his velocity is 8km/h.
- 15 An engine of mass 30 tons and force 51 ton.wt pulls a number of train cars each of 10 tons to ascend a slope inclined at 30° to the horizontal with a uniform velocity. If the resistance of the motion of the engine and the cars is 10 kg.wt per ton of the mass, find the number of cars.
- 16 A train of mass 300 tons ascends a slope inclined to the horizontal with an angle of sine $\frac{1}{240}$ in the direction of line of the greatest inclination. If the train's maximum velocity is 108 km/h and the force of the engine is equal to 3500 kg.wt and if the magnitude of the resistance is proportional to the square of the magnitude of the velocity, find the resistance which the train meets when it moves with velocity of magnitude 72 km/h.

Newton's Second Law

Unit Two

2 - 3



Think and discuss

From Newton's first law, you know that the resultant of the forces acting on a uniformly moving body vanishes, but if the resultant of the forces acting on a body is not equal to zero, the body moves with an acceleration.

- Is there a relation between the magnitude of the resultant acting on a body and the magnitude of the acceleration of motion?
- Are you able to figure out such a relation?



Learn

1 - Newton's second law

The rate of change of momentum with respect to the time is proportional to the acting force and takes place in the direction in which the force is acting

$$\frac{d}{dt} (m \vec{v}) \propto \vec{F} \quad \text{i.e.} \quad \frac{d}{dt} (m \vec{v}) = K \vec{F}$$

(where K is the proportionality constant)

As the mass of body is constant during motion, then:

$$m \frac{d\vec{v}}{dt} = K \vec{F} \quad (\text{where } K \text{ is the proportionality constant})$$

$$m \vec{a} = K \vec{F}$$

If we define the unit of forces as the force if it acts on a body of mass a unit of masses, it acquires the body the unit of acceleration. By substituting in the equation above, we find that:

$$1 = K \times 1 \times 1 \quad \therefore K = 1$$

and the equation above takes the form $m \vec{a} = \vec{F}$

This equation is called **the equation of motion of a constant mass body**, it is considered the basic equation in dynamics. It can be applied on all the moving bodies of a constant mass by considering.

From the equation of motion above, we find that \vec{F} and \vec{a} are in the same direction. If \vec{a} is measured in a certain direction, it is necessary to measure \vec{F} in the same direction so, it is better to write down the equation of motion in the form:

$$m \vec{a} = \vec{F}$$

to determine the direction of the acceleration first

You will learn

- ↪ Newton's second law
- ↪ Force units
- ↪ Weights and mass

Key terms

- ↪ Newton's second law
- ↪ Equation of motion
- ↪ Force
- ↪ Mass
- ↪ Weight

Materials

- ↪ Scientific calculator

Newton's Laws

If a and F express the algebraic measure of each of \vec{a} and \vec{F} respectively, then the equation of motion of a constant mass body is written in the form:

$$m a = F$$

where m is the mass of the moving body, a is the acceleration of motion and F expresses the algebraic measure of the resultant of the system of forces acting on the body. i.e.:

$$m a = \Sigma F$$

But if the mass m of the body is variable, then the equation of motion is written in the form

$$\vec{F} = \frac{d}{dt} (m \vec{V})$$

$\therefore \Sigma F = \frac{d}{dt} (m V)$ where each of m and v are differentiable function in t

Equation of motion using differentiation

The equation of motion of a constant mass body is given in the form

$$F = ma \quad \text{If } a = \frac{dV}{dt} \quad , \text{ then } F = m \frac{dV}{dt} \quad \therefore \int_{v_1}^{v_2} F dt = m \int_{v_1}^{v_2} dV$$

$$\text{If } a = V \frac{dV}{dS} \quad , \text{ then } F = mV \frac{dV}{dS} \quad \therefore \int_{s_1}^{s_2} F dS = m \int_{v_1}^{v_2} dV$$

Units of force and units of mass

As we deduce the equation of motion of a moving body, we choose certain units for each of the force, mass and acceleration until the constant of proportionality is equal to 1 (unity). The equation of motion takes the form $m a = F$. As a result, when we use the equation of motion, we use the absolute units of force such as newton and dyne.

$$m \times a = F$$

$$1 \text{ kg} \times 1 \text{ m/ sec}^2 = 1 \text{ newton}$$

$$1 \text{ gm} \times 1 \text{ cm/ sec}^2 = 1 \text{ dyne}$$

Remember 

$$1 \text{ kg.wt} = 9.8 \text{ newton}$$

$$1 \text{ gm.wt} = 980 \text{ dyne}$$

Weight and mass

The weight of a body is the Earth's attraction force to the body. If we have a body of mass 1 kg, then its weight according to the equation of motion is equal to 1 kg.wt.

$$\therefore m a = F \quad \therefore 1 \times 9.8 = F \quad F = 9.8 \text{ newton} = 1 \text{ kg.wt}$$



Example

- ① A force of magnitude 10 newtons acts on a body at rest of mass 8 kg to move it in its direction with a uniform acceleration. Calculate the distance traveled after 12 sec and its velocity.

Solution

$$F = 10 \text{ newton} \quad v_0 = 0$$

$$m = 8 \text{ kg} \quad t = 12 \text{ sec}$$

The equation of motion of the body

$$m a = F \quad \therefore 8 a = 10$$

$$a = \frac{5}{4} \text{ m/sec}^2$$

$$\therefore V = v_0 + a t \quad \therefore V = 0 + \frac{5}{4} \times 12 = 15 \text{ m/sec}$$

$$\therefore S = v_0 t + \frac{1}{2} a t^2 \quad \therefore S = 0 + \frac{1}{2} \times \frac{5}{4} \times 144 = 90 \text{ meter}$$

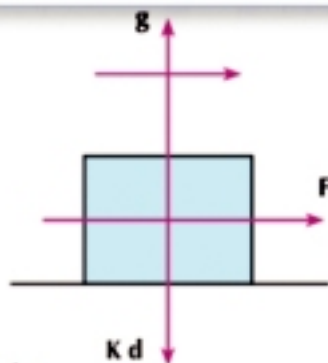


Figure (20)

Try to solve

- 1 If the last car of a train of mass 24.5 tons is separated from the train when its velocity was 54 km/h to move in a uniform retardation and stopped after 125m, find the magnitude of the resistance acting on the separated car in kg.wt.



Figure (21)

Example

- 2 A body of mass 3 kg is let to fall from a height of 10 meters on a sandy ground to sink (embed) for a distance of 5 cm. Find the sand resistance to the body in kg.wt in position constant given that the body moves with a uniform acceleration within the sand.

Solution
The phase of free falling

$$v^2 = V_0^2 + 2 g S$$

$$v^2 = 0 + 2 \times 9.8 \times 10$$

$$V = 14 \text{ m/sec}$$

The phase of sinking (embedding) in sand

$$v^2 = v_0^2 + 2a S$$

$$0 = (14)^2 + 2a \times 0.05$$

$$a = -1960 \text{ m/sec}^2$$

Equation of motion

$$m a = m g - r$$

$$3 \times -1960 = 3 \times 9.8 - r$$

$$\therefore r = 3 \times 9.8 + 3 \times 1960$$

$$r = 5909.4 \text{ newton}$$

$$r = 603 \text{ kg.wt}$$

Try to solve

- 2 A box of mass 100 kg, is lifted off vertically upwards by a string with a uniform acceleration of magnitude 25 cm/sec^2 . Find the force of the tension in the string while neglecting the resistance.

Example

- 3 A train of mass 220 tons moves a long horizontal straight railroad with a uniform velocity of magnitude 29.4 m/sec . During the train's motion, the last car of mass 24 tons is separated and moves with a uniform retardation to stop completely after one minute of the separation moment.



Figure (23)

Find:

First: the magnitude of the resistance per ton of the train's mass supposing it is constant.

Second: the magnitude of the force of the train's engine.

Third: The distance between the remaining part of the train and the separated car at the moment the car is completely at rest given that the remaining part of the train moves with a uniform acceleration.

Solution

First: Study the motion of the separated car:

$$\begin{aligned}
 V &= V_0 + at & m &= 24 \text{ ton} & v_0 &= 29.4 \text{ m/sec} \\
 0 &= 29.4 + 60a & V &= 0 & t &= 60 \text{ sec} \\
 \therefore a &= -0.49 \text{ m/sec}^2
 \end{aligned}$$

From the equation of motion of the separated car

$$\begin{aligned}
 ma &= -r & 24 \times 1000 \times -0.49 &= -r \\
 r &= 11760 \text{ newton.} & \therefore r &= 1200 \text{ kg.wt} \\
 r &= 50 \text{ kg.wt/ton}
 \end{aligned}$$

the distance traveled by the separated car within a minute after separation

$$\begin{aligned}
 S &= V_0 t + \frac{1}{2} a t^2 \\
 &= 29.4 \times 60 - \frac{1}{2} \times 0.49 \times (60)^2 \\
 S &= 882 \text{ meters}
 \end{aligned}$$

Second: study the motion of the train before separation

$$\begin{aligned}
 \therefore \text{The train was moving with a uniform velocity before separation} & \quad m = 220 \text{ ton} \\
 \therefore \text{The moving force} = \text{the total resistances} & \quad \text{resistance/ton} = 50 \text{ kg.wt} \\
 \therefore F = 50 \times 220 = 11000 \text{ kg.wt}
 \end{aligned}$$

Third: study the motion of the remaining part of the train after the last car has been separated

Equation of motion

$$m a = F - r$$

$$196 \times 1000 a = (11000 - 50 \times 196) \times 9.8$$

$$a = \frac{3}{50} \text{ m/sec}^2$$

From the laws of motion

$$S = V_0 t + \frac{1}{2} a t^2$$

$$= 29.4 \times 60 + \frac{1}{2} \times \frac{3}{50} \times (60)^2$$

$$S = 1872 \text{ meters}$$

The distance between the remaining part of the train and the separated car at the moment of its rest

$$= 1872 - 882$$

$$= 990 \text{ meters}$$

Critical thinking: Draw a graph representing the distance between the remaining part of the train and the separated car from the separation moment until this car stops, then find the following from the graph:

- When is the distance between them equal to 110 meters?
- The distance between them after 40 seconds from the separation moment.

P Try to solve

- A zeppelin of mass 105 kg, moves vertically downwards with a uniform acceleration of magnitude 98 cm/sec^2 . Find the magnitude of the air rising force acting on the zeppelin in kg. If a body of mass 35 kg is let to fall from the zeppelin when the velocity of the zeppelin was 490 cm/sec , find the distance between the zeppelin and the fallen body after $\frac{20}{7}$ seconds from the separation moment.



Figure (24)

Example

- A body of a unit mass under the action of three forces $\vec{F}_1 = a \hat{i} + \hat{j}$, $\vec{F}_2 = \hat{i} + b \hat{j} + 3 \hat{k}$, $\vec{F}_3 = \hat{i} + 2 \hat{j} - e \hat{k}$, if the displacement vector \vec{S} is given by the relation $\vec{S} = t \hat{i} + (\frac{1}{2} t^2 + t) \hat{j} + 5 \hat{k}$ find the value for each of a, b and e

Solution

$$\therefore \vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = (a + 2) \hat{i} + (b + 3) \hat{j} + (3 - e) \hat{k}$$

$$\therefore \vec{S} = t \hat{i} + (\frac{1}{2} t^2 + t) \hat{j} + 5 \hat{k}$$

$$\vec{V} = \frac{dS}{dt} = \hat{i} + (t + 1) \hat{j}$$

$$\vec{a} = \frac{d\vec{V}}{dt} = \hat{j}$$

$$\begin{aligned} \therefore m \vec{a} &= \vec{F} & \therefore \hat{j} &= (a + 2) \hat{i} + (b + 2) \hat{j} + (3 - e) \hat{k} \\ a + 2 &= 0 & , b + 3 &= 1 & 3 - e &= 0 \\ a &= -2 & , b &= -2 & e &= 3 \end{aligned}$$

P Try to solve

- 4 A body of mass 3 kg moves under the action of three coplanar forces $\vec{F}_1 = a \hat{i} + \hat{j}$, $\vec{F}_2 = 2 \hat{i} - \hat{j}$, $\vec{F}_3 = 3 \hat{i} + b \hat{j}$ where \hat{i} , \hat{j} are two unit vectors perpendicular to the plane of the forces. If the displacement vector is given as a function of time by the relation $\vec{S} = (t^2 + 1) \hat{i} + (2t^2 + 3) \hat{j}$, identify the value for each of a and b.

Example

- 5 A force \vec{F} acts on a body of mass 1 kg at rest, moving in a straight line starting from the origin point "o" on the straight line and $F = 5x + 6$ where x is the distance between the body and point "o" measured in meter and F in newton.

Find:

First: the velocity of the body V when $x = 4$ meters

Second: the displacement of the body when $V = 9$ m/sec

Solution

$$\therefore F = 5x + 6$$

$$\therefore ma = 5x + 6$$

$$\therefore a = v \frac{dv}{dx}, m = 1 \text{ kg}$$



Figure (25)

First:

$$\therefore V \frac{dV}{dx} = 5x + 6$$

$$\begin{aligned} \therefore \left[\frac{1}{2} v^2 \right]_0^v &= \left[\frac{5}{2} x^2 + 6x \right]_0^x \\ v^2 &= 128 \end{aligned}$$

$$\therefore \int^v V dV = \int_0^4 (5x + 6) dx$$

$$\therefore \frac{1}{2} v^2 = \left(\frac{5}{2} \times 16 + 6 \times 4 \right) - 0$$

$$\therefore V = \pm 8\sqrt{2} \text{ m/sec}$$

Second:

$$\therefore V \frac{dV}{dx} = 5x + 6$$

$$\therefore \left[\frac{1}{2} v^2 \right]_0^9 = \left[\frac{5}{2} x^2 + 6x \right]_0^x$$

$$\therefore 5x^2 + 12x - 81 = 0$$

$$(5x + 27)(x - 3) = 0$$

$$\therefore \int^9 V dv = \int^x (5x + 6) dx$$

$$\therefore \frac{81}{2} = \frac{5}{2} x^2 + 6x - 0$$

$$\therefore x = 3, \quad x = -\frac{27}{5}$$

Try to solve

- 5 A force F acts on a body of mass 3 kg, moving in a straight line with initial velocity of a magnitude 2 m/sec. If $F = \frac{3}{2v + 1}$ where V is the velocity of the body after a time of a magnitude t , when is the velocity of the body 6 m/sec.

Example

- 6 A force acts upon a body of mass 250 gm moving in a straight line starting from rest at the origin point "o" on the straight line and $\vec{F} = (5t - 2) \hat{i} + 4t \hat{j}$. If F is measured in newton unit and t in second, find the velocity \vec{V} and the displacement \vec{S} in terms of t .

Solution

$$\begin{aligned} \therefore \vec{F} &= (5t - 2) \hat{i} + 4t \hat{j} & \therefore \vec{F} &= m \vec{a} \quad \text{where } m = \frac{1}{4} \text{ kg} \\ \therefore \frac{1}{4} \vec{a} &= (5t - 2) \hat{i} + 4t \hat{j} & \therefore \vec{a} &= (20t - 8) \hat{i} + 16t \hat{j} \\ \therefore \vec{a} &= \frac{d\vec{V}}{dt} & \therefore \frac{d\vec{V}}{dt} &= (20t - 8) \hat{i} + 16t \hat{j} \\ \therefore \int_0^V d\vec{V} &= \int_0^t [(20t - 8) \hat{i} + 16t \hat{j}] dt \\ \therefore \vec{V} &= (10t^2 - 8t) \hat{i} + 8t^2 \hat{j} \\ \therefore \vec{V} &= \frac{d\vec{S}}{dt} & \therefore \frac{d\vec{S}}{dt} &= (10t^2 - 8t) \hat{i} + 8t^2 \hat{j} \\ \therefore \int_0^S d\vec{S} &= \int_0^t [(10t^2 - 8t) \hat{i} + 8t^2 \hat{j}] dt \\ \therefore \vec{S} &= \left(\frac{10}{3}t^3 - 4t^2\right) \hat{i} + \frac{8}{3}t^3 \hat{j} \end{aligned}$$

Try to solve

- 6 A force \vec{F} acts upon a body of mass $\frac{1}{2}$ kg at rest starting its motion from a constant point "O" on the straight line and $\vec{F} = (4t - 1) \hat{i} + 4 \hat{j}$ where t is the time measured in second and F in newton. Find the velocity of the body when $t = 2$ second and the distance between the body and point "O".

Example

- 7 A body of a variable mass of $m = 2t + 1$ moves in a straight line and its displacement vector is given by the relation: $\vec{S} = \left(\frac{1}{2}t^2 + t\right) \hat{i}$ where \hat{i} is the unit vector parallel to the straight line. Find the momentum of this body and deduce the magnitude of the force acting on it.

Solution

$$m = 2t + 1$$

The velocity vector $\vec{V} = \frac{d\vec{S}}{dt} = \frac{d}{dt} \left(\frac{1}{2} t^2 + t \right) \hat{i} = (t + 1) \hat{i}$

The momentum vector $\vec{H} = m \vec{V}$
 $= (2t + 1)(t + 1) \hat{i} = (2t^2 + 3t + 1) \hat{i}$

From Newton's second law, we find that:

$\vec{F} = \frac{d}{dt} (m \vec{V}) = \frac{d\vec{H}}{dt} = \frac{d}{dt} (2t^2 + 3t + 1) \hat{i} = (4t + 3) \hat{i}$

i.e. the force acting upon the body is in the direction of vector \hat{i} and its magnitude is equal to $(4t + 3)$

Try to solve

- 7 A metal ball of mass 100 gm moves with a uniform velocity 10 m/sec in a dusty medium such that the dust sticks to its surface at a constant rate equal to 0.6 gm per second. Find the mass of the ball and the force acting on it at any moment in dyne.



Exercises 2-3



Choose the correct answer:

- 1 A body of mass 5 kg then its weight is:
 a $\frac{25}{49}$ newton b 5 newton c 49 newton d 49 kg.wt
- 2 A body of mass m kg moves under the action of the force $\vec{F} = 3m \hat{i} + 4m \hat{j}$, where F is in newton, then the magnitude of the acceleration of motion in m/sec^2 unit is:
 a 3 b 4 c 5 d 7
- 3 A body of mass unit moves under the action of the force $\vec{F} = 5 \vec{C}$ if its velocity vector $\vec{V} = (a t^2 + b t) \vec{C}$, then $a + b$ is:
 a 0 b $\frac{5}{2}$ c $\frac{7}{2}$ d 5
- 4 A body of mass 8 kg moves vertically upwards with a uniform acceleration a under the action of a force acts at the direction of motion of a magnitude 12 kg.wt. Find a in m/sec^2 :
 a $\frac{1}{2}$ b $\frac{3}{2}$ c 4.9 d 14.7
- 5 A bullet of mass 7 gm is shot vertically from the barrel of a pistol with velocity 245 m/sec on a vertical barrier of wood to embed in it for 12.25 cm before being at rest. Calculate the wood resistance to the bullet given that it moves in a retarded motion.
 a 17.15 newton b 175 newton c 175 kg.wt d 1715 kg.wt
- 6 If a body of mass $m(2t + 3)$ kg moves in a straight line and its displacement vector as a function of time is given by the relation $\vec{S} = \left(\frac{3}{2} t^2 + 2t \right) \vec{C}$, S is measured in meter and, t in second, then the magnitude of the force acting upon the body in newton is:
 a $2t + 3$ b $12t + 3$ c $12t + 13$ d $6t + 13$

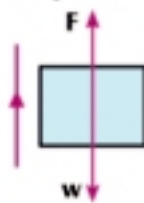
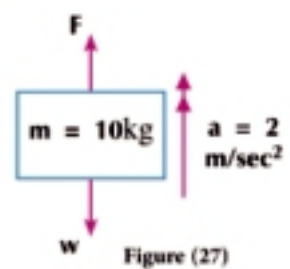
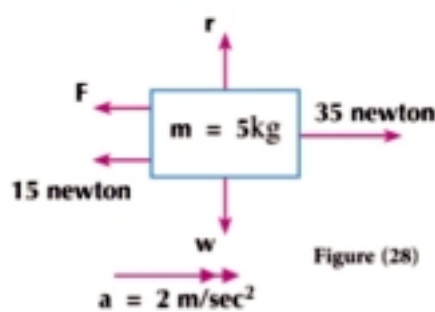
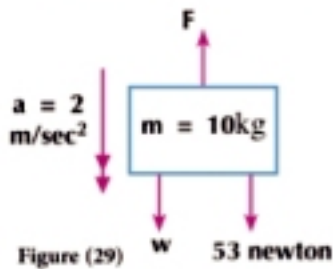
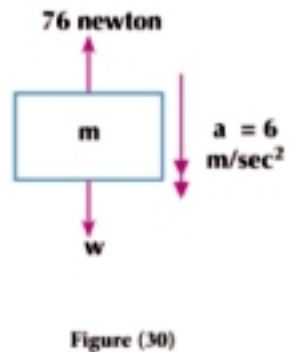
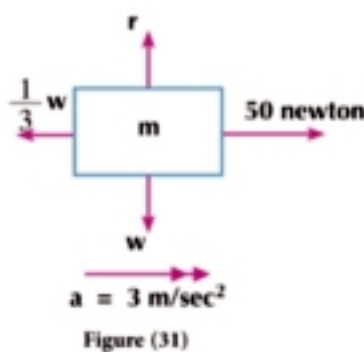
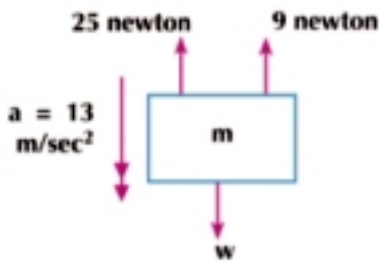


Figure (26)

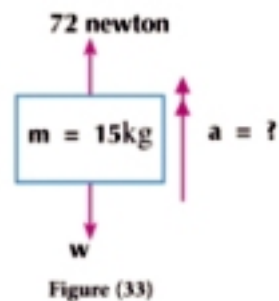
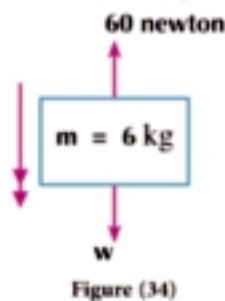
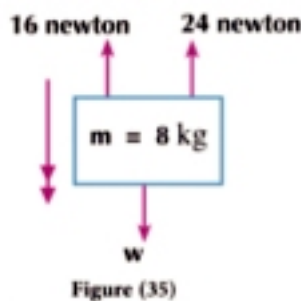
- 7 In each of the following cases; the force F acts upon the body whose mass is m and acquires it a uniform acceleration shown in the figures, both magnitude and direction, find F



- 8 In each of the following cases; the force F acts upon the body whose mass is m and acquires it a uniform acceleration shown in the figures both magnitude and direction, find F



- 9 In each of the following cases; the force F acts upon the body whose mass is m and acquires it a uniform acceleration a measured in m/sec^2 , find a



- 10 A body mass 150 gm is acted upon by a force of a magnitude 4500 dyne. Find the acceleration resulted.
- 11 A mass of a magnitude 20 kg is placed on a smooth horizontal plane. A horizontal force of a magnitude F acts on it to move it with a uniform acceleration of a magnitude $49 m/sec^2$. Find F .
- 12 A car at rest of mass 4.9 tons, is acted upon by a force and its velocity gets 27 km/h within one minute, find the force acted upon the car in kg.wt.

- 13 If the force of the engine is equal to 2.5 ton.wt and the mass of the train and the engine is 200 tons and if the train starts to move from rest, find the velocity of the train after half a minute in pose negligence resistors.
- 14 Find the resistance force of the brakes to the motion of the train in kg.wt per each ton of its mass if the velocity is 72 km/h and the brakes stop the train after it traveled 250 meters, find the time needed for doing that.
- 15 A man has pushed a car at rest of mass 980 kg with a constant force. If the velocity of the car is 45 cm/sec after 5 seconds, find in kg.wt the force by which the man has pushed the car if the resistance is 50 kg.wt.
- 16 Find the horizontal force by which an engine of a train of mass 245 tons is pulled to speed up its velocity into 18 km/h after it covers a distance of one kilometer on a horizontal railroad if the resistance force is 4 kg.wt ton.
- 17 A constant horizontal force of a magnitude 1 ton.wt acts upon a car of mass 4 tons moving on a horizontal road. If the car starts to move from rest and reaches a velocity of 4.9 m/sec in 10 seconds. Find the magnitude of the resistance acting on the car.
- 18 A body of mass 2 kg is acted upon by the forces $\vec{F}_1 = 4 \hat{i} + 2 \hat{j}$, $\vec{F}_2 = -\hat{i} + \hat{j}$ where the magnitude of F is in newton. Find the magnitude of the acceleration \vec{a}
- 19 The force \vec{F} acts upon a body of mass 500 gm to acquire it an acceleration \vec{a} where $\vec{a} = 5 \hat{i} + 2 \hat{j}$ if a is measured in m/sec², find the magnitude of \vec{F} in newton.
- 20 Find the acceleration vector \vec{a} which a body of mass 2 kg, acquires if the next forces act upon it $\vec{F}_1 = 2 \hat{i} - 3 \hat{j} + 4 \hat{k}$, $\vec{F}_2 = \hat{i} + 5 \hat{j} + 2 \hat{k}$ if F is in newton unit.
- 21 The forces $\vec{F}_1 = a \hat{i} + b \hat{j} + c \hat{k}$, $\vec{F}_2 = 2 \hat{i} - 3 \hat{j} + \hat{k}$ act on a body of a mass 2 kg, to acquire it an acceleration $\vec{a} = 4 \hat{i} + \hat{k}$. Find a, b, c, if F in newton and a is in m/sec².
- 22 A body of mass 3 kg moves under the action of 3 coplanar forces $\vec{F}_1 = 2 \hat{i} - b \hat{j}$, $\vec{F}_2 = a \hat{i} + \hat{j}$, $\vec{F}_3 = 3 \hat{i} + 2 \hat{j}$ where \hat{i} , \hat{j} are two unit vectors perpendicular in the plane of the forces. If the displacement vector is given as a function of time by the relation $\vec{S} = (t^2 + 1) \hat{i} + (2t^2 + 3) \hat{j}$, identify the two constants a and b.
- 23 A body of mass $m = (2t + 5)$ kg and position vector $\vec{r} = (\frac{1}{2}t^2 + t - 5)\vec{C}$ where vector \vec{C} is a constant unit, vector r is measured in meter and t is the time in second, find:

First: the two vectors of velocity and acceleration of the body at any moment t.

Second: the magnitude of the force acting upon the body when $t = 10$ seconds

- 24 A metal ball of mass 150 kg moves with a uniform velocity 12 m/sec within a dusty medium such that the dust sticks to its surface at a constant rate of 0.5 gm per second. Find the mass of the ball and the force acting upon it at any moment t in dyne.
- 25 A metal ball of mass 10 gm moves in a straight line within a dusty medium such that the dust sticks to its surface at a rate of 1 gm per second. If the displacement of this ball at the end of a time interval is $\vec{S} = (t^2 + 3t) \hat{i}$ where \hat{i} is a unit vector in the direction of its motion, find the force acting upon the ball at any moment t and calculate its magnitude when $t = 3$ seconds given that the magnitude of the displacement is measured in cm.
- 26 A body of variable mass moves in a straight line and its mass at any moment t is equal to $m = (4t + 1)$ gm and its displacement vector is given by the relation $\vec{S} = (t^2 + 2t) \hat{i}$ where \hat{i} is a constant unit vector parallel to the straight line, t is the time in second and S is the distance in cm, find:
- the momentum vector of this body.
 - the magnitude of the force acting upon the body when $t = 4$.
- 27 A force $F = 3t + 1$ acts upon a body at rest of mass, 4 kg starting its motion at the origin point "O" on the straight line.
- Find V when $t = 2$ seconds.
 - Find S when $t = 2$ seconds, given that F is in newton unit.
- 28 Find the least acceleration by which a man of mass 75 kg can slide on a survival rope from a fire if the rope cannot stand tension more than 50 kg.wt. Find the velocity of the man after landing 30 meters given that the acceleration is uniform.
- 29 A bullet of mass 20 gm collides with a constant barrier of wood when its velocity was 700 m/sec to embed in it for a distance of 5 cm. Calculate the resistance of wood supposing it is constant in kg.wt.
- 30 A body of mass 2 kg is let to fall from a height of 10 meters toward a sandy ground to embed in it for a distance of 5cm. Find the sand resistance supposing it is constant in kg.wt.
- 31 A train of mass 245 tons (including the engine) moves with a uniform acceleration of a magnitude 15 cm/sec^2 on a horizontal straight rail road. If air resistance and friction are 75 kg.wt per each ton of the train's mass, find the force of the engine in kg.wt. If the last car of the train of mass 49 tons is separated after the train moved from rest for 4.9 minutes, find the time taken by the separated car until it stops.



Cooperative work

Work with a classmate to bring a pressure scale and place it on the roof of a lift. Then stand on the scale while the lift is at rest and have your classmate record the scale readings as you stand on the pressure scale. Let the lift move upwards and have your classmate record any change occurring in the scale readings. After that, stop the lift and record the readings once more. Then let the lift descend and have your classmate record the readings of the scale when any change occurs in the readings. Repeat this experiment alternately with your classmate. Record the scale readings as you and your classmate stand on the scale in each phase of the scale; at rest, ascending and descending.



Figure (36)

What is your interpretation of the scale readings in each phase?



Learn

1 - Newtons third law

To every action, there is a reaction equal in magnitude and opposite in direction.

2 - Pressure and reaction

When we place a body of mass m on a rested horizontal plane, then the body acts on the plane with a pressure force equal, in this case, to the weight of the body and a force of reaction is generated for the plane acting upon the body completely equal to the pressure exerted by the body on the plane and the two forces are opposite in direction but equal in the magnitude completely. The pressure of the body on the plane changes as the plane moves up or down. The pressure in this case is known as the apparent weight.

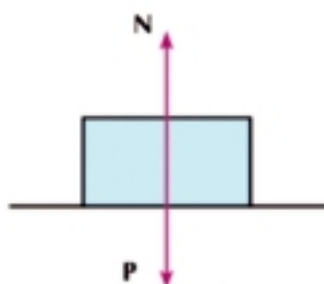


Figure (37)

Lift motion

The lift motion is considered the most well known application of the reaction when a person of mass m stands in a lift of mass m' , then there is a system of forces acting upon each of them.



Figure (38)

You will learn

- ↪ The pressure and reaction
- ↪ Lift motion

Key terms

- ↪ Newtons third law
- ↪ Pressure
- ↪ Reaction
- ↪ Lift motion
- ↪ Spring scale
- ↪ Pressure scale
- ↪ Balance

Materials

- ↪ Scientific calculator
- ↪ Balance
- ↪ Spring scale
- ↪ Pressure scale

The forces acting on the person inside the lift

There are two forces acting upon the person inside the lift:

- 1 - The weight of the person = mg (it acts vertically downwards whatever the direction of the lift is)
- 2 - The reaction of the lift on the person = R
(it acts vertically upwards whatever the direction of the lifts is).

The equation of a person's motion

When the lift is at rest or moves uniformly (constant velocity upwards or downwards) then $mg = N$

When the lift moves up with acceleration of a magnitude a , the equation of the person's motion is $ma = N - mg$

When the lift moves down with acceleration of a magnitude a , the equation of the person's motion is $ma = mg - N$

Critical thinking: What do you expect about the reaction of the lift on the person if the lift falls with an acceleration equal to the gravitational acceleration?

The forces acts on the lift only if the person is inside it (Figure 40)

There are three forces acting on the lift as a person is inside it:

- 1 - The weight of the lift only = $m'g$ (it acts vertically downwards whatever the direction of the lift is)
- 2 - The pressure of the person on the floor of the lift = P
(it acts vertically downwards whatever the direction of the lift is)
- 3 - The tension in the wire carrying the lift = T
(it acts vertically upwards whatever the direction of the lift is)

The equation of the lift's motion

As moving up with an acceleration of a magnitude a , the equation of the lift's motion is $m'a = T - P - m'g$

As moving down with an acceleration of a magnitude a , the equation of the lift's motion is $m'a = m'g + P - T$

The forces acting on a system (the lift and the person together) (Figure 41)

There are two forces acting on both the lift and the person:

- 1 - the weight of the system (lift and person) = $(m + m')g$
(it acts vertically downwards whatever the direction of the lift is)
- 2 - The tension in the wire carrying the lift = T
(it acts vertically upwards whatever the direction of the lift is)



Figure (39)



Figure (40)



Figure (41)

Note:

The pressure of the person on the floor of the lift is equal and opposite to the reaction of the lift on the person

The equation of the motion of the a system

As moving up with an acceleration of a magnitude a, the equation of the motion of the lift is

$$(m + m') a = T - (m + m')g$$

As moving down with an acceleration of a magnitude a, the equation of the motion of the lift is

$$(m + m') a = (m + m') g - T$$

Spring Scale

When a body of mass m is suspended in a spring scale fixed in the ceiling of a lift, then the reading of the scale expresses the tension occurring in the spring.



Figure (42)

Pressure scale

When a body of mass m is placed on a pressure scale fixed in the floor of a lift, then the reading of the scale expresses the pressure of the body on the scale.

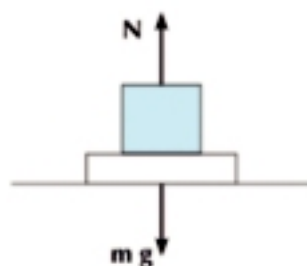


Figure (45)

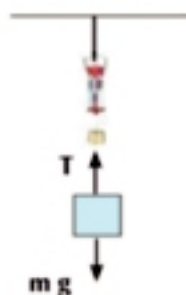


Figure (44)



Figure (43)

- 1 - If the reading of the scale $>$ the real weight $T > mg$, $N > mg$, then the lift is moving upwards with an positive acceleration or moving downwards with negative acceleration.
- 2 - If the reading of the scale $<$ the real weight $T < mg$, $N < mg$, then the lift is moving downwards with an positive acceleration or moving upwards with a negative acceleration.
- 3 - If the reading of the scale $=$ the real weight $T = mg$, $N = mg$, then the lift is either at rest or moving with a uniform velocity. The readings of the spring scale or pressure scale are called the apparent weight.

Notice that

If the lift moves upwards with a uniform acceleration and moves downwards with the same acceleration, then:

the reading of the scale when moving upwards + reading of the scale when moving down wards = twice the real weight.

The balance

The balance is the only machine that can measure the real weight in all conditions.



Figure (46)

Example

- 1 A man of mass 80 kg is inside a lift. Calculate in kg.wt the pressure of the man on the floor of the lift in each of the following:
- 1 - Moving upwards with a uniform acceleration of magnitude 49 cm/sec^2 .
 - 2 - Moving with a uniform velocity of magnitude 80 cm/sec .
 - 3 - Moving downwards with a uniform acceleration of magnitude 98 cm/sec^2 .

Solution

The pressure exerted by the man on the floor of the lift is equal in magnitude to the reaction of the lift on the man.

- 1 - The lift moves upwards with an acceleration of magnitude 0.49 m/sec^2 .

$$\therefore ma = N - mg$$

$$80 \times 0.49 = N - 80 \times 9.8$$

$$\therefore N = 80 \times 0.49 + 80 \times 9.8$$

$$N = 823.2 \text{ newton}$$

$$N = 84 \text{ kg.wt}$$

- 2 - The lift moves with a uniform velocity.

$$\therefore a = 0$$

$$\therefore N = mg \quad g = 80 \text{ kg.wt}$$

- 3 - The lift moves downwards with a uniform acceleration of magnitude 0.98 m/sec^2 .

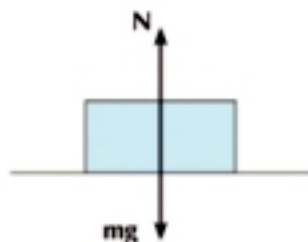
$$ma = mg - N$$

$$80 \times 0.98 = 80 \times 9.8 - N$$

$$N = 80 \times 9.8 - 80 \times 0.98$$

$$N = 705.6 \text{ newton.}$$

$$N = 72 \text{ kg}$$



Try to solve

- 1 A person of mass 60 kg is inside a lift. Calculate in kg.wt the pressure of the person on the floor of the lift in each of the following cases:
- 1 - If the lift is at rest.
 - 2 - The lift moves upwards with a uniform acceleration of magnitude 49 cm/sec^2 .
 - 3 - The lift moves downwards with a uniform acceleration of magnitude 49 cm/sec^2 .

Example

- 2 A body is suspended by a string in a spring scale fixed at the top of a lift moving upwards. If the magnitude of tension in the string during ascending with an uniform acceleration of magnitude 2.45 m/sec^2 is equal to 50 kg.wt , find the mass of the body. What is the magnitude of tension if the lift moves down with the same acceleration?

Solution

First: the lift moves upwards with an acceleration 2.45 m/sec^2

the equation of motion: $ma = T - mg$

$$m \times 2.45 = 50 \times 9.8 - m \times 9.8$$

$$m(2.45 + 9.8) = 50 \times 9.8 \quad m = 40 \text{ kg}$$

Second: the lift moves downwards with an acceleration 2.45 m/sec^2 .

the equation of motion: $ma = mg - T$

$$40 \times 2.45 = 40 \times 9.8 - T$$

$$T = 40(9.8 - 2.45)$$

$$T = 294 \text{ newton}$$

$$T = 30 \text{ kg.wt}$$

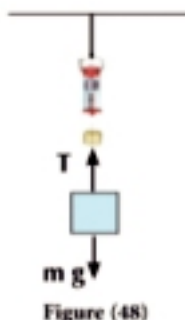


Figure (48)

Try to solve

- 2 A body of real weight 240 kg.wt is suspended in a spring scale fixed at the top of a lift and its apparent weight is 276 kg.wt according to the reading of the spring scale. Show that the acceleration of motion of the lift has two values, then find them and determine the direction of the motion.

Example

- 3 A lift moves vertically upwards with a uniform acceleration of a 140 cm/sec^2 , and a person is inside the lift. If the pressure of the person on the floor of the lift is equal to 72 kg.wt , calculate the mass of the person, then find the magnitude of his pressure on the floor of the lift as he moves downwards with the same acceleration.

Solution

First: The lift moves up with acceleration $a = 1.4 \text{ m/sec}^2$.

the equation of motion: $ma = g - m g$

$$m \times 1.4 = 72 \times 9.8 - m \times 9.8$$

$$\therefore m = 63 \text{ kg}$$

Second: the lift moves down with acceleration $a = 1.4 \text{ m/sec}^2$.

the equation of motion: $ma = m g - N$

$$63 \times 1.4 = 63 \times 9.8 - N$$

$$N = 63(9.8 - 1.4) \quad \therefore N = 529.2 \text{ newton}$$

$$N = 54 \text{ kg.wt}$$

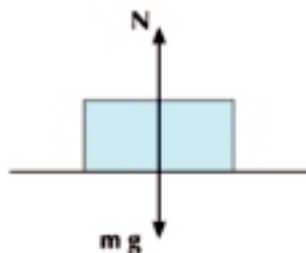


Figure (49)

P Try to solve

- 3 A man of mass 70 kg is inside an electrical lift of mass 420 kg. If the lift moves vertically upwards with an acceleration of uniform 70 cm/sec^2 ,
find in kg.wt the magnitude for each of the tension in the rope carrying the lift and the pressure of the man on the floor of the lift.

Example

- 4 A body is suspended in a spring scale fixed at the top of a lift. It is noticed that when the lift moves up with acceleration $a \text{ m/sec}^2$, the reading of the scale is 8 kg.wt and when the lift moves down with acceleration $2 a \text{ m/sec}^2$ the reading of the scale is 5 kg.wt.
Calculate of a if the steel wire carrying the lift cannot stand tension more than 1.2 ton.wt. Find the maximum load that the lift can stand as it moves up with acceleration a given that the mass of the lift as it is empty is 600 kg.

Solution

First: The lift moves up with acceleration a
the equation of motion: $ma = T - mg$

$$ma = 8 \times 9.8 - m \times 9.8$$

$$ma = (8 - m) \times 9.8 \quad (1)$$

Second: The lift moves down with acceleration $2a$

the equation of motion $ma = mg - T$

$$2ma = m \times 9.8 - 5 \times 9.8$$

$$2ma = (m - 5) \times 9.8 \quad (2)$$

From (1) and (2) we find that

$$\frac{2ma}{ma} = \frac{(m - 5) \times 9.8}{(8 - m) \times 9.8}$$

$$\frac{2}{1} = \frac{m - 5}{8 - m}$$

$$m - 5 = 16 - 2m$$

$$3m = 21$$

$$\therefore m = 7 \text{ kg}$$

from (1) we find that

$$7a = 9.8 \quad a = 1.4 \text{ m/sec}^2.$$

Third:

Suppose the maximum load that can be placed inside the lift of mass $m \text{ kg}$

Hence, the tension in the wire carrying the lift is equal to 1200 kg.wt

The equation of motion: $(m + m')a = T - (m + m')g$

$$\therefore (m + 600) \times 1.4 = 1200 \times 9.8 - (m + 600) \times 9.8$$

$$\therefore (m + 600) \times 11.2 = 1200 \times 9.8$$

$$m + 600 = 1050 \quad m = 450 \text{ kg}$$

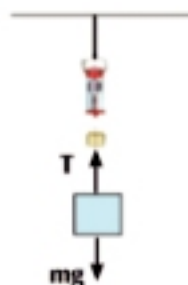


Figure (48)

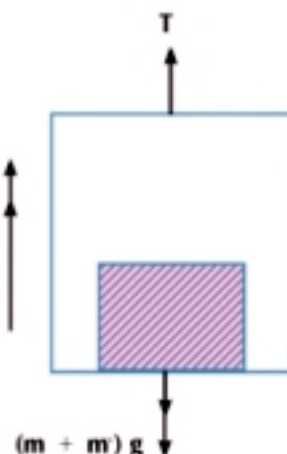


Figure (51)

Try to solve

- 4 A body is suspended in a spring scale fixed at the top of a lift to record 17 kg.wt. When the lift moves up with a uniform acceleration 1.5 m/sec^2 and the scale records 16 kg.wt as the lift moves down with a the acceleration negative uniform of magnitude $a \text{ m/sec}^2$. Find the mass of the body and the magnitude of a .



Exercises 2 - 4



Complete each of the following:

- 1 A body of mass 70 kg is placed on a pressure scale on the floor of a lift moving with a uniform acceleration 1.4 m/sec^2 downwards, then the reading of the scale is..... kg.wt.
- 2 A body is suspended in the hook of a spring scale fixed at the top of a lift to record 390 kg.wt as the lift moves up:
if the acceleration of motion is -70 cm/sec^2 , then the mass of the body is..... gm.
If the mass of the body is 350 gm, then the acceleration of motion is..... cm/sec^2 .
- 3 A person stands on a pressure scale fixed at the floor of a lift the scale reads 75 kg.wt. As the lift moves up with acceleration $a \text{ m/sec}^2$, and reads 69 kg.wt as the lift moves down with the same acceleration, then the real weight of the person is..... kg.wt.
- 4 A child stands on a pressure scale inside a lift moving downwards with acceleration 1.4 m/sec^2 .
If the reading of the scale is 30 kg.wt, then the weight of the child is =..... kg.wt
If the weight of the child is 49 kg.wt, then the reading of the scale is..... kg.wt

Answer the following questions:

- 5 A person of mass 80 kg stands on a pressure scale fixed in the floor of a lift. Find the scale reading in each of the following cases:
- a The lift moves with a uniform velocity.
 - b The lift moves upwards with negative acceleration of magnitude 44.1 cm/sec^2 .
 - c The lift moves downwards with an uniform acceleration of magnitude 29.4 cm/sec^2 .

- 6 A body of mass m is suspended in a spring scale fixed at the top of a lift. Find m in each of the following cases:
- The lift moves upwards with a uniform acceleration of magnitude 98 cm/sec^2 and the scale reading is 44 gm.wt .
 - The lift moves downwards with a uniform acceleration of magnitude 140 cm/sec^2 and the scale reading is 210 gm.wt .
 - The lift is at rest and the scale reading is 100 gm.wt .
- 7 An electric lift moves vertically upwards in a retarded motion with a uniform acceleration of magnitude $a \text{ m/sec}^2$, and there is a spring scale fixed at its top carrying a body of weight 35 kg . If the apparent weight which the scale shows is of magnitude 30 kg.wt , find the value of a .
- 8 A body is placed on a pressure scale on the floor of a lift to record reading 14 kg.wt as the lift was at rest. Find in kg.wt the scale reading when it moves vertically upwards with a uniform acceleration of 70 cm/sec^2 .
- 9 A body of mass 94.5 kg is placed in a box of mass 52.5 kg , then raised vertically upwards by a movable rope with an acceleration of magnitude 1.4 m/sec^2 . Find the magnitude of the pressure of the body on the box base and the magnitude of the tension in the rope carrying the box. If the rope is cut, find the pressure of the body on the box base on that time.
- 10 An electrical lift of mass 350 kg.wt moves vertically downwards with a negative uniform acceleration of magnitude 49 cm/sec^2 and there is a person of weight 70 kg.wt inside it. Find the magnitude for each of the pressure of the person on the floor of the lift and the tension in the rope carrying the lift in kg.wt .
- 11 A body is suspended in a spring scale fixed at the top of a lift. The scale reading is 7 kg.wt when the lift is at rest. Then the scale reads 8 kg.wt when the lift moves vertically with a uniform acceleration. Find the magnitude and direction of the acceleration in which the lift moves.
- 12 A body is suspended in a spring scale fixed at the top of a lift if the scale reads 16 gm.wt , when the lift moves up with acceleration of uniform $a \text{ cm/sec}^2$, and the reads 11 gm.wt when the lift moves downwards with acceleration of uniform $1.5 a \text{ cm/sec}^2$. Find the mass of the body and acceleration a , then calculate the scale reading when the lift moves downward with a acceleration uniform negative of magnitude $\frac{1}{2} a \text{ cm/sec}^2$.

Motion of a body on a smooth inclined plane



Think and Discuss

You will learn

- ▷ The motion of a body on an inclined plane

Key terms

- ▷ inclined plane
- ▷ Smooth plane

Materials

- ▷ scientific calculator

If a body of mass m kg is placed on a smooth inclined plane, it inclines to the horizontal with an angle of measure θ . If this body is acted upon by a force of magnitude F newton in the direction of the line of the greatest slope upwards the plane, then determine the direction of the motion of this body and what acts on the direction of the motion

Motion of a body on a smooth inclined plane

Let a body of mass m moves on a smooth plane inclined to the horizontal with an angle of measure θ under the action of a force of magnitude F acting in the direction of the line of the greatest slope of the plane upwards, then we notice that the body is under the action of the following three forces:

- 1 - The given force F acting in the direction of the line of the greatest slope to the plane upwards and its magnitude is F .
- 2 - The weight of the body acting vertically downwards of magnitude $m g$.
- 3 - The reaction of the plane acting in a perpendicular direction to the plane upwards of magnitude N .

By analyzing the weight into two component; one of them is in the direction of the plane downwards and the other is in the perpendicular direction to it.

The component is in the direction of the plane = $m g \sin \theta$.

The component is in the perpendicular direction to the plane = $m g \cos \theta$ then, we have three cases that rely on the comparison among F , $m g \sin \theta$ with the same unit .

First case: If $F > m g \sin \theta$, then the body moves with a uniform acceleration upwards the plane and the equation of its motion is $m a = F - m g \sin \theta$

If the action of the force F is ceased after passing time t from the beginning of the motion, then the body moves upwards the plane (the same previous direction) but with a retarded acceleration a' where $a' = - g \sin \theta$

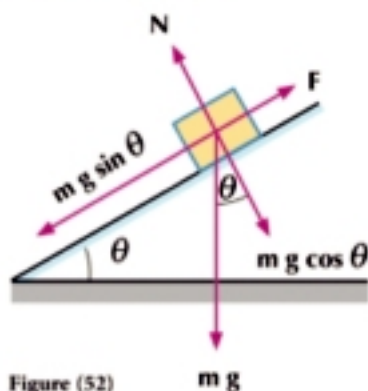


Figure (52)

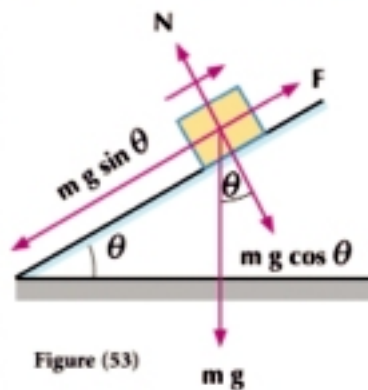


Figure (53)

The body inevitably reaches an instantaneous rest, then it changes the direction of its motion downwards the plane with an increasing acceleration of magnitude $g \sin \theta$.

Second case: If $F < mg \sin \theta$ the body moves with a uniform acceleration a downwards the plane and its equation of motion

$$m a = mg \sin \theta - F$$

Third case: If $F = mg \sin \theta$, then the body preserves its rest state on the plane. But if the body has acquired a uniform velocity V in the direction of the plane upwards or downwards, then the body moves on the plane in the direction of \vec{V} with a uniform velocity according to Newton's first law.

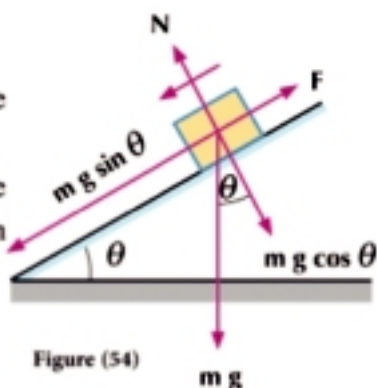


Figure (54)

Example

- ① A body of mass 12 kg is placed on a smooth plane inclined at 30° , to the horizontal. A force of magnitude 88.8 newton acts in the direction of the line of the greatest slope upwards the plane. Find the velocity of this body after 14 seconds from the beginning of the motion. If the force acting on the body is ceased at this moment, find the distance which the body moves on the plane after that until it is at rest.

Solution

$$\therefore F = 88.8 \text{ newton}$$

$$\begin{aligned} \therefore mg \sin \theta &= 12 \times 9.8 \times \frac{1}{2} \\ &= 58.8 \text{ newton} \end{aligned}$$

$$F > mg \sin \theta$$

\therefore The body moves upwards the plane with a uniform acceleration a

Equation of motion:

$$m a = F - mg \sin \theta$$

$$12 a = 88.8 - 58.8$$

$$a = 2.5 \text{ m/sec}^2$$

$$\therefore V = V_0 + a t = 0 + 2.5 \times 14 = 35 \text{ m/sec}$$

After ceasing the action of the force, the body moves in the same previous direction with a uniform retardation a'

Equation of motion:

$$m a' = -mg \sin \theta$$

$$a' = -9.8 \times \frac{1}{2} = -4.9 \text{ m/sec}^2$$

the body travels a distance S until it reaches the instantaneous rest where

$$v^2 = v_0^2 + 2 a' S$$

$$0 = (35)^2 - 2 \times 4.9 S$$

$$S = 125 \text{ meters}$$

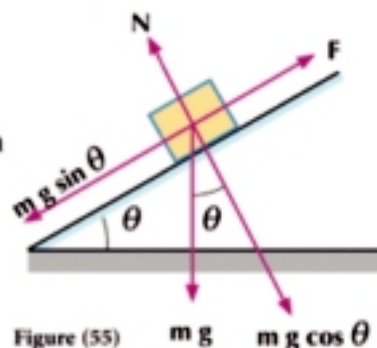


Figure (55)

Try to solve

- 1 A body of mass 32.5 kg is placed on a smooth plane inclined with an angle of measure θ , where $\cos \theta = \frac{12}{13}$. A force of magnitude 83.5 newtons acts in the direction of the line of the greatest slope to the plane upwards, find the magnitude and direction of the acceleration of motion, then find the velocity of the body after 8 seconds from the beginning of the motion.

Example

- 2 A body of mass 25 kg is placed on a smooth plane inclined with an angle of measure θ where $\tan \theta = \frac{4}{3}$. A horizontal force of magnitude 30 kg.wt , acts in the direction of the plane and its line of action lies in the vertical plane passing through the line of the greatest slope to the plane. Find the acceleration generated acceleration and the magnitude of the reaction force of the plane.

Solution

$$F \cos \theta = 30 \times \frac{3}{5} = 18 \text{ kg.wt}, \quad w \sin \theta = 25 \times \frac{4}{5} = 20 \text{ kg.wt}$$

$$\therefore F \cos \theta < w \sin \theta$$

\therefore The body moves downwards the plane with a uniform acceleration a where

$$m a = m g \sin \theta - F \cos \theta$$

$$25 a = (20 - 18) \times 9.8$$

$$a = \frac{98}{125} \text{ m/sec}^2$$

$$N = F \sin \theta + m g \cos \theta \\ = 30 \times \frac{4}{5} + 25 \times \frac{3}{5} = 39 \text{ kg.wt}$$

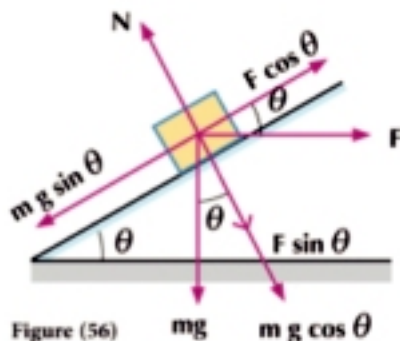
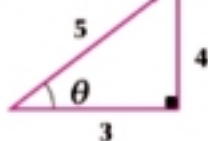


Figure (56)

Try to solve

- 2 A body of mass 2 kg moves on the line of the greatest slope of a smooth plane inclined at 60° to the horizontal under the action of a force of magnitude 1 kg.wt directed towards the plane and makes an angle of measure 30° to the horizontal upwards. Find the magnitude of the reaction force of the plane on the body and the acceleration of motion.

Example

- 3 A body of mass 30 kg moves upwards a smooth inclined plane inclined at 30° to the horizontal under the action of a force of magnitude $F \text{ newtons}$ in the direction of the line of the greatest slope upwards with an acceleration of magnitude 1.5 m/sec^2 . Find the acceleration by which this body moves on the same plane under the action of a force of magnitude $\frac{1}{2} \text{ newton}$ and acts in the direction of the line of the greatest slope upwards.

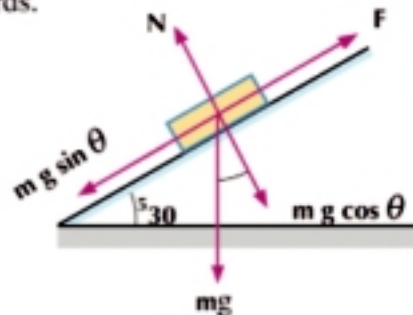
Solution

The equation motion of the first case

$$m a = F - m g \sin \theta$$

$$\therefore 30 \times 1.5 = F - 30 \times 9.8 \times \frac{1}{2}$$

$$\text{then } F = 192 \text{ newton} \quad \therefore \frac{1}{2} F = 96 \text{ N}$$



$$m g \sin \theta = 147 \text{ N} \quad \therefore \frac{1}{2}F < m g \sin \theta \quad \therefore \text{The motion downward of plane}$$

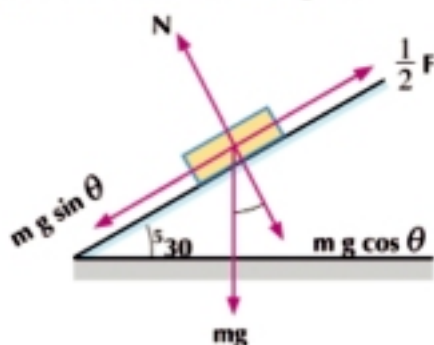
The equation motion of the second case

$$m a' = \frac{1}{2}F - m g \sin \theta$$

$$\therefore 30 a' = 96 - 30 \times 9.8 \times \frac{1}{2} \quad \therefore a' = -1.7 \text{ m/sec}^2$$

Try to solve

- 3 A body of mass 200 kg moves upwards a smooth inclined plane inclines at 30° to the horizontal under the action of a force F of magnitude F newton in the direction of the line of the greatest slope upwards with an acceleration of magnitude 2 m/sec^2 . If this force is reduced into a half, then the body . Find the magnitude and direction with an acceleration which the moves of the body on the same place.



Exercises 2 - 5

Complete the following:

- 1 In the Figure drawn: the mass of the body placed on the smooth plane is 2 kg and starts to move from rest under the action of the force F whose magnitude is 1.5 kg.wt
- The acceleration of motion = m/sec^2 and its direction is
 - The velocity of the body after 4 seconds of starting to move
 - The reaction of the plane = kg.wt.

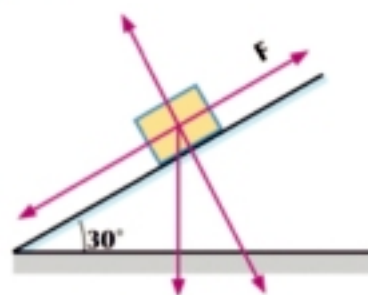


Figure (58) $m g$

- 2 In the Figure drawn: the mass of the body placed on a smooth plane is $m = 12 \text{ kg}$. It starts to move from rest under the action of the force F whose magnitude is 8 kg.wt.
- The acceleration of motion = m/sec^2 , and its direction is
 - The distance which the body travels on the plane in 3 seconds from the beginning of the motion is meters
 - The reaction of the plane = kg.wt

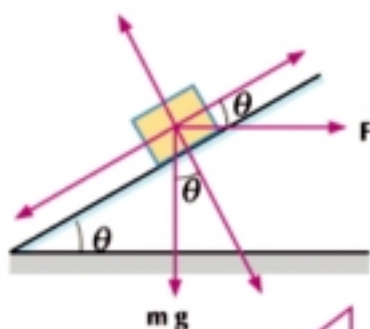


Figure (59)

Choose the correct answer:

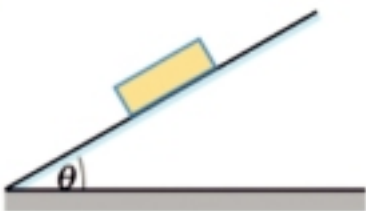
- 3 A cyclist and his cycle of mass 85 kg move with a uniform acceleration of magnitude 0.5 m/sec^2 , then the force used to occur this acceleration is:
- 42.5 kg.wt
 - 42.5 newton



- c 170 kg.wt d 170 newton.
- 4 A car on a road of negligible resistances with acceleration of magnitude 1.47 m/sec^2 . If the force of the engine is 150 kg.wt, then the mass of the car is equal to:
- a 102 kg b 100 kg
c 1000 kg d 220.5 kg.



- 5 If a body moves on a smooth inclined plane and inclines with an angle of measure θ under the action of its weight only, then its acceleration of motion is equal to:
- a g b $g \cos \theta$
c $g \sin \theta$ d zero.
- 6 If a body moves on a smooth inclined plane under the action of its weight only, then its acceleration is only based on:
- a its mass b its weight
c the angle of inclination of the plane d the reaction of the plane.

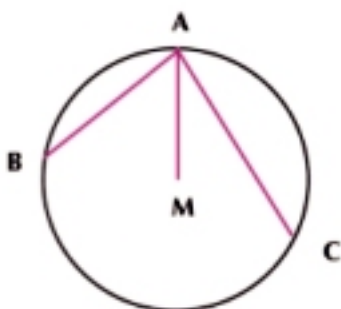


Answer the following questions:

- 7 A body of mass 10 kg is placed on a smooth plane inclines with an angle of $\sin^{-1} \frac{3}{5}$ to the horizontal. A force of magnitude 80 newtons acts in the direction of the line of the greatest slope upwards. Find the magnitude and direction of the acceleration generated and the magnitude of the reaction of the normal.
- 8 A body of mass 1 kg is placed on a smooth plane inclines at 30° to the horizontal. A force of magnitude 10 newtons acts in the direction of the line of the greatest slope of the plane upwards. Find the acceleration of motion and the reaction of the plane on the body.
- 9 A body of mass 16 kg is placed on a smooth plane inclines at 45° , to the horizontal. A horizontal force of magnitude 24 newtons acts toward the plane and its line of action lies in the vertical plane passing through the line of the greatest slope to the plane. Find the magnitude of the acceleration of the motion and the reaction of the plane.

10 Creative thinking:

In the opposite Figure: \overline{AM} is a vertical radius, \overline{AB} , \overline{AC} are two chords representing two smooth rods in the circle where $AC > AB$. Two beads slid from rest from point A such that one bead on the chord \overline{AB} to reach B after time t_1 and the other bead on the chord \overline{AC} to reach C after time t_2 , find the value of the ratio $t_1 : t_2$.



Motion of a body on a rough plane

Unit Two

2 - 6

Introduction:

In your previous study of friction, you knew that when you try to move a body on a rough plane, the friction force appears as a resistance force acting in an opposite direction to the direction which the body intends to move in this force keeping completely equal. The tangential force acting to move the body increases, the more the friction force increases until it remains equal to this force until it reaches the maximum value. At this time, the body is about to move. If the tangential force acting to move the body increases, it can move the body and the friction force then changes and its value decreases as the body starts to move on that time. The friction force is called the kinetic friction and the coefficient of friction in this case is the coefficient of the kinetic friction.



Learn

Motion on a rough plane

If the body is in equilibrium on a rough plane under the action of a force acting to move it, then the friction force is the static friction force and the coefficient of friction in this case is the coefficient of the static friction μ_s . But if the body moves on a rough plane, then the coefficient of the friction is the coefficient of the kinetic friction μ_k .



Example

- 1 A rough inclined plane of length 250 cm and height 150 cm, another body at rest is placed on it to slide downwards the plane and the acceleration of motion is equal to 196 cm/sec^2 . Find the coefficient of the kinetic friction, then find the velocity of the body after it travels (cuts) 200 cm on the plane.

You will learn

- Motion on a rough plane.

Key terms

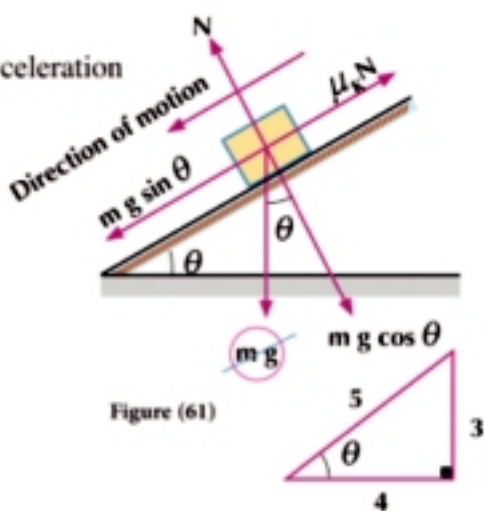
- Rough plane
- Kinetic friction
- Static friction

Materials

- Scientific calculator
- Computer graphics

Solution

$$\begin{aligned}
 N &= m g \cos \theta = \frac{4}{5} m g \\
 \therefore \text{The body moves downwards with a uniform acceleration} \\
 m a &= m g \sin \theta - \mu_k N \\
 196 m &= \frac{3}{5} m g - \mu_k \times \frac{4}{5} m g \\
 196 &= \frac{3}{5} \times 980 - \mu_k \times \frac{4}{5} \times 980 \\
 \therefore \mu_k &= \frac{1}{2} \\
 \therefore V^2 &= V^2 + 2 a S \\
 V^2 &= 0 + 2 \times 196 \times 200 \\
 \therefore V &= 280 \text{ cm/sec}
 \end{aligned}$$


Try to solve

- 1 The boxes at a factory are transported by sliding them on an inclined plane of length 15 m and height 9 m. Find the velocity of the box which starts its motion from rest at the top of the plane at the base of the plane if the plane is smooth and the coefficient of the kinetic friction is equal to $\frac{1}{4}$.

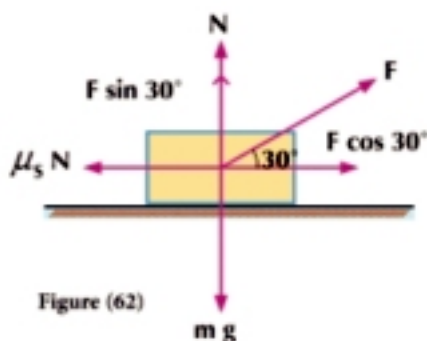
Example

- 2 A body of mass 12 kg. is placed on a rough horizontal plane. If the coefficient of the static friction between the body and the plane is equal to $\frac{\sqrt{3}}{3}$ whereas the coefficient of the kinetic friction is equal to $\frac{\sqrt{3}}{4}$, then calculate the force which makes the body about to move, then find the force which makes the body move with an acceleration of magnitude $\frac{49\sqrt{3}}{20} \text{ m/sec}^2$ if the force inclines at 30° to the horizontal.

Solution

First: The force makes the body about to move

$$\begin{aligned}
 N + F \sin 30 &= W \\
 N &= (12 - \frac{1}{2} F) \text{ kg.wt} \\
 \therefore F \cos 30 &= \mu_s N \\
 \therefore \frac{\sqrt{3}}{2} F &= \frac{\sqrt{3}}{3} (12 - \frac{1}{2} F) \\
 3 F &= 24 - F \\
 4 F &= 24 \\
 F &= 6 \text{ kg.wt}
 \end{aligned}$$



Second: The force moves the body with an acceleration of magnitude $\frac{49\sqrt{3}}{20}$ m/sec²
 $\therefore N = mg - F \sin 30$ i.e. $N = (12 \times 9.8 - \frac{1}{2} F)$ newton

$$\begin{aligned} \therefore m a &= F \cos 30 - \mu S N \\ 12 \times \frac{49\sqrt{3}}{20} &= F \times \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{4} (12 \times 9.8 - \frac{1}{2} F) \\ 12 \times \frac{49\sqrt{3}}{20} &= \frac{5\sqrt{3}}{8} F - 3\sqrt{3} \times 9.8 \\ F &= 94.08 \text{ newton} \end{aligned}$$

Try to solve

- 1 In the previous example, calculate the magnitude of the force F if the force acting on the body is horizontal.

Example

- 3 A body of weight 800 newtons is placed on a rough inclined plane inclined at 25° to the horizontal and the coefficient of the static friction between the body and the plane is equal to 0.35 and the coefficient of the kinetic friction is equal to 0.25. Find the force which the acts on the body and in the line of the greatest slope upwards in each of the following cases:

- F the force which that makes the body starting of moves.
- F the force which that force P the body of moves.
- F prevents the body from sliding.
where F acts in the direction of the line of the greatest slope upwards the plane.

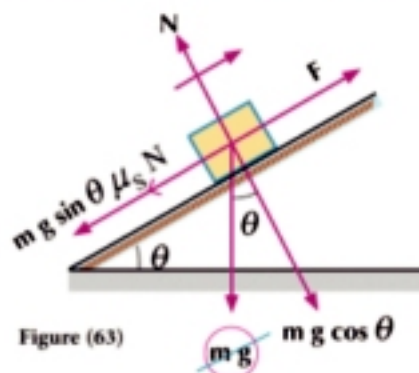


Figure (63)

Solution

- a \therefore Power that make the body starts movement is the same power that will make it on the verge of movement

$$\begin{aligned} g &= mg \cos \theta \\ F &= \mu_s N + mg \sin \theta \\ &= 0.35 \times mg \cos \theta + mg \sin \theta \\ &= 0.35 \times 800 \cos 25 + 800 \sin 25 \\ &= 591.86 \text{ newton} \end{aligned}$$

- b The force that keeps the body moving is the force that keeps the movement after the a start and then be regular movement

$$\begin{aligned} F &= \mu_s N + mg \sin \theta \\ &= 0.25 \times mg \cos \theta + mg \sin \theta \\ &= 0.25 \times 800 \cos 25 + 800 \sin 25 \\ &\approx 519.36 \text{ newton} \end{aligned}$$

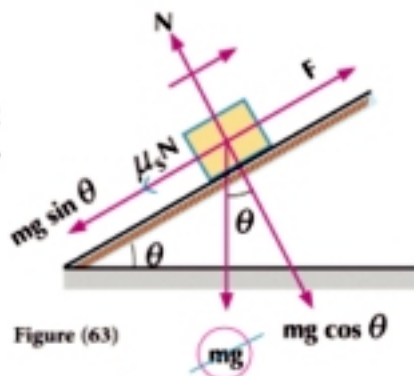


Figure (63)

- c The force F prevents the body from sliding
- $$N = mg \cos \theta$$
- $$F + \mu_s N = mg \sin \theta$$
- $$F + 0.35 \times mg \cos \theta = mg \sin \theta$$
- $$F = 800 \sin 25 - 0.35 \times 800 \cos 25$$
- $$F = 84.33 \text{ newton}$$

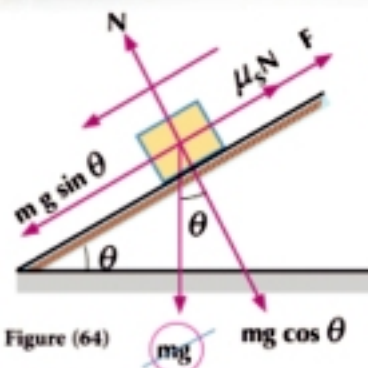


Figure (64)

P Try to solve

- 2 In the previous example, calculate the magnitude of the force F if it is horizontal in all cases.

Exercises 2 - 6

- 1 In each of the following figures, a body of mass 5 kg is placed on a rough horizontal plane and the coefficient of the kinetic friction between the body and the body μ_k , calculate μ_k in each case, F is the kinetic force.

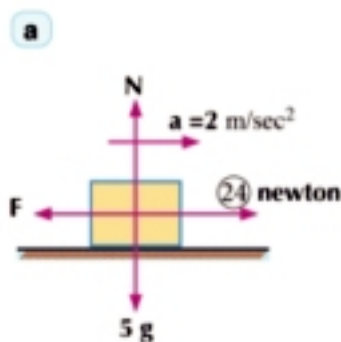


Figure (65)

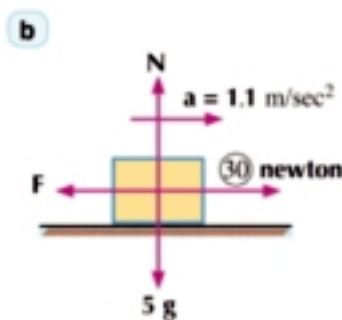


Figure (66)

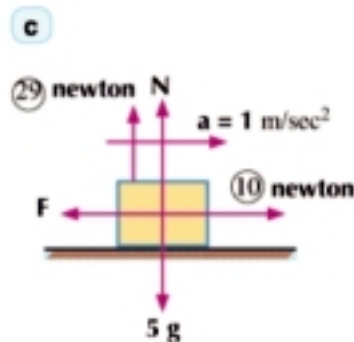


Figure (67)

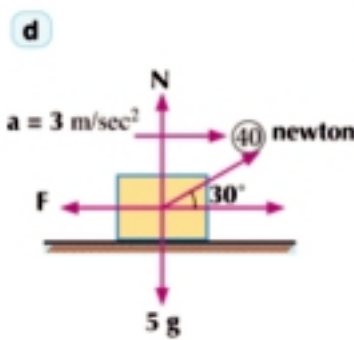


Figure (68)

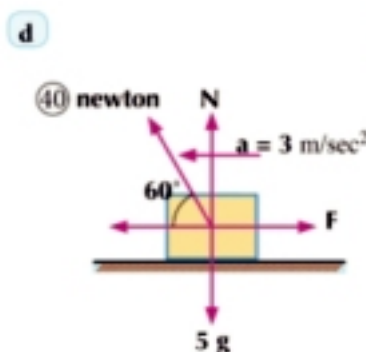


Figure (69)

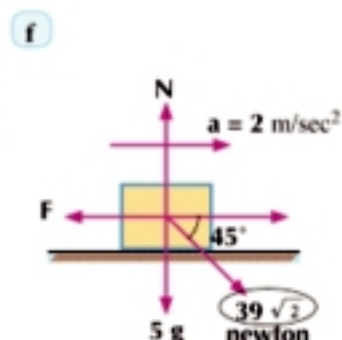


Figure (70)

You will learn

- Simple pulleys
- Motion of a system of two bodies suspended vertically by a string passing over a smooth pulley
- Motion of a system of two bodies one of which moves on a smooth horizontal table and the other moves vertically
- Motion of a system of two bodies one of which moves on a rough horizontal table and the other moves vertically
- Motion of a system of two bodies one of which moves on a smooth inclined plane and the other moves vertically.
- Motion of a system of two bodies one of which moves on a rough inclined plane and the other moves vertically.

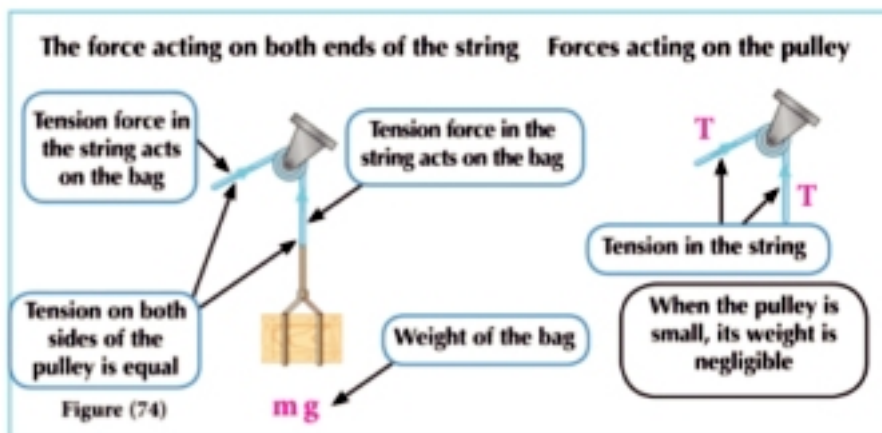
Materials

- Scientific calculator
- Computer graphics

Introduction

Pulleys are used for a lot of purposes such as reducing the force needed to lift a body, ease the motion and to change the direction of a force. There are fixed pulleys and movable pulleys, in this unit. You are going to learn a system of pulleys of a constant pulley.

When the pulley is small and smooth, then the tension on both sides of the pulley is equal. The following Figure shows the forces acting as we lift a bag (body) using a pulley.



Learn

Motion of a system of two bodies suspended vertically by a string passing over a smooth pulley

The motion of two bodies connected by a string passing over a smooth pulley and suspended vertically

If two bodies of masses m_1 , m_2 are connected by the two ends of an inelastic light string passing over a smooth small pulley, suspended vertically and $m_1 > m_2$, then the system starts to move from rest with a uniform acceleration of magnitude a .

Equation of motion

$$m_1 a = m_1 g - T \qquad m_2 a = T - m_2 g$$

By adding the two equations by eliminating the tension, then acceleration of motion can be calculated:

$$(m_1 + m_2) a = (m_1 - m_2) g \quad \therefore a = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) g$$

Thus, from any two equations, we find the tension T in the string

When the string is cut:

If the string connecting two bodies is cut after time t , then both bodies move in their same previous directions before the string is cut.

- 1) The mass m_1 moves downwards with initial velocity V (it is the same velocity at the moment the string is cut) and under the action of the gravitational acceleration.
- 2) The mass m_2 moves upwards with initial velocity V (it is the same velocity at the moment the string is cut) until it reaches an instantaneous rest under the action of the gravitational acceleration, then it falls freely.

The pressure on the pulley

When the two masses are hanged up by the ends of the string passing over the pulley, the string gets tensioned due to the tension exerted on the string and a pressure force generated on the axis of the pulley is equal to the resultant of the two forces of the tension in the string. $P = 2T$

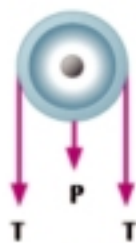


Figure (76)

Similar cases (1)

In the drawn case:

$(m_1 + m_3) > m_2$, if $m_1 < m_2$ then the equation of motion

$$(m_1 + m_3) a = (m_1 + m_3) g - T$$

$$m_2 a = T - m_2 g$$

When the additional mass is separated

If the mass m_3 is separated after time of magnitude t seconds, then the system moves in the same previous direction, but with a retarded acceleration (deceleration) until it instantaneously rests, then changes its direction to find the deceleration motion after the mass m_3 is separated, we find the equation of motion

$$m_1 a' = m_1 g - T$$

$$m_2 a' = T - m_2 g$$

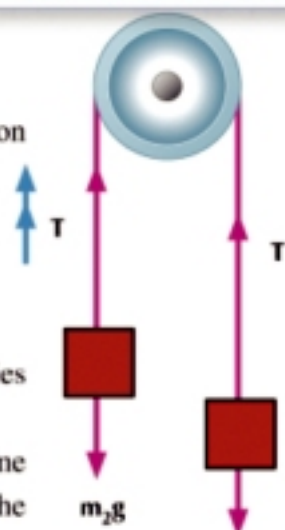


Figure (75)

Notice that

If the system starts the motion and the two masses are in one horizontal plane and the distance covered after time of magnitude t is equal to S unit length, then the vertical distance between the two masses at the same time is equal to $2S$ unit length

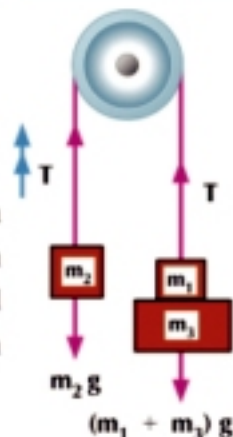


Figure (77)

After the mass m_3 gets separated, the system moves with initial velocity which is the velocity it acquired at the moment of separation and reached the instantaneous rest, then changes the direction of its motion and rebounds to get mass m_2 to be the leading.

The tension in the string between the two masses

In the previous Figure, if the two masses m_1, m_3 are tied by another string then the tensions are as shown in Figure (78) and the equations of motion are:

$$m_1 a^{\uparrow} = m_1 g + T^{\uparrow} - T$$

$$m_3 a^{\downarrow} = m_3 g - T$$

Similar cases (2)

If $m_1 = m_2 = m$ Figure (79)

i.e. the two masses are equal in this case, the system does not move. But if a mass of magnitude m' is added to one mass of both, then the system moves in the direction of the two masses $(m + m')$ and the equations of motion

$$(m + m') a = (m + m') g - T$$

$$m a = T - mg$$

when the additional mass is separated:

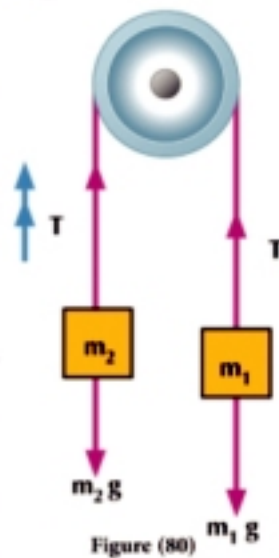
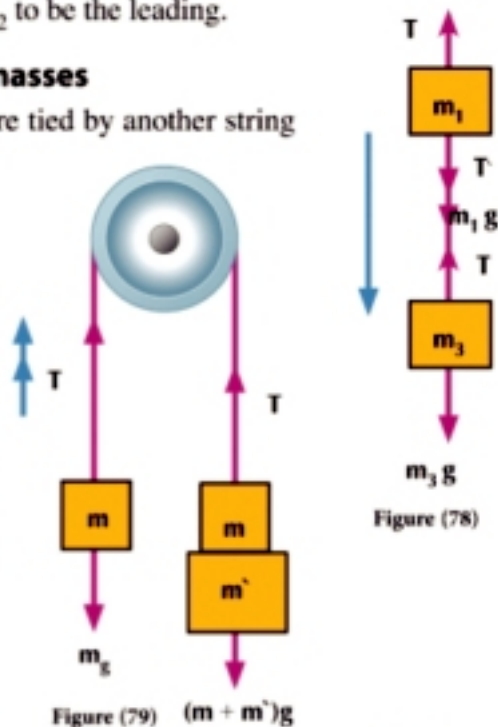
If the additional mass m' is separated after time of magnitude t second, then the system moves in the same previous direction with a uniform velocity which is the velocity it acquired during t second t (the velocity at the moment of separating mass m')

Similar cases (3) Figure (80)

If the two masses m_1, m_2 are hanged up in the ends of a string and we do not know which of the two masses is heavier and we let the mass m_1 acquire a velocity of magnitude V downwards and the system moves, then we have three cases

- 1) If the system returns back to its original position after time of magnitude t then we deduce that $m_1 < m_2$ and the system moves with deceleration until it instantaneously rests, then changes the direction of its motion the acceleration of motion can be deduced from the given data where the initial velocity is the velocity which the mass m_1 , acquired and the final velocity = 0, time = $\frac{t}{2}$
- 2) If the system moves uniformly with a constant velocity which is the velocity the mass m_1 acquired, we can deduce that the two masses are equal $m_1 = m_2$, and the motion belongs to Newton's first law.
- 3) If the system moves uniformly with an increasing acceleration, we deduce that $m_1 > m_2$, and the motion can be studied to find the equations of motion

$$m_1 a = m_1 g - T \qquad m_2 a = T - m_2 g$$



Example

- 1 Two bodies of masses m_1, m_2 , where $m_1 > m_2$ are connected by the ends of a string passing over a smooth pulley and the two bodies are at the same height from the ground at the beginning of motion, after one second, the vertical distance between them is 20 cm. Find $m_1 : m_2$

Solution

At the beginning of motion, the two bodies were in a one horizontal plane and after a second the vertical distance between them was 20 cm.

$$\therefore S = \frac{20}{2} = 10 \text{ cm}$$

$$10 = 0 + \frac{1}{2} \times a \times 1$$

equation of motion

$$m_1 a = m_1 g - T$$

By adding, we find that

$$(m_1 + m_2) a = (m_1 - m_2) g$$

$$20 (m_1 + m_2) = 980 (m_1 - m_2)$$

$$m_1 + m_2 = 49 (m_1 - m_2)$$

$$m_1 + m_2 = 49 m_1 - 49 m_2$$

$$50 m_2 = 48 m_1$$

$$\frac{m_1}{m_2} = \frac{25}{24}$$

$$\therefore S = Vt + \frac{1}{2} a t^2$$

$$a = 20 \text{ cm/sec}^2$$

$$m_2 a = T - m_2 g$$

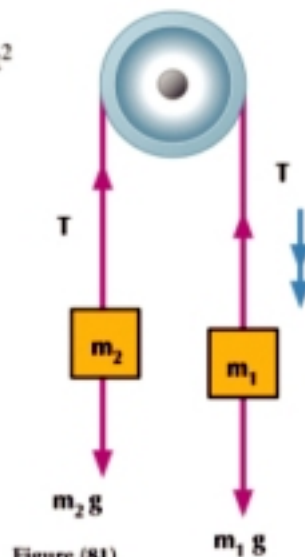


Figure (81)

$$m_1 : m_2 = 25 : 24$$

Try to solve

- 1 Two bodies of masses 21 gm and 28 gm are connected by the two ends of a string passing over a smooth small pulley. If the system moves from rest, find the acceleration of the set, the magnitude of the tension in the string, and the velocity of the system after two seconds from the beginning of motion.

Example

- 2 Two bodies of masses 105 gm and 70 gm are connected by the two ends of a light string of constant length passing over a smooth small pulley and suspended vertically. If the system starts to move from rest when the two masses are on one horizontal plane, find the magnitude of the acceleration of motion of the system. If the first body is impinging against the ground after it traveled 50 cm, find the total time taken by the second body from the beginning of motion until it instantaneously rests.

Solution
Equation of motion:

$$105 a = 105 \times 980 - T$$

$$70 a = T - 70 \times 980$$

By adding the two equations, we find that

$$175 a = 35 \times 980$$

$$a = 196 \text{ cm/sec}^2$$

 At the moment the body of mass 105 gm impings against the ground, the body of mass 70 gm, moves vertically upwards with a gravitational acceleration beginning with velocity $V_0 = 140 \text{ cm/sec}$. to rest instantaneously after time t_2

$$V^2 = V_0^2 + 2 a S$$

$$V^2 = 0 + 2 \times 196 \times 50$$

$$V = 140 \text{ cm/sec}$$

$$V = V_0 + a t$$

$$140 = 0 + 196 t$$

$$t = \frac{5}{7} \text{ seconds}$$

 When the body of mass 105 gm impings against the ground, the body of mass 70 gm, moves vertically upwards with a gravitational acceleration beginning with velocity $V_0 = 140 \text{ cm/sec}$. to rest instantaneously after time t_2

$$\therefore V = V_0 + g t$$

$$\therefore 0 = 140 - 980 t$$

$$t = \frac{1}{7} \text{ seconds}$$

 \therefore The body of mass 70 gm takes time of magnitude t to reach the instantaneous rest from the beginning of motion

$$\text{where } t = t_1 + t_2 = \frac{5}{7} + \frac{1}{7} = \frac{6}{7} \text{ seconds}$$

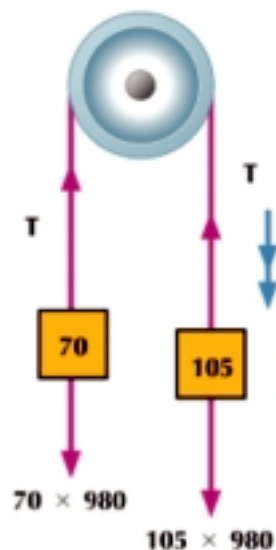


Figure (82)

Try to solve

- 2 A light string passes over a fixed smooth pulley and a body of mass 90 gm is suspended in one end of the string while the other end is connected by a body of 70 gm. If the system starts moving from rest when the mass 90 gm is at a height of 245 cm from the ground, find:
- The time taken until the mass 90 gm reaches the ground.
 - The time taken until the string gets tensioned once more.

Example

- 3 Two bodies of mass 5 kg and 3 kg are connected by the two ends of a light string passing over a smooth pulley. The system starts to move from rest when the two bodies are in the same horizontal plane at a height of 245 cm on the ground and after one second from the beginning of motion, the string is cut. Find the acceleration of motion and the velocity of the two bodies as they reach the ground.

Solution

Equations of motion:

$$5a = 5 \times 9.8 - T \quad (1)$$

$$3a = T - 3 \times 9.8 \quad (2)$$

By adding, we find that

$$8a = 2 \times 9.8$$

$$\therefore a = 2.45 \text{ m/sec}^2$$

at the moment the string is cut

$$V = V_0 + at$$

$$= 0 + 2.45 \times 1 = 2.45 \text{ m/sec}$$

$$S = V_0 t + \frac{1}{2} a t^2$$

$$= 0 + \frac{1}{2} \times 2.45 \times 1 = 1.225 \text{ meters}$$

after the string is cut

the body of mass 5 kg moves vertically downwards

$$V_0 = 2.45 \text{ m/sec}, \quad g = 9.8 \text{ m/sec}^2, \quad S = 2.45 - 1.225 = 1.225 \text{ meters}$$

$$\therefore V^2 = V_0^2 + 2gS$$

$$\therefore V^2 = (2.45)^2 + 2 \times 9.8 \times 1.225$$

$$\therefore V = \frac{49\sqrt{5}}{20} \text{ m/sec}$$

the body of mass 3 kg moves vertically upwards free from a point distant S from the ground surface to reach an instantaneous rest, then it turns back passing through the starting point of motion, then to the ground surface.

$$V_0 = 2.45 \text{ m/sec}, \quad g = -9.8 \text{ m/sec}^2, \quad S = -(2.45 + 1.225) = -3.675$$

$$\therefore V^2 = v_0^2 + 2gS$$

$$= (2.45)^2 + 2 \times -9.8 \times -3.675$$

$$V = \frac{49\sqrt{13}}{20} \text{ m/sec}$$

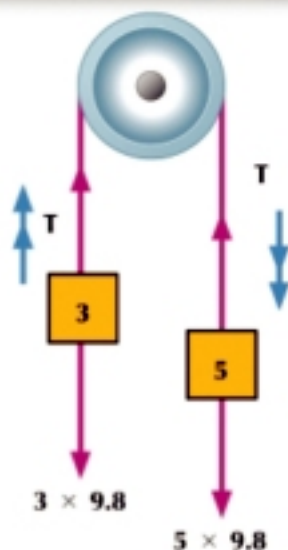


Figure (83)

Try to solve

- 3 A light string of constant length passes over a fixed smooth small pulley and is connected to two masses 20, 12 gm suspended vertically. Find the acceleration of the motion of the system and the tension in the string if the system starts moving from rest and the string is cut after two minutes from the beginning of the motion. Determine the maximum height the mass 12 gm can reach from its original position at the beginning of motion.

Example

- 4 A light string passes over a smooth vertical pulley and a body of mass 40 gm is connected by one of its ends and two bodies the mass of each is 30 gm are connected by the other end of the string. The system is let to move from rest. After one second from the beginning of the motion, one of the two small masses is separated from the system. Find the distance traveled by the mass 49 gm from the beginning of the motion until it reaches an instantaneous rest.

Solution
Equations of motion:

$$60 a = 60 \times 980 - T$$

$$40 a = T - 40 \times 980$$

By adding the equations, we find that

$$100 a = 20 \times 980$$

$$a = 196 \text{ cm/sec}^2$$

The moment of separating the small mass

$$V = V_0 + a t$$

$$= 0 + 196 \times 1 = 196 \text{ cm/sec}$$

$$S_1 = V_0 t + \frac{1}{2} a t^2$$

$$= 0 + \frac{1}{2} \times 196 \times 1 = 98 \text{ cm}$$

After the separation of the small mass - equations of motion

$$40 a' = T' - 40 \times 980$$

$$30 a' = 30 \times 980 - T'$$

By adding the equations, we find that

$$70 a = -10 \times 980$$

$$a = -140 \text{ cm/sec}^2$$

i.e. the system moves in the same previous direction before the small mass is separated but with deceleration until it reaches the instantaneous rest after it covers a distance S_2 , then changes its motion.

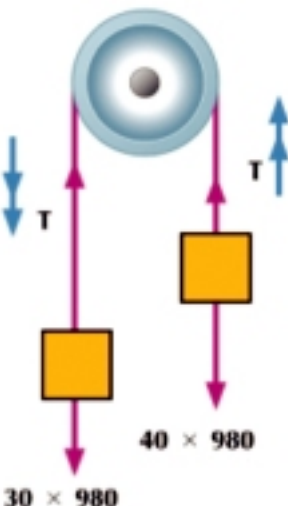
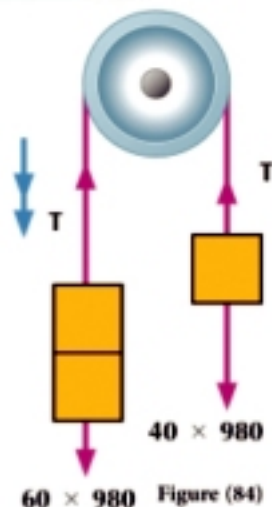
$$\therefore V^2 = V_0^2 + 2 a S$$

$$0 = (196)^2 - 2 \times 140 S_2 \quad \therefore S_2 = 137.2 \text{ cm}$$

\therefore the mass 40 gm moves upwards for a distance S before it rests instantaneously where $S = S_1 + S_2 = 235.2 \text{ cm}$

Try to solve

- 4 A light string passes over a smooth small pulley and carries two weights 235 gm and 20 gm connected together by a string in an end such that the weight 20 is placed below the weight 235 and in the other end of the string a weight of magnitude 235 gm is connected. Calculate the common acceleration if the system moves from rest. If the string carrying the weight 20 gm is cut after the system covers a distance of 45 cm and the weight 235 descending at a distance of 90 cm above the ground then, calculate the time taken by this weight to reach the ground.



Motion of a system of two bodies one of which is on a horizontal table and the other moves vertically downwards

If two bodies of masses m_1, m_2 are connected by the two ends of a light inelastic string passing over a smooth small pulley such that the body of mass m_2 is placed on a horizontal plane and the body of mass m_1 is suspended vertically.

First: the smooth horizontal plane

Equations of motion

$$m_1 a = m_1 g - T$$

$$m_2 a = T$$

By adding the two equations, the tension is eliminated and then the acceleration of motion can be calculated

$$(m_1 + m_2) a = m_1 g \quad \therefore a = \left(\frac{m_1}{m_1 + m_2} \right) g$$

then, from any two equations, we find the tension T in the string

When the string is cut:

If the string connecting two bodies is cut after time t , then each of the two bodies moves in the same previous direction before the string is cut.

- 1- The mass m_1 moves downwards with initial velocity V (it is the same velocity at the moment the string is cut) and under the action of the gravitational acceleration.
- 2- The mass m_2 moves on the plane with a uniform velocity V (it is the same velocity at the moment the string is cut).

The pressure on the pulley

When the two masses are connected by the ends of the string passing over the pulley, the string gets tensioned due to the tension occurring in the string and a pressure force is generated on the axis of the pulley equal to the resultant of the two forces of tension in the string. The pressure on the pulley $= \sqrt{2} T$

Second: The rough horizontal plane

If μ_k is the coefficient of the kinetic friction, then

$$N = m_2 g$$

Equations of motion

$$m_1 a = m_1 g - T$$

$$m_2 a = T - \mu_k N$$

By adding the two equation and eliminating the tension then the acceleration of motion can be calculated

$$(m_1 + m_2) a = m_1 g - \mu_k N$$

then, from any two equations, we find the tension T in the string **when the string is cut.**

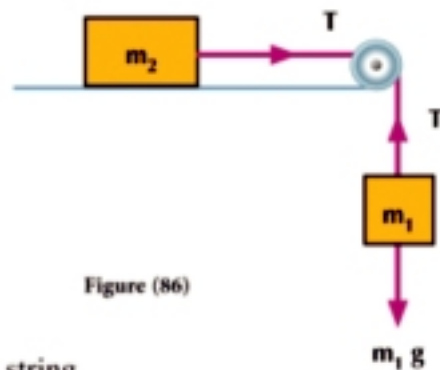


Figure (86)

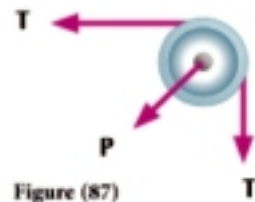


Figure (87)

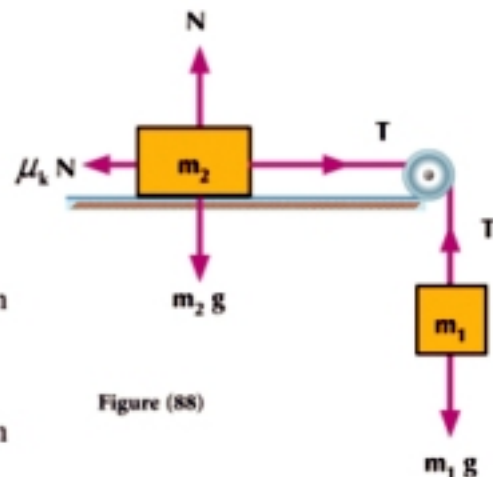


Figure (88)

If the string connecting the two bodies after time t is cut, then each of the two bodies moves at the same previous direction before the string is cut.

- 1 The mass m_1 moves down with initial velocity V (it is the same velocity at the moment the string is cut) under the action of the gravitational acceleration.
- 2 The mass m_2 moves upon the plane with initial velocity V (it is the same velocity at the moment the string is cut) with uniform deceleration until it rests the deceleration motion can be deduced from the equation of motion.

$$m_2 a' = -\mu_k N$$

Example

- 5 A body of mass 45 gm is placed on a horizontal table and is connected by a string whose other end is connected by a body of 4 gm suspended vertically and the string passes over a smooth pulley at the edge of the table. Find the common acceleration for the system, the tension in the string and the pressure on the pulley.

Solution

Equations of motion

$$4a = 4 \times 980 - T \quad (1)$$

$$45a = T \quad (2)$$

By adding the two equations, we find that

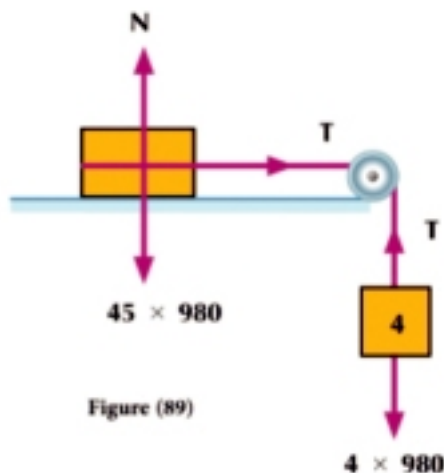
$$49a = 4 \times 980$$

$$a = 80 \text{ cm/sec}^2$$

$$\text{from (2) } T = 45 \times 80 \text{ dyne}$$

$$\therefore T = 3600 \text{ dyne}$$

$$\begin{aligned} \text{the pressure on the pulley} &= \sqrt{2} T \\ &= 3600\sqrt{2} \text{ dyne} \end{aligned}$$



Try to solve

- 5 A body of mass 400 gm is placed on a smooth horizontal table, then connected by a light string passing over a smooth small pulley at the edge of the table. Another body of weight 90 gm suspended vertically is connected by the end of the string. Find the common acceleration of the two bodies, the tension in the string, and the pressure on the pulley.

Example

- 6 A body of 60 gm is placed on a rough horizontal table, then connected by a string passing over a smooth pulley at the edge of the plane and a body of mass 38 gm is connected by the other end of the string. If the system moves from rest to travel a distance of 70 cm in one second, calculate the coefficient of friction if the string is cut at this moment. Calculate the distance which the first mass moves after that on the plane until it rests.

Solution

$$\therefore S = V_1 t + \frac{1}{2} a t^2$$

$$\therefore 70 = 0 + \frac{1}{2} \times a \times 1$$

$$a = 140 \text{ cm/sec}^2$$

$$N = 60 \times 980 \text{ dyne}$$

Equations of motion

$$38 a = 38 g - T$$

$$38 \times 140 = 38 \times 980 - T \quad (1)$$

$$60 a = T - \mu_k N$$

$$60 \times 140 = T - \mu_k \times 60 \times 980 \quad (2)$$

from (1), (2) by adding, we find that

$$98 \times 140 = 38 \times 980 - \mu_k \times 60 \times 980$$

$$\therefore \mu_k = \frac{2}{5}$$

At the moment the string is cut

$$V = V_0 + a t$$

$$= 0 + 140 \times 1$$

$$V = 140 \text{ cm/sec}$$

After the string is cut

the mass 60 gm moves with deceleration on the rough plane until it rests.

Equation of motion

$$m a' = -\mu_k N$$

$$60 a' = -\frac{2}{5} \times 60 \times 980$$

$$a' = -392 \text{ cm/sec}^2$$

$$V^2 = V_0^2 + 2 a S$$

$$\therefore 0 = (140)^2 - 2 \times 392 S$$

$$\therefore S = 25 \text{ cm}$$

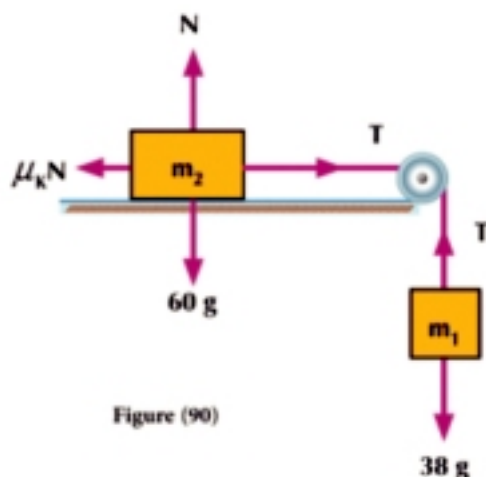


Figure (90)

Try to solve

- 6 A body of mass 63 gm is placed on a rough horizontal table and connected by a horizontal string passing over a smooth small pulley fixed at the edge of the table. Another body of mass 35 g at a height of 280 cm above the ground is connected by the other end of the string. If the coefficient of the kinetic friction between the body and the plane is equal to $\frac{1}{3}$, find the velocity by which the mass 35 gm reaches the ground surface and the distance the mass 63 gm moves until it rests.

Example

- 7 A body of mass 400 gm is placed on a smooth horizontal table connected from two sides by two strings one of them passes over a smooth pulley fixed at the edge of the table distant 150 cm from the body and a body of 200 gm is suspended vertically from it and the other string passes over another smooth pulley fixed at the opposite edge of the table and at distance 80 cm from the mass 400 gm and a body of 100 gm is suspended vertically from it such that the two pulleys and the body between them are colinear. If the system starts to move from rest and the string carrying the mass 200 gm is cut after one second from the motion, find when the mass 400 gm reaches the edge of the table.

Solution
Equations of motion

$$200 a = 200 \times 980 - T_1$$

$$400 a = T_1 - T_2$$

$$100 a = T_2 - 100 \times 980$$

By adding the equation, we find that

$$700 a = 100 \times 980$$

$$a = 140 \text{ cm/sec}^2$$

At the moment the string is cut off

$$\begin{aligned} V &= v_0 + a t \\ &= 0 + 140 \times 1 = 140 \text{ cm/sec} \end{aligned}$$

$$\begin{aligned} S &= v_0 t + \frac{1}{2} a t^2 \\ &= 0 + \frac{1}{2} \times 140 \times 1 = 70 \text{ cm} \end{aligned}$$

i.e. the mass 400 gm is distant $150 - 70 = 80$ cm from the edge of the table.

After the string is cut, the system moves with deceleration which can be deduced by the new equations of motion which are:

$$400 a' = -T_2$$

$$100 a' = T_2 - 100 \times 980$$

$$\therefore 500 a' = -100 \times 980$$

$$a' = -196 \text{ cm/sec}^2$$

$$\therefore V^2 = V_0^2 + 2 a S$$

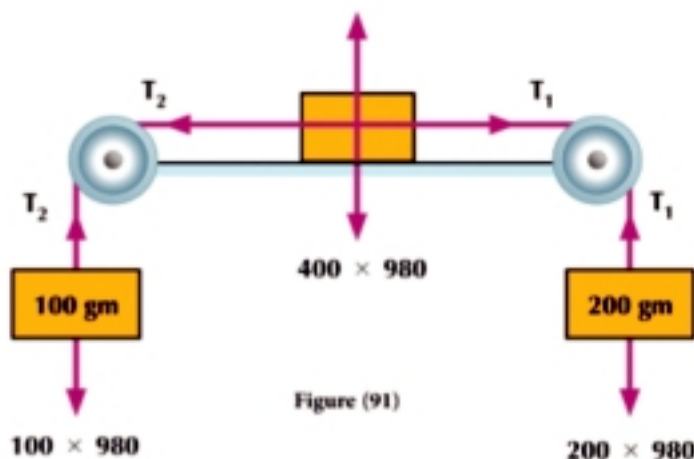
$$\therefore 0 = (140)^2 - 2 \times 196 S$$

$$S = 50 \text{ cm}$$

$$V = V_0 + a t$$

$$0 = 140 - 196 t$$

$$t_2 = \frac{140}{196} = \frac{5}{7} \text{ sec}$$



i.e. the mass 400 gm instantaneously rests after $\frac{5}{7}$ second from the moment the string is cut and it is distant $80 - 50 = 30$ cm from the edge and $80 + 70 + 50 = 200$ cm from the other edge. Now, the system changes the direction of its motion and moves in the opposite direction with acceleration starting from rest.

$$\therefore S = v_1 t + \frac{1}{2} a t^2$$

$$\therefore 200 = 0 + \frac{1}{2} \times 196 t^2$$

$$\therefore t_3 = \frac{10}{7}$$

\therefore the mass 400 gm reaches the edge after time of magnitude $1 + \frac{5}{7} + \frac{10}{7} = \frac{22}{7}$ seconds

P Try to solve

- 7 A body of mass 14 kg is placed on a rough horizontal plane and the coefficient of the kinetic friction between them is $\frac{1}{7}$, the body is connected from two sides by two light strings one of them passes over a smooth pulley at the edge of the plane and a body of 35 kg is suspended vertically from it and the second string passes over another smooth pulley at the opposite edge of the plane and a body of 21 kg is suspended vertically from it such that the two pulleys and the body between them are colinear. If the system moves from rest and all the parts of the string are tensioned when the mass 35 kg is at a height of 21 cm above the ground surface, find its velocity when it impings against the ground.

The motion of two bodies connected by a string passing over a smooth pulley one of them on an inclined plane and the other suspended vertically

Two bodies of masses m_1, m_2 are connected by two ends of an inelastic light string passing over a smooth small pulley such that the body of m_2 , is placed on an inclined plane inclines with an angle of measure θ to the horizontal and the other body is suspended vertically. If $m_1 > m_2$ then the body placed on the plane moves up the plane and the equations of motion are:

$$m_1 a = m_1 g - T$$

$$m_2 a = T - m_2 g \sin \theta$$

Adding the equations by eliminating the tension, the acceleration a of motion can be obtained and so can the tension in the string when the string connecting the two bodies is cut, the mass m_1 moves vertically downwards with a gravitational acceleration starting with the velocity at the moment the string is cut. The mass m_2 moves on the inclined plane at the same previous direction with deceleration starting with the velocity at the moment the string is cut until it instantaneously rests, then changes the direction of its motion.

The vertical distance between two masses:

If the system starts to move, the two masses m_1, m_2 are at a singular horizontal plane and the system traveled a distance S , then the vertical distance between the two masses is equal to $S(1 + \sin \theta)$ where θ is the angle of inclination of the plane to the horizontal.

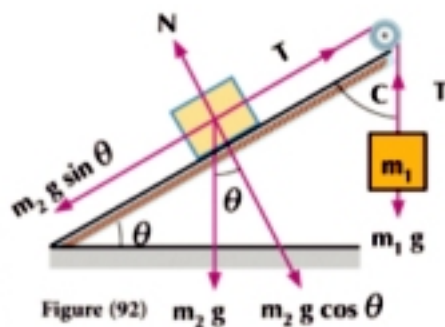


Figure (92) $m_2 g$ $m_2 g \cos \theta$

The pressure on the pulley.

If the tension T in the string and e is the measure of the angle of inclination of the plane to the horizontal, then the pressure on the axis of the pulley is the resultant of the two equal tensions in the string.

$$P = 2T \cos \frac{C}{2} = 2T \cos \left(\frac{90 - \theta}{2} \right) = T \sqrt{2 + 2\sin \theta}$$

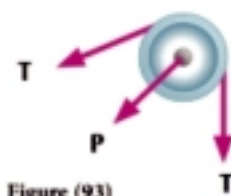


Figure (93)

Example

- 8 A body of mass 3 kg is placed at the lowest point of a smooth inclined plane of length 210 cm and height 140 cm. This body is connected with another body of mass 4 kg by a string of length 210 cm, coincident to the line of the greatest slope of the plane and the other body is suspended at the upper edge of the plane. If the system starts to move from rest until the greater mass reaches the ground and gets at rest, find the distance which the small mass moves on the plane before it stops supposing that its motion is not acted as the greater mass impinges against the ground.

Solution

$$\because 4g > 3g \sin \theta$$

\therefore The direction of motion as shown in Figure (94)

The equations of motion of the system:

$$4a = 4g - T$$

$$3a = T - 3g \sin \theta$$

By adding the two equations, we find that

$$7a = \left(4 - 3 \times \frac{140}{210} \right) \times 9.8$$

$$a = 2.8 \text{ m/sec}^2$$

Calculate the velocity of the body 4 kg to reach the ground

$$V^2 = V_0^2 + 2aS$$

$$= 0 + 2 \times 2.8 \times 1.4$$

$$V = 2.8 \text{ m/sec}$$

After the body 4 kg reaches the ground, the body 3 kg moves on the plane with deceleration.

The equation of the motion of the body moving on the inclined plane

$$3a' = -3g \sin \theta$$

$$\therefore a' = -\frac{98}{15} \text{ m/sec}^2$$

We find the distance which the body moves on the plane until it rests:

$$V^2 = V_0^2 + 2aS$$

$$0 = (2.8)^2 - 2 \times \frac{98}{15} S$$

$$S = 0.6 \text{ meters}$$

\therefore The mass 3 kg rests instantaneously at a distance of two meters from the base of the inclined plane.

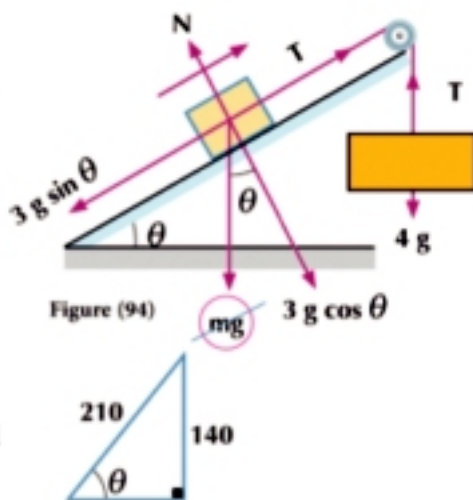


Figure (94)

Try to solve

- 8 A smooth inclined plane inclines to the horizontal with an angle of $\sin^{-1} \frac{2}{3}$ and a body of mass 210 gm is placed on this plane and connected by a light string passing over a smooth small pulley at the top of the plane and its other end carries a pan of mass 70 gm, on which a body of mass 210 gm. If the system starts to move from rest, find the tension in the string and the pressure on the pan in kg.wt. If the body is placed far from the pan after seven seconds from the beginning of the motion, prove that the system instantaneously rests after 8 seconds more.

Example

- 9 A body of mass 1 kg is placed on a rough inclined plane inclines to the horizontal with an angle of measure θ where $\sin \theta = \frac{1}{3}$ and coefficient of the kinetic friction between the body and the plane is equal to $\frac{\sqrt{2}}{2}$. The body is connected by a string coincident on the line of the greatest slope of the plane and passes over a smooth pulley at the top of the plane and suspends vertically to carry a body of mass 3 kg, in its end. Find the pressure on the axis of the pulley, if the system starts to move from rest and after the mass 1 kg travels a distance of 1.8 meters on the plane, the string connecting the two masses is cut. Find the total distance which mass 1 kg traveled on the plane before it rests instantaneously.

Solution

$$\because 3g > g \sin \theta$$

\therefore The direction of motion as shown in Figure (95)

$$\begin{aligned} N &= g \cos \theta \\ &= 9.8 \times \frac{2\sqrt{2}}{3} \\ &= \frac{98\sqrt{2}}{15} \text{ newton} \end{aligned}$$

The equations of motion

$$3a = 3g - T \quad (1)$$

$$a = T - \mu_k N - g \sin \theta \quad (2)$$

By adding the two equations, we find that

$$4a = 3g - \mu_k N - g \sin \theta$$

$$\begin{aligned} 4a &= 3 \times 9.8 - \frac{\sqrt{2}}{2} \frac{98\sqrt{2}}{15} - 9.8 \times \frac{1}{3} \\ a &= 4.9 \text{ m/sec}^2 \end{aligned}$$

From (1) we find that $T = 14.7$ newtons

$$P = \sqrt{2} T \sqrt{1 + \sin \theta} = 14.7 \sqrt{2} \times \frac{2}{\sqrt{3}} = \frac{49\sqrt{6}}{5} \text{ newtons}$$

At the moment the string is cut

$$V^2 = V_0^2 + 2aS = 0 + 2 \times 4.9 \times 1.8$$

$$V = 4.2 \text{ m/sec}$$

After the string is cut, the body moving on the plane moves with deceleration until it instantaneously rests.

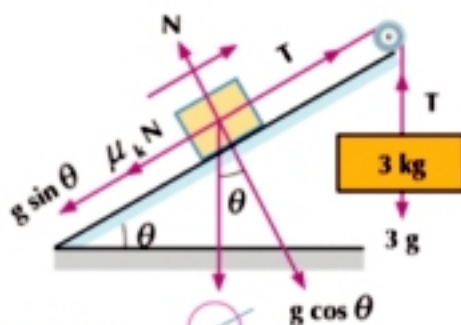
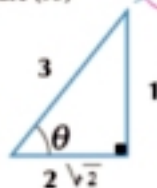


Figure (95)



The equation of motion of the body moving on an inclined plane.

$$m a = -\mu_k N - m g \sin \theta$$

$$1 \times a = -\frac{\sqrt{2}}{2} \times \frac{98\sqrt{2}}{15} - 1 \times 9.8 \times \frac{1}{3}$$

$$a = -9.8 \text{ m/sec}^2$$

$$V^2 = V_0^2 + 2 a S$$

$$\therefore 0 = (4.2)^2 - 2 \times 9.8 S \quad \therefore S = 0.9 \text{ meters}$$

\therefore The mass travels a distance of magnitude $1.8 + 0.9 = 2.7$ meters until it instantaneously rests.

Try to solve

- 9 A body of mass 1 kg is placed on a rough plane inclines to the horizontal with an angle of measure θ where $\sin \theta = \frac{4}{5}$ and is connected by a light string passing over a smooth pulley at the top of the plane such that a pan of mass 400 gm holds a mass of magnitude 100 gm, is suspended from the other end. If the coefficient of friction between the body and the plane is equal to $\frac{1}{3}$, and the system is let to move from rest and the string is coincident on the line of the greatest slope of the plane, find the pressure of the mass on the pan. If another mass of magnitude 100 gm is placed in the pan after one second from the beginning of motion, find the pressure exerted on the pan at this moment and the distance which the system travels within the successive three seconds:

Exercises 2 - 7

Complete the following:

- 1 Two bodies the mass of each 3 kg, are connected by the ends of an inelastic light string passing over a smooth small pulley. If the system acquires a velocity of magnitude 2 m/sec, then:
- The acceleration $a =$ _____
 - The tension in the string $T =$ _____ kg.wt
 - The distance which one of the two masses travels within one second from the beginning of motion is _____ meters.
- 2 In the opposite Figure: if the system moves from rest, then:
- The acceleration of the system $=$ _____ m/sec²
 - The velocity of the system after 2 sec $=$ _____ m/sec
 - If the mass $2m$ is separated from the system after 2 seconds, then the systems moves after that with acceleration $=$ _____
 - The distance traveled by the mass m within 5 seconds from the beginning of motion $=$ _____

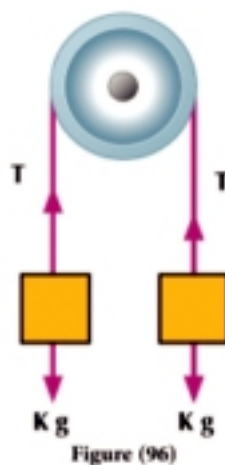


Figure (96)

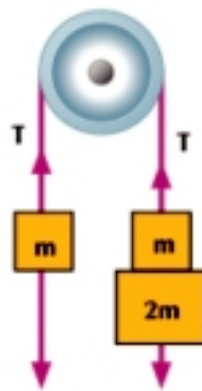


Figure (97)

- 3 Two masses each of a magnitude 420 gm one of them is placed in a pan of mass 140 gm. and the system moves from rest, then:
- Acceleration = cm/sec^2
 - Tension in the string = gm.wt
 - Pressure on the axis of the pulley = gm.wt
 - Pressure on the pan = gm.wt

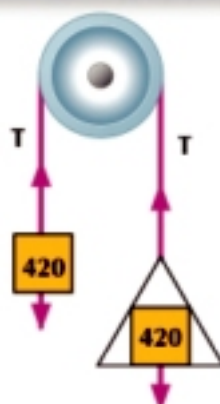


Figure (98)

- 4 In the opposite Figure: two bodies of masses m and $2m$ are connected by the ends of a string passing over a smooth small pulley and the system moves from rest when the two bodies are in a horizontal plane.
- Acceleration = m/sec^2 .
 - Pressure on the pulley = kg.wt.
 - Velocity of the system after $\frac{3}{2}$ seconds from the beginning of the motion = m/sec .
 - Vertical distance between the two bodies after $\frac{3}{2}$ seconds from the beginning of motion = meters.
 - If the string is cut after $\frac{3}{2}$ seconds from the beginning of motion, then the mass m reaches the instantaneous rest after time of magnitude seconds.
 - If the distance between the two bodies after time t second as the string is cut becomes 12.25 meters, then t = seconds.

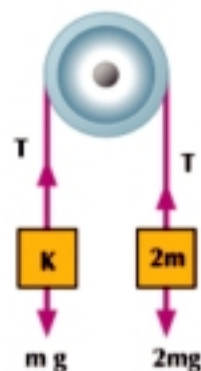


Figure (99)

If a smooth horizontal plane group moved from sleep

- 5 In the opposite Figure:
- a = m/sec^2
 - T = kg.wt
 - The pressure on the pulley = kg.wt
 - The distance traveled after 2 seconds = meters
 - The velocity of the system after 2 seconds = m/sec

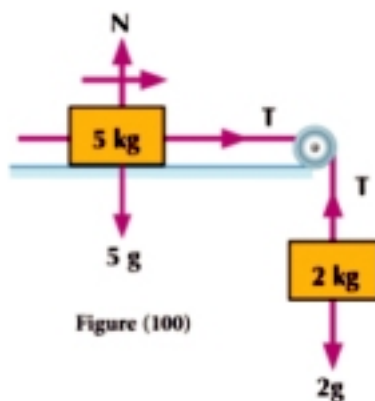


Figure (100)

6 In the opposite Figure:

The body 3 kg is placed on the inclined smooth plane and connected by the body 4 kg suspended vertically. Complete:

- a The acceleration of the system = m/sec^2
- b Tension in the string = newton
- c The pressure on the pulley = newton

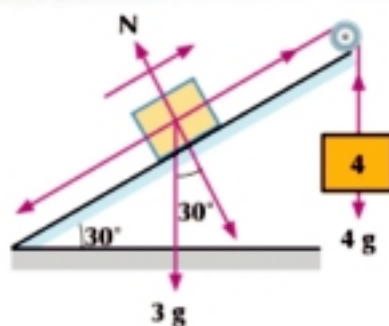


Figure (101)

Answer the following questions:

7 In each of the following figures find:

- a The acceleration.
- b The tension in the string.
- c The pressure on the pulley.

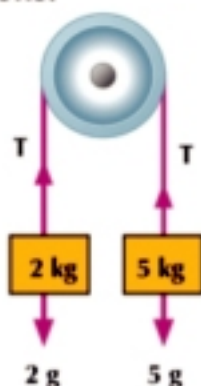


Figure (104)

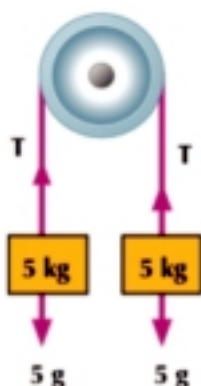


Figure (103)

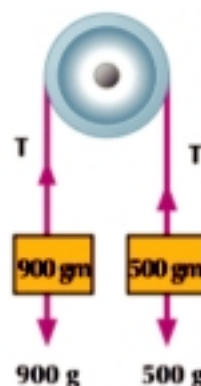


Figure (102)

- 8** Two bodies of masses 5 kg and 3 kg are connected by the ends of a string passing over a smooth small pulley and the system is kept in equilibrium and the two parts of the string are vertical. If the system is let to move, find the magnitude of its acceleration, and the pressure exerted on the pulley, then identify the velocity of the body whose mass is 5 kg when it lands 40 cm.
- 9** Two bodies of masses m_1, m_2 where $m_1 > m_2$ are connected by the ends of a string passing over a smooth pulley. If the system moves with acceleration 196 cm/sec^2 , find $m_1 : m_2$.
- 10** Two masses $3m, m$ are connected by the ends of a light string passing over a smooth pulley and the system is kept in an equilibrium state and the two parts of the string are vertical. If the system is let to move from rest when the vertical distance between the two masses is 160 cm and the mass m is below the mass $3m$, find the time at which the two masses are in one horizontal plane.
- 11** Two pans each of mass 210 gm are connected by the ends of a light string passing on a smooth small pulley and suspended vertically. A body of mass 700 gm is placed in a pan and a body of mass 840 gm is placed in the other pan. Find the acceleration of the system and the pressure on both pans.

- 12 Two masses $5m$, $2m$ are connected by the ends of a string passing over a smooth small pulley and the system is kept in an equilibrium state and the two parts of the string are vertical. If the system is let to move from rest, find the acceleration of the system. If the pressure on the axis of the pulley is equal to 112 newtons, find the value of m .
- 13 Two bodies of masses 420 gm and 560 gm are connected by the ends of a light string passing over a smooth small pulley. The system starts to move from rest when the two bodies are in the same horizontal plane. After passing one second, the string connecting them is cut. Calculate the distance between the two masses after passing another second from the moment of cutting the string.
- 14 A body of mass 4 kg is placed on a rough plane inclined at 30° to the horizontal and connected with a string passing over a smooth small pulley at the top of the plane. A body of mass m suspends from the other end of the string. If the mass 4 kg moves from rest on the plane upwards for a distance of 560 cm in 2 seconds. Find the magnitude of m given that the coefficient of the kinetic friction between the body and the plane is equal to $\frac{\sqrt{3}}{2}$. Then find the magnitude of the pressure on the axis of the pulley.
- 15 A body of mass 400 gm, is placed on a smooth horizontal table, then connected by a light string passing over a smooth pulley fixed at the edge of the table and carries a body of mass 90 gm in its end. Find the acceleration of the system, the tension in the string, and the pressure on the pulley.
- 16 A, B are two bodies of masses 200 gm and 45 gm respectively. The body A is placed on a smooth horizontal table of height 90 cm and distance 270 cm from the edge of the table and connected by a light string of length 270 cm passing over a small pulley fixed at the edge of the table. The body B is connected by the other end of the string at the edge of the table. If the body B is displaced quietly to fall from the edge of the table, find the time taken by the body A after that to reach the edge of the table.
- 17 A body of mass 200 gm is placed on a rough horizontal table and the coefficient of the kinetic friction between them is equal to $\frac{1}{2}$. It is then connected by a light string passing over a smooth pulley fixed at the edge of the table and a body of mass 200 gm is suspended from the other end of the string and a height of 1 meter above the ground surface. If the system moves from rest, calculate:
- The pressure on the pulley in newton.
 - The velocity of the impact of the suspended mass against the ground surface.
 - The distance which the mass, placed on the table, moves until it rests.

UNIT SUMMARY

- 1 **The momentum of a body at a moment is a vector quantity** whose magnitude is equal to the product of the mass of this body by its velocity at this moment and its direction is the direction of the velocity itself.

$$\therefore \vec{H} = m \vec{V}$$

- 2 **The change of the momentum of a body** $= m(\vec{v}_2 - \vec{v}_1)$

$$\Delta H = m \int^t a \, dt$$

If the acceleration a is a function of time t

- 3 **Newton's first law:** Every body preserves in its state of rest or of moving uniformly except in so far as it is made to change that state by an external effect,

- 4 **Inertia principle:** Every body, as much as in it lies, endeavors to preserve its present state, whether it be of rest or of moving uniformly forward in a straight line.

- 5 **Force:** Newton's first law includes a definition to the force as it is the effect which changes or acts on changing the state of the body whether of rest or of moving uniformly forward in a straight line.

- 6 **Newton's second law:** The rate of change of momentum with respect to the time is proportional to the acting force and takes place in the direction in which the force is acting.

- 7 **The equation of motion of a body** whose mass is m and moves with a uniform acceleration a

$$m \vec{a} = \Sigma \vec{F}$$

where $\Sigma \vec{F}$ is the resultant of the forces acting on the body

- If $a = \frac{dV}{dt}$ then the equation of motion is in the form:

$$\int^t F \, dt = m \int^t dV$$

- If $a = V \frac{dV}{dS}$ then the equation of motion is in the form:

$$\int^s F \, dS = m \int^s V \, dV$$

- If the mass is variable, then the equation of motion is in the form:

$$F = \frac{d}{dt} (mv)$$

- 8 **The units used with the equation of motion**

$$m(\text{kg}) \cdot a \, (\text{m/sec}^2) = F \, (\text{newton})$$

$$m(\text{gm}) \cdot a \, (\text{cm/sec}^2) = F \, (\text{dyne})$$

- 9 **Newton's third law**

To every action, there is a reaction equal in magnitude and opposite in direction.

UNIT SUMMARY

10 Lift motion



the forces acting on the lift and the person together

Figure (107)



the forces acting on the lift and the person is inside

Figure (106)



the forces acting on the person inside the lift

Figure (105)

Simple pulleys:

- Equations of motion

$$m_1 a = m_1 g - T$$

$$m_2 a = T - m_2 g$$

$$\text{The pressure on the pulley} = 2T$$

- Equations of motion

$$m_1 a = m_1 g - T$$

$$m_2 a = T$$

$$\text{The pressure on the pulley} = T\sqrt{2}$$

- Equations of motion

$$m_1 a = m_1 g - T$$

$$m_2 a = T - m_2 g \sin \theta$$

$$\text{The pressure on the pulley} = 2T \cos \frac{\alpha}{2} = T\sqrt{2 + 2\sin \theta}$$

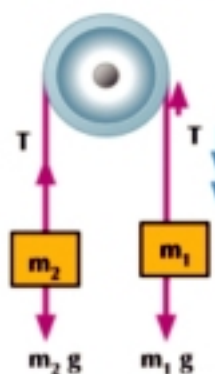


Figure (108)

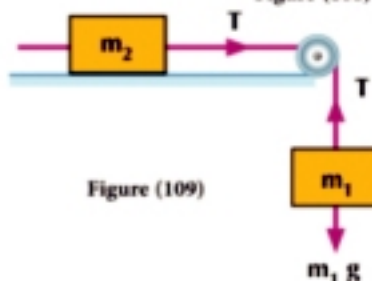


Figure (109)

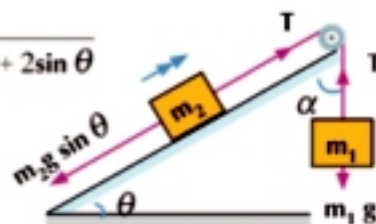


Figure (110)

GENERAL EXERCISES

Complete the following:

- 1 A body of mass 40 kg, then its weight is:
 - a In kg.wt. _____
 - b In newton. _____
- 2 A body moves with velocity of magnitude 135 km/h then it travels. _____ meters per second.
- 3 An inclined plane of length 200 cm and height 150 cm then the two sines of its angle of inclination to the horizontal are. _____
- 4 A body of mass 8 tons moves with a uniform velocity and the resistance per ton mass is 4.5 kg.wt then the force in newton is _____
- 5 A body of mass 35 kg, is placed on a pressure scale fixed in the roof of a lift moving with velocity of magnitude 4 m/sec and the scale reading is 343 newton, then the distance traveled by the lift in 7 seconds is... meters.
- 6 147 newton = _____ kg.wt
- 7 1 kg.wt = _____ newton

Answer the following questions:

- 8 A rubber ball of mass 200 gm is vertically thrown with velocity 30 m/sec to collide with a vertical wall and rebound with velocity 26 m/sec. Find the change in the momentum of the ball due to the impact in kg m/sec



- 9 A car of mass 6 tons moves under the action of a resistance proportional to the square of the velocity. If the resistance is 5 kg.wt per ton when the car's velocity is 36 km/h. Find the force of the engine if the maximum velocity of this car is 40 m/sec.



- 10 A train of mass 160 tons starts to move from rest from a train station. The force of the engine increases with a magnitude 4 ton.wt more than the total resistance of the train when the velocity reaches 44.1 km/h. The train keeps moving with this velocity a period of time, then the driver presses the brakes to acquire the train deceleration of magnitude 17.5 cm/sec^2 . The train stops at the next station at a distance of 4998 meters from the station from which the train moves. Find the time taken to travel the distance between the two stations.



GENERAL EXERCISES

- 11 A light string passes over a smooth small pulley. A body of mass 800 gm is suspended at an end of the string and a spring scale of mass 400 gm, is connected by a body of mass m gm is suspended at the other end of the string. If the system moves from rest and the scale reading during the motion is 160 gm.wt, find the value of m .
- 12 Two masses of 1300 and 600 gm are placed on a smooth horizontal plane and connected by a string tensioned between them of length 50 cm. Then a mass of 600 gm is connected by another string along the first string and passes over a smooth pulley fixed at the edge of the plane near to the second mass. A mass of magnitude 100 gm suspended vertically is connected by the other end. Find the magnitude of the acceleration of the system and the magnitude of tension in each of the two strings, if the string connecting the first two bodies after 2 seconds from beginning the motion; what is the distance between the two bodies after 1 second from the moment the string is cut?
- 13 Two bodies of masses 350 gm, and mg are connected by the ends of a string passing over a smooth small pulley and suspended vertically. The system moves from rest when the two masses are in a horizontal plane and the pressure on the axis of the pulley is 200 gm.wt. Find m and the vertical distance between the two bodies after one second from the beginning of motion.
- 14 Two bodies each of mass m kg are connected by the ends of a light string passing over a smooth small pulley fixed vertically and the two parts suspended vertically. When a body of mass 2 kg is added to one body, the value of tension in the string gets $\frac{8}{7}$ of its value in the first case, find m .
- 15 A light string of a constant length passing over a smooth small pulley is fixed in one of its ends. A body of mass 60 gm and two bodies of masses 40, 50 gm are fixed in the other end. If the system moves from rest, find the acceleration and the tension in the string connecting the two masses 40 gm, 50 gm. If the body of mass 50 gm is separated after 2 seconds from the beginning of motion, prove that the system instantaneously rests after two seconds from the moment of separation.
- 16 Two bodies of masses 260 gm and 230 gm are connected by the ends of a string passing over a smooth small pulley and suspended vertically. The system moves from rest when the greater mass is at a height of 270 cm above the ground surface. Find the acceleration of the system, the tension in the string, and the time passing until the greater mass reaches the ground.
- 17 Two bodies of mass 260 gm and 230 gm are connected by the ends of a string passing over a smooth small pulley and suspended vertically in one horizontal plane at a height of 70 cm above the ground surface. If the system moves from rest and the string is cut after a second from the beginning of the motion, calculate the velocity by which each of the two bodies reach the ground surface.

For more activities and exercises

www.sec3mathematics.com.eg

Impulse and Collision

Unit 3



Introduction

You have previously learned some physical quantities by which you can describe the motion of the bodies such as velocity, acceleration, and mass.

In this unit, you are going to identify other quantities useful to describe the motion of bodies.

Some times, you find it is quite easier to stop a small bead when it rolls over than to stop the bowling ball of the same velocity. We find that the two bodies move retroactively in a straight line.

There is another observation, the truck does much work to stop suddenly although its velocity may not be high. While small sized-cars can stop within a short distance. However its velocity may be higher than that of the truck. From the observations above, it is clear that the velocity and mass are closely related and this is called the momentum. Since the mass of the body is constant in most cases, the velocity always changes. The change of velocity means acceleration of motion and, in turn, the acceleration means that there is a resultant force, i.e the change of velocity changes the momentum.

There is another essential factor which is the time through which the force acts upon the moving body. The force and time are two necessary factors to cause change in the momentum and their product is called impulse. The collision of bodies is considered a practical application to the momentum. When two bodies collide in the absence of any external effects, we find the resultant of momentum for each of the two bodies before and after collision is equal. As a result, the resultant of the momentum before collision is equal to the resultant of the momentum after the collision.

There are various forms of collisions such as the elastic and inelastic collisions and you are going to learn about in this unit.

Unit objectives

By the end of this unit and by doing all the activities involved, the student should be able to:

- ◆ Identify the concept of impulse.
- ◆ Deduce the relation between the impulse and the change of momentum.
- ◆ Identify the elastic collision.
- ◆ Deduce that the sum of momentums of two bodies before collision is equal to the sum of momentums of the two bodies after collision.

Key Terms

- Impulse
- Momentum
- Impulsive forces
- Elastic
- Collision of smooth balls

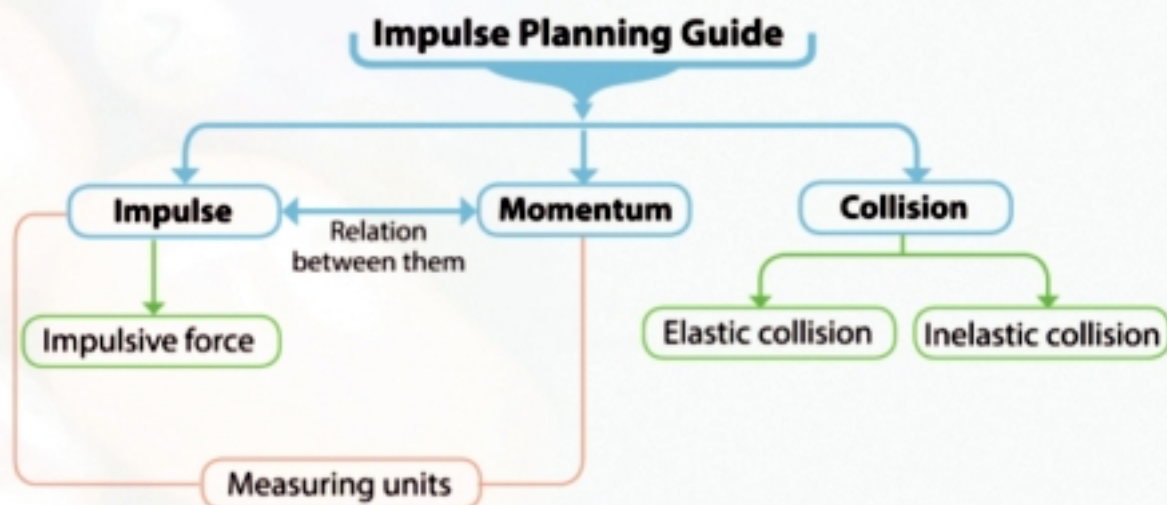
Unit lessons

- (3 - 1) Impulse
- (3 - 2) Collision

Materials

Scientific calculator

Unit Chart



Impulse

You will learn

- The concept of impulse.
- Deduce the relation between the impulse and the change of momentum.

Key terms

- Impulse
- Momentum
- Impulsive forces

Materials

- Scientific calculator

Preface:

- Throwing a ball in the direction of a vertical wall.
- Colliding of cars on the highways.
- Colliding the wheel of planes like landing in airports.



In such cases, the study of the motion of the body is an extremely difficult process due to the overlapping of the acting factors on them and the shortness of the infinitesimal time intervals. In this unit, you are going to learn some information specially related to this topic to relate the position of a body before and after the occurrence of change of its velocity vector through this activity.



Activity

Materials: a wooden ruler of length more than 1 meter and a group of different balls such as a golf ball, tennis ball, billiard ball, clay ball,...

Construction: Let these balls fall one after another from a constant height. Let it be 2 meters on the roof of marble or ceramics, then record the height which each ball rebounds back.

Observation and deduction: Have you noticed differences in the heights in which the different balls rebounded? Can you order the balls according to the rebound for each in a descending order?

The difference of the distance of rebound is related to several factors such as the change of momentum due to the impact (collision) of the ball with the ground.

Search in the internet for impulse and momentum.

First: impulse

If force \vec{F} of a constant magnitude acts on a body during a time interval t , then the impulse of this force

It is denoted by the symbol \vec{I} - is known as the product of force vector by the time of its action i.e

$$\vec{I} = \vec{F} t$$

Type of ball	Height	Maximum rebound
Glass ball	2 m
Billiard ball	2 m
Tennis ball	2 m
Wood ball	2 m
Golf ball	2 m
Bowling ball	2 m
Tennis ball	2 m
Clay ball	2 m

According to such a definition, it is clear that the impulse \vec{I} is a vector in the same direction as the force vector \vec{F} :

$$I = F t$$

The relation between the algebraic measure of impulse \vec{I} and the algebraic measure of the force \vec{F} can be written as follows:

The measuring units of the magnitude of impulse:

From the definition of the impulse, we find that:

The measuring unit of magnitude of impulse = measuring unit of magnitude of force \times time measuring unit

In the international system of units, the magnitude of the impulse is measured in newton. sec (N. sec) It can also be measured by the product of any force by any time unit.

Furthermore, the measuring units of the magnitude of impulse can be expressed in a different method with regarding that:

kg.wt.sec , gm.wt.sec , ...

So, we find that: if the mass is in kg and the velocity is in m /sec, then the impulse magnitude unit is kg.m/sec and it is the same N. sec unit.

When the mass is in gm and the velocity is in cm/sec then the impulse magnitude unit is gm.cm/ sec and it is the same dyne.sec unit

Example Definition of the impulse

- ① A force of magnitude 25 kg.wt acts on an object for a time interval of magnitude $\frac{1}{10}$ of second. Find the force impulse on the body in N.sec unit.

Solution

$$\text{Impulse} = F \cdot t = 25 \times 9.8 \times \frac{1}{10} = 24.5 \text{ N.sec}$$

Try to solve

- ① A force of magnitude 10^{12} dyne acts on a body for a time interval 10^{-5} seconds, find the force impulse on the body in N.sec

Example Finding the magnitude of impulse

- ② The forces $\vec{F}_1 = 4\hat{i} - 3\hat{j} + \hat{k}$, $\vec{F}_2 = \hat{i} + 2\hat{k}$, $\vec{F}_3 = 4\hat{j} - \hat{k}$ act on a body for a time interval of magnitude 5 seconds. Find the magnitude of the forces impulse on the body if the magnitude of the forces is measured in newton unit.

Solution

$$\begin{aligned}\vec{F} &= \vec{F}_1 + \vec{F}_2 + \vec{F}_3 \\ &= (4\hat{i} - 3\hat{j} + \hat{k}) + (\hat{i} + 2\hat{k}) + (4\hat{j} - \hat{k}) = 5\hat{i} + \hat{j} + 2\hat{k}, \\ \therefore \vec{I} &= \vec{F} \times t = 5(5\hat{i} + \hat{j} + 2\hat{k})\end{aligned}$$

$$\text{Magnitude of impulse} = \sqrt{(25)^2 + (5)^2 + (10)^2} = 5\sqrt{30} \text{ N}\cdot\text{sec}$$

Try to solve

- 2 The force $\vec{F}_1 = 2\hat{i} + 3\hat{j}$, $\vec{F}_2 = \hat{j} - 5\hat{i}$ act on a body for one second. Find the force impulse on the body if the force magnitude is measured in newton unit.

Second: impulse and momentum

Since the impulse of a force F of a constant magnitude on a body for a time interval t is equal to Ft and from Newton's second law, we find that:

$$\text{impulse} = m a \cdot t$$

$$\therefore \text{impulse} = m (v - v_0)$$

Where v_0 and v are the two algebraic measures to the two vectors of the initial velocity and velocity after time t respectively.

i.e the impulse is equal to the change of momentum. But if the force is variable, then the impulse is given by the following integration :

$$\text{impulse} = \int_0^t F dt$$

$$\therefore \int_0^t F dt = \int_0^t m a dt$$

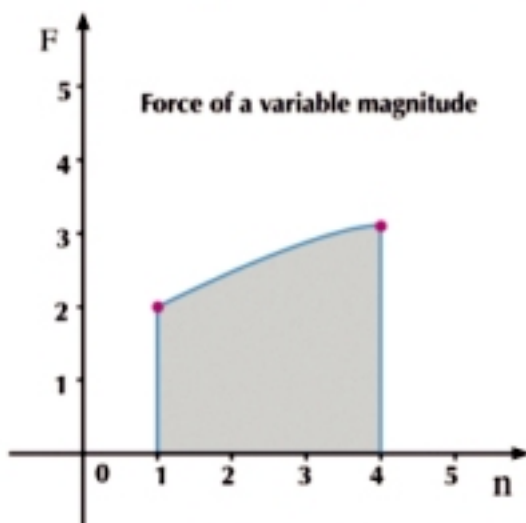
$$\int_0^t F dt = m \int_0^t \left(\frac{dv}{dt}\right) dt$$

$$= m \int_{v_0}^v dv$$

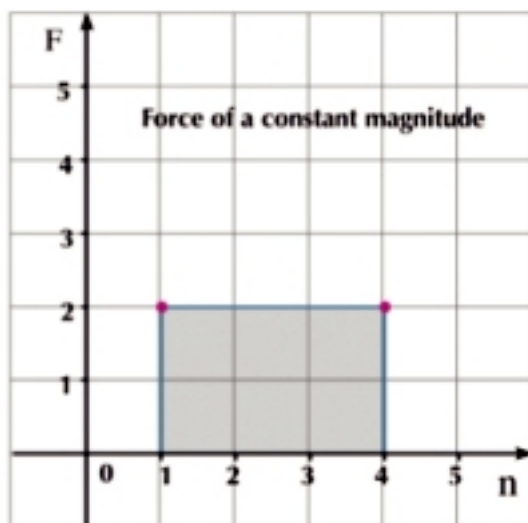
$$\int_0^t F dt = m [v]_{v_0}^v$$

$$\int_0^t F dt = m (v - v_0)$$

In general, impulse is equal to the change of momentum



$$\text{Impulse} = \int_1^4 F dt$$



$$\text{Impulse} = \int_1^4 F dt$$

Remember

$$\therefore v = v_0 + a t$$

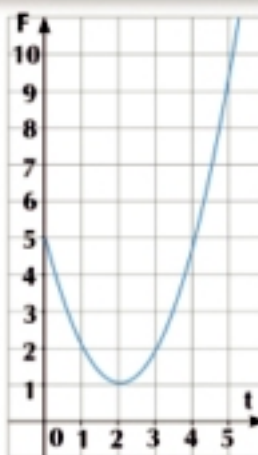
$$\therefore v - v_0 = a t$$

Example

- 3 The opposite figure represents the force-time graph where $F = 1 + (t - 2)^2$. Find:

- a The impulse of the force F within the first three second.
 b The impulse of the force F in the fifth second.

Where the magnitude of the force F is in newton and the time t in second



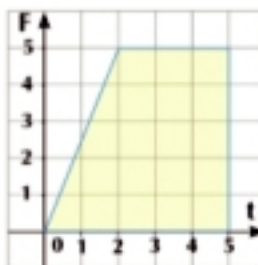
Solution

$$F = 1 + (t - 2)^2 \qquad F = t^2 - 4t + 5$$

- a Impulse within the first three seconds $= \int_0^3 F \, dt$
 $= \int_0^3 (t^2 - 4t + 5) \, dt$
 $= \left[\frac{1}{3} t^3 - 2t^2 + 5t \right]_0^3 = 6 \text{ N}\cdot\text{sec}$
- b Impulse within the first fifth seconds $= \int_4^5 F \, dt$
 $= \int_4^5 (t^2 - 4t + 5) \, dt$
 $= \left[\frac{1}{3} t^3 - 2t^2 + 5t \right]_4^5 = \frac{22}{3} \text{ N}\cdot\text{sec}$

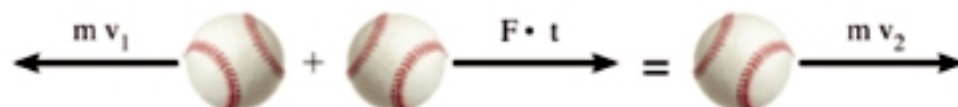
Try to solve

- 3 The opposite figure represents the force-time graph. Find using the integration:
- a The impulse of the force F within the first second.
 b The impulse of the force F within the first five seconds where the force F is in newton and the time t is in second.



The impulsive forces

The impulse forces are extremely tremendous forces acting for infinitesimal time interval and causing an extremely tremendous change in the momentum of the body without any acting on its position. The motion resulted at the action of these forces is called an impulsive motion. For example, the baseball when hit by a bat, the contact time between the bat and the ball is extremely infinitesimal although the average of the force acting on the ball is extremely tremendous. The impulse is tremendous enough to change the momentum of the ball with out any change in the position of the ball. When an impulsive force acts on a body then $m v_1 + F \cdot t = m v_2$ where t is an extremely infinitesimal time interval.



Example Impulse and momentum

- 4 A body of mass 4 kg placed at rest on a smooth horizontal plane and is acted upon by a horizontal force of magnitude 5 newtons for 8 seconds. Find the impulse magnitude on the body and the velocity magnitude of the body after 8 seconds.

Solution

$$\begin{aligned} \therefore \text{impulse} &= F \cdot t \\ \therefore \text{impulse} &= 5 \times 8 = 40 \text{ N}\cdot\text{s} \\ \therefore \text{impulse} &= \text{The change of momentum} \\ 40 &= m(v - v_0) \\ 40 &= 4(v - 0) \\ v &= 10 \text{ m/sec} \end{aligned}$$

Try to solve

- 4 A constant force of magnitude F acts on a body of mass m for $\frac{1}{49}$ of a second to change its velocity from 3 m/sec to 54 km/h in the force direction and the force impulse is equal to 4.8 N.sec. Find the mass of the body and the force magnitude in kg.wt.

Example Expressing the impulse and momentum using vectors

- 5 The force $\vec{F} = 2\hat{i} + 7\hat{j}$ acts on a body of mass 5 kg for 10 seconds when its velocity vector $\vec{v} = \hat{i} - 2\hat{j}$. Find its velocity after the action of the force if the force magnitude is in newton unit and the velocity is in m/sec unit.

Solution

$$\begin{aligned} \therefore \text{impulse} &= \text{change of momentum} \\ \therefore \vec{F} \cdot t &= m(\vec{v} - \vec{v}_0) \\ \therefore 10(2\hat{i} + 7\hat{j}) &= 5(\vec{v} - \vec{v}_0) & \therefore \vec{v} - \vec{v}_0 &= 2(2\hat{i} + 7\hat{j}) \\ \therefore \vec{v} &= 2(2\hat{i} + 7\hat{j}) + (\hat{i} - 2\hat{j}) \\ \therefore \vec{v} &= 4\hat{i} + 14\hat{j} + \hat{i} - 2\hat{j} & \therefore \vec{v} &= 5\hat{i} + 12\hat{j} \\ \therefore \|\vec{v}\| &= \sqrt{25 + 144} = 13 \text{ m/sec} \end{aligned}$$

Try to solve

- 5 A body of mass 3 kg moving with velocity $\vec{v} = 5\hat{i} - 2\hat{j}$, is acted upon a constant force for a time interval t and the force impulse on the body is equal to $6\hat{i} + 9\hat{j}$. Find the velocity of the body after the action of the force if the velocity is in m/sec, unit and the impulse magnitude is in N.sec unit

Notice that

- When a body of weight «w» falls vertically on the ground, **then:**
at the moment of contact

The pressure of the body on the ground = reaction of the ground on the = $F + w$

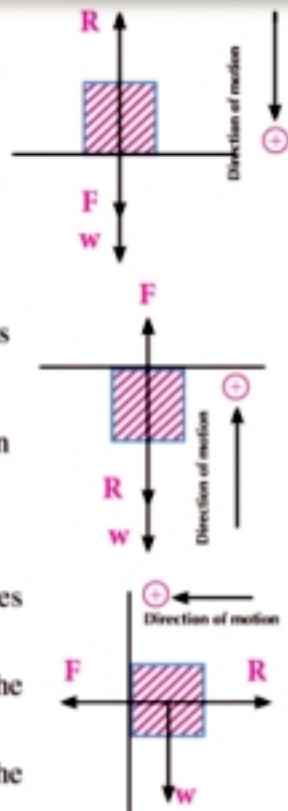
- When a body of weight «w» is projected vertically and collides with the ceiling of a room, **then:**

The pressure of the body on the ceiling = reaction of the ceiling on the body = $F - w$

- When a body of weight «w» is projected horizontally and collides with a vertical wall, **then:**

The pressure of the body on the wall = reaction of the wall on the body = F

Where F is the magnitude of the impulsive force in each of the cases above.



Example The vertical motion

- 6 A rubber ball of mass $\frac{1}{4}$ kg fell down from a height of 10 meters above the ground and rebounded after collided with the ground for a height of 2.5 meters. Find the impulse resulted from the collision (impact) of the ball with the ground and identify the reaction of the ground on the ball if the contact time of the ball with the ground is $\frac{1}{10}$ of second.

Solution

Studying the phase of falling down

$$\therefore v^2 = v_1^2 + 2g s$$

$$\therefore v_1^2 = 0 + 2 \times 9.8 \times 10$$

$$\therefore v_1 = 14 \text{ m/sec}$$

It is the velocity of the ball be for it contacts directly with the ground.

$$\text{impulse} = \text{Change of momentum} = m(v_2 - v_1)$$

$$= \frac{1}{4} [7 - (-14)] = 5.25 \text{ kg} \cdot \text{m/sec}$$

$$\therefore \text{impulse} = F \cdot t \quad \therefore 5.25 = F \times \frac{1}{10}$$

$$\therefore F = 52.5 \text{ newton}$$

The reaction of the ground on the ball = impulsive force + weight of the ball

$$= 52.5 + \frac{1}{4} \times 9.8 = 54.95 \text{ newton}$$

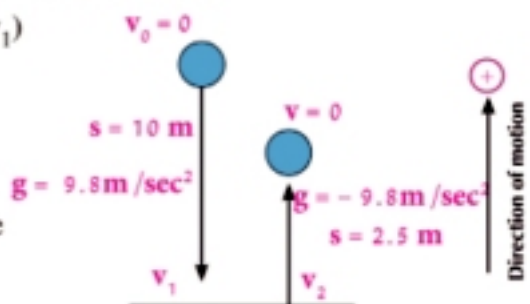
Studying the phase of rebound back

$$\therefore v^2 = v_2^2 + 2g s$$

$$\therefore 0 = v_2^2 - 2 \times 9.8 \times 2.5$$

$$\therefore v_2 = 7 \text{ m/sec}$$

\therefore The velocity of rebound back = 7m/sec vertically up wards



Try to solve

- 6 A body of mass 300 gm is projected vertically upwards with velocity 840 cm/sec from a point placed under the ceiling of a room of magnitude 110 cm to collide (impact) with the ceiling and bounds down to the roof of the room after $\frac{1}{2}$ of second of the rebound. Find the impulse of the ceiling to the body given that the ceiling height is 272.5 cm. If the contact time is $\frac{1}{10}$, find the impulsive force.

Critical thinking: A clay ball of mass 1kg fell down from a height of 40cm on a pressure scale and the collision (impact) time is $\frac{1}{7}$ of seconds, find the scale reading given that the ball did not rebound after the impact.

Example Horizontal motion

- 7 A ball of mass 100 gm moves horizontally with velocity 9 m/sec to collide with a vertical wall and rebound back with velocity of magnitude 7.2 km/h. If the contact time of the ball with the wall is $\frac{1}{10}$ of second, find the impulse of the wall to the ball and then find the pressure of the ball on the wall.

Solution

Let the rebound direction be the positive direction of motion

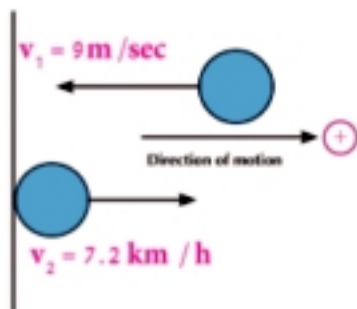
$$\therefore v_1 = -9 \text{ m/sec}, v_2 = 7.2 \times \frac{5}{18} = 2 \text{ m/sec}$$

$$\therefore I = m(v_2 - v_1)$$

$$\therefore I = \frac{100}{1000} [2 - (-9)] = 0.11 \text{ gm. m/sec}$$

$$\therefore I = F \times t \quad \therefore 0.11 = F \times \frac{1}{10}$$

$$\therefore F = 1.1 \text{ newton} = \text{the pressure of the ball on the wall}$$



Try to solve

- 7 A tennis ball of mass 40 gm moves horizontally with velocity of 50 cm/sec to collide with the bat and rebounds in the opposite direction with velocity of 110 cm/sec. Find the impulse magnitude of the bat on the ball. What is the magnitude of the impulsive force of the bat on the ball. If the contact time of the ball with the bat is $\frac{1}{49}$ of a second?

Exercises 3 - 1

First : Choose the correct answer:

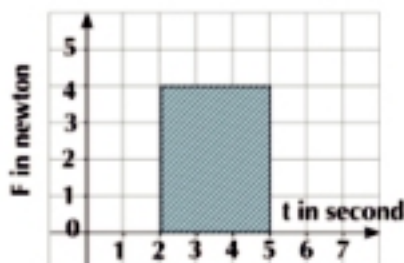
- 1 If a force of magnitude 16 kg.wt acts on a body for four seconds, then the magnitude of the force impulse in N.sec unit is equal to.
 a 4 b 39.2 c 49 d 64
- 2 If the magnitude of the impulse of the force F on a body for 10^{-4} seconds is equal to 10 N.sec, then the magnitude of F is equal to:
 a 10^3 dynes b 10^5 dynes c 10^3 newton d 10^5 newton

- 3 If the two forces $\vec{F}_1 = \hat{i} + 5\hat{j} + 7\hat{k}$ and $\vec{F}_2 = 2\hat{i} - \hat{j} - 2\hat{k}$ which F_1, F_2 of measured Newton unit act on a body for a time interval of magnitude 2 seconds, then the magnitude of the impulse of the force in N.sec unit is equal to:

- a $5\sqrt{2}$ b $10\sqrt{2}$
 c $50\sqrt{2}$ d $100\sqrt{2}$

- 4 If a force of constant magnitude acts up on a body for a time interval as given in the figure, then the impulse magnitude in N.sec unit is equal to:

- a 8 b 12
 c 20 d 50

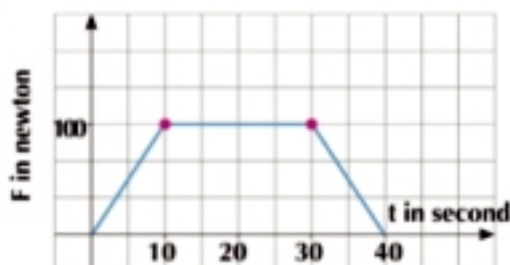


- 5 If a force of magnitude 90 newton acts upon a body of mass 10 kg for five seconds, then the magnitude of the change of the velocity of the body in the same direction of the force is equal to:

- a 45m/sec b 50m/sec
 c 90m/sec d 120 m/sec

- 6 A body of mass 20 kg is placed on a smooth horizontal plane. If it moves under the action of a force in a constant direction and its magnitude changes over time as shown in the figure, then the impulse magnitude of this force after 40 seconds in N.sec unit is equal to:

- a 1000 b 2000
 c 3000 d 4000



Second : Answer the following questions :

- 7 A bullet of mass 20 gm is shot horizontally from a gun. If its path inside the gun continues for 0.5 of a second and the magnitude of the impulsive force of the gun on the bullet is 20 newton, find the velocity of the bullet's exit from the barrel of the gun.
- 8 A fast shooting mortar shoots bullets each of mass 500 gm vertically upwards. If the average of the impulsive force of the gas in the mortar's cylinder on the bullet is 250 newton and acts on the bullet for 0.2 of a second until the moment the bullet exits from the barrel of the mortar, calculate the speed of the bullets of the barrel of a gun.

- 9 A rubber ball of mass 20 gm fell down from a height of 6.4 meters above the ground to rebound vertically upwards. If the average force which the ground exerts (does) on the ball is 182×10^4 dynes and the contact time of the ball with the ground is 0.02 of seconds, find:
- The magnitude of the impulse of the ground to the ball
 - The maximum height the ball reaches after it rebounds
- 10 A smooth ball of mass 200 gm moves in a straight line on a smooth horizontal ground with velocity 10 m/sec. If the ball collided with a smooth vertical wall and rebounded with velocity 4m/sec, Find:
- The magnitude of impulse of the wall on the ball.
 - The magnitude of the impulsive force of the wall if the contact time of the ball on the wall is 0.05 of a second.
- 11 A train car of mass 10 tons moves with velocity 18 km/h to collide (impact) with a barrier and rebounds with velocity 9 km/ h. Find the magnitude of the impulse of the barrier on the train car.
- 12 A car at rest, of mass 1 ton is pushed in the direction of its motion with a force of 200 kg.wt for 5 seconds, then it is released freely to become at rest again after 15 seconds. Find the magnitude of the resistance supposing it is constant in the two cases. Then find the maximum velocity that the car reached using the relation between the impulse and momentum.
- 13 A ball of mass 1 gm is projected vertically upwards and in the direction of a ceiling of height 360 cm from the projection point with velocity of magnitude 14 m/sec. If the ball collides with the ceiling and rebounds back with velocity 10 m/sec, find the impulsive force of the ceiling on the ball if the contact time of the ball with the ceiling is 0.02 of a second.
- 14 A fast shooting mortar shoots horizontal 600 bullets each of mass 39.2 gm a minute with velocity 1260 km/h. Calculate the force of reaction acting on the mortar in kg.wt.
- 15 A ball of mass 1500 gm falls down from a height of 2.5 meters on a viscous liquid surface to embed in it with a uniform velocity and travels for a distance of 70 cm in 0.2 of a second. Calculate the magnitude of the impulse of the liquid on the ball.
- 16 The forces $\vec{F}_1 = a \hat{i} - \hat{j}$, $\vec{F}_2 = 3 \hat{i} + b \hat{j}$, $\vec{F}_3 = a \hat{i} + 2 \hat{j}$ act on a body for $\frac{1}{2}$ of a second and the impulse of this force on the body is given by the relation $\vec{T} = 2 \hat{i} + 4 \hat{j}$, Find the value of a , b.
- 17 A body of mass 20 gm falls down from a height of 40 cm above a pond surface to embed in water for a distance 210 cm within one second with acceleration 2.1m/sec^2 . Find the impulse of water on the body. As a result of colliding surface water.

Collision

Introduction:

The collision of bodies is considered a practical application of the momentum. When two bodies collide in the absence of external effects, then each body changes the momentum of the other body. According to Newton's third law, the two forces are equal in magnitude and opposite in direction. Due to the collision of the two bodies, the change of the motion of the two bodies remains constant. This is known as momentum conservation law. By considering that the collision is instantaneous (**it takes extremely infinitesimal time**), then the impulse of the first body on the second body is equal in magnitude and opposite in direction to the impulse of the second body on the first one.

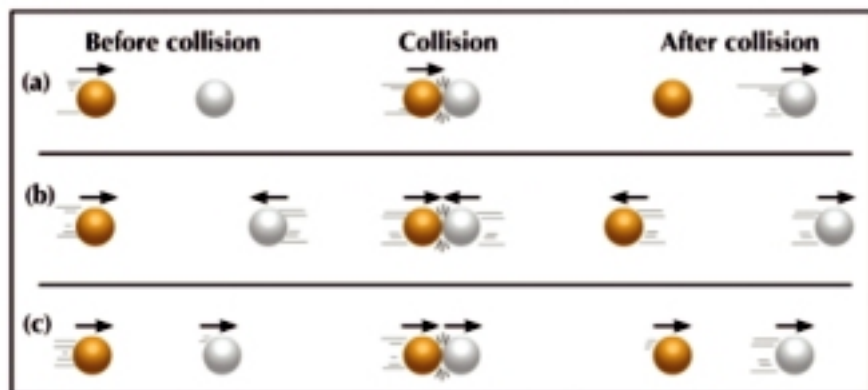
There are several forms of collision such as the elastic collision and inelastic collision.

First: Elastic Collision

If the deformation and generating heat due to the collision between the two bodies do not occur (there is no loss of the kinetic energy), it is said the collision is elastic. For example, when a moving billiard ball collides with a ball at rest of the same mass, we find that the first ball rests. Meanwhile, the second ball moves with initial velocity equal to the initial velocity of the first ball. In regard to this example, we notice that the momentum has totally traveled from the first ball to the second one.

Collision of smooth balls

Through the process of the collision among bodies, you can notice that the directional sum of the momenta before and after the collision is equal.



You will learn

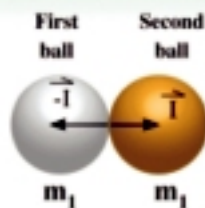
Key terms

Materials

In the figure above:

Let the mass of the first ball be m_1 , the second ball be m_2 , and \vec{I} be the impulse of the first ball on the second one, then $-\vec{I}$ is the impulse of the second ball on the first one.

Let \vec{v}_1, \vec{v}_2 be the two vectors of velocity of the two balls before collision directly and \vec{v}'_1, \vec{v}'_2 are the two vectors of the velocity of the two balls after the collision directly.



In direct collision, the two velocities before the collision directly are parallel to the line of the line of the centers at the moment of collision.

With respect to the first ball:

\therefore The change of momentum of the ball = the impulse acting on it

$$\therefore m_1 \vec{v}'_1 - m_1 \vec{v}_1 = -\vec{I} \quad (1)$$

\therefore The two vectors $\vec{v}'_1, -\vec{I}$ are parallel to the line of centers
(Since the collision is direct)

\therefore The vector \vec{v}'_1 is also parallel to the line of the centers.

With respect to the second ball:

\therefore The change of momentum of the ball = the impulse acting on it

$$\therefore m_2 \vec{v}'_2 - m_2 \vec{v}_2 = \vec{I} \quad (2)$$

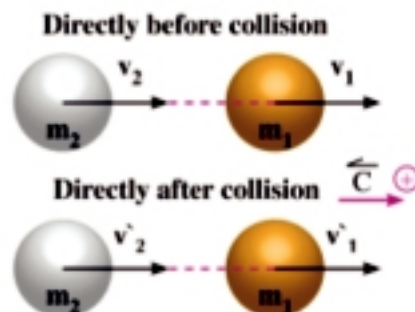
\therefore The two vectors \vec{v}'_2, \vec{I} are parallel to the line of centers
(Since the collision is direct)

\therefore The vector \vec{v}'_2 is also parallel to the line of centers by adding (1), (2):

$$\therefore (m_1 \vec{v}'_1 - m_1 \vec{v}_1) + (m_2 \vec{v}'_2 - m_2 \vec{v}_2) = \vec{0}$$

$$\therefore m_1 \vec{v}'_1 + m_2 \vec{v}'_2 = m_1 \vec{v}_1 + m_2 \vec{v}_2 \quad (3)$$

ie: the sum of the two momentums after the collision directly = the sum of the two momentums before the collision directly. If two smooth balls collide, then the sum of the their two momentums doesn't change due to the collision.


Using the algebraic measures:

The algebraic measures can be used for the vectors of velocity and impulse. This means that the three previous can be reformulated as follows:

$$m_1 v'_1 - m_1 v_1 = -I$$

$$m_2 v'_2 - m_2 v_2 = I$$

$$m_1 v'_1 + m_2 v'_2 = m_1 v_1 + m_2 v_2$$


Example

- Two smooth balls each of mass 200 gm move in a straight line on a smooth horizontal plane; the first with velocity 4 m/sec and the second with velocity 6 m/sec in the same direction of the first ball. If the two balls collide, identify the velocity for each of them directly after collision given that the impulse magnitude of the second ball on the first is equal to 5×10^4 dyne.sec.

Solution

We take a constant unit vector \vec{C} in the direction of the two velocities directly before collision and consider it in the positive direction.

\therefore The impulse of the second ball on the first ball is in the positive direction.

$$\therefore I = 5 \times 10^4 \text{ dynes.sec.}$$

The impulse of the first ball on the second is in the negative direction.

$$\therefore I = -5 \times 10^4 \text{ dynes.sec.}$$

With respect to the first ball:

$$\therefore I = m_1 (\vec{v}'_1 - v_1) \quad \therefore 5 \times 10^4 = 200 (\vec{v}'_1 - 400)$$

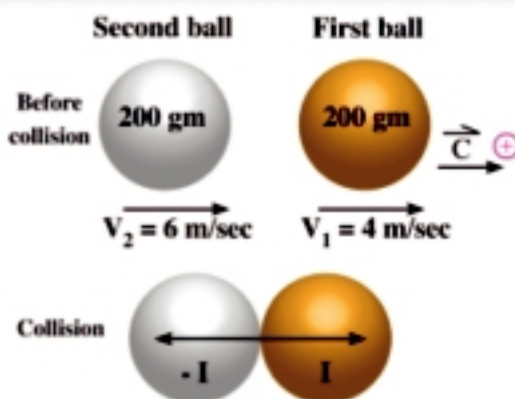
$$\therefore 250 = \vec{v}'_1 - 400 \quad \therefore \vec{v}'_1 = 650 \text{ c m/sec} = 6.5 \text{ m/sec}$$

With respect to the second ball:

$$\therefore I = m_2 (\vec{v}'_2 - v_2) \quad \therefore -5 \times 10^4 = 200 (\vec{v}'_2 - 600)$$

$$\therefore -250 = \vec{v}'_2 - 600 \quad \therefore \vec{v}'_2 = 350 \text{ c m/sec} = 3.5 \text{ m/sec}$$

i.e : the two balls move directly after the collision in the same direction; the first with velocity 6.5 m/sec and the second with velocity 3.5m/sec



Try to solve

- Two smooth balls each of mass 200 gm move in one straight line on horizontal ground; the first with velocity 5 m/sec and the second with velocity 9 m/sec in the same direction of the first. If the two balls collide, identify the velocity of each directly after collision given that the impulse magnitude of the second ball on the first is equal to 0.6×10^5 dyne.sec

Example (Collision of balls)

- Two balls of masses 100 gm, 50 gm move in one horizontal straight line in two opposite directions. The two balls collided when the velocity of the first was of magnitude 50cm/sec and the velocity of the second was of magnitude 30 cm/sec. If the second ball rebounded back directly after collision with velocity = 40 cm/sec, find the magnitude and direction of the velocity of the first ball directly after collision and the impulse magnitude for any of the two balls on the other.

Solution

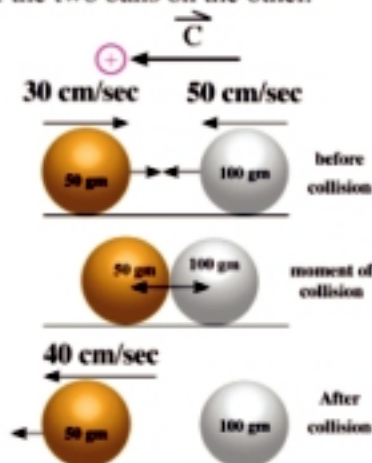
First: Let the direction of the velocity of the first ball before collision in the direction of the unit vector \vec{C}

\therefore The sum of the two momentums before collision = the sum of the two momentums after collision

$$\therefore m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2$$

$$100 \times 50 - 50 \times 30 = 100 v'_1 + 50 \times 40$$

$$v'_1 = 15 \text{ cm/sec in the direction of the unit vector } \vec{C} \text{ it self}$$



Second:

We find the impulse of the first ball on the second ball.

The impulse of the first ball on the second ball = the change of momentum of the second ball

$$I = m (v_2' - v_2)$$

$$I = 50 (40 - (-30)) = 3500 \text{ gm. cm/sec}$$

Second: Inelastic collision

The inelastic collision means that a deformation takes place. Heat is generated or the bodies get contacted due to the collision process (loss of kinetic energy occurs).

In spite of it all, the momentum after and before collision remains as it is without change.

And the momentum conservation equation is in form: (in case the two bodies contacted)

$$m_1 \vec{v}_1 + m_2 \vec{v}_2 = (m_1 + m_2) \vec{v} \quad (\text{by using vectors})$$

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v \quad (\text{by using algebraic measures})$$

Common practical examples of the inelastic collision are the collision of train cars and a hammer falling down on the wedge used to drill the foundations.

 **Example** Collision of train cars

- 3 A train car of mass 10 tons moves with velocity 20 m/sec. It collides with another train car at rest of mass 10 tons. If both cars move directly after collision as one body, calculate their common velocity at this moment.

Solution

Let the mass of the moving car be m_1

Then $m_1 = 10 \text{ tons} = 10000 \text{ kg}$ with velocity v_1

where: $v_1 = 20 \text{ m/sec}$

Let the mass of the car at rest be m_2

$m_2 = 10 \text{ tons} = 10000 \text{ kg}$ with velocity $v_2 = \text{zero}$.

Consider the velocity of the first body before collision is positive and the common velocity of the two bodies directly after collision is v

\therefore Sum of the two momentums before collision = sum of two momentums after collision

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v$$

$$10000 \times 20 + 10000 \times \text{zero} = (10000 + 10000) \times v$$

$$200000 = 20000v$$

$\therefore v = 10 \text{ m/sec}$ in the direction of the motion of the first car itself

**Try to solve**

- 2 A train car of mass 6 tons moves with velocity 25 m/sec and collides with another train car of mass 3 tons. If the two cars move directly after collision as one body, calculate their common velocity at this moment.

Example

- 4 An iron hammer of mass 210 kg falls down from a height of 90 cm on a foundation pole of mass 140 kg to push it in the ground for a distance of 18 cm. If the hammer and pole move as one body directly after collision, find the common velocity of each, then find in kg.wt the average of the ground resistance supposing it is constant.

Solution

First: the velocity of the hammer hitting the foundation pole

$$v^2 = v_0^2 + 2 g s$$

$$v^2 = 2 \times 9.8 \times 0.9$$

$$v = 4.2 \text{ m/sec}$$

Second: the moment of collision

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v$$

$$210 \times 4.2 + 140 \times \text{zero} = 350 v$$

$$\therefore v = 2.52 \text{ m/sec}$$

Third: after collision

The two bodies are one body moving with acceleration a for a distance 0.18m and initial velocity $v = 2.52 \text{ m/sec}$, and its final velocity = zero

$$v^2 = v_0^2 + 2 a s \quad \therefore \text{zero} = (2.52)^2 + 2 \times a \times 0.18$$

$$a = -17.64 \text{ m/sec}^2$$

To find the average of the ground resistance:

$$(m_1 + m_2) g - R = m a$$

$$350 \times 9.8 - R = 350 \times -17.64$$

$$R = 350 \times 9.8 + 350 \times 17.64$$

$$\therefore R = 9604 \text{ newton}$$

$$\therefore R = 980 \text{ kg.wt}$$



Try to solve

- 3 An iron hammer of mass 2.1 tons falls down from a height of 1.6 m on a foundation pole of mass 350 kg to push it in the ground for a distance of 12 cm. If the hammer and the pole move as one body vertically downwards directly after collision, calculate the magnitude of the common velocity of each after collision. Then calculate the magnitude of the ground resistance supposing it is constant.



Exercises 3 - 2

**First: Complete:**

- ① If a force \vec{F} acts upon a body of a constant mass within a time interval t , then the impulse of this force is equal to _____.
- ② If a constant force acts on a body for an extremely infinitesimal time interval, then the change of the momentum of the body with this interval is equal to _____.
- ③ If the mass is measured in kg and the magnitude of the velocity in m/sec, then the impulse unit is measured in _____ or _____.
- ④ If two smooth balls collide and their velocities before collision directly were parallel to the line of centers at the moment of collision, then this collision is called _____.
- ⑤ If two smooth balls collide, then the sum of their momentums before collision is equal to _____.

Second: choose the correct answer:

- ⑥ The magnitude of impulse in (dyne .sec) unit acting on a body of mass 20 gm to change its velocity from 10 cm/sec to 18 cm/sec in the same direction is equal to:
 a 80 b 160 c 280 d 560
- ⑦ If a force of magnitude 8 newton acts on a body at rest of mass 4 kg, then the velocity which the body acquires by the end of five seconds from the beginning of the motion is equal to:
 a 6.4 m/sec b 10 m/sec c 20 m/sec d 40 m/sec
- ⑧ If a force acts on a body of mass 700 gm to change its velocity from 30 cm/sec to 65 cm/sec in the same direction and the time of its action is 10 seconds, then the magnitude of this force in kg.wt unit is equal to:
 a 2.5 b 25 c 1225 d 2445
- ⑨ A body of mass 400 gm is acted upon by a force that changes its velocity from 25 cm/sec to 55 cm/sec in the same direction, find the magnitude of the impulse of this force.
- ⑩ A force acts on a body of mass 150 gm moving with velocity 20 cm/sec and changes the velocity from 10 cm/sec in the opposite direction of its first motion. Find the impulse magnitude of this force on the body.
- ⑪ A ball of mass 800 gm fell down from a height of 2.5 m on a viscous liquid surface to embed in with uniform velocity of magnitude 2 m/sec. Calculate the impulse of the liquid on the ball.

- 12 A smooth ball of mass 300 gm moves in a straight line on a horizontal ground with velocity 8 m/sec. If this ball collides with a smooth vertical wall and rebounds with velocity 5m/sec, find the impulse magnitude of the wall on the ball. What is the magnitude of impulsive force of the wall on the ball if the contact time of the ball with the wall is $\frac{1}{20}$ of a second.
- 13 Two balls of masses 30 gm and 90 gm move in a straight line on a horizontal table and in two opposite directions to collide when their velocities are 50 cm/sec and v cm/sec respectively to form one body moving directly after collision with velocity 10 cm/sec in the direction of the heavier ball. Calculate the magnitude of v . If the motion resistance of the new body is 300 dynes, find the distance it travels before resting.
- 14 A hammer of mass 1 ton fell down from a height of 4.9 m vertically on a foundation pole of mass 400 kg to embed it vertically in the ground for a distance 10 cm . If the hammer and pole move as one body directly after collision, find their common velocity, then find the ground resistance supposing it is constant in kg.wt.
- 15 Two balls moving in a horizontal straight line in two opposite directions. The first ball is of mass 5 kg and velocity 40 cm/sec while the second ball of mass 6 kg and velocity 50 cm/sec collide. If the first ball moves in the opposite direction to its motion with velocity 20 cm/sec, prove that the second ball rests directly after collision. What is the impulse magnitude of the second ball on the first one?
- 16 Two smooth balls; the mass of the first is 50 gm, the mass of the second is 40 gm, the displacement of the first is $\vec{s}_1 = 300t \hat{i}$ and the displacement of the second is $\vec{s}_2 = -150t \hat{i}$ where s is measured in cm and timed in second. If the two balls collide and form one body directly after collision, calculate the common velocity of this body, then calculate the force of pressure between the two balls if the collision time is $\frac{1}{6}$ of a second.
- 17 A small ball of mass 30 gm moves in a straight line with a uniform velocity of magnitude 13 m/sec After 4 seconds of its passing through a certain position, another ball of mass 10 gm moves from this position in the same direction of the first ball with initial velocity 4 m/sec and acceleration 2 m/sec^2 to form one body after collision directly. Calculate the common velocity of the body. When does this body rest if this body is acted on by a constant resistance on the horizontal plane of magnitude 4 gm.wt?
- 18 A body of mass 1 kg is placed on a smooth horizontal surface. A force of magnitude 8 newton acts on this body for $\frac{1}{2}$ second During the absence of the force action, this body collides with another body of mass 2 kg and the first body rebounds with velocity 2 m/sec. Find the velocity of the second body directly after collision.

UNIT SUMMARY

1 Impulse: If force \vec{F} of a constant magnitude acts on a body during a time interval t , then the impulse of this force - denoted by the symbol \vec{T} is known as the product of force vector by the time of its action i.e. : $\vec{T} = \vec{F} t$

2 Measuring unit of impulse magnitude: The measuring unit of impulse magnitude = measuring unit of force magnitude \times time measuring unit

In the international system of unit, the impulse magnitude is measured in N.sec unit. It can also be measured by any force unit in time unit.

impulse and momentum : impulse = change of momentum. i.e $F \times t = m (v_2 - v_1)$

The force - time graph:

The impulse can be represented by the area under the force - time graph and it can be identified by the relation : impulse = $\int_{t_1}^{t_2} F dt$

3 The impulsive force: The impulsive force are extremely tremendous force act for infinitesimal time interval and cause an extremely tremendous change in the momentum of the body without any acting on its position and the motion resulted at the action of these forces is called an impulsive motion. For example, the baseball when hit by the bat, the contact time between the bat and the ball is extremely infinitesimal. Although the average of the force acting on the ball is extremely tremendous. The impulse is tremendous enough to change the momentum of the ball without any change in the position of the ball.

4 Elastic collision : If the deformation doesnot occur, heat is not generated and there is not loss in the kinetic energy. $m_1 \vec{v}_1 + m_2 \vec{v}_2 = m_1 \vec{v}_1 + m_2 \vec{v}_2$

i.e: the sum of the two momentums after the collision directly = the sum of the two momentums before the collision directly. If two smooth balls collide, then the sum of the their two momentums does not change due to the collision.

Algebraic measures can be used as follows :

$$m_1 v_1 - m_1 v_1 = -I, \quad m_2 v_2 - m_2 v_2 = I, \quad m_1 v_1 + m_2 v_2 = m_1 v_1 + m_2 v_2$$

Since I is the algebraic measure to the impulse of the second ball on the first ball v_1, v_2 are the algebraic measure of the velocities before collision v_1, v_2 are the velocities after collision.

The direct collision : The two velocities before and after the collision directly are parallel to the line of the two centers at the moment of collision.

5 Inelastic collision: The inelastic collision is meant that a deformation takes place, heat is generated or the bodies get contacted due to the collision process (loss of kinetic energy occurs). In spite of it all, the momentum after and before collision remains as it is without change.

and the momentum conservation equation is form: (in case the two bodies contacted):

$$m_1 \vec{v}_1 + m_2 \vec{v}_2 = (m_1 + m_2) \vec{v} \quad \text{(by using vectors) (in case of soldering)}$$

$$m_1 v + m_2 v_2 = (m_1 + m_2) v \quad \text{(by using algebraic measures)}$$

GENERAL EXERCISES

First :

- 1 Define each of the impulse and momentum and mention the relation between them.
- 2 Define each of elastic collision and inelastic collision, then give an example for each.
- 3 Show how the concept of momentum can be used to cut down the traffic accident.

Second: Choose the correct answer

- 4 If the mass is measured in kg and velocity in m/sec then the measuring unit of impulse is :
a kg.sec **b** newton.sec **c** dyne.sec **d** newton. m/sec.
- 5 The impulse is:
a The change of the force acting on the body.
b The interval in which the force acts on the body.
c The change of the velocity of the body.
d The change of the momentum of the body.
- 6 The momentum is known as the product of:
a mass of body and its velocity **b** mass of body and its acceleration
c mass of body and the force acting time **d** mass of body and the distance it traveled.
- 7 If a force acts upon a body of mass 300gm, to change its velocity from 20cm/sec to 45cm/sec in the same direction, then the impulse magnitude of this force on the body with unit in gm . cm/sec is equal to:
a 7.5×10^2 **b** 7.5×10^3 **c** 2.7×10^5 **d** 2.94×10^6
- 8 A ball of mass 300 gm moving on horizontal ground with velocity 60 cm/sec directly collides with a vertical wall. What is the velocity of the ball's rebound from the wall in cm/sec unit if the wall acts on the ball with impulse of magnitude 48000 dyne.sec?
a 100 **b** 120 **c** 220 **d** 500
- 9 If the two forces $\vec{F}_1 = 2\hat{i} - 14\hat{j}$, $\vec{F}_2 = 3\hat{i} + 2\hat{j}$ act up on a body for a time interval of magnitude $\frac{1}{2}$ of second, then the magnitude of the impulse of the force in N.sec unit. is equal:
a $6\frac{1}{2}$ **b** $7\frac{1}{2}$ **c** 9 **d** 13

Third: Answer the following questions:

- 10 A rubber ball of mass 500 gm moves horizontally in a straight line to collide with a vertical wall and rebound with velocity 150 cm/sec on the same straight line. If the average of the force between the ball and wall is 10 kg.wt and the contact time between them is $\frac{1}{50}$ of a second, find the velocity of the ball directly before the moment of collision with the wall.
- 11 A rubber ball of mass 200 gm falls down from a height of 3.6 m above the ground and it rebounds back after collision at a height of 2.5 m. Find the ground reaction on the ball in kg.wt given that the collision time with the ground is $\frac{1}{7}$ of a second.

GENERAL EXERCISES

- 12 A rubber ball of mass 1 kg falls down a height of 4.9 m above solid horizontal ground to rebound back to a maximum height of 2.5 m. Calculate the magnitude of the change of its momentum due to its collision with the ground, then find the reaction magnitude of the ground on the ball in newton if the contact time with the ground is 0.1 of a second.
- 13 A ball of mass 400 gm moves with velocity 70 cm/sec. It collides with another ball at rest of mass 800gm and the ball starts to move directly after the collision with velocity 35 cm/sec in the same direction of the first ball. Prove that the first ball rests after the collision, then find the collision force on any of the two balls in wgm if the contact time is $\frac{1}{7}$ of a second
- 14 Two balls of masses 100 gm, 50 gm move in a horizontal straight line in two opposite directions. The two balls collide when the velocity of the first ball was of magnitude 50 cm/sec and the velocity of the second ball was of magnitude 30 cm./sec and if the second ball rebounded directly after collision with velocity of magnitude 40 cm/sec. Find the magnitude and direction of the velocity of the first ball directly after the collision, then find the impulse magnitude of any of the two balls to the other.
- 15 A bullet of mass 20 gm is shot with horizontal velocity of magnitude 50.5 m/sec towards a piece of mass 2 kg placed on a horizontal table and embedded in it to form one body. Find the velocity of this body directly after collision. If this body rebound with velocity 2cm/sec after collision with a fixed barrier on the table and perpendicular to the direction of the motion, find the barrier impulse on the body given that the total resistance is equal to 1.01 newton and the barrier is a distance of 24 cm from the piece of wood before shooting the bullet.
- 16 An iron hammer of mass 210 kg falls down from a height of 90 cm on a foundation pole of mass 140 kg to push it in the ground for a distance of 18 cm. If the hammer and pole move as one body directly after collision, find the common velocity of both, then find in kg.wt the average of the ground resistance supposing it is constant.
- 17 A body A of mass 10 gm moves vertically downwards to collide with another body B of mass 4 gm moving vertically upwards when the velocity of A was 200 cm/sec and the velocity of B is 800 cm/sec. The body B rebounds vertically downwards with velocity 100 cm/sec while the body A rebounds vertically upwards and after $\frac{1}{7}$ of a second the body A collided with another body C of mass 100 gm moving vertically downwards with velocity 13 cm/sec to form one body. Find the common velocity between the two bodies A, C after collision.
- 18 \overline{AB} is the line of greatest slope for a smooth inclined plane of length 9.8 m inclines at 30° , to the horizontal, A is the highest point on the plane and B is the lowest point. A smooth ball of mass 700 gm is placed at A to move from rest on \overline{AB} and collide with a smooth perpendicular vertical barrier at B to act on it with impulse of magnitude 11.76 newton. second and the ball rebounded. Calculate the maximum distance the ball moves up along \overline{BA} .

For more activity and exercises, visit: www.sec3mathematics.com.eg

ACCUMULATIVE TEST

- 1 If a body is projected vertically upwards with velocity 49m/sec, find the time taken for the body to reach the maximum height and the distance it has already reached.
- 2 A car moves on a straight lined road with velocity 75 km/h. If a motorcycle moves on the same road with velocity 35 km/h, find the relative velocity of the motorcycle with respect to the car in each of the cases.
First: the motorcycle moves in the same direction of the car
Second: The motorcycle moves in the opposite direction of the car.
- 3 A cyclist traveled 30 km on a straight lined road with velocity 18 km/h , then returns on the same road to travel 20 km in the opposite directive with velocity 15 km/h . Find the average of his velocity vector through the entire trip.
- 4 A particle moves from rest with a uniform acceleration in a constant direction and by the end of 20 seconds its velocity reaches 36 km/h. Find the magnitude of its acceleration in. m /sec².
- 5 If the position vector of a particle is given as a function of time by the relation: $\vec{R} = (t^3 + 2)\vec{C}$ find the vectors of displacement, velocity and acceleration, then prove that the motion is accelerated at any moment $t > 0$. When is the magnitude of the acceleration equal to 12 units?
- 6 A particle is projected vertically up wards with velocity \vec{v} . Write down the law which gives its velocity in terms of the time and deduce the rate of change of momentum with respect to time being a constant vector and find its magnitude.
- 7 A particle of unit mass moves under the action of two force: $\vec{F}_1 = a\hat{i} + \hat{j}$, $\vec{F}_2 = 2\hat{i} + b\hat{j}$ where \hat{i} , \hat{j} are the fundamental unit vectors A, B are two constants. If given that the displacement vector of the body as a function of time is: $\vec{s} = \hat{i} + (t^2 + t)\hat{j}$, find the constants A, B.
- 8 A lift contains a pressure scale on its floor where a person stands on this scale to record 75 kg when the lift was moving up with a uniform acceleration of magnitude a and the scale recorded 60 wkg when the lift was moving down with a uniform acceleration of magnitude 2a. Find the magnitude for each of the acceleration a and the mass of the person.
- 9 Two bodies of masses 125 gm 120 gm respectively are connected by the ends of a string passing over a smooth small pulley. Identify the acceleration of the system and the pressure on the axis of the pulley. What is the vertical distance between. the two bodies after one second from the beginning of motion if the system moves from rest and the two bodies are in one horizontal plane?
- 10 A body is placed at the top of a rough inclined plane of length 4.5 m and height 2.7m and the body starts to move from rest. Calculate the velocity of the body as it reaches the base of the plane and the time required to do this where the coefficient of friction = $\frac{1}{2}$.
- 11 A car A of mass 4 tons moves with a uniform velocity of magnitude 5 m/sec in a straight line on a smooth horizontal plane to collide with another car B at rest of mass 3 tons. If the velocity of car B with respect to car A is 2m/sec directly after collision, find the magnitude of the actual velocity for each of the two cars after collision.

Work, Power & Energy

Unit 4



Introduction

In our study to the previous units, you found that when the resultant of a system of forces acts on a body, it moves in different forms. If you ask about the benefits of moving a body, the answer would be in two parts; first, humans are naturely curious and always eager to interpret the natural phenomena, their reasons and results.

Second, the humans need to benefit from the blessing which God gifts the people. People for example need cars to travel from a place to another, they need the light bulbs to lighten cities and villages and so on. Of course, these matter can only be achieved when people know how to control the objects and benefit from their motion whether these objects are electric or electronic devices, means of transportation or even celestial bodies cause the Earth's revolution and rotation and the succession of day and night. In this unit, you are going to learn the motion of the bodies and identify the work to know how to get the best benefit of moving the bodies. You also identify the kinetic energy and potential energy to relate these scalar quantities (non - vectored) of different measuring units and the relation among them. Finally you identify the force which conserve the energy and the forces which do not conserve it to explore the principle of work and energy, then you identify the simple machines which humans had used and compare them by calculating the power resulted in each one and their different benefits in your daily life.

Unit objectives

By the end of the unit and carrying out the involved activities, the student should be able to:

- ⊕ Identify the work done by a force and the work measuring units.
- ⊕ Identify the concept of the power and its measuring units.
- ⊕ Identify the kinetic energy of a particle and its measuring units.
- ⊕ Identify the principle of work and energy.
- ⊕ Identify the potential energy, its measuring unit, and applications.

Key terms

- Work
- Constant force
- Scalar quantity
- Displacement vector
- Position vector
- Joule
- Variable force
- Erg
- Kinetic energy
- Potential energy
- Change in potential energy
- The work – energy principle
- Power
- Horse power
- Conservation of energy

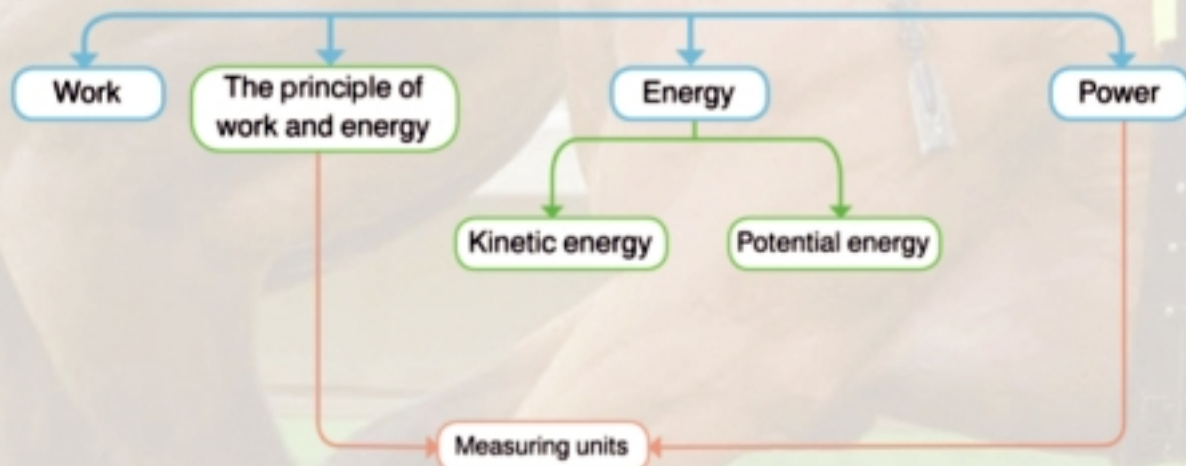
Lessons of the unit

- (4 -1): Work
- (4 -2): Kinetic energy
- (4 -3): Potential energy
- (4 -4): Power

Materials

Scientific Calculator .

Unit planning guide



You will learn

- ◊ The work done by a constant force.
- ◊ Some different cases of the two vectors of force and displacement.
- ◊ Work measuring units.
- ◊ Work done by a variable force.

Key terms

- ◊ Work
- ◊ Constant force
- ◊ Scalar quantity
- ◊ Displacement vector
- ◊ Position vector
- ◊ Joule
- ◊ Erg

Materials

- ◊ Scientific calculator

Introduction:

The concept of work is one of the important topics in Kinetic since it relies on the concept of the force stated by Newton in his three laws. It is worth to mention that the work and energy are scalar quantities and then it is easier to deal with them than using Newton's laws of motion, especially when the force vector is variable and in turn, the acceleration vector is also variable. In this lesson, we are going to clarify the concept of work which is considered the link between the force and energy. work may be resulted from a constant force or a variable force. in this lesson, you are going to learn both types.

First: The work done by a constant force

Let a body move in a straight line under the action of a constant force \vec{F} and it moves from position A to position B and its displacement vector is $\vec{AB} = \vec{S}$ as shown in figure (1)

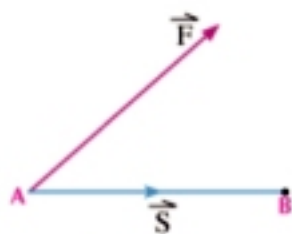


Figure (1)

Definition

The work done by the constant force \vec{F} to move a particle from an initial position to a final position and denoted by the symbol (W) is known as the scalar product of the force vector by the displacement vector between the two position

$$W = \vec{F} \cdot \vec{S}$$

It is clear that the work is a scalar quantity that may be positive, negative or zero according to the magnitude and direction of both vectors \vec{F} , \vec{S}

Example

- ① A particle moves in a straight line under the action of the force $\vec{F} = 6\hat{i} + 8\hat{j}$ from point A (3, -4) to point B (7, 2), calculate the work done by this force.

Solution

$$\begin{aligned} \text{The displacement vector } \vec{S} &= \vec{AB} = \vec{B} - \vec{A} \\ &= (7\hat{i} + 2\hat{j}) - (3\hat{i} - 4) \\ \vec{S} &= 4\hat{i} + 6\hat{j} \end{aligned}$$

Apply the definition of work and notice that the force given is constant

$$W = \vec{F} \cdot \vec{S}$$

$$W = (6 \hat{i} + 8 \hat{j}) \cdot (4 \hat{i} + 6 \hat{j}) = 6 \times 4 + 8 \times 6 = 72 \text{ work measuring unit}$$

5 Try to solve

- 1 A body moves in a straight line under the action of the force $\vec{F} = 5 \hat{i} + 2 \hat{j}$ from the point A (5, 2) to the point B (3, 1), calculate the work done by this force

Some different cases of the vectors of the force and displacement

Since the equation of defining the work $W = \vec{F} \cdot \vec{S}$ can be rewritten in another form which is $W = \|\vec{F}\| \|\vec{S}\| \cos \theta$ where θ is the measure of the least angle between the force vector \vec{F} and the displacement vector \vec{S} by considering them drawn from the same point.

- a If the force is constant and its direction is parallel to the direction of displacement i.e $\theta = \text{zero}$, on that moment, the work $W = \|\vec{F}\| \|\vec{S}\| \cos \text{zero} = \|\vec{F}\| \|\vec{S}\|$

and written: $W = F \times S$

Figure (2) illustrates that



Figure (2)

- b If the force is constant and its direction inclines on the direction of the displacement with an angle of measure less than 90° , then the work

$$W = \|\vec{F}\| \|\vec{S}\| \cos \theta$$

The work in this case is positive and equal to the horizontal component of the force F multiplied by the distance S .

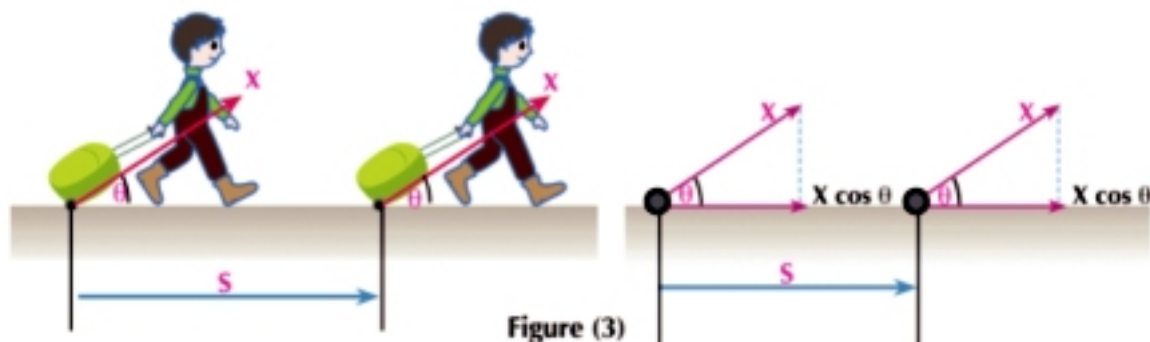
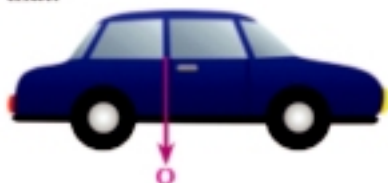


Figure (3)

- a If the force is constant and its direction is perpendicular to the direction of displacement i.e. $\angle \theta = 90^\circ$, then the work $W = \|\vec{F}\| \|\vec{S}\| \cos 90 = \text{zero}$. Figure (4) illustrates that.



The car moves horizontally, its weight does not do any work in the motion pathway

- d If the force is constant and its direction inclines at 90° to the direction of displacement, then the work $W = \|\vec{F}\| \|\vec{S}\| \cos \theta$ and the work is negative and called work resistant. For example, the work done by the resistance force and friction force.

Example

- 2 A particle moves in a straight line under the action of the force $\vec{F} = 5\hat{i} - 3\hat{j}$ from point A (1, 0) to point B (3, 3) where coordinates are referred to a system of rectangular cartesian coordinates \vec{OX} , \vec{OY} . identify the work

Solution

Figure (5) illustrates the position of the two point A, B referred to the axes.

To calculate the displacement vector \vec{S} :

$$\begin{aligned} \vec{S} &= \vec{OB} - \vec{OA} && \text{(Rule of subtracting vectors)} \\ \therefore \vec{S} &= (3 - 1)\hat{i} + (3 - 0)\hat{j} \\ &= 2\hat{i} + 3\hat{j} \\ \therefore W &= \vec{F} \cdot \vec{S} && \text{(Work definition)} \\ &= (5\hat{i} - 3\hat{j}) \cdot (2\hat{i} + 3\hat{j}) \\ &= 5 \times 2 + (-3) \times (3) = 1 \text{ Work measuring unit.} \end{aligned}$$

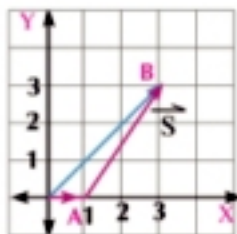


Figure (5)

Try to solve

- 2 A particle moves under the action of two forces $\vec{F}_1 = 2\hat{i} - 3\hat{j}$, $\vec{F}_2 = 5\hat{i} + \hat{j}$ from point A (2, 1) to point B (3, 0) where \hat{i} , \hat{j} are the main two unit vectors. Calculate the work done.

Critical thinking:

Prove that if two subsequent displacements occur to a body under the action of a force, then the work done within the resultant displacement is equal to the sum of the two works done within the two displacements.

Example

- 3 The force $\vec{F} = 3\hat{i} + 5\hat{j}$ acts on a body and moves from point A (2, 4) on a straight line to point B (5, 3), then to point C (8, -2). Calculate the work done by this force during each of the two displacements, then check if the sum of the two works is equal to the work done during the resultant displacement.

Solution

First: The first displacement vector $\vec{AB} = \vec{B} - \vec{A} = (5, 3) - (2, 4) = (3, -1)$

The work done during the first displacement

$$W_1 = \vec{F} \cdot \vec{S}_1 = (3\hat{i} + 5\hat{j}) \cdot (3\hat{i} - \hat{j})$$

$$W_1 = 9 - 5 = 4 \text{ a work measuring unit}$$

The second displacement vector $\vec{BC} = \vec{C} - \vec{B}$

$$= (8, -2) - (5, 3) = (3, -5)$$

The work done during the second displacement

$$W_2 = \vec{F} \cdot \vec{S}_2 = (3\hat{i} + 5\hat{j}) \cdot (3\hat{i} - 5\hat{j})$$

$$W_2 = 9 - 25 = -16 \text{ a work measuring unit}$$

The resultant work = sum of two works

$$W = W_1 + W_2 = 4 - 16 = -12 \text{ a work measuring unit}$$

Second: The resultant displacement $\vec{AC} = \vec{C} - \vec{A} = (8, -2) - (2, 4) = (6, -6)$

\therefore The work during the resultant displacement

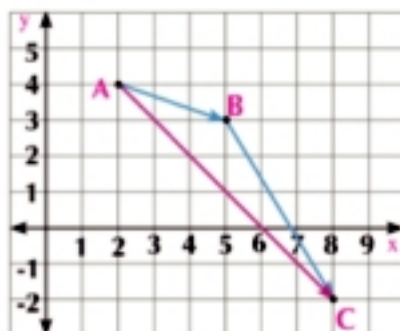
$$W = \vec{F} \cdot \vec{S} = (3\hat{i} + 5\hat{j}) \cdot (6\hat{i} - 6\hat{j})$$

$$W = 18 - 30 = -12 \text{ a work measuring unit}$$

Try to solve

- 3 The force $\vec{F} = 5\hat{i} - 7\hat{j}$ acts on a body and moves it from point A (5, -1) on a straight line to point B (-1, 3), then to point C (4, 6). Calculate the work done by this force during each of the two displacement, then check if the sum of the two works is equal to the work done during the resultant displacement.

Oral expression: What is the magnitude of the work done in the pathway if a particle moves on a straight line from a position, then it turns back to this position under the action of the same force?



Example

- 4 The force $\vec{F} = 2\hat{i} + 3\hat{j}$ acts on a particle and the position vector at a moment t is identified by the relation: $\vec{r}(t) = (t + 5)\hat{i} + (t^2 + 4)\hat{j}$ where \hat{i}, \hat{j} are the main unit vectors, Calculate the work done by this force from $t = 1$ to $t = 5$

Solution

The displacement occurring from $t = 1$ to $t = 5$ is

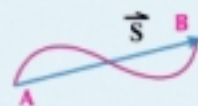
$$\vec{S} = \vec{r}_5 - \vec{r}_1$$

$$\therefore \vec{S} = (10\hat{i} + 29\hat{j}) - (6\hat{i} + 5\hat{j}) = 4\hat{i} + 24\hat{j}$$

$$\therefore W = \vec{F} \cdot \vec{S} \quad (\text{From work definition})$$

$$\therefore W = (2, 3) \cdot (4, 24) = 8 + 72 = 80 \text{ work unit.}$$

Note that



The work is not based on the pathway which the body take from position A to position B but it is based on the displacement \vec{AB}

5 Try to solve

- 4 If the position vector of a particle is given as a function of time by the relation: $\vec{r}(t) = (t + 4)\hat{i} + (t^2 + 3)\hat{j}$ where \hat{i}, \hat{j} are the basic unit vectors. A force $\vec{F} = 3\hat{i} + 2\hat{j}$ acts on the particle, calculate the work done by the force \vec{F} from $t = 1$ to $t = 3$

Work measuring units:

From the work definition, we deduce that:

The work measuring unit = the magnitude of the force measuring unit \times displacement measuring unit

From the work measuring units are:

- ✦ **Joule:** It is known as the magnitude of work done by a force of magnitude 1 newton to move a body a distance one meter .

If we take $\|\vec{F}\| = 1 \text{ newton}$, $\|\vec{S}\| = 1 \text{ meter}$ then:

$$\text{Joule} = 1 \text{ newton} \times 1 \text{ meter i.e. Joule} = \text{newton} \cdot \text{meter (N.m)}$$

The joule is the international unit to measure the work

- ✦ **Erg:** It is known as the magnitude of the work done by a force of magnitude one dyne to move a body a distance one centimeter.

If we take $\|\vec{F}\| = 1 \text{ dyne}$, $\|\vec{S}\| = 1 \text{ cm}$ then,

$$\text{Erg} = 1 \text{ dyne} \times 1 \text{ cm i.e. Erg} = \text{dyne} \cdot \text{cm}$$

- ✦ **Kg.wt.m:** it is the magnitude of the work done by a force of a magnitude 1 kg.wt to move a body a distance one meter.

If we take $\|\vec{F}\| = 1 \text{ kg.wt}$, $\|\vec{S}\| = 1 \text{ m}$, then $\text{kg.wt.m} = 1 \text{ kg.wt} \times 1 \text{ m}$

You can convert the work units as follows:

$$\begin{aligned} 1 \text{ kg.wt. m} &= 1 \text{ kg.wt} \times 1 \text{ m} \\ &= 9.8 \text{ N} \cdot \text{m} \end{aligned}$$

$$\text{kg.wt.m} = 9.8 \text{ Joule}$$

$$\begin{aligned} 1 \text{ Joule} &= 1 \text{ newton} \times 1 \text{ m} \\ &= 10^5 \text{ dyne} \times 100 \text{ cm} \\ &= 10^7 \text{ dyne} \times \text{cm} \end{aligned}$$

$$\text{Joule} = 10^7 \text{ Erg}$$

Example

- 5 A body moves on a straight line. A resistance force of magnitude 100 newtons acts on it. Calculate the work done by this force during a displacement of magnitude 300 m.

Solution

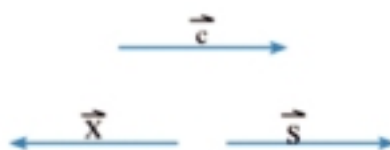
Since the force is a force of resistance, then it acts in an opposite direction to the displacement vector, and if \hat{c} is a unit vector in the direction of the displacement, we can express each of the displacement and force vectors in terms of their algebraic measures.

$$\vec{S} = S \hat{c}, \quad \vec{F} = -F \hat{c}$$

In our case:

$$S = +300 \text{ m}, \quad F = 100 \text{ newton}$$

$$\begin{aligned} \text{From figure (6) } W &= -F S \\ &= (-100) \times (300) \\ &= -3 \times 10^4 \text{ N} \cdot \text{m} \\ &= -3 \times 10^4 \text{ Joule} \end{aligned}$$



(figure 6)

Example Work of weight, perpendicular reaction and friction

- 6 A body of mass 10kg slides a distance 6 m on a rough plane and the coefficient of the kinetic friction between them is 0.2 and inclined at 30° to the horizontal, Find in kg.wt.m unit the work done by:

First: The force of the weight of the body

Second: The normal reaction of the plane

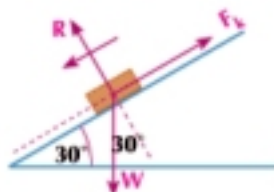
Third: Friction force

Solution

First: the work done by the force of weight

$$\text{Weight of the body } (W) = 10 \text{ kg.wt}$$

\therefore The angle included between \vec{W} , \vec{S} is equal to 60°



From the work definition:

$$\text{Work} = W_1 \times S \cos 60^\circ$$

$$\therefore \text{Work} = 10 \times 6 \times \frac{1}{2} = 30 \text{ kg.wt. m}$$

Another Solution:

The weight component acting in the displacement direction, so the work $W = W_1 \sin \theta \times S$

$$\therefore W = 10 \times \frac{1}{2} \times 6 = 30 \text{ kg.wt. m}$$

Second:

\therefore The normal reaction force of the plane (R) is always perpendicular to the plane on which the body moves, then the angle between R, S is equal to 90° .

\therefore The worked done from R = zero.

Third: The work done by the friction force:

We know that the force of the kinetic friction $\mu_k R$ (where μ_k is the coefficient of the kinetic friction)

$$\therefore F_k = 0.2 \times 10 \times 9.8 \times \cos 30^\circ = 49\sqrt{3} \text{ newton}$$

\therefore The work done by friction force $W = -\mu_k R \times S$

$$\therefore W = -49\sqrt{3} \times 6 = -294\sqrt{3} \text{ Joule} = -30\sqrt{3} \text{ kg.wt.m}$$

5 Try to solve

- 5 A car of mass 6 tons ascends a slope making an angle of $\sin \frac{1}{98}$ to the horizontal against resistances equal to 10 kg.wt per each ton of the mass of the car. If the car acquires velocity 54 km/h within 30 seconds and starts to move from rest, calculate the magnitude of the work done in joule by:

First: The engine force

Second: Resistance force

Third: Weight of car

Work done by a variable force

You have previously used the concept of work as you deal with the motion when the force is uniform. This can be clear through the following example:

Explanatory Example:

Let a constant force of magnitude 10 newtons act on a body to move from A to B as illustrated in figure (8)

Then, the displacement from A to B = 20 m. To represent that graphically, draw the force axis and displacement axis as illustrated in the figure, then the force represented on a horizontal straight line parallel to the displacement axis S.

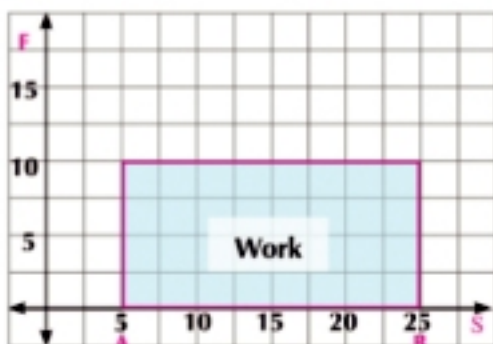


Figure (8)

$$\text{Work} = F S = 10 (25-5) = 200 \text{ Joules}$$

It is the area under the curve and represented by the area of the rectangle whose width is 10 newton and length is 20 meters.

In figure (9), then the area under the curve is determined by the relation:

$$W = \int_A^B F \, ds$$

In this case, we take a small displacement of magnitude Δs until the force acting on this displacement is uniform and the work done is given by the relation:

$$\Delta W = F_s \Delta s$$

If we divide the force curve into small parts and calculate the work done during each part, and find their sum, it can be expressed by the relation:

$$W = \sum_A^B F_s \Delta s$$

When the displacement Δx is as smallest as possible (it reduces to zero) to get accurate values in the previous equation, then the previous equation is converted into:

$$W = \int_A^B F_s \, ds$$

This is the general form of work (notice that: $F_s = F \cos\theta$ (represents the component of the force in the direction of the displacement))

$$W = \int_A^B F_s \, ds$$

Example

- 7 Figure (10) illustrates the action of a variable force on a body, calculate the work done in Erg by this force in the following cases: where the force magnitude with dayn, S by cm

First: When the body moves from

$$S = 0 \quad \text{to} \quad S = 8$$

Second: When the body moves from

$$S = 8 \quad \text{to} \quad S = 12$$

Third: When the body moves from

$$S = 0 \quad \text{to} \quad S = 12$$

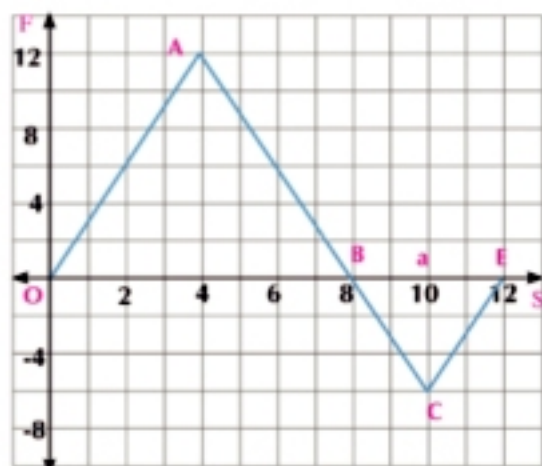


Figure (10)

Solution

$$\begin{aligned} \text{First: } W_1 &= \int_0^8 F \, ds = \text{the area under the curve from } S = 0 \text{ to } S = 8 \\ &= \text{area of } \triangle OAB = \frac{1}{2} \times 8 \times 12 = 48 \text{ Erg} \end{aligned}$$

$$\begin{aligned} \text{Second: } W_2 &= \int_8^{12} F \, ds = \text{-the area under the curve from } S = 8 \text{ to } S = 12 \\ &= \text{-area of } \triangle BCD = \frac{1}{2} \times 4 \times 6 = -12 \text{ Erg} \end{aligned}$$

$$\begin{aligned} \text{Third: } W_3 &= \int_0^{12} F \, ds = \text{the area under the curve from } 0 \int_0^8 F \, ds + \int_8^{12} F \, ds \\ &= \text{area of } \triangle OAB - \text{area of } \triangle BCD \\ &= \frac{1}{2} \times 8 \times 12 - \frac{1}{2} \times 4 \times 6 = 36 \text{ Erg} \end{aligned}$$

Try to solve

- 6 The opposite figure illustrates the action of a variable force on a body, calculate the total work done by this force in the following cases:

First: from $S = 0$ to $S = 10$

Second: from $S = 8$ to $S = 14$

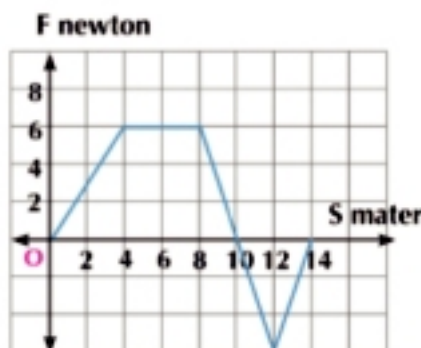


Figure (11)

Example

- 8 A variable force F (measured in newton) acts up on a body where $F = 3s^2 - 4$, S measured in meter S Algebraic measured of displacement find the work done by this force in the interval from $S = 2$ m to $S = 5$ m.

Solution

$$\begin{aligned} \therefore f &= 3s^2 - 4, \quad W = \int_a^b F \, ds \\ \therefore W &= \int_2^5 (3s^2 - 4) \, ds = [s^3 - 4s]_2^5 \\ \therefore W &= [(125 - 20) - (8 - 8)] = 105 \text{ Joules} \end{aligned}$$

Try to solve

- 7 A variable force F (measured in dyne) acts upon a body where F is given by the relation: $F = 4s^3 - 2s + 1$, S The algebraic measured of displacement at measured in with cm find the work done by this force in the interval from $S = 0$ to $S = 4$

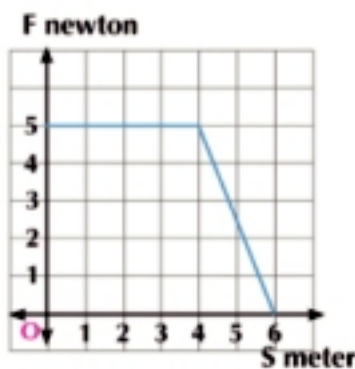


Exercises 4 - 1



First: Choose the correct answer:

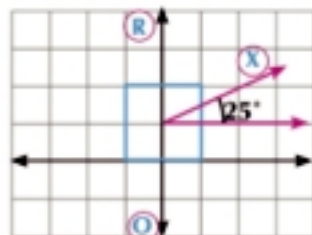
- 1 If a body moves in a straight line from the origin point to the point A (3, 2) under the action of the force $\vec{F} = 3 \hat{i} - 5 \hat{j}$, then the work done by this force = _____ work unit .
 a -4 b -1 c zero d 1
- 2 If a body moves in a straight line from point A (-3, 2) to point B (5, -3) under the action of the force $\vec{F} = 5 \hat{i} + 8 \hat{j}$, then the work done by this force = _____ work unit
 a zero b -40 c 40 d 80
- 3 The opposite figure illustrates the action of the force (F) on a body moving a distance (S), then the work done by this force to make the body move from $S = 0$ to $S = 6$ m is equal to _____ Joule
 a Zero b 40
 c 80 d 25
- 4 The work done to lift a mass of magnitude 200 gm placed on the ground surface a distance of 10 meters above the ground is equal to _____ Joules.
 a zero b 9.8 c 19.6 d 29.4
- 5 If a body moves in a straight line and a resistance force of magnitude 400 newton acts on it, then the work done by this force during the displacement \vec{S} where $\|\vec{S}\| = 350$ m is equal to Joule.
 a -14×10^4 b -7×10^4 c 7×10^4 d 14×10^4



Second: Complete:

- 6 A person goes shopping and pushes a shopping cart with a force of magnitude 35 newtons inclined at 25° to the horizontal to move the cart a distance 50 meters, then the work done by the person = _____ Erg
- 7 The work done to move a mass of magnitude 600 gm a distance 4m with acceleration of magnitude 20 cm / w^2 is equal to _____ Joule

- 8 The figure opposite illustrates a force of magnitude 16 newtons inclined at 25° to the horizontal acts on a body of magnitude 2.5 kg to move on a smooth horizontal table a distance 220 cm then:



- The work done by the force = _____ Joule
- The work done by the reaction of the table = _____
- The work done by the weight of the body = _____
- The total work done by the forces acting on the body = _____ Joule

Third: Answer the following questions:

- A particle moves in a straight line under the action of the force $\vec{F} = 6\hat{i} - 3\hat{j}$ from point A (-1, 2) to point B (3, 4) where \hat{i} , \hat{j} are the main unit vectors, calculate the work done by this force.
- The forces $\vec{F}_1 = 4\hat{i} + 3\hat{j}$, $\vec{F}_2 = 2\hat{i} - 4\hat{j}$, $\vec{F}_3 = 3\hat{i} - \hat{j}$ act up on a body to move from point A (2, 3) to point B (4, 4), calculate the work done by the resultant of these forces during the displacement \vec{AB}
- A body of mass 1kg and its displacement vector $\vec{S} = (3t^2)\hat{i} + (3t^2 + t)\hat{j}$ moves. what is the moving force? Calculate the work done by the moving force during five second from the beginning of motion given that S is measured in meter, F in newton and t in second.
- The position vector of a particle of mass 3 kg is given as a function of time by the relation $\vec{r} = (3t^2 + 2)\hat{i} + (4t^2 + 3)\hat{j}$ where \hat{i} , \hat{j} are two unit vectors perpendicular to the plane. Prove that the particle moves under the action of a constant force, then calculate the work done by this force from $t = 1$ to $t = 5$
- A tram car is pulled by a rope inclined at 60° to the railroad. If the tension force is 500 kg.wt and the car moved with acceleration 5 cm/sec^2 for 30 seconds, calculate the work done by the tension force.
- A construction worker of mass 70 kg carries an amount of bricks and ascends a ladder whose top is 12 meters high from the ground. If work done with magnitude 11760 Joules until the worker reaches the ladder top, find the mass of bricks.
- A force acts on a body at rest of mass 50 kg to acquire it a uniform acceleration 0.7 m/sec^2 to move distance S in direction. If the work done by this force is equal to 350 kg.wt. m, find the distance which the body moves.
- A stone of mass 4 kg is projected vertically upwards from the ground surface. If the work done to reach the maximum height is 1176 Joule, find the maximum height the stone reached.

- 17 Calculate in Joule the magnitude of the work needed to be done to lift 50 cubic meters to a height of 10 meters.
- 18 A woman pushes a baby stroller with a baby from rest on a horizontal road with a force of magnitude 2 kg.wt inclined at 60° to the horizontal downwards against resistances of magnitude 0.95 kg.wt. If the mass of the stroller and the baby is 18 kg, find in kg.wt.m the magnitude of the work done during one minute from :
- The weight of the stroller and the baby
 - The force of the woman
 - The resistance of the road.
- 19 A train of mass 200 tons ascends a slope inclined at an angle of $\sin^{-1} \frac{1}{100}$ to the horizontal with a uniform velocity. If the work done by the train engine is equal to 15×10^5 kg.wt.m unit the train reaches the top of the slope and the work done against resistances is 5×10^5 kg.wt.m find:
- First:** The slope length
Second: The resistance per each ton of the train's mass
- 20 A car of mass 4 tons ascends a slope inclined at an angle of $\sin^{-1} \frac{1}{100}$ to the horizontal against a resistance of magnitude 5 kg.wt per each ton of the car's mass to acquire velocity 54km/h during $\frac{1}{2}$ minute. If the car starts moving from rest, calculate in Joule the work done by:
- First:** the force of the car's engine **Second:** the resistance force
Third: the car's weight **Fourth:** against the car's weight
- 21 A particle moves in a straight line under the action of the force F(newton) where $F = 0.4 S$ and S is measured in meter, calculate the work done by the force F when the particle moves from :
- $S = 0$ to $S = 10$
 - $S = 1$ to $S = 5$
- 22 A particle moves in a straight line under the action of the force F(newton) where $F = \sin 2 S$ where S is measured in meter, Calculate the work done by the force F when the particle moves from:
- $S = 0$ to $S = \frac{\pi}{2}$
 - $S = \frac{-\pi}{4}$ to $S = \frac{\pi}{4}$
 - $S = \frac{\pi}{4}$ to $S = \frac{3\pi}{4}$



Preface

You will learn

- Kinetic energy
- Measuring units of kinetic energy
- Energy-work principle

Key terms

- Kinetic energy

Materials

- Scientific calculator

In the last lesson, you learned that the force is the main reason for motion. In this lesson, you will learn the source from which the force is derived to move the bodies.

This source is simply the energy. In turn, the energy can be defined as the ability of a system to perform (do) work.

Energy appears in our practical life in different forms such as mechanical energy, electrical energy, light energy and so on.

In this lesson, you namely learn the mechanical energy represented in moving the bodies. It is divided into two types; **the kinetic energy** and **potential energy**.

Kinetic energy

The kinetic energy of the body is the energy which the body acquires due to its velocity and it is estimated at a moment as the product of half the mass of the particle times the square of the magnitude of its velocity and it is denoted by the symbol T .

If m is the mass of a particle, \vec{v} is its velocity vector and V is the algebraic measure of this vector, then:

$$T = \frac{1}{2} m \|\vec{v}\|^2 = \frac{1}{2} m v^2 \quad (1)$$

Since $\|\vec{v}\|^2 = \vec{v} \odot \vec{v}$, then the kinetic energy can be expressed as follows:

$$T = \frac{1}{2} m (\vec{v} \cdot \vec{v}) \quad (2)$$

From the definition above, it is quite clear that the kinetic energy of a particle is a non-negative scalar quantity and vanishes when the velocity vanishes. According to the definition, the kinetic energy of a particle may change from an instant to another during the particle's motion in regard to the magnitude of its velocity.

Measuring units of kinetic energy:

Since the work is a form of the energy forms:

Measuring unit of kinetic energy = Measuring unit of work

For example, if the mass is measured in kg and velocity in meter/sec, then:

$$\text{The measuring unit of kinetic energy} = \text{kg} \times \frac{\text{meter}}{\text{sec}} \times \frac{\text{meter}}{\text{sec}}$$

$$= \text{kg} \frac{\text{m}^2}{\text{sec}^2} \times \text{m}^2 = \text{newton} \cdot \text{m}^2 = \text{joule}$$

If the mass is measured in gm and velocity in cm/sec then:

$$\text{The measuring unit of kinetic energy} = \text{gm} \times \frac{\text{cm}}{\text{sec}} \times \frac{\text{cm}}{\text{sec}} = \text{gm} \times \frac{\text{cm}^2}{\text{sec}^2} \times \text{cm} = \text{deyan} \times \text{cm} = \text{Erg}$$

Example

- ① A body of mass 100 gm moves with velocity $\vec{v} = 5 \hat{i} + 12 \hat{j}$ where \hat{i} , \hat{j} are two perpendicular unit vectors and the magnitude of the velocity is measured in cm/sec, unit, calculate the kinetic energy of this body. **First:** in erg **Second:** Joule

Solution

$$\text{Find the } \vec{v} = 5 \hat{i} + 12 \hat{j}$$

$$\|\vec{v}\| = \sqrt{5^2 + 12^2} = 13 \text{ cm/sec} \quad \therefore \|\vec{v}\|^2 = 169$$

$$\text{First: The kinetic energy of the body} = \frac{1}{2} m \|\vec{v}\|^2 = \frac{1}{2} \times 100 \times 169 = 8450 \text{ erg}$$

$$\text{Second: The kinetic energy} = \frac{8450}{10^7} = 8.45 \times 10^{-4} \text{ Joule}$$

Try to solve

- ① A body of mass 200 gm moves with velocity $\vec{v} = 60 \hat{i} - 80 \hat{j}$ where \hat{i} , \hat{j} are two perpendicular unit vectors and the magnitude of the velocity is measured in cm/sec unit, calculate the kinetic energy of this body.

First: In erg

Second: In Joule.

Example

- ② A body of mass 1kg is projected vertically upwards with velocity 49m/sec, find
- The kinetic energy of the body after 6 seconds of throwing
 - The kinetic energy of the body when it is at a height of 102.9 meters from the point of projection

Solution

$$\text{a } \therefore v = v_0 + g t \quad \therefore v = 49 - 9.8 \times 6 = -9.8 \text{ m/sec}$$

$$\therefore \text{The body descends with velocity of magnitude } 9.8 \text{ m/sec}^2, T = \frac{1}{2} m V^2$$

$$T = \frac{1}{2} \times 1 \times (9.8)^2 = 48.02 \text{ Joule}$$

$$\text{b } \therefore v^2 = v_0^2 + 2 g s \quad \therefore v^2 = (49)^2 - 2 \times 9.8 \times 102.9$$

$$\therefore v^2 = 384.16 \quad T = \frac{1}{2} m v^2 = \frac{1}{2} \times 1 \times 384.16 = 192.08 \text{ Joule}$$

Try to solve

- ② A body of mass 500 gm is projected vertically downwards from a height of 78.4 meters above the ground surface, find:

a The kinetic energy of the body after 2 seconds from the instant of fell.

b The kinetic energy of the body at the instant it contacts the ground surface.

Example

- 3 A body of mass 200 gm is projected with velocity 280 cm/sec on the line of the greatest slope to a smooth inclined plane inclined at sine $\frac{1}{140}$ to the horizontal upwards the plane. Find the kinetic energy of the body in the erg unit for each of the following:
- After half a minute of throwing it.
 - When the body is at distant 24.5 meter from the throwing point.

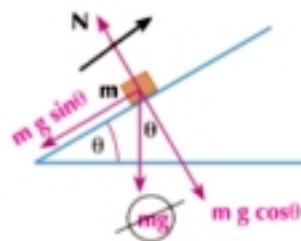
Solution

The equation of motion of the moving body

$$m a = -m g \sin \theta$$

$$\therefore a = -980 \times \frac{1}{140} = -7 \text{ cm/sec}^2$$

- $v = v_0 + a t$
 $v = 280 - 7 \times 30 = 70 \text{ cm/sec}$
 $T = \frac{1}{2} m v^2 = \frac{1}{2} \times 200 \times (70)^2 = 49 \times 10^4 \text{ Erg}$
- $v^2 = v_0^2 + 2 a s$
 $v^2 = (280)^2 - 2 \times 7 \times 2450 = 44100$
 $v = \pm 210 \text{ cm/sec}$
 $T = \frac{1}{2} m v^2 = \frac{1}{2} \times 200 \times (210)^2 = 441 \times 10^4 \text{ Erg}$



Try to solve

- 3 A car of mass 1 ton ascends a slope inclined at sine $\frac{1}{20}$ to the horizontal. Its engine is turned off to stop after it traveled a distance of 20 meters from the instant of turning the engine off. If the resistance force of the slope is $\frac{1}{5}$ of the cars weight, calculate the kinetic energy of the car in joule unit.

The Principle of work and energy

If F is constant :

Let a body of mass (m) move a distance (S) under the action of the resultant of forces (F) such that its velocity changes from (v_1) to (v_2) then the work done by the resultant of these force:

$$W = F \times S$$

$\therefore v^2 = v_0^2 + 2 a s$ and let v_1, v_2 be the initial and final velocities respectively.

$\therefore v_2^2 - v_1^2 = 2 a s$ by multiplying both sides of the relation by $\frac{1}{2} m$

$$\frac{1}{2} m (v_2^2 - v_1^2) = m a s$$

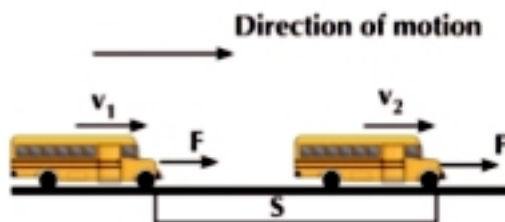


Figure (1)

- $\therefore \frac{1}{2} m (v_2^2 - v_1^2) = F \cdot S$ where F is a constant force
 \therefore The change of the kinetic energy is equal to the work done
 If F is a variable force, then:
 $\therefore T = \frac{1}{2} m v^2$
 $\therefore \frac{d}{dt}(T) = m v \frac{dv}{dt}$ $\frac{d}{dt}(T) = m a v$ $\frac{d}{dt}(T) = F \frac{dS}{dt}$
 $\therefore \int_{T_1}^{T_2} d(T) = \int_{s_1}^{s_2} F ds$ i.e $T - T_1 = W$
 \therefore The change of the kinetic energy = work done

The last relation expresses the principle of work and energy which states the following:

«The change of the kinetic energy of a particle as it transfers from an initial position to a final position is equal to the work done by the force acting on it during the displacement between those two positions».

It is noticed that when the previous relations are used, the measuring units of kinetic energy are to be the same measuring units of work.

Critical thinking:

Prove that If a particle starts its motion from a certain position, then it turns back to the same position, then its final kinetic energy is equal to its initial kinetic energy. Deduce that in the motion of the vertical projectile under the action of the gravitational force, the velocity of the projectile during the ascending phase at a point is equal to its velocity during its descending phase at the same point.

Example

- 4 A bullet of mass 200 gm is fired with velocity 400 m/sec on a thick barrier to embed in depth 20 cm. Find the magnitude of the resistance force of the barriers material to the motion of the bullet supposing this force is constant.

Solution

Let A be the position at which the bullet entered the barrier, B the position at which the bullet rested in the barrier and R is the resistance force measured in dyne unit.

we obtain $AB = 20$ cm, since the resistance force works in the opposite direction of the displacement,

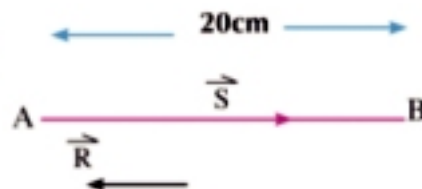


Figure (6)

Then the work done by this force is negative and be calculated as follows:

$$W = -AB \times R = -20 R$$

The kinetic energy as it enters the barrier :

$$T_A = \frac{1}{2} \times 200 \times (400 \times 100)^2 = 1.6 \times 10^{11} \text{ Erg}$$

(Notice the velocity is converted into cm/sec).

The kinetic energy of the bullet at position B: $T_B = \text{Zero}$ since the bullet is at rest in this position.

The change in the kinetic energy of the bullet : $T_B - T_A = -1.6 \times 10^{11}$ Erg

$$\therefore T_B - T_A = W$$

$$\therefore -1.6 \times 10^{11} = -20 R$$

$$\therefore R = \frac{-1.6 \times 10^{11}}{-20} = 8 \times 10^9 \text{ dyne}$$

Try to solve

- 4 A bullet is fired on a target of thickness 9 cm and exits from the other side with half velocity before it enters the target. What is the least thickness needed for a target of the same material in order the bullet not to exit from if it is fired with its previous velocity?

Example

- 5 A body of mass 20kg is let to descend on the line of the greatest slope to a smooth plane inclined at 30° to the horizontal. Find the velocity of the body after it travels 5 meters on the plane using the principle of work and energy.

Solution

The only force doing work is the component of the weight force parallel to the line of the greatest slope on which the motion takes place. The force is directed down the plane and its magnitude is $mg \sin 30$ where m is the mass of the body and g is the magnitude of the gravitational acceleration figure (7).

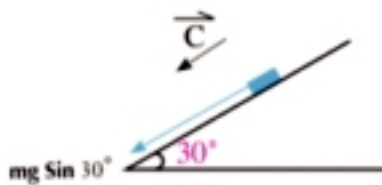


Figure (7)

The work done by this force during the given displacement:

$$W = (m g \sin 30^\circ) \times S = (20 \times 9.8 \times \frac{1}{2}) \times 5 = 490 \text{ Joule}$$

\therefore The change in kinetic energy = work done

$$\therefore \frac{1}{2} m v^2 - \text{zero} = W$$

$$\therefore \frac{1}{2} \times 20 \times v^2 = 490$$

$$\therefore v^2 = 49$$

$\therefore v = 7 \text{ m / sec}$ and this is the velocity of the body after it traveled 5 meters from its initial position.

Try to solve

- 5 A body of mass 2 kg is thrown with velocity 3 m/sec downward on the line of the greatest slope to a smooth plane of length 10 meters and height 2 meters. Find the kinetic energy of this body as it reaches the base of the plane.

Example

- 6 A body of mass 300 gm is placed at the top of an inclined plane whose height is 1m. Find the velocity with which the body reaches the bottom of the plane if the work effort against by the resistance force of the plane to the motion is equal to 1.59 Joules.

Solution

Let S be the length of the plane measured in meters, θ the inclination of the plane to the horizontal. Two forces acting on the body in a direction parallel to the direction of motion, the component of weight acting in a line of greatest slope downwards and of magnitude $mg \sin \theta$ and the resistance force, let its magnitude be R and acting along a line of greatest slope upwards.

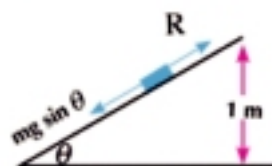


Figure (8)

The work done during the motion of the body from the top of the plane to its base:

$$W = (m g \sin \theta - R) \times S = (0.3 \times 9.8 \times \frac{1}{S} - R) \times S = 0.3 \times 9.8 - R S$$

But $RS = 1.59$ Joule is work effort against the resistance.

$$\therefore W = 0.3 \times 9.8 - 1.59 = 1.35 \text{ Joule}$$

$$\therefore T - T_1 = W$$

$$\therefore v^2 = 9$$

$$\therefore \frac{1}{2} \times 0.3 v^2 - \text{zero} = 1.35$$

$$\therefore v = 3 \text{ m/sec}$$

Try to solve

- 6 A body of mass 200 gm is placed at a top of an inclined plane of height 3 meters. Calculate the velocity by which the body reaches the bottom of the plane given that the work effort against the resistance force of the plane to the motion is 4.48 Joule.

Example

- 7 A body of mass 1 kg moves with a uniform velocity of magnitude 12 m/sec. A resistance force of magnitude $6x^2$ (newton) where x is the distance which the body travels under the action of the resistance (meter) acts on it.
- Find the work done by the resistance when $x = 4$
 - Find the velocity of the body and its kinetic energy $x = 2$

Solution

$$\begin{aligned} \text{a } W &= \int_0^4 F dx \\ &= \int_0^4 -6x^2 dx = [-2x^3]_0^4 \\ &= -128 \text{ Joule} \end{aligned}$$

$$\begin{aligned} \text{b } \therefore \text{Change in kinetic energy} &= \text{Work done} \\ \frac{1}{2} m (v^2 - v_1^2) &= \int_0^2 F dx \\ \frac{1}{2} \times 1 (v^2 - 144) &= \int_0^2 -6x^2 dx \\ \frac{1}{2} (v^2 - 144) &= [-2x^3]_0^2 \\ \frac{1}{2} (v^2 - 144) &= -16 \\ v^2 &= 112 \\ v &= 4\sqrt{7} \text{ m/sec} \\ T &= \frac{1}{2} m v^2 = \frac{1}{2} \times 1 \times 112 = 56 \text{ Joule} \end{aligned}$$

- 11 A particle of mass 5 kg is thrown on the line of the greatest slope inclined to the horizontal at an angle whose sine is $\frac{1}{10}$, upwards with velocity 4 m/sec. Calculate the change in the kinetic energy of the particle after passing one second from throwing instant, then when it returns to the throwing position.
- 12 A rough inclined plane of length 20 m and height 5 m. Find the minimum velocity by which a body is thrown from the lowest point of the inclined plane and in the direction of the line of the greatest slope to the plane in order to hardly reach the highest point in the plane given that the body encounters resistances equal to $\frac{1}{4}$ of its weight.
- 13 A cannon shell is fired with velocity $\vec{v} = 105 \hat{i} + 360 \hat{j}$ where \hat{i} , \hat{j} are two perpendicular unit vectors and the magnitude of the velocity is measured in m/sec unit. If the kinetic energy of the shell is equal to 1.125×10^6 Joules, Find the mass of the shell in kg.
- 14 A body of mass 2 kg moves under the action of the forces $\vec{F}_1 = \hat{i} + 2\hat{j}$, $\vec{F}_2 = 2\hat{i} + \hat{j}$, $\vec{F}_3 = 3\hat{i} + 5\hat{j}$ each is measured in newton where \hat{i} , \hat{j} are two perpendicular unit vectors. If the displacement vector as a function of time is given by the relation $\vec{S} = At^2\hat{i} - B(t^2 - t)\hat{j}$ and the magnitude of the displacement is in meter. Find:
First: The value of the two constants A, B
Second: The work done by such force after 2 seconds from the beginning of motion.
Third: The kinetic energy at the end of time of magnitude 2 seconds
- 15 A bullet is horizontally fired with velocity 540 km/h on a piece of wood to embed inside it at depth 20 cm. If the same bullet is fired with the same velocity on a constant target of the same kind of wood of thickness 15 cm. What is the velocity by which the bullet exits from the target supposing that the resistance is constant.
- 16 A vertical target made up of two layers of two different metals; the thickness of the first is 7 cm and the second is 14 cm. If two bullets of the same masses are fired in opposite directions and perpendicular to the target with one velocity, the first bullet penetrated the first layer and rested in the second layer after it embedded a distance of 5 cm while the second bullet penetrated the second layer and rested in the first layer after it embedded a distance of 1 cm. Find the ratio between the resistance of the two metals.

- 17 Two smooth balls of masses 100 gm, 200 gm move in a straight line in opposite directions to collide when their velocities are 8 m/sec, 12 m/sec respectively. If the first ball rebounds directly after collision with velocity 2 m/sec, calculate the kinetic energy lost due to the collision in joule.
- 18 A ball of mass 100 gm is let to fall down of a height 3.6 m on horizontal ground to collide with it and rebounds vertically upwards. If the loss in the kinetic energy of the ball due to the collision with the ground is 1.96 Joules, calculate the distance by which the ball rebounded back after it collided with the ground.
- 19 A rubber body is let to fall from rest from a top of a tower. If its momentum reaches 1092 gm. m/sec and kinetic energy reaches 1014 gm.wt.m directly before collision, calculate the mass of this body and the height of the tower. If the body rebounds back as it collides with the ground a distance of 4.9 m, find the impulse magnitude of the ground on the body.
- 20 A body (A) of mass 1.8 kg is let to fall down from rest of a height above the ground. At the same time, a body (B) of mass 1.14 kg is thrown vertically upwards from the ground surface with velocity 49 m/sec to collide with body (A) and form one body. If given that the velocity of body (A) is 28 m/s before the collision directly, calculate:

First: the common velocity of the two bodies after collision directly.

Second: the kinetic energy lost by collision.

Third: the impulse on body (A)

- 21 A hammer of 800 kg is let to fall vertically from a height 6.4 m on a foundation pool of mass 320 kg to dig it vertically downwards in the ground a distance of 10 cm. Find:

First: the common velocity of the hammer and body after collision directly.

Second: the kinetic energy lost due to collision.

Third: the ground resistance to the body measured in kg.wt.

Potential energy

Unit Four

4 - 3



Preface

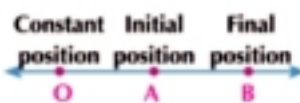
You have previously learned that the energy of a body is related to its motion and this called the kinetic energy. In this lesson, you learn the potential energy of the body which is related to the place of its existence. Several types of the potential energy can be defined such that each type is related to a type of forces. The potential energy of the Earth's attraction to the bodies is one of the most common potential energies.



Learn

potential energy

When a particle moves in a straight line under the action of a constant force \vec{F} parallel to this line, then the potential energy of the particle P at an instant is the work done by the force acting on this particle if it's motion from its position to another constant position on the straight line \overrightarrow{AB} as shown in the opposite figure. If the force \vec{F} is parallel to \overrightarrow{AB} , (O) is the constant position, A, B are two different positions of the body on this line, then:



the potential energy at A is $P_A = \vec{F} \cdot \overrightarrow{AO}$, and the potential energy at B is $P_B = \vec{F} \cdot \overrightarrow{BO}$, by using the symbol P to express the potential energy, We find that:

✦ Potential energy at (O) = 0 since potential energy at O = $\vec{F} \cdot \vec{0} = 0$

✦ Let A, B be the initial and final positions of the moving body. P_A, P_B , are the two potential energies at A, B respectively, then:

$$\begin{aligned} P_B - P_A &= (\vec{F} \cdot \overrightarrow{BO}) - (\vec{F} \cdot \overrightarrow{AO}) \\ &= \vec{F} \cdot (\overrightarrow{BO} - \overrightarrow{AO}) = (\vec{F} \cdot \overrightarrow{BA}) \\ &= -\vec{F} \cdot \overrightarrow{AB} \quad \text{①} \end{aligned}$$

But: $\vec{F} \cdot \overrightarrow{AB} = W$

②

from ① ②

$$P_B - P_A = -W$$

You will learn

- ⊙ Potential energy
- ⊙ The relation between work and the change in the potential energy
- ⊙ The principle of work and energy - the conservation of energy
- ⊙ The potential energy of a body acted on by a variable force

Key terms

- ⊙ Potential energy
- ⊙ Change in potential energy
- ⊙ The work - energy principle
- ⊙ conservation of energy

Materials

- ⊙ Scientific calculator.
- ⊙ Computer graphics.

i.e.: The change in the potential energy of a body when it moves from an initial position to a final position is equal to negative the work done by the force during the motion.

Conservation of energy

If a body moves from position A to another position B without encountering any resistance, then the sum of the kinetic and potential energies at A is equal to the sum of the kinetic and potential energies at B.

From the principle of work and energy, we find that:

$$T_B - T_A = W$$

From the previous relation relating the work to the potential energy, we find that:

$$P_B - P_A = -W$$

$$\therefore T_B - T_A = -[P_B - P_A]$$

$$\therefore T_B + P_B = T_A + P_A$$

The sum of the kinetic and potential energies remains constant during the motion.

Measuring units of the potential energy: in regard to the definition of the potential energy, we find that its measuring units are the same measuring units of work and kinetic energy.

Example

- ① The force $\vec{F} = 6\hat{i} + 2\hat{j}$ acts on a body to move it from position A to position B in two seconds and the position vector of the body is given by the relation: $\vec{r} = (3t^2 + 2)\hat{i} + (2t^2 + 1)\hat{j}$. Calculate the change in the potential energy of the body where the magnitude of F is measured in newton and r in m and t in second.

Solution

$$\begin{aligned} \therefore \vec{s} &= \vec{r} - \vec{r}_i &&= (3t^2 + 2)\hat{i} + (2t^2 + 1)\hat{j} - (2\hat{i} + \hat{j}) \\ &&&= 3t^2\hat{i} + 2t^2\hat{j} = \vec{AB} \\ \therefore \text{The change in the potential energy} &= \vec{F} \cdot \vec{BA} = -(\vec{F} \cdot \vec{AB}) \\ &= -(6, 2) \cdot (3t^2, 2t^2) \\ &= -(18t^2 + 4t^2) = -22t^2 \\ &= -22 \times 4 = -88 \text{ joule} \end{aligned}$$

Try to solve

- ① The force $\vec{F} = 4\hat{i} + 5\hat{j}$ acts on a particle to move it from position A to position B within two seconds and the position vector of the body is given as a function of time by the relation $= (2t^2 + 3)\hat{i} + (4t + 1)\hat{j}$. Calculate the change in the potential energy of the particle where the magnitude of F is measured in newton and r in m and t in second.

Example

- 2 A body of mass 300 gm is placed at a height of 10 m above the ground surface. Find the potential energy of the body. If it fell vertically, find the sum of the kinetic and potential energies of the body at any instant during its fall, then find the energy of its motion when it is 3 m above the ground surface.

Solution

The potential energy of the body at A:

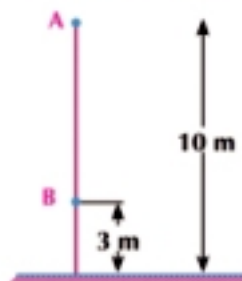
$$\begin{aligned} \text{The potential energy of the body at A} &= mg \times h \\ &= 0.3 \times 9.8 \times 10 = 29.4 \text{ joule} \end{aligned}$$

\therefore The body rests at A \therefore its kinetic energy = zero

$$\therefore T_A + P_A = 29.4 \text{ joule}$$

\therefore The sum of kinetic and potential energies remains constant during motion

\therefore The sum of kinetic and potential energies of the body at any instant during its falling = 29.4 joule



Kinetic energy and potential energy at B:

$$\begin{aligned} \therefore \text{The potential energy of the body} &= mg \times h \\ &= 0.3 \times 9.8 \times 3 = 8.82 \text{ joule} \end{aligned}$$

$$\therefore T_B + P_B = T_A + P_A$$

$$\therefore T_B + 8.82 = 29.4 \quad \therefore T_B = 29.4 - 8.82 = 20.58 \text{ joule}$$

Try to solve

- 2 A particle of mass 100 gm is let to fall down from a height of 4 m above the ground surface. Find the sum of kinetic and potential energies of the body at any instant of its falling, then find the energy of its motion when it is 1 m above the ground surface.

Example

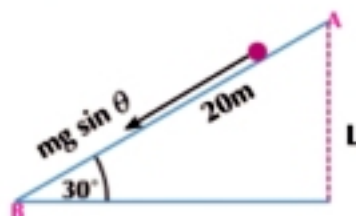
- 3 A body of mass 3 kg is placed at A which is the highest point of a smooth inclined plane of length 20 m and inclined at 30° to the horizontal. Calculate the potential energy of the body and if it descends in the direction of the line of the greatest slope of the plane. Calculate the velocity of the body at the instant it reaches the lowest point of the plane.

Solution

The potential energy of the body at A:

$$\begin{aligned} P_A &= mg \times h \\ &= 3 \times 9.8 (20 \sin 30^\circ) \\ &= 294 \text{ joule} \end{aligned}$$

$$T_A + P_A = 0 + 294 = 294 \text{ joule} \quad (\text{since the body rests at A})$$



The kinetic and potential energies at B:

$$T_B + P_B = 294 \text{ joule}$$

$$\frac{1}{2} mv^2 + 0 = 294$$

$$\therefore V^2 = \frac{294 \times 2}{3} = 196$$

$$\therefore \frac{1}{2} \times 3 \times v^2 = 2994$$

$$\therefore v = 14 \text{ m/sec}$$

5 Try to solve

- 3 A, B are two points on the line of the greatest slope in a rough inclined plane such that B is down A. A body of mass 500 gm starts its motion from rest from point A. If the vertical distance is equal to one meter and the velocity of the body as it reaches B is equal to 4 m/sec, find in joule:

First: the lost potential energy

Second: the work done by the resistances

Example

- 4 A simple pendulum is made up of a light rod of length 80 cm carrying a body of mass 4 kg suspended vertically and is oscillatory an angle of measure 120° . Find:

First: the increase of the potential energy at the end of the pathway more than at the middle of the pathway.

Second: the velocity of the body at the middle of the pathway.

Solution

From the geometry of the figure:

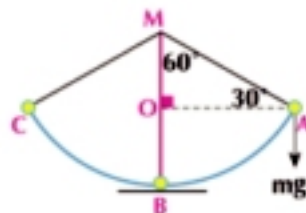
The mass moves in a circular arc whose center is point M and its radius length = 80 cm.

$$\therefore m(\angle AMC) = 120^\circ$$

$$\therefore m(\angle AMD) = 60^\circ$$

\therefore Triangle AOM is thirty -sixty triangle

$$\therefore MO = 40 \text{ cm}, BO = 40 \text{ cm}$$



The increase of the potential energy at A is more than at B:

$$\begin{aligned} P_A - P_B &= mg h_1 - mg h_2 = mg (h_1 - h_2) = mg \times BO \\ &= 4 \times 980 \times 40 = 156800 \text{ erg} \end{aligned}$$

To find the velocity of the body at the middle of the passway:

from the principle of the conservation of energy $T_B + P_B = T_A + P_A$

$$\therefore \frac{1}{2} \times 4 \times V^2 + 0 = 0 + 156800$$

$$\therefore V^2 = 78400$$

$$\therefore V = 280 \text{ cm/sec}$$

Motion on a rough inclined plane

If a body descends on a rough inclined plane under the action of its weight only from position A to position C, then the change in the potential energy = the change in kinetic energy + the work done against resistances.

Proof:

Let the distance traveled by the body on the plane be (S), then the vertical distance AB which the body descends $AB = S \sin \theta$

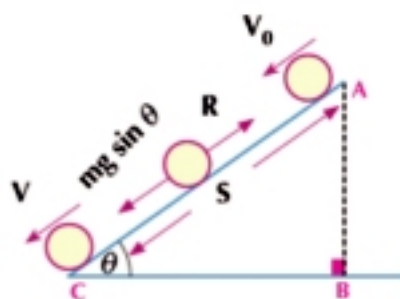
The change in the kinetic energy from A to B =

the work done by $(mg \sin \theta - R)$

$$\frac{1}{2} m (V^2 - V_0^2) = (mg \sin \theta - R) \times S \qquad \frac{1}{2} m (V^2 - V_0^2) = mg \sin \theta \times S - R \times S$$

$$\frac{1}{2} m (V^2 - V_0^2) = mg \times AB - R \times S \qquad mg \times AB = \frac{1}{2} m (V^2 - V_0^2) + R \times S$$

The change in the potential energy = the change in the kinetic energy + the work done against the resistances.



Note: the previous rule can be generalized whether the motion is vertical or on an inclined plane as follows:

If a body is fallen or thrown vertically in a medium containing a resistance or descended on a rough inclined plane, then:

The change in potential energy = change in kinetic energy + the work against resistance

Example Motion on a rough plane

- 5 In the opposite figure, a cube of wood of mass 2 kg at A, slides on a surface (as illustrated in the figure) where \widehat{AB} , \widehat{CD} are two smooth surfaces. The horizontal plane BC is rough, its length is 30 m, and its coefficient of the kinetic friction between the cube and the horizontal surface $\frac{1}{5}$. If the cube starts motion from rest and it is 4 m high, at which distance does the cube rest on \widehat{BC} .



Solution

The cube slides on the arc \widehat{AB}

According to the principle of conservation of energy $T_A + P_A = T_B + P_B$

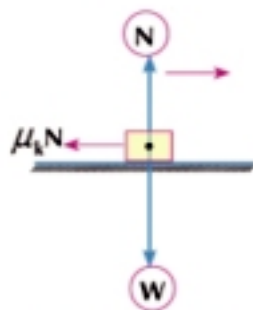
$$\text{zero} + 2 \times 9.8 \times 4 = T_B + \text{zero}$$

$$\therefore T_B = 78.4 \text{ joule.}$$

Since the cube moves on the rough plane \overline{BC} .

The change in potential energy = change in kinetic energy + the work against resistances

$$\begin{aligned} \text{zero} &= (0 - 78.4) + \mu_k N \times S \\ \frac{1}{5} \times 9.8 \times 2 \times S &= 78.4 \qquad \therefore S = 20 \text{ m} \end{aligned}$$



5 Try to solve

- 4 A car descends a slope from rest. The length of 180 m, its height of 10 m. If it is given that $\frac{3}{4}$ of the potential energy is lost due to overcoming the resistances against motion and these resistances remain constant during the motion of the car, find the velocity of the car after it traveled the previous distance of 180 m.

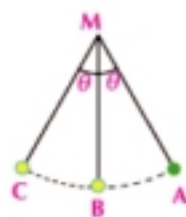
Exercises 4 - 3

First complete:

- 1 A body of mass 0.2 kg is let to fall from a height 5 m above the ground .
 - a The potential energy of the body at the instant it fell = _____
 - b The kinetic energy of the body at the instant it fell = _____
 - c The sum of the kinetic and potential energies at the instant the body reached the ground = _____ joule
- 2 A body of mass 350 kg and height 20 m above the ground, then its potential energy = _____ joule.
- 3 A helicopter of weight 3500 kg.wt descends vertically downwards from height 250 m to height 150 m above the ground, then the magnitude of loss in its potential energy = _____ joule.
- 4 A body of weight 2 kg.wt ascends a distance of 200 cm on the line of the greatest slope to a smooth plane inclined at 30° to the horizontal, then the increase of its potential energy = _____ joule
- 5 A body is placed on the top of a smooth inclined plane of height 90 cm , then its velocity as it reaches the bottom of the plane = _____ m/sec
- 6 A body moves from position A(2, 3) to position B (7, 6) under the action of the force $\vec{F} = 3\hat{i} + 4\hat{j}$, then the change in the potential energy of the body = _____ Erg; where S is measured in cm and \vec{F} in dyne.
- 7 The force $\vec{F} = 4\hat{i} + 5\hat{j}$ acts on a body to move it from position A to position B in two seconds and the position vector of the body is given as a function of time by the relation $\vec{R} = (2t^2 + 3)\hat{i} + (4t + 1)\hat{j}$, then the change in the potential energy of the body = _____ joule; where F in newton and $\|\vec{R}\|$ in m and t in seconds.

Answer the following questions:

- 8 A body of mass 300 gm is placed on a height of 10 m above the ground. Find the potential energy of the body. If the body falls vertically, find its kinetic energy when it is at a height of 3 m above the ground.
- 9 A body of mass 140 gm is projected vertically upwards from the top of a tower whose height is 25 m above the ground. Calculate the change in the kinetic energy of the body from the instant it is projected until it reaches the ground in joule.
- 10 A body of mass 2 kg from the Earth's surface is projected vertically upwards with velocity 70 m/sec. Find the sum of the kinetic and potential energies after 5 seconds. If the kinetic energy of the body after time is 125.44 joule. Find this time and find its potential energy.
- 11 A body of mass 100 gm is let to fall from a height of 5 m above flabby ground and embeds in it 20 cm. Find:
First: the magnitude of what is lost of the potential energy in joule before the instant the body collides with the ground directly.
Second: the average resistance of the ground in kg.wt.
- 12 A person of mass 72 kg ascends a road inclined at an angle of $\sin^{-1} \frac{1}{6}$ to the horizontal. he covers a distances of 120 m. Calculate the change in the potential energy of the person.
- 13 Calculate the velocity of a body of mass 300 gm placed at the top of an inclined plane of height 2 m reaches the bottom of the plane if the magnitude of work done against the resistance is equal to 2.13 joule.
- 14 A, B are two points on the line of the greatest slope to a rough inclined plane such that B is under A. A body of mass 500 gm starts moving from rest from point A. If the vertical distance is equal to 1 m and the velocity of the body when it reaches B is equal to 4 m/sec Find in joule:
First: the lost potential energy.
Second: the work done by the resistances.
- 15 **In the figure opposite:** A simple pendulum of a string of string length 130 cm, If the pendulum starts its motion from rest from point A to move freely to oscillate at angle of measure 2θ where $\tan \theta = \frac{5}{12}$. Find the velocity of the sphere at the midpoint of the pathway.
- 16 A ring of mass $\frac{1}{2}$ kg slides on rough vertical cylindric pool. If its velocity is 6.3 m/sec after it traveled 4.8 m from the beginning of its motion, use the work-energy principle to calculate the work done by the resistance during the motion.



You will learn

↪ Power

↪ Constant power and variable power

Key terms

↪ Power

↪ Horse power

Materials

↪ Scientific calculator



Think and Discuss

A machine does work of magnitude 200 kg.wt. m in 4 minutes and another machine does another work of magnitude 100 kg.wt.m in a minute.

Which machine is more efficient?

You may think that the first machine is more efficient than the second machine since it did more work.

But what the first machine did in a minute = $\frac{200}{4} = 50$ kg.wt.m and what the second machine did in a minute = 100 kg.wt.m. Thus, we can deduce that when we measure the power of a machine, it is necessary to know the work this machine does in a time unit.

Definition

Power : the time rate to do work

This definition is formulated as follows:

«The power is the work done in a time unit»

$$\begin{aligned} \text{Power} &= \frac{d}{dt} (w) \\ \therefore W &= \int \vec{F} \cdot d\vec{S} \\ \therefore \frac{d}{dt} (w) &= \frac{d}{dt} \int \vec{F} \cdot d\vec{S} \\ &= \frac{d}{dt} \int (\vec{f} \cdot \frac{d\vec{S}}{dt}) dt \\ \frac{d}{dt} (w) &= \frac{d}{dt} \int (\vec{F} \cdot \vec{V}) dt \\ \frac{d}{dt} (w) &= \vec{F} \cdot \vec{V} = \end{aligned}$$

If \vec{V} has the same direction of \vec{F} then power = $F V$

From this relation; we can find that the power is a scalar quantity identified at any instant in terms of F , V and its value is determined by the time rate to do work at this instant.

Notice that: the power is identified instantaneously (at a certain instant) otherwise the work which is always calculated between two instants.

The average power:

If the force does work of magnitude w during a time interval $\Delta t = t_2 - t_1$ then:

$$\text{The average power} = \frac{w}{\Delta t} = \frac{w}{t_2 - t_1}$$

Using the integration to find the work

$$\therefore \text{Power} = \frac{d}{dt} (w), \quad \therefore w = \int_{t_1}^{t_2} (\text{power}) dt$$

The variable power and maximum power

When the magnitude of the force F is constant, then the magnitude of power changes directly with the magnitude of the velocity V of the body and F is the arbitrary constant where

$$\text{power} = F V \qquad \text{power} \propto V \quad \text{when } F \text{ is constant}$$

As long as the magnitude of the velocity changes, the magnitude of the power changes and we obtain the maximum power when the velocity is as maximum as possible. In this case, the power is called the power of machine (in general)

Measuring units of power:

Since the power is equal to the time rate to do work.

$$\therefore \text{The measuring unit of power} = \frac{\text{measuring unit of work}}{\text{measuring unit of time}} = \text{measuring unit of force} \times \text{measuring unit of velocity}$$

The measuring units of power are : watt (newton. m/s), kg.wt .m/sec - erg /sec, horse

- ✎ **Newton - meter/second** : is known as the power of a force doing work at a constant time rate measured in newton one meter per second. The newton-meter /second (joule/second) are called "Watt".
- ✎ **(Kg.wt .meter /sec)** : is known as the power of a force doing work at a constant time rate of magnitude kilogram-one meter per second.
- ✎ **Erg/second** : is known as the power of a force doing work at a constant time rate of magnitude one erg per second.
- ✎ **Horse**: It is known as the power of the machine doing work of magnitude 75 kg.wt.m per second.

Here are the rules to convert the measuring units of power.

- 1 kg.wt. meter/sec = 9.8 newtons. meter/sec
- 1 newton. meter/sec = 1 watt = 10^7 erg/sec

There are other measuring units for the power such as kilowatt and horse.

$$\nearrow 1 \text{ kilowatt} = 1000 \text{ watt} = 1000 \text{ newton. meter/sec} = 10^{10} \text{ erg/sec}$$

$$\begin{aligned} \nearrow 1 \text{ horse} &= 75 \text{ kg.wt. meter/sec} \\ &= 75 \times 9.8 \text{ newton. meter/sec} \\ &= 735 \text{ newton. meter/sec (watt)} \\ &= 0.735 \text{ kilowatt} \end{aligned}$$

Example

- ① A person of mass 50 kg ascends the stairs of a tower of height 441 meters in time of magnitude 15 minutes. Calculate the average power in watt unit.

Solution

$$\text{Force (F)} = mg = 50 \times 9.8 = 490 \text{ newton}$$

$$\text{The person's average velocity} = \frac{\text{distance}}{\text{time}} = \frac{441}{15 \times 60} = 0.49 \text{ m/sec}$$

$$\text{The average power} = \text{force} \times \text{velocity} = F \times V = 490 \times 0.49 = 240.1 \text{ watt}$$

Try to solve

- ① A plane engine produces a force of magnitude 32.2×10^4 newtons when the plane's velocity is 900 km/h. Calculate the power of the engine in horse.

Example

- ② A car of mass 2 tons moves on a horizontal road with a uniform velocity of magnitude 108 km/h against resistances equivalent to 150 kg.wt per each ton of the mass. Calculate the power of its engine in horse.

Solution

The body moves with a uniform velocity (according to newton's first law) then $F =$

$$R = 150 \times 2 = 300 \text{ kg.wt}$$

$$\text{Car's velocity} = 108 \times \frac{5}{18} = 30 \text{ m/sec}$$

$$\therefore \text{Power} = F \times V = 300 \times 30 = 9000 \text{ kg.wt. m/sec}$$

$$\therefore \text{Power} = \frac{9000}{75} = 120 \text{ horse}$$

Try to solve

- ② A truck of mass 6 tons moves on a horizontal road with a uniform velocity of magnitude 54 km/h when the power of its engine is 300 horses. Calculate the resistance of the road in kg.wt per ton of mass.

Example

- 3 A car of mass 9 tons ascends a slope inclined at an angle of $\sin^{-1} \frac{1}{125}$ to the horizontal with maximum velocity of magnitude 45 km/h against resistances equivalent to 200 kg.wt per each ton of the mass. Calculate the power of its engine in horse.

Solution

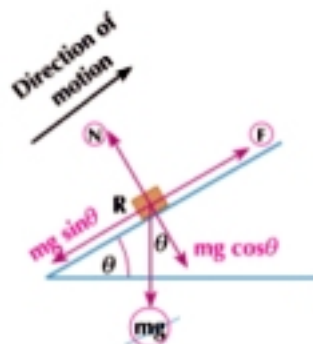
The motion is upwards the plane

$$\begin{aligned} \text{Equation of motion } F &= R + mg \sin \theta \\ F &= 200 \times 9 \times 9.8 + 9 \times 10^3 \times 9.8 \times \frac{1}{125} \text{ newtons} \\ F &= 2520 \text{ kg.wt} \end{aligned}$$

The maximum velocity the car can ascend the slope

$$V = 45 \times \frac{5}{18} = \frac{25}{2} \text{ m/sec}$$

$$\therefore \text{ The maximum power of the car } = F \times V = \frac{2520 \times \frac{25}{2}}{75} = 420 \text{ horse}$$



Try to solve

- 3 In the previous example, if the car descends on the same plane after loading it with goods of mass 3 tons, calculate the maximum velocity to descend in km/h given that the resistance per ton of mass does not change.

Note: If the rate of doing work is uniform (constant), then:

$$\text{Power} = \frac{\text{Work}}{\text{Time}} = \frac{\text{Force} \times \text{Distance}}{\text{Time}}$$

Example

- 4 A worker whose job is to load boxes each of mass 30 kg on a truck. If the height of the truck is 0.9 meter, calculate the number of boxes which the worker can load in time of magnitude 1 minute if his average power is equal to 0.6 horse.

Solution

$$\text{Power} = \frac{\text{Total work}}{\text{Time}} = \frac{\text{Number of boxes} \times \text{Work needed to load a box}}{\text{Time}}$$

$$\therefore \text{ Number of boxes needed to be loaded in a second} = \frac{\text{Power} \times \text{Time}}{\text{The work to one box}}$$

$$\text{Number of boxes} = \frac{0.6 \times 735}{30 \times 9.8 \times 0.9} = \frac{5}{3} \text{ boxes per second}$$

$$\text{Number of boxes} = \frac{5}{3} \times 60 = 100 \text{ boxes per minute}$$

Try to solve

- 4 In the previous example, calculate the number of boxes if the power of the man is 352.8 watt

Example

- 5 A train of mass 200 tons ascends a slope whose inclination to the horizontal is an angle of $\sin^{-1} \frac{1}{200}$ with a uniform velocity of magnitude 27 km/h against resistances to the motion parallel to the direction of the line of the greatest slope of the plane at a rate of 18 kg.wt per each ton of the mass. What is the power of the engine in horse? If the train descends on the same slope with the same velocity; what is the power of the engine in this case supposing the resistances are constant in both cases?

Solution

First: when the train ascends the slope:

we take the unit vector \vec{e} in the direction of motion.
i.e. upwards the plane

$$\therefore \text{Motion resistances} = 200 \times 18 = 3600 \text{ kg.wt}$$

component of the train weight in the direction of the plane = $200 \times 1000 \times \frac{1}{200} = 1000 \text{ kg.wt}$

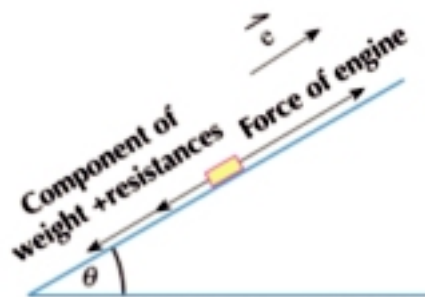
\therefore The train ascends with a uniform velocity

$$\therefore \text{Force of the engine} = \text{resistances} + \text{component of weight} = 3600 + 1000 = 4600 \text{ kg.wt}$$

\therefore Power = $F_1 v$ where F_1 force of the engine, v velocity

$$\therefore \text{Power} = 4600 \times 27 \times \frac{5}{18} \text{ kg.wt . meter/sec}$$

$$= 4600 \times 27 \times \frac{5}{18} \times \frac{1}{75} = 460 \text{ horses}$$



Second: when the train descends the slope:

We take the unit vector \vec{e} in the direction of motion. i.e. downwards the plane

\therefore The train descends with a uniform velocity

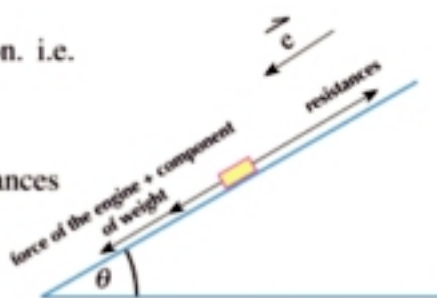
$$\therefore \text{Force of the engine} + \text{component of weight} = \text{resistances}$$

$$\therefore \text{Force of the engine} + 1000 = 3600$$

$$\therefore \text{Force of the engine} = 2600 \text{ kg.wt}$$

\therefore Power = $F_2 v$ where F_2 is the force of the engine, v is the velocity (since it doesn't change)

$$\therefore \text{Power} = 2600 \times 27 \times \frac{5}{18} \times \frac{1}{75} = 260 \text{ horses}$$



Try to solve

- 5 A locomotive of mass 28 tons pulls a train car of mass 56 tons with a uniform acceleration downwards a plane inclined at $\sin^{-1} \frac{1}{100}$ to the horizontal. When the power of the engine reaches 84 horses, its velocity becomes 21m/sec. Calculate the acceleration if given that the resistance is 10 kg.wt per each ton of mass.

Example

- 6 A particle moves under the action of a force $\vec{F} = 3\hat{i} + 4\hat{j}$ such that its displacement \vec{S} is given as a function of time by the relation $\vec{S} = 3t^2\hat{i} + 6t\hat{j}$ where \hat{i} , \hat{j} are two perpendicular unit vectors. Find the work done by the force then find the power when $t = 2$ seconds if F is measured in newton, S in m and t in second.

Solution

$$\therefore w = \vec{F} \cdot \vec{S}$$

$$\therefore w = (3, 4) \cdot (3t^2, 6t) = 9t^2 + 24t$$

$$\therefore \text{power} = \frac{d}{dt} (W)$$

$$\text{when } t = 2 \text{ second}$$

$$\therefore \text{power} = 18t + 24$$

$$\text{power} = 60 \text{ watt}$$

Try to solve

- 6 A constant force \vec{F} acts on a particle such that its displacement vector is given as a function of time t by the relation $\vec{S} = (3t^2 + t)\hat{i} - 4t\hat{j}$ where \hat{i} , \hat{j} are two perpendicular unit vectors. Find \vec{F} if the power of the force \vec{F} is equal to 75 erg/sec when $t = 4$ seconds and the power of force \vec{F} is equal to 165 erg/sec when $t = 9$ seconds given that S is measured in cm and F in erg dayn.

Example

- 7 If the power of an engine at any time measured in seconds is equal to $(9t^2 + 4t)$, find the work done by the engine during the first three minutes, then find the work done during the fourth second.

Solution

$$\therefore \text{Power} = \frac{dw}{dt}$$

$$\therefore w = \int_0^t (\text{power}) dt$$

$$\text{The work done during the first three seconds} = \int_0^3 (9t^2 + 4t) dt$$


$$= [3t^3 + 2t^2]_0^3$$

$$= 99 \text{ work unit}$$

$$\text{The work done during the fourth second} = \int_3^4 (9t^2 + 4t) dt$$

$$= [3t^3 + 2t^2]_3^4$$

$$= 125 \text{ work unit}$$

 **Example**

- 8 Find the time taken by a car of mass 1200 kg to reach the velocity 126 km/h from rest. If the power of the engine is constant and equal to 125 horses.

 **Solution**

$$\begin{aligned} \therefore w &= \int^t (\text{power}) dt & \therefore w &= \int^t (125 \times 735) dt \\ w &= 125 \times 735 t \end{aligned}$$

$$\therefore \text{work} = \text{change in kinetic energy} \quad \therefore \frac{1}{2} m (v^2 - v_0^2) = 125 \times 735 t$$

$$\therefore \frac{1}{2} \times 1200 \left(\left(126 \times \frac{5}{18}\right)^2 - 0 \right) = 125 \times 735 t$$

$$\therefore 735 \times 1000 = 125 \times 735 t \quad \therefore t = 8 \text{ sec}$$

 **Try to solve**

- 7 The force of a car's engine does work at a rate given during the time interval $t \in [0, 5]$ by the relation $144t - 26t^2$. If the mass of the car is 980 kg and its velocity by the end of the third second is 90 km/h, find its velocity by the end of the fourth second.


Exercises 4 - 4

Complete

- A particle moves under the action of force $\vec{F} = 3 \hat{i} + 4 \hat{j}$ where its displacement $\vec{S} = t \hat{i} + (t^2 + t) \hat{j}$ where F in dyne, and S in cm then the power of the force \vec{F} at instant $t = 3$ seconds is equal to _____ dyne. cm/sec.
- A train of mass 375 tons and the power of its engine is 625 horses moves on horizontal ground with maximum velocity of magnitude 90 km/h, then the resistance which it encounters per ton of the train's mass = _____ kg.wt.
- A car of mass 4 tons and the power of its engine is 10 horses moves in a straight line on horizontal ground. If the maximum velocity of the car is 75 km/h, then the resistance magnitude of the road to the car's motion = _____ kg.wt.
- A train of mass 108 tons moves with a uniform velocity on a horizontal railroad with velocity 30 km/h. If the resistances are equivalent to 10.5 kg.wt per each ton of mass, find the power of the engine in horse.
- The power of a train's engine is 504 horses and its mass is 216 tons moves on a horizontal railway with its maximum velocity against resistances equivalent to 5 kg.wt per each ton of its mass, find its maximum velocity in km/h.
- A horizontally train moves under the action of a resistance proportional to the square of its velocity. If the resistance is equivalent to 800 kg.wt when its velocity is 20 km/h and the power of the train is 200 horses when it moves with a maximum velocity, find this velocity in km/h.

- 7 A car of mass 1500 kg and the power of its engine is 120 horses moves on a horizontal straight road with maximum velocity of magnitude 72 km/h. What is the maximum velocity this car can ascend a straight road inclined to the horizontal with an angle of sine $\frac{1}{10}$ given that the resistance is the same on the two roads?
- 8 A car of mass 3 tons moves on a horizontal road with a uniform velocity of magnitude 37.5 km/h to reach the top of a slope inclined at an angle of sine 0.03 to the horizontal. The driver stops the engine and the car moved down the slope with its previous velocity. If the slope resistance is $\frac{2}{3}$ the horizontal road resistance, find:
first: the slope resistance in kg.wt. **second:** the engine power on the horizontal road.
- 9 A car of mass 6 tons moves with maximum velocity of magnitude 27 km/h ascending a slope inclined at an angle of sine $\frac{1}{10}$, to the horizontal, then the car starts to descend the same road with maximum velocity of magnitude 135 km/h. Identify the magnitude of the resistance force of the road to the motion supposing it did not change along the time, then find the power of the car's engine.
- 10 The engine power of a plane is 1350 horses when it moves horizontally with a uniform velocity of magnitude 270 km/h. Find the air resistance to the plane's motion. If the air resistance is proportional to square of its velocity. Find the power of its engine when it fly horizontally with a uniform velocity of magnitude 180 km/h.
- 11 A train engine of power 400 horses pulls a train with a maximum velocity of magnitude 72 km/h on horizontal ground. Calculate the resistance to the train's motion. If the mass of the engine and the train was 200 tons. Find the maximum velocity by which the train ascends a slope inclined at an angle of sine $\frac{1}{200}$ to the horizontal, supposing the road resistance to the motion didn't change.
- 12 The mass of a cyclist and the bike together is 80 kg, and the greatest power to the cyclist is $\frac{4}{5}$ of horse. If the maximum velocity for this cyclist in a horizontal road is 18 km/h, calculate the road resistance in kg.wt. If given that the cyclist ascends a slope inclined at an angle of sine $\frac{3}{40}$ to the horizontal with maximum velocity, calculate this velocity in km/h.
- 13 A truck of mass 5 tons moves on a horizontal road with a uniform velocity of magnitude 144 km/h when the power of its engine is 120 horses. Find the road resistance per ton of the truck's mass in kg.wt. If the resistance is proportional to the velocity, find the power of the engine in horse when the truck ascends a slope inclined at an angle of sine $\frac{3}{200}$ to the horizontal with a uniform velocity of magnitude 96 km/h.
- 14 A truck of 2 tons descends a road inclined at an angle of sine $\frac{1}{100}$ to the horizontal from site (A) to site (B) with maximum velocity of magnitude 90 km/h. Calculate the power of the truck's engine if given that the road resistance to its motion is estimated with ratio 13 % of the truck's weight. If the car is loaded with a mass of $\frac{1}{2}$ ton as it reaches site (B), then it ascends the road to site (A) with maximum velocity, find this velocity if the resistance remains with its same ratio of the weight.
- 15 A train of mass (m) ton moves on a horizontal road with the maximum velocity of magnitude 60 km/h. The last car of mass 15 tons is separated from the train and the maximum velocity of the train increases at a magnitude of 7.5 km/h. Find the power of the engine in horse and the mass of the train given that the resistance is equal to 9 kg.wt per each ton of mass.

- 16 A particle moves under the action of the force $\vec{F} = 3\hat{i} + 4\hat{j}$ and its displacement vector \vec{S} is given as a function of time by the relation $\vec{S} = t\hat{i} + (\frac{1}{2}t^2 + t)\hat{j}$, if F is measured in newton, s in meter and t in second, find:
- The work done during the first three seconds
 - The average power during the first three seconds
 - The power of the force \vec{F} when $t = 3$ sec
- 17 A particle moves under the action of the force $\vec{F} = (2t - 1)\hat{i} + (5t + 2)\hat{j}$ where its displacement vector is given as a function of time by the relation $\vec{S} = (3t^2 + t)\hat{i} + 4t\hat{j}$, if F is measured in newton, S in meter and t in seconds, find:
- The work done during the third, fourth and fifth seconds
 - The average power during the third, fourth and fifth seconds.
 - The force power when $t = 5$ sec
- 18 A body of 3 kg moves under the action of the force \vec{F} and the position vector of the body at any instant t is given by the relation $\vec{r}(t) = t^3\hat{i} + t^2\hat{j}$. If F is measured in newton, r in meter and t in seconds, find:
- The acting force \vec{F} in terms of t.
 - The power of force \vec{F} in terms of t.
 - The work done by force \vec{F} during the time interval $0 \leq t \leq 2$
- 19 If the power of an engine (in horse) is equal to $(6t - \frac{1}{20}t^2)$ where t is time in second, $t \in [0, 120]$ find:
- Power of engine when $t = 90$ sec.
 - The work done during the time interval $[0, 30]$.
 - The maximum power of the engine.
- 20 A body of mass 5 kg moves under the action of force \vec{F} such that its position vector at the time t is given buy the relation $\vec{r}(t) = t\hat{i} + t^2\hat{j}$. If F is measured in newton, r in meter and t in seconds, find :
- using the integration the work done by force F in the time interval $[0, 2]$.
- 21 A body of mass 3 kg moves under the action of force \vec{F} such that its velocity vector \vec{V} is given by the relation $\hat{v} = (1 - \sin 2t)\hat{i} + (-1 + \cos 2t)\hat{j}$. If F is measured in newton, V in meter /second, find:
- Force \vec{F} in terms of t.
 - The kinetic energy at time t.
 - Prove that the rate of change of T is equal to the power resulted form force \vec{F} .

GENERAL EXERCISES

- 1 A particle of mass 200 gm is projected upwards a smooth plane inclined at an angle of $\sin^{-1} \frac{8}{49}$ to the horizontal in the direction of the line of the greatest slope with velocity 30 cm/sec. Calculate the change occurring to the potential energy of this particle when its velocity gets 18 cm/sec.
- 2 A force of magnitude 48 gm.wt acts on a rested body placed on a horizontal plane for a time interval to acquire kinetic energy of magnitude 18900 gm.wt.cm by the end of this interval. Its momentum reaches 176400 gm.cm/sec at the instant, then the force is ceased and the body gets rested once again after it traveled a distance of 10.5 m from the instant of ceasing the force. Find the mass of the body and the magnitude of the plane resistance to its motion supposing it is constant and then find the action time of the force.
- 3 A car of mass 1800 kg moves on a horizontal road with a uniform velocity of magnitude 54 km/h. If the magnitude of the resistance to the car's motion is equivalent to 0.25 of the car's weight, find the power of the engine in this case in horse.
- 4 A hammer of mass 1 ton falls down from height 4.9 m vertically on an iron body of mass 400 kg to embed it vertically in the ground for a distance of 10 cm. Identify the common velocity for both the hammer and the body after collision directly. Identify the kinetic energy lost due to collision and the magnitude of the ground resistance supposing it is constant.
- 5 A body of mass 2 kg is projected on an inclined plane inclined at an angle of $\sin^{-1} \frac{1}{98}$ to the horizontal in the direction of the line of the greatest slope to the plane upwards with velocity 1.4 m/sec. Calculate the work done by the weight until the body rests instantaneously.
- 6 A body of mass 2 kg moves under the action of a constant force \vec{F} where $\vec{F} = 4\hat{i} + 8\hat{j}$ where F is measured in newton. If the body starts to move from rest from the point of the position vector $2\hat{i} + 5\hat{j}$, find the position vector of the body after 3 seconds, then find the magnitude of the work done by this force during this time interval and the power generated when $t = 3$ sec.
- 7 A cyclist and the bike of mass 98 kg move on a rough horizontal ground from rest to reach the maximum velocity of magnitude 7.5 m/sec after time of magnitude 1 minute when the cyclist stop peddling. The bike gets rested after it traveled a distance of magnitude 15 m. Calculate the maximum power in horse for the cyclist during this trip.
- 8 A body of mass 60 kg ascends from rest on the line of the greatest slope to an inclined plane of length 20 m and height 12 m. If the body starts its motion from the highest point on the plane and the coefficient of kinetic friction between the body and the plane is $\frac{3}{16}$, find the kinetic energy of the body when it reaches the plane base.

GENERAL EXERCISES

- 9 A body of mass 5 kg is placed on a rough inclined plane inclined at an angle of tangent $\frac{7}{24}$ to the horizontal. A force in the direction of the line of the greatest slope to the plane upwards acts on the body to move it with a uniform velocity a distance of 75 cm. if the coefficient of kinetic friction between the body and the plane is $\frac{5}{12}$ find :
- The magnitude of the work done against the resistance of the plane.
 - The magnitude of the work done by this force.
- 10 A car's engine does work at a constant rate of magnitude 5 kilowatt and the car's mass is 1200 kg. If the car moves on a horizontal road against a constant resistance of magnitude 325 newtons, find:
- The magnitude of the car's acceleration when its velocity is 8m/sec.
 - The car's maximum velocity.
- 11 A car of mass 5 tons moves with uniform velocity of magnitude 36 km/h ascending a slope inclined at angle of sine $\frac{1}{40}$ to the horizontal against the resistance equivalent to 2.5% of the car's weight. Find the power of the engine in horse. If the power of engine suddenly increases to 50 horses, find the magnitude of the car's acceleration directly after this increase.
- 12 A train moves with uniform velocity of magnitude 72 km/h, the last car is of mass 16 tons is separated and the velocity of the train increases to be 96 km/h. If the power of the train engine is constant, find the power of the engine and the train's mass given that the train encounters a constant resistance of magnitude 6 kg.wt per each ton of the moving mass.
- 13 A particle moves in a straight line under the action of a variable force F where $F = \frac{1}{5}x$ (newton) where x in meter is the distance between the particle and a constant origin point on the straight line. Find the work done by the force F in the following cases :
- When the particle moves from $x = 0$ to $x = 10$.
 - When the particle moves from $x = 1$ to $x = 5$.
- 14 A body of mass 1 kg is let to fall vertically downwards from rest under the action of the gravitational acceleration against resistances of magnitude $\frac{24}{25}x$ (newtons) where x is the distance between the body and the falling point in meter at any instant. Find the work done by the body against the resistance from the instant of falling until the body travels a distance of 10m under the falling point and find its velocity at this instant.
- 15 A constant force of magnitude F Newton inclined at an angle of tangent $\frac{4}{3}$ pulls a stalled car of mass 1400 kg with uniform velocity of magnitude 22.5 m/sec on a rough horizontal road and the coefficient of kinetic friction between the road and the car is 0.3, find :
- The force power in this case.
 - The work done by this force to move the car for a minute.

For more activities and exercises visit www.sec3mathematics.com.eg

ACCUMULATIVE TEST

First: Statics

Test 1

Answer the following question:

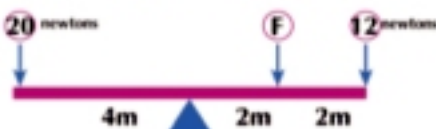
Question 1 : Choose the correct answer

- 1 If θ is the measure of the angle between the final limiting force and the resultant reaction, then the coefficient of the static friction is equal to:

a $\tan \theta$ b $\sin \theta$ c $\cos \theta$ d $\cot \theta$

- 2 The opposite figure represents a rod in equilibrium, then $F =$

a 28 newtons b 16 newtons
c 2 newtons d 4 newtons



- 3 The force $\vec{F} = 3\hat{i} - 5\hat{j}$ acts at point A (-1, 1) then the moment of the force \vec{F} about the origin point is equal to:

a $-2\hat{k}$ b $2\hat{k}$ c $8\hat{k}$ d $-8\hat{k}$

- 4 Two forces form a couple; the magnitude of one of them is 15 newtons and the moment of the couple is 45 newtons. cm, then the perpendicular distance between them is equal to

a 675 cm b 60 cm c 3 cm d 30 cm

- 5 If a system of coplanar forces is in equilibrium, then the algebraic measure of the sum of its moments about any point in the plane is equal to:

a non-zero constant b zero
c resultant of these force d the unity

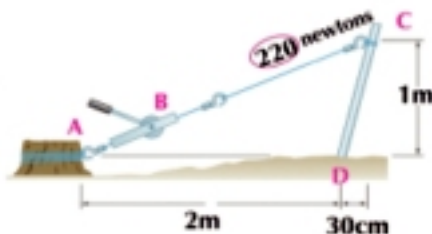
- 6 The center of gravity of two physical bodies of masses 3 newtons and 6 newtons and the distance between them is 15 cm is at distance cm from the 3 newtons body

a 5 b 10 c 7.5 d 9

Second: answer three questions of the following:

Question 2

- 1 The opposite figure illustrates a winch puller AB acting on an inclined fence CD. Find the magnitude of the moment of the tension force about point D.



- 2 A body of weight (W) is placed on a rough plane inclined at an angle of measure (θ). If the measure of the angle of friction is (Y), find the magnitude and direction of the least force making the body move upwards.

ACCUMULATIVE TEST

Question 3

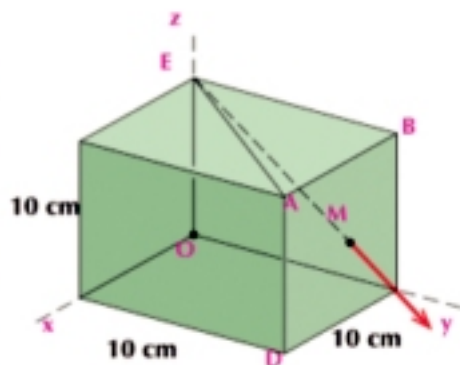
- Two like forces each of magnitude 10, 15 newtons act at the two points A, B where $AB = 75$ cm. Find the resultant of the two forces.
- ABC is an isosceles triangle in which $AB = AC = 13$ cm and $BC = 10$ cm forces of magnitudes 65, F, 65 newtons act along \vec{AB} , \vec{BC} , \vec{CA} respectively. if the system is equivalent to a couple 3. What is the value of F and the magnitude of the moment of the couple ?

Question 4

- AB is a light fine rod of length $2L$ connected in a vertical plane at its two ends A, B by two strings inclined at $30^\circ, 60^\circ$ to the horizontal respectively, two weights of 2, 8 newtons are suspended on the rod distant $\frac{1}{5}L, \frac{6}{5}L$ from A.
Find in the position of equilibrium, and tension magnitude in the two strings and the measure of the angle of inclination of the rod to the horizontal.
- ABC is an equilateral triangle of side length 10 cm the weights 3,6,9 act at its vertices. Identify the position of the center of gravity of the system.

Question 5

- In the opposite figure, A force of $25\sqrt{6}$ newton acts at \vec{EM} where M the center of face ABCD. Find the components of the moment of the force about the coordinate axes.
- A fine lamina of uniform density in the form of a rectangle ABCD in which $AB = 5$ cm, $BC = 12$ cm, $E \in \overline{AD}$ such that $\overline{AE} = 5$ cm. The triangle ABE is bent about side \overline{BE} until \overline{AB} is coincident on \overline{BC} totally. Identify the center of gravity of the lamina after bending it with respect to \vec{CB} , \vec{CD} .



Test 2

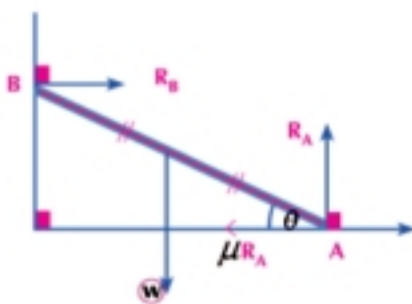
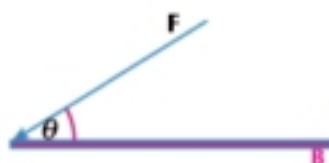
First : Answer the following question:

Question 1: Choose the correct answer

- Two couples act on a body; the magnitude of one of the two forces of the first couple is 20 kg.wt, the moment arm is $\frac{1}{2}$ m and the direction of its rotation is in anti clockwise direction while the magnitude of one of the two forces of the second couple is 30 kg.wt, moment arm is 1 m and the direction of its rotation is in the clockwise direction, then the resultant couple is equal to:
 - 20 kg.wt.m and the direction of its rotation is in the clockwise direction.
 - 20 kg.wtm and the direction of its rotation is in the anti-clockwise direction.
 - 40 kg.wt.m and the direction of its rotation is in the clockwise direction.
 - 40 kg.wt.m and the direction of its rotation is in the anti-clockwise direction.

ACCUMULATIVE TEST

- 2 The angle of friction is:
- The angle included between the normal reaction and the resultant reaction in the case of limiting friction.
 - The ratio between the force of limiting friction and the normal reaction.
 - The ratio between the coefficients of static and kinetic friction.
 - The angle included between the force of the limiting friction and the resultant reaction.
- 3 The opposite figure illustrates a force of magnitude F acts on an end of a rod. The measure of angle θ which generates the maximum moment about point B is:
- 0°
 - 90°
 - 45°
 - 30°
- 4 Two unlike forces; the magnitude of one of them is 7 newtons and the magnitude of their resultant is 10 newtons, then the magnitude of the other force is.
- 3 newtons
 - 17 newtons
 - 27 newtons
 - 6 newtons
- 5 In the opposite figure:
If λ is the angle of friction between the ground and the rod, then $\tan \theta \cdot \tan \lambda =$
- 2
 - $\frac{1}{2}$
 - 1
 - 3
- 6 A mass of 5 kg acts at point $(2, -1)$ and a mass of 7 kg acts at point $(1, 2)$ then the center of gravity of the two masses acts at point
- $(17, 9)$
 - $(\frac{17}{12}, \frac{3}{4})$
 - $(19, 13)$
 - $(\frac{19}{12}, \frac{1}{4})$



Second: answer three questions:

Question 2

- 1 If the force $\vec{F} = 5\hat{i} - \hat{j} + 3\hat{k}$ acts at point $A(-1, 2, 1)$ find:
First: the moment of the force \vec{F} about the origin point.
Second: the length of the perpendicular drawn from the origin point on the line of action of \vec{F} .
- 2 Prove that if a body is placed on a rough inclined plane, and the body is about to slide, then the measure of the angle of friction is equal to the measure of the angle of inclination of the plane to the horizontal.

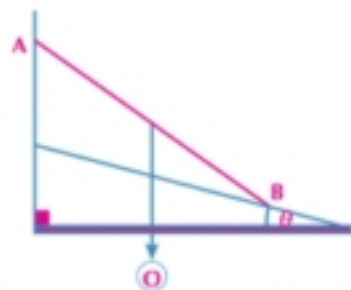
ACCUMULATIVE TEST

Question 3

- Three bodies of weights 5, 7, 11 kg.wt are placed on a light rod as shown in the figure. Identify the suspension point on the rod such that the rod remains horizontal.
- ABCD is a rectangle in which $AB = 6$ cm, $BC = 8$ cm, $E \in \overline{BC}$, $F \in \overline{AD}$ such that $BE = DF = 6$ cm. The forces of magnitudes 5, 5, 7, 7, F, F kg.wt act at the direction of \vec{AB} , \vec{CD} , \vec{BC} , \vec{DA} , \vec{EA} , \vec{FC} , respectively if the system is equivalent to a couple whose moment magnitude is 10 kg.wt.cm in the direction of CBAD, find F.

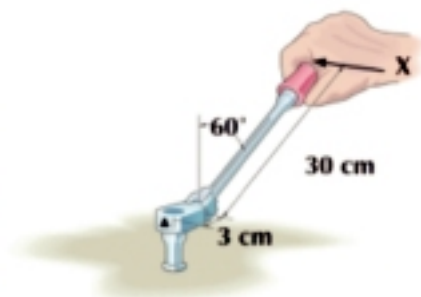
Question 4

- In the opposite figure, the top of a uniform ladder weight (W) is leaning against a smooth vertical wall and its base is leaning against rough ground inclined to the horizontal at an angle of measure (θ) upwards. If the ladder is about to slide while it is in a vertical plane perpendicular to the intersection line of the wall and ground, prove that the ladder is inclined at an angle of measure α to the vertical where $\tan \alpha = 2 \tan (\lambda - \theta)$ where λ is the angle of friction.
- A uniform rod of the length $15L$ is bent from point B where $\overline{AB} = 15L$ such that $m(\angle ABC) = 90^\circ$ and a rod is suspended freely from end A. Prove that \overline{BC} is inclined at an angle of θ where $\tan \theta = \frac{4}{5}$ to the horizontal.



Question 5

- In the opposite figure:
If the moment of the force F perpendicular to the rotation arm about point A is equal to 620 newtons cm, find F
- ABC is an equilateral triangle of side length 2 cm, M is the intersection point of its medians, D is the midpoint of \overline{BC} , masses of magnitudes 15, 30, 75, 45, 45 are fixed at the points A, B, D, C, M respectively. Identify the center of gravity of this system. Where does the center of gravity of the remaining system lie if the mass existed at B is lifted.

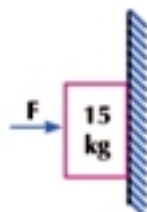


Test 3

First : Answer the following question:

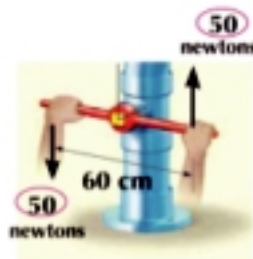
Question 1: Complete

- The magnitude of the least horizontal force \vec{F} needed to equilibrate a body of mass 15 kg.wt on a rough vertical plane the coefficient of the static friction between it and the body is equal to $\frac{1}{5}$ is equal to _____ kg.wt.



ACCUMULATIVE TEST

- 2 A force of magnitude 70 newtons acts at \overline{AB} where ABCD is a square of side length 10cm, then the magnitude of the moment of the force about the center of the square is equal to.....
- 3 If $\vec{F}_1 \parallel \vec{F}_2$, $\vec{F}_1 = \hat{i} - 2\hat{j}$, $\|\vec{F}_2\| = 4\sqrt{5}$ unit, then $\vec{F} =$
- 4 In the opposite figure: the moment of the couple resulted from the two forces 50, 50 is equal to.....
- 5 When a rod is placed in a smooth spherical container, it is in equilibrium when the line of action of weigh passes
- 6 The center of gravity of the triangular uniform lamina lies at



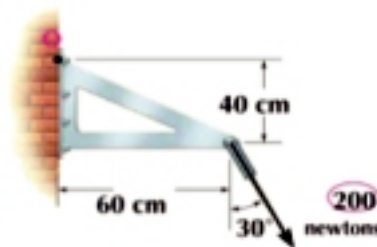
Second : Answer three question:

Question 2

- 1 A body of weight $66\frac{2}{3}$ newtons is placed on a rough horizontal plane. The coefficient of friction between them is equal to $\frac{3}{4}$. A force of magnitude 40 newtons acts on the body and inclined at an angle of measure θ where θ is acute. What is the value of θ if the body is about to move?
- 2 In the opposite figure, find the moment of the force 200 newtons about O.

Question 3

- 1 a AB is a non uniform rod of length 1m is rests in a horizontal position on two supports at C, D where AC = 20 cm, BD = 10 cm. If the heaviest weight can be suspended at point A or point B without disturbing the rod is 5,4 kg.wt respectively, find the weight of the rod and its point of action.



- 2 ABCDEF is a uniform hexagon of side length 10 cm. The forces of magnitudes 2, 5, 4, 6, 1, 3 newtons acts at \overline{AB} , \overline{CB} , \overline{CD} , \overline{ED} , \overline{EF} , \overline{AF} respectively. Find the magnitude and direction of the force which should act at the center of the hexagon in order to reduce the system to a couple, then identify its moment

Question 4

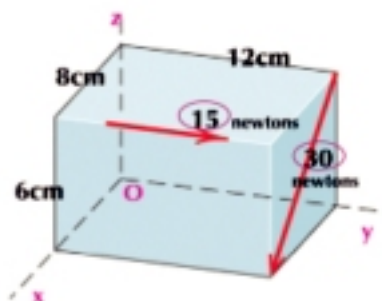
- 1 AB is a uniform rod of weight (W) is leaning by one of its ends A on smooth horizontal ground and its end B on a smooth plane inclined to the horizontal at an angle twice the measure of the angle of inclination of the rod to the horizontal in an equilibrium. The rod is being kept by a string one of its ends is connected to the end of the rod leaning on the horizontal ground and the other end of the string is in a point on the intersection line of the inclined plane with the horizontal plane. Find the magnitude of tension in the string and the two reactions at the two ends of the rod when the rod inclines at 30° to the horizontal.
- 2 ABCD is A fine uniform lamina in the form of a square of side length L. E, F, N are the midpoints of \overline{AB} , \overline{AD} , \overline{BC} respectively. The triangle AEF is bent about \overline{EF} such that A is coincident on the center of the square M and the triangle BEN is bent on \overline{EN} such that the vertex B is coincident on the center of the square M. Identify the center of gravity of the

ACCUMULATIVE TEST

lamina in its new position.

Question 5

- In the opposite figure: find the sum of moments of the forces about O
- Find the center of gravity for the following distributions:
 $W_1 = 20$ newtons acts at $(2, 1)$, $W_2 = 15$ newtons acts at $(-3, 1)$ $W_3 = 25$ newtons acts at $(1, -1)$

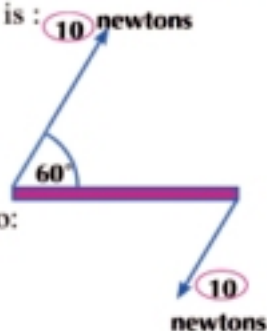
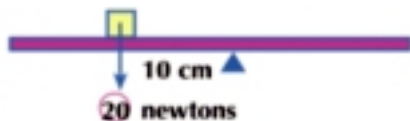


Test 4

First: Answer the following question:

Question 1: choose the correct answer

- The coefficient of friction is based upon:
 - The area of the contact surface between two bodies
 - Shape of the two bodies
 - Nature of the two bodies
 - All mentioned
- The opposite figure represents a uniform rod leaning on a support at its midpoint. A body is placed on the rod as shown in the figure which of the following forces makes the rod be in equilibrium:
 - A force of magnitude 10 newtons upwards act on a distance 20 cm on right of the rod midpoint.
 - A force of magnitude 10 newtons downwards acts on a distance 20 cm on the right of the rod midpoint.
 - A force of magnitude 30 newtons upwards acts on a distance 5 cm on the left of the rod midpoints.
 - A force of magnitude 30 newtons downwards acts on a distance 5 cm on the left of the rod midpoint.
- The force $\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$ acts at point A whose position vector about the origin point is $A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$ then the component of moment of \vec{F} about x-axis is:
 - $F_x \times A_z - F_z \times A_x$
 - $F_y \times A_z + F_z \times A_y$
 - $F_x \times A_y + F_y \times A_x$
 - $F_y \times A_y + F_z \times A_z$
- The algebraic measure of the moment of the opposite couple is equal to:
 - 800 newtons . cm
 - 800 newtons .cm



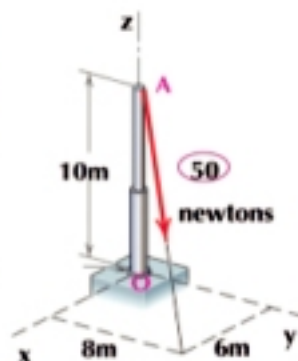
ACCUMULATIVE TEST

Question 4:

- AB is a uniform rod. Its end A is connected to a hinge fixed in a vertical wall and end B is connected by an end of a string and the other end of the string is connected by a point in the horizontal plane passing through the hinge such that each of the rod and the string inclined at an angle of measure θ to the horizontal. If (W) is the weight of the rod, show that the reaction of the hinge at A is equal to $\frac{W}{4} \sqrt{8 + \csc^2 \theta}$.
- ABCD is a square of side length 20 cm. Four masses equal in magnitudes are placed at its vertices:
First: Identify the center of gravity of the system
Second: Where does the center of gravity of the remaining system lie if the mass placed at a one of the vertices is ceased?

Question 5:

- ABC is a lamina in the form of an equilateral triangle whose weight is 3 kg and M is its center of gravity. Forces of magnitudes 2, 2, 11 kg are placed at the vertices A, B, C respectively. Prove that the center of gravity of the system lies at the midpoint of \overline{MC} .
- In the opposite figure: A force of magnitude 50 newtons acts at point A. Find the moment of the force about point O.



Test 5

First: Answer the following question

Question 1: Complete:

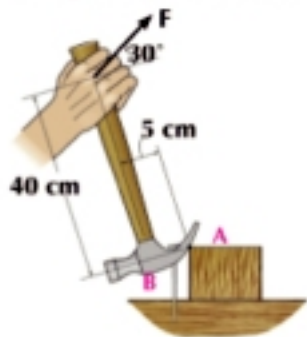
- The coefficient of the static friction is the ratio between _____
- If the force $\vec{F} = 2\hat{i} - \hat{j} + 5\hat{k}$ acts at point A whose position vector is $\hat{i} - 3\hat{k}$ then the moment of \vec{F} about point B whose position vector is $\hat{j} + 3\hat{k}$ is equal to _____
- Two like forces; the magnitude of one of them is twice the magnitude of the second and the magnitude of their resultant is 31 newtons, then the magnitude of the smaller force is equal to _____
- If the two forces $\vec{F}_1 = A\hat{i} + 5\hat{j}$, $\vec{F}_2 = 3\hat{i} - B\hat{j}$ form a couple, then $A + B =$ _____
- The necessary and sufficient condition for the equilibrium of a system of coplanar forces _____
- The center of gravity of the rigid body suspended freely on the vertical straight line passing through _____

ACCUMULATIVE TEST

Second: Answer three questions of the following:

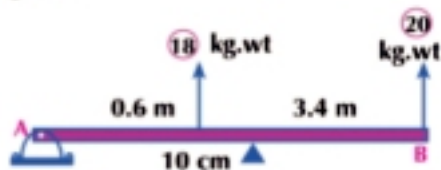
Question 2:

- 1 A body of weight 50 newtons is placed on a rough inclined plane inclined at an angle of measure θ to the horizontal. The least and greatest forces parallel to the line of the greatest slope and makes the body in equilibrium on the plane are 10, 40 newtons respectively. Find the coefficient of friction and the measure of the angle of inclination of the plane to the horizontal.
- 2 The opposite figure illustrates the force F needed to remove a nail at B. If the magnitude of the moment of the force about point A needed to remove the nail is equal to 200 newtons.cm. Find the magnitude of the force F .



Question 3

- 1 If the resultant of three forces act on the rod AB of negligible weight in the figure is 13.6 kg.wt and acting upwards distant 3 meters on the right of A. Find the magnitude direction and point of action of the third force
- 2 ABCE is a rectangle which $AB = 12$ cm, $BC = 9$ cm, $M \in \overline{BC}$ such that $BM = 4$ cm. Forces of magnitudes $F_1, 8\sqrt{10}, 26, F_2, 18$ newtons in the directions $\overrightarrow{BA}, \overrightarrow{AM}, \overrightarrow{MD}, \overrightarrow{DC}, \overrightarrow{DA}$ respectively. If the system of forces is in equilibrium, find the value for each of F_1, F_2 .



Question 4

- 1 AB is a uniform ladder of length 5 m and weight 20 kg.wt rests with its end A on a smooth vertical wall and on rough horizontal ground with its end B and the coefficient of friction between them is $\frac{1}{4}$ and the end B is distant 3m from the wall. Prove that the ladder is not in equilibrium in such a state, then find the smallest weight of the body which the coefficient of friction between it and the ground is $\frac{1}{5}$ such that if it is placed at the end B of the ladder, it stops the ladder from sliding.
- 2 A uniform wire of length 100 cm is bent in the form of five sides of a uniform hexagon ABCDEF and starts from point A. Identify the distance between its center of gravity and that of the hexagon. If the wire is freely suspended from end A, identify the measure of the angle of inclination of \overline{AB} to the vertical in the equilibrium state.

Question 5:

- 1 \overline{AB} is a uniform rod of length 2 m and weight 5 newtons. C,D are two trisection points from direction A. Weights of magnitudes 1, 2, 3, 4 newtons are connected at point A,C,D,B respectively. Identify the center of gravity of the system.
- 2 Two forces $\vec{F}_1 = 2\hat{i} - \hat{j}$, $\vec{F}_2 = \hat{j} - 2\hat{i}$ act at two points $A(1, 1)$, $B(0, -4)$ respectively. Find the moments of the system about any point on the plane.

ACCUMULATIVE TEST

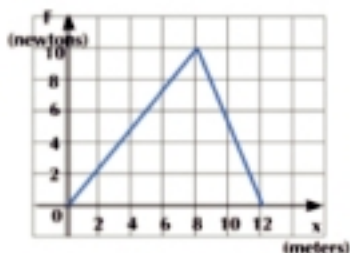
Second : Dynamics

Test 6

First: Answer the following question:

Question 1: Complete:

- The momentum of a body of mass 700 gm moving in a straight line starting with velocity of magnitude 15 m/sec and with a uniform acceleration 2.5 m/sec^2 in the same direction of its initial velocity after 12 sec from the beginning of motion is equal to _____ kg.m/sec.
- A body of mass unity moves under the action of the force $\vec{F} = (A + 3)\hat{i} + B\hat{j}$ if its displacement vector $\vec{S} = t^2\hat{i} + \frac{1}{2}t^2\hat{j}$ then $A = \text{_____}$, $B = \text{_____}$.
- If A child of mass 35 kg stands on a pressure scale inside a lift moving downwards with acceleration of magnitude 1.4 m/sec^2 then the scale reading = _____ kg.wt.
- The opposite figure illustrates the relation between the force \vec{F} by which a child acts horizontally on a box of mass 10 kg to move on a smooth surface with the distance component which the box travels in the direction of x , then the work done by \vec{F} on the box from $x = 0$ to $x = 8$ is equal to _____ the work done by \vec{F} on the box from $x = 8$ to $x = 12$.
- A body is projected horizontally with velocity 2.8 m/sec on a rough horizontal plane and the coefficient of friction between it and the body is $\frac{1}{10}$, then the distance traveled by the body on the plane before it rests is equal to _____ meters.
- In the opposite figure: the small pulley and the plane are smooth. If the system moves from rest, then the magnitude of the acceleration of the system = _____ m/sec^2 .



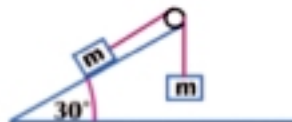
Second: Answer three questions

Question 2:

- An locomotive of mass 30 tons starts to move from rest on a horizontal plane with a uniform acceleration against resistances $\frac{1}{100}$ of its weight. When its velocity reaches 90 km/h, its magnitude becomes 441 kilowatt. Find:
 - The force of the engine in kg.wt.
 - The magnitude of the uniform acceleration.
- A force of magnitude 20 newtons and its direction making acute angle of $\sin^{-1}\frac{3}{5}$ to the vertical downwards acts on a body of mass 2 kg placed on a smooth horizontal table. Find the acceleration of the body generated from this action and the perpendicular reaction of the table.

ACCUMULATIVE TEST

- 2 A body of mass 500 gm is projected vertically upwards from a point on the ground surface with velocity 14.7 m/sec, then its potential energy after one second from projection = joule
- 3 A body moves with a uniform velocity in a straight line under the action of the forces $\vec{F} = 2a \hat{i} - 3 \hat{j}$, $\vec{F}_2 = 6 \hat{i} + b \hat{j}$, $\vec{F}_3 = a \hat{i} + 5 \hat{j}$ then $a = \dots\dots\dots$, $b = \dots\dots\dots$
- 4 In the opposite figure: the plane and pulley are smooth. When this system moves, then its acceleration = m/sec².
- 5 If the work done by the force $\vec{F} = m \hat{i} + 4 \hat{j}$ during the displacement of its point of action $\vec{S} = -3 \hat{i} + (m + 1) \hat{j}$ is equal to 0.05 joule, $\|\vec{S}\|$ in cm where m is constant, then the value of $m = \dots\dots\dots$
- 6 A body is suspended in a hook of a spring scale fixed at the top of a lift moving vertically upwards and the apparent weight of the body is twice the actual weight, then the acceleration $a = \dots\dots\dots$ m/sec²



Answer three questions of the following:

Question 2:

- 1 A man of weight 72 kg.wt ascends a road inclined at an angle of $\sin^{-1} \frac{1}{4}$ to the horizontal to travel 100 m. Calculate the change in the potential energy of the man.
- 2 A locomotive of mass 30 tons and force 56 ton.wt pulls a number of wagons each of mass 10 tons to ascend a slope inclined at 30° to the horizontal with a uniform acceleration 49 cm/sec². How many cars are there if the resistance to the motion of the engine and cars is 10 kg.wt per each ton of mass?

Question 3:

- 1 A worker at a factory pushes a box of mass 30 kg a distance of 4 m with a uniform velocity on a horizontal surface. If the coefficient of friction between the box and the surface is $\frac{1}{4}$, calculate the work done by the worker on the box, then calculate the work done by the reaction.
- 2 A body of mass 35 gm is placed on a smooth horizontal table and connected by a light string passing over a smooth pulley fixed at the edge of the table and the other end of the string is connected by another body of mass 14 gm vertically. Find:
 First : the common acceleration of the system and the tension in the string, then identify the pressure on the axis of the pulley in gm.wt unit.
 Second : if the string is cut off after $1\frac{1}{2}$ seconds from the beginning of the motion, find the distance which both the two bodies travel after $\frac{1}{2}$ of a second from the instant of cutting the string.

ACCUMULATIVE TEST

Question 4:

- 1 A train car of mass 20 tons descends a slope from rest making an angle of $\sin^{-1} \frac{1}{70}$ to the horizontal against resistances of magnitude 14 kg.wt per each ton of its mass to reach the bottom of the slope after it traveled a distance of 350 m on it. At the bottom of the slope, this car collided with another rested car of the same mass to move together as one body on a horizontal road. If the two cars rest after one minute from the collision instant, find the horizontal distance which both cars moved together.
- 2 A zeppelin moves vertically upwards when it was at a height of 40.4 m above the ground. A body of mass 5 kg fell. If the kinetic energy of the body at the instant it collides with the ground is equal to 2940 joules, supposing the air resistance is negligible, calculate:
First: the velocity of the zeppelin at the instant the body fell.
Second: the distance traveled by the body from the instant of collision until it rests.

Question 5:

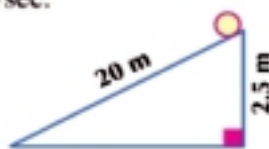
- 1 A car of mass 3 tons moves with its maximum velocity of magnitude 27 km/h ascending a slope inclined at an angle of $\sin^{-1} \frac{1}{30}$ to the horizontal. Then the car descends the same slope with its maximum velocity of magnitude 72 km/h. Find the resistance, supposing it is constant, then calculate the power of the engine in horse.
- 2 A simple pendulum is made up of a string of length $1\frac{1}{2}$ m. Its upper end is fixed and its lower end is connected by a body of mass 500 gm and suspended vertically. If the body is pulled by a horizontal force until it becomes inclined at 60° to the vertical, find:
First: The change in the potential energy of the body in joule
Second: The work done by the force in joule
Third: The velocity of the body at the midpoint of the pathway if the horizontal force is ceased and the body is let to oscillate.

Test 8

First : Answer the following questions

Question 1: Complete:

- 1 In an instant, the momentum of a body is 112 kg.m/sec and its kinetic energy is 80 kg.wt.m then the mass of the body = _____ kg, its velocity = _____ m/sec.
- 2 A body of mass 300 gm moves in a straight line whose displacement vector is $(t^2 + t + 1) \vec{e}$ where $\|\vec{S}\|$ in cm, t in seconds, then the magnitude of the force acting on it = _____ dyne.
- 3 A body of actual weight 28 newtons and apparent weight 32 newton in regard to the spring scale reading inside a lift moving in a uniform deceleration, then the direction of the motion is _____ and the direction of the acceleration is _____.
- 4 The vertical distance between two bodies connected by the end of a light string passing over a smooth pulley fixed and suspended vertically is 100 cm after 2 seconds from the beginning of the motion. Then the velocity of each on that instant = _____ cm/sec.
- 5 In the opposite figure: A smooth inclined plane of length 20 m and height 2.5 m. A body is placed at the top of the plane and is let to descend on the plane, then it reaches the base of the plane with velocity = _____ m/sec².



ACCUMULATIVE TEST

- 6 A body of mass 200 gm is projected vertically upwards with velocity 49 m/sec, then its potential energy at the maximum height the body reaches = _____ joule

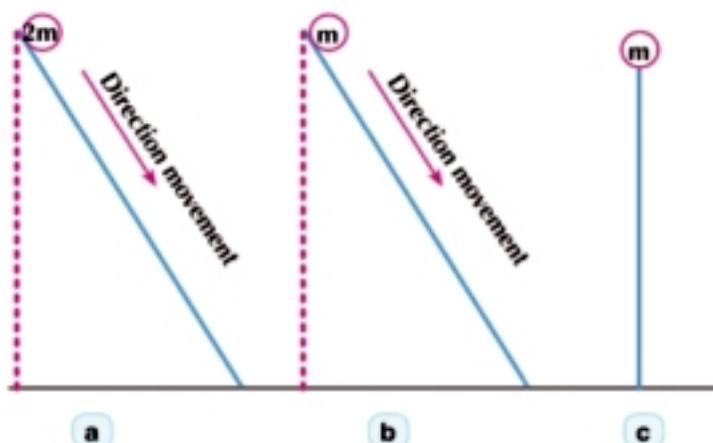
Second: Answer three questions of the following:

Question 2:

- 1 The opposite figure shows three masses m , m , $2m$ move from up to down from rest (suppose the air resistance and friction are negligible).

First: Which of the three masses reaches the ground with greatest velocity?

Second: Which of the three masses does the most work to reach the ground?



- 2 A force of 5 kg.wt acts on a mass of 196 kg moving in a horizontal straight line in the direction of the force to travel a distance 2.8 m. Calculate the magnitude of the increase in the kinetic energy of the body in joule. If the kinetic energy of the body at the end of the distance is 141.12 joule, calculate the initial velocity of the body.

Question 3:

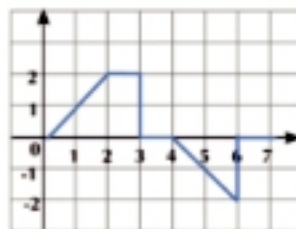
- 1 A body of mass 170 gm is placed on a rough inclined plane inclined at an angle of $\sin^{-1} \frac{8}{17}$ to the horizontal is connected by a string passing over a smooth pulley at the top of the plane and a weight is suspended from the other end. If the least needed weight to be hanged from this end of the string to keep the equilibrium of the body on the plane is 70 gm.wt, find the resistance of plane in gm.wt. if a weight of magnitude 194 gm is connected by the other end. Find the acceleration of the system supposing the resistance is constant in both cases.
- 2 The power of a car's engine is constant and its maximum velocity as it ascends a slope is 54 km/h and its maximum velocity as it descends the same slope is 108 km/h. Find the maximum velocity by which the car moves on the horizontal plane given that the resistance of the road to the motion of the car is constant in the three cases.

Question 4:

- 1 A ball of mass 200 g moves with velocity 7 m/ sec collides with a rested ball of mass 300 gm and both move together as one body.
- Find the common velocity for both balls directly after collision.
 - The kinetic energy lost by collision.
 - The distance at which the body rests if it encounters resistance 200 w.gm.

ACCUMULATIVE TEST

- 2 In the opposite figure, \vec{F} acts on a baby stroller of mass 2 kg moving in a straight line parallel to x-axis. The component x changes by the change of the force as illustrated in the figure. Calculate the work done by this force when:



- a $x = 0$ to $x = 3$ m b $x = 3$ m to $x = 4$ m
c $x = 4$ m to $x = 7$ m d $x = 7$ m to $x = 2$ m

Question 5:

- 1 A body of variable mass moves in a straight line and its mass at any instant t is $m = (4t + 1)$ gm and its displacement vector is given by the relation $\vec{S} = (t^2 - 2t)\vec{e}$, t in second, $\|\vec{S}\|$ in cm. Find its momentum in the time interval $[3, 5]$.
- 2 To identify the magnitude of the gravitational acceleration in a place, a body of mass 1.5 kg is suspended in the hook of a spring scale fixed at the top of a lift. If the scale reading recorded 15.5 newtons when the lift was ascending with acceleration a m/sec² and 12.75 newtons when it was descending with acceleration a m/sec², calculate the gravitational acceleration in this place and the acceleration of the lift ascending.

Test 9

First: Answer the following question

Question 1: Complete:

- 1 A body of mass 5 mass units moves under the action of the force $\vec{F} = (a + 1)\hat{i} + (b - 2)\hat{j}$ and its displacement vector at any instant is given by the relation $\vec{S} = t^2\hat{i} + (\frac{1}{2}t^2 + 3t)\hat{j}$ then $a = \dots$, $b = \dots$
- 2 In the opposite figure: A smooth horizontal plane, then the pressure on the pulley = gm.wt
- 3 A bullet of mass 98 gm moves horizontally with velocity 720 km/h to embed in a vertical barrier a distance of 10 cm before it rests, then the average resistance of the barrier = kg.wt.
- 4 A ship of mass 441 tons moves with velocity 72 km/h, then its kinetic energy = kilowatt. h
- 5 An engine does work of magnitude 15000 kg.wt.m during 10 seconds, then the power of the engine in horse
- 6 A force of magnitude 80 newtons acts in the direction of 30° northeast, then the work done by this force during a displacement of magnitude 40 m towards north is equal to joule.



Second: Answer three question of the following:

Question 2:

- 1 A cyclist moves on a rough horizontal road with a uniform acceleration. The kinetic energy is changed with magnitude 107800 joules during $\frac{1}{2}$ km. Then the cyclist stops pedaling to travel 100 m during which the kinetic energy is lost with magnitude 7840 joules. Find each of resistances and force in kg.wt.

ACCUMULATIVE TEST

- ② Two pans each of mass 35 gm connected by a light inelastic string passing over a smooth small pulley. A body of mass 280 gm is placed in a pan and another body of mass m is placed in the second pan. If the pan holding the mass of 280 gm is pulled down a distance of 560 cm from rest in two seconds, find:

First: The acceleration of the system **Second:** The tension in the string and the value of m

Third: The pressure exerted on both pans

Question 3:

- ① A ball of mass 200 gm is projected with velocity 21m/sec on a horizontal plane against resistances equivalent to $\frac{1}{14}$ of its weight. After 10 seconds, the ball collides with another ball equal in mass and moves with velocity 7m/sec in the opposite direction. If the two balls move as one body after collision, calculate:

First: The common velocity of the two balls after collision

Second: The impulse of each ball on the other

Third: The kinetic energy lost due to collision

- ② At a factory, the boxes are transferred by sliding them on an inclined plane ends in a horizontal plane. If the length of the inclined plane is 40 m, inclined at 30° to the horizontal and the resistance for each of the two planes is equivalent to $\frac{1}{5}$ of the weight of the body, find the velocity of the box at the end of the pathway supposing its velocity does not change by transferring on the horizontal plane if the length of the horizontal part is 10 m.

Question 4:

- ① A force of magnitude 12.6 newtons acts on a rested body placed on a horizontal plane for a period of time to acquire kinetic energy of magnitude 9 kg.wt by the end of this time. At this instant, the momentum of the body reaches 42 kg.m/sec, then this force is ceased and the body returns back to rest once again after it traveled a distance of 21m from the instant of ceasing the force. Find the mass of the body and the resistance of the plane to the motion of the body in newton supposing it is constant, then find the time of action of this force.
- ② A body is suspended in a spring scale attached in the top of a lift to record 80 kg.wt when the lift was ascending with a uniform acceleration c m/sec² and records 60 kg.wt when it was ascending with uniform deceleration of magnitude c m/sec², find the mass of the body and the value of a .

Question 5:


- ① The power of an engine is 1080 horses and its mass is 50 tons pulling a train of mass 130 tons on a rough horizontal plane with acceleration 49 cm/sec². If the air resistance and friction are equivalent to 10 kg.wt per each ton of the mass, calculate the maximum velocity the train can travel in km/h.
- ② A worker pushes a car of mass 20 km to ascend a plane inclined at 25° upwards with force of magnitude 140 newtons to the horizontal. If the coefficient of friction between the plane and the car is $\frac{3}{10}$ and the car moves a distance of 3.8 m. Calculate the total work done by the car if the car moves downwards the plane from rest. Calculate the velocity of the car when it is distant 3.8 on the plane.

ACCUMULATIVE TEST

Test 10


First: Answer the following questions:

Question 1: Complete

- 1 A horse pulls a wooden mass on horizontal ground with force of magnitude 100 kg.wt and inclined at 30° to the horizontal. If this mass moves with a uniform velocity, then the magnitude of the ground resistance to its motion = _____ kg.wt.
- 2 If a constant force of magnitude 5 kg.wt acts on a rested body of mass 49 kg for 3 seconds, then the velocity of the body by the end of this time = _____ m/sec.
- 3 In the opposite figure, 3 m and 3m are two masses connected by two ends of a string passing over a smooth small pulley. An additional mass m is attached to one of the two masses. If the system is let to move from rest, then the velocity of the system after 2 seconds = _____ cm/sec. 
- 4 A shell of mass 45 gm moves with a uniform velocity of magnitude 1440 km/h, then its kinetic energy = _____ joules.
- 5 A machine does work at a uniform rate = 18000 kg.wt. m per minute, then the power of the machine = _____ horses.
- 6 A ball of mass 300 gm moves horizontally to collide with a vertical wall when its velocity is 60 m/sec. If it rebounds after losing $\frac{2}{3}$ of its velocity, then the change of its momentum due to the collision with the wall = _____ gm.cm/sec.

Second : Answer three questions of the following:

Question 2:

- 1 A body of mass 1 kg moves under the action of the forces $\vec{F}_1 = b \hat{i} + 2 \hat{j}$, $\vec{F}_2 = 2 \hat{i} + \hat{j}$, $\vec{F}_3 = -3 \hat{i} + a \hat{j}$ where \hat{i} , \hat{j} are two perpendicular unit vectors, $\|\vec{F}_1\|$, $\|\vec{F}_2\|$, $\|\vec{F}_3\|$ are measured in newton, a, b are two constants. If the displacement vector $\vec{S} = t^2 \hat{i} + (2t^2 - t) \hat{j}$ where S is in meter and t in second, find:
 First: The value of the two constants a, b
 Second: The work done by the resultant of the mentioned forces during the first ten seconds of the body's motion.
- 2 In the opposite figure, two masses of 40 gm, 30 gm connected by the two ends of a light string passing over a smooth small pulley fixed at the top of two smooth opposite planes inclined at 30° to the horizontal as shown in the figure. The system is being kept in equilibrium when the two bodies are on one horizontal line and the two parts of the string are tensioned. If the system is let to move from rest, find the acceleration and the vertical distance between the two bodies after one second from the beginning of the motion. 

Question 3:

- ① A locomotive moves horizontally under the action of a resistance proportional to the square of its velocity and the resistance is equal to 450 kg.wt. When the velocity of the locomotive is 30 km/h, calculate the maximum velocity of the locomotive if the power of the engine is 400 horses.
- ② An armored shield is made of two soldered layers of uniform thickness of iron and copper. The iron thickness is 1 cm and the copper thickness is 3 cm and the shield is in a vertical plane when two bullets of equal masses are fired in two opposite directions and perpendicular to the shield plane with the same velocity. The first bullet penetrated the iron layer and embedded in the copper layer $\frac{5}{4}$ cm, where as the second bullet penetrated the copper and embedded in the iron $\frac{3}{4}$ cm. Prove that the iron resistance = 7 times the copper resistance.

Question 4:

- ① When doing the foundations of a building, a hammer of mass 480 kg is used to fall vertically from a height of 2.5 m down on a construction column of mass 120 kg to form one body embedded in the ground for a distance of 24 cm. Find:

First: The common velocity of the hammer and the column directly after collision.

Second: The impulse of the hammer to the cylinder.

Third: The average resistance of the ground surface to the hammer and the column
- ② A body is placed on the highest point of a slope of height 125 cm and inclined at 30° to the horizontal. The body moves in the direction of the line of the greatest slope to the plane downwards against a constant resistance estimated $\frac{1}{4}$ the weight of the body. Calculate the velocity by which the body reaches the lowest point of the plane. What is the velocity by which the body is projected by to hardly reach the top of the plane?

Question 5:

- ① A body of mass 42 gm placed on a rough plane is pulled by a rope inclined at an angle of $\sin^{-1} \frac{4}{5}$ to the horizontal. If the tension force in the rope is 10 gm.wt performed work of magnitude 84 gm.wt.cm during 2 seconds from the beginning of the motion, find:

First: The acceleration of the body

Second: The ratio between the plane resistance and the perpendicular reaction.
- ② A child stands on a pressure scale inside a lift ascending upwards with acceleration 1.96 m/sec^2 and the scale records 24 kg.wt. Find the weight of the child. If the lift descends downwards, in the same acceleration on, find the scale reading in this case.

First: Statics:**Unit 1: Friction****Answers of exercises (1 - 1)**

- ① Friction ② Smooth
 ③ is about to move
 ④ Normal reaction
 ⑤ The resultant reaction
 ⑥ Normal reaction
 ⑦ 18 kg.wt ⑧ $\frac{2}{3}$
 ⑨ $R = 56$, $\mu = 0.75$ ⑩ 70.86 kg.wt
 ⑪ $F = 13$ gm.wt, $m = \frac{1}{3}$
 ⑫ The body is about to move
 ⑬ $R =$ weight of the box = 40 kg.wt
 ⑭ $F = 15$ newtons, $F_s = 9$ newtons

Answers of exercises (1 - 2)

- ① \times ② \checkmark ③ \checkmark ④ \times
 ⑤ \checkmark ⑥ \times ⑦ A ⑧ A
 ⑨ A
 ⑩ 32 newtons
 ⑪ The body is about to move
 ⑫ The friction is not limiting, 5 kg.wt
 ⑬ 25.0453 newtons ⑭ $R = W$, $\theta = \lambda$
 ⑮ $\theta = 30^\circ$, $\mu = \frac{\sqrt{3}}{15}$ ⑯ 19 kg.wt
 ⑰ $\frac{1}{\sqrt{3}}$, $F = 4\sqrt{3}$ kg.wt, $2\sqrt{3}$ kg.wt
 ⑱ $\frac{5}{3}\sqrt{3}$ kg.wt, $\frac{4}{3}\sqrt{3}$ kg.wt
 ⑲ $\tan^{-1}\frac{3}{4}$ ⑳ $F = O \times \sin(\theta + L)$

Answers of general exercises

- ① a ② b ③ c
 ④ The body is not about to move.
 ⑤ a $15\sqrt{3}$ kg.wt b $R = 30\sqrt{3}$ kg.wt
 ⑥ $\frac{1}{2}$ ⑦ 10 kg.wt
 ⑧ $r <$ the component of weight, the body is in equilibrium
 $F = 1.2$ newtons

Unit 2 : (Moments)**Answers of exercises (2 - 1)**

- ① 400 ② 540 ③ $20\hat{V}$
 ④ the force passes through this point
 ⑤ the distance from the point to the line of action of the force
 ⑥ 35 ⑦ b ⑧ a $L \sin \theta$
 ⑨ $\vec{X} \parallel \overline{AB}$ ⑩ $L = \frac{-7}{9}$, $m = \frac{13}{9}$
 ⑪ the line of action of the resultant is parallel to the straight line passing through the two points (2, 1), (6, 4)
 ⑫ 149.7689 newtons, meters
 ⑬ a -1200 newtons .cm
 b -1175.877 newtons .cm
 c $600\sqrt{2}$ newtons .cm
 ⑭ (-46, 28) newtons

Answers of exercises (2 - 2)

- ① a $-11\hat{i} + 9\hat{j} + 5\hat{V}$
 b $-11\vec{X} + 5\vec{Y} - 7\vec{V}$, 3.73
 ② $L = 3$ ③ $120\vec{Y} + 480\vec{V}$
 ④ $-120\vec{x} + 60\vec{V}$ ⑤ $\vec{X} = 7\vec{X}$
 ⑥ $B = -3$, perpendicular length = $\frac{\sqrt{27}}{\sqrt{14}}$
 ⑦ $480\vec{Y} + 360\vec{V}$ ⑧ $V = 1$, $x = 1$
 ⑨ 150 ⑩ 1328.43

Answers of general exercises

- ① 18.199 newtons
 ② $-56\vec{X} - 28\vec{Y} + 28\vec{V}$
 ③ 11 newtons ④ $-28\vec{V}$
 ⑤ $-7\vec{V}$ ⑥ $K = 2$
 ⑦ -3260.77 newtons, m
 ⑧ The resultant of the force passes through point C
 ⑨ $200\sqrt{2}$ newtons, m

Answer of accumulative test

- 1 $10 \cos \theta$ 2 5
 3 $(2, \frac{\pi}{3})$ 4 $\frac{-2}{3} \cdot \frac{2}{3} \cdot \frac{1}{3}$
 5 $8 \sin 70$ 6 4
 7 $(-4, 7, -6)$
 8 a -130 b -30 c -80
 9 $\frac{-75\sqrt{3}}{2}$ newtons . m
 10 6485.19 newtons . cm
 11 The least value of the force F when $\sigma = 90$ is 20 newtons

Unit 3 : Parallel coplanar forces

Answers of exercises (3 - 1)

- 1 d 2 a 3 a
 4 a 26 newtons 8.5 cm
 b 38 newtons 22.5 cm
 c 26 newtons 18.75 cm
 5 b 8 newtons 14 newtons
 c 12.75 newtons
 6 a 35 newtons 40 cm
 b $F_1 = 20$ newtons $r = 14$
 7 a 14 newtons 2.5 cm at A
 b 3 newtons 21 cm
 c 2 newtons 12 cm
 8 27 cm 9 4 meter
 10 30 cm 11 20 newtons 12cm
 12 $\frac{5}{8}$ 1 meter 13 6.5 newtons
 14 The magnitude of the resultant = $\frac{3}{2}$ newtons and acts vertically upwards $\frac{5}{3}$ m from point A

Answers of exercises (3 - 2)

- 1 11 newtons, X = 28cm
 2 m = 10 newtons X = 17 newtons
 3 x = 4cm
 4 K = 10 newtons X = 15 newtons
 5 $r_2 = 14.25$ kg $r_1 = 48.75$ kg
 6 1 meter

- 7 x = 56 cm O = 15 newtons
 8 a 100 kg.wt 100 kg.wt
 b 160 kg.wt 124 kg.wt
 9 first: W = 100 gm.wt
 R = 300 gm.wt
 second: o = 200 gm.wt
 10 x lies between 40cm, 60cm
 the maximum value of tension is where A = 12 w.gm
 and the minimum value = 7w.gm
 the maximum value of tension is where B = 15 w.gm
 and the minimum value = 10 w.gm
 11 8 newtons the maximum distance from A is 33 km and the reaction at A, B respectively is 60 kg.wt / 40 kg.wt
 12 $\vec{X}_3 = -9 \hat{i} - 12 \hat{j}$
 $\vec{X}_4 = -6 \hat{i} - 8 \hat{j}$

Answers of general exercises

- 1 105 cm
 2 the moment of the resultant about the same point
 3 26 cm
 4 First: 100 newtons and the distance between the line of action of newton is 100 cm
 Second: 400 newtons and the distance between the line of action of newton is 25cm
 5 8 newtons 32 cm
 6 $\frac{6}{7}$ 3cm
 7 4, 2, 12
 8 First: 30cm from a
 Second: F = 80 kg.wt
 9 weight of rod = 90 gm.wt the distance between the two supports is 22cm
 10 First: 10 newtons 70 newtons
 Second: 100 newtons

Unit 4 : General equilibrium

Answers of exercises (4 - 1)

- 1 ✗ 2 ✗ 3 ✓ 4 ✗
- 5 The vector of the sum of the forces vanishes
the moment of the system vanishes about one point
- 6 Perpendicular to the rod
- 7 2 newtons
- 8 $W = 5$ newtons $R = \sqrt{65}$
 $L = 19'' 15' 60''$
- 9 $W = 4$ newtons $R = 2\sqrt{7}$
- 10 $\frac{5}{12}$
- 11 $W = 80$, $r_1 = 128$, $r_2 = 80$
- 12 $r_2 = 12$, $m = \frac{\sqrt{3}}{2}$, reaction of the ground $12\sqrt{3}$
- 13 first: $W = 65$ kg.wt
second: $W = 85$ kg.wt
- 14 10 newtons 15 $m = \frac{1}{3}$
- 16 first: 70 newtons second: 14 newtons
- 17 proof

Answers of general exercises

- 1 80 cm from end B
- 2 34 newtons
- 3 $m = \frac{5}{8}$ $F = 110$ kg.wt
- 4 $F = 20$ kg.wt $r_2 = 12.5$ kg.wt
- 5 proof
- 6 $\frac{8}{11}$ 7 $\theta = 2L$
- 8 $r_2 = 5$ kg.wt
 $F = 2.5$ kg.wt

Answer of accumulative test

- 1 $x = 26'' 35' 138''$
- 2 $W = 30\sqrt{3}$ S gm $F = 300$ gm.wt
- 3 $W = 24$ newtons $w_2 = 10$ newtons

- 4 $\theta = 22^\circ$ $O = 18.9$ newtons
- 5 $r_1 = 10\sqrt{3}$ w.gm, $r_2 = 20\sqrt{3}$ w.gm
- 6 $w_2 = 12$ newton, $w_1 = 16$ newtons
- 7 $F = \frac{1}{2\sqrt{3}} O$ 8 $\tan \theta = 2$
- 9 $F = 18$ kg.wt
- 10 proof
- 11 $F = \frac{13}{4}$ kg.wt $r_1 = 3$ kg.wt
 $r_2 = \frac{15}{2}$ kg.wt

Unit 5 : Couple

Answers of exercises (5 - 1)

- 1 d 2 d 3 b 4 d
- 5 c 6 b 7 d 8 b
- 9 a 160 newtons . cm
b $240\sqrt{2}$ newtons . cm
c $100\sqrt{3}$ newtons . cm
d $120\sqrt{3}$ newtons . cm
e 520 newtons . cm
- 10 a 100 newtons cm
b 100 newtons . cm
- 11 10 moment units
- 12 $A = -5$, $B = 2$, $L = \frac{10}{\sqrt{29}}$
- 13 6, 6 are the two vertices of the two forces
6, 6
- 14 $X = 40$ newtons
- 15 $W = 12\sqrt{3}$ newtons $R = 12\sqrt{3}$ newtons
- 16 30° or 150°
- 17 The two forces $30\sqrt{2}$, $30\sqrt{2}$ newtons
one of them acts at the direction \overrightarrow{BD} and
the other in the direction of \overrightarrow{DB}

Answers of exercises (5 - 2)

- 1 a is equivalent to a couple
10 newtons 0 meter
- b is not equivalent to a couple
- c is not equivalent to a couple
- d The system is equivalent to a couple
-15 newtons 0 m

e is equivalent to a couple 17 kg.wt . m

f is not equivalent to a couple

g is equivalent to a couple
16 newtons 0 cm

h is not equivalent to a couple

i The system is equivalent to a couple
 $= 75\sqrt{3}$ newtons 0 cm

2 9 newtons . cm 3 98 kg.wt 0 cm

4 The magnitude of the moment of the couple = 300 gm.wt 0 cm and acts in the direction of A B C D

the two forces are 6, 6 gm.wt

5 $175\sqrt{3}$ kg.wt 0 cm

the two forces are 17.5 , 17.5 kg.wt and act at B , D

6 400 newtons.

7 1134 newtons. cm

8 $300\sqrt{3}$ newtons. cm

9 516 kg.wt. cm 10 $54\sqrt{3}$ gm.wt. cm

11 4800 newtons . cm

12 $F = 200$ newtons

Answers of general exercises

1 a 2000 newtons 0 cm
b 1000 newtons 0 cm 0 c zero

2 a 70 newtons . cm
b $40\sqrt{3}$ newtons 0 cm
c $\theta = 30^\circ$ d $\theta = 3^\circ 61'$

3 weight of rod = 100 newtons downwards
reaction of pathway = 100 newtons upwards

4 $M_A = M_C = M_B = 220$ w.gm . cm

5 a 200 newtons 0 cm
b 176 newtons 0 meter
c 650 newtons

6 -800 newtons 0 meter
- 700 newtons 0 meter
 $= 300(\sqrt{3} - 5)$ newtons 0 meter

7 120 kg.wt 0 meter

8 $A = 5$, $B = 3$ $\|\vec{C}\| = 11$ gm unit
the perpendicular distance between the

two forces = $\frac{11}{34}\sqrt{34}$ length unit

9 $-12\vec{V}$

10 $\vec{C}_C = \vec{BC} = \vec{C}_C = 8\vec{B}$

11 $F = 15$ newtons, $K = 5$ newtons

Answer of accumulative test

1 C 2 A 3 B

4 A 5 D

6 $F = 10$ newtons $m = 10$ newtons

7 $m = 4$, $L = -6$ 8 $C = 0$ $B = 1$

9 -54 newtons 0 cm

10 $F_2 = 8$ newtons $F_1 = 16$ newtons

11 $K = -2$, $L = 3$

Unit 6: Statics : Center of gravity

Answers of exercises (6 - 1)

1 ✓ 2 × 3 ✓ 4 ✓

5 ✓ 6 × 7 ✓ 8 ×

9 ✓ 10 ✓

11 3cm from A 12 $(\frac{9}{4}, 2)$

13 $(\frac{1}{3}, 2)$ 14 $(\frac{1}{2}, \frac{37}{20}, \frac{5}{2})$

15 a $(5, \frac{16}{3})$ b (44, 6)

c $(\frac{13}{2}, \frac{3}{2}\sqrt{3})$ d $(\frac{130}{3}, 10)$

e (5, 6) f $(\text{zero}, \frac{\sqrt{3}}{5}L)$

let the hexagon side length = 2 L

16 49cm from A

17 12cm from A

18 the center of gravity of the system lies at the midpoint of \overline{MC}

19 $19^\circ 11'$

20 The center of gravity is (4.8, 3), $4^\circ 28'$

21 $(\frac{19}{30}L, \frac{1}{2}L)$ 22 $(\frac{152}{15}, 6)$

Answers of exercises (6 - 2)

1 Center of gravity

2 Suspension point

3 Its midpoint

4 Its geometrical center (the intersection

- point of the diagonals)
- 5 medians of the triangle
 - 6 Line of this axis
 - 7 this plane
 - 8 center of the circle
 - 9 center of the sphere
 - 10 Its geometrical center
 - 11 midpoint of its axis
 - 12 **first:** it lies in the center of the square
second: it lies at (50, 50) from A
 - 13 $30 - 27.5 = 2.5$ cm from the center of gravity of the original disc
 - 14 the center of gravity of the remaining part is $(6, \frac{8\sqrt{3}}{3})$
 - 15 it is distant 7cm at E
 - 16 **first:** it is distant $2\sqrt{3}$ cm from the center of the hexagon
second: $11^{\circ} 54'$
 - 17 $\frac{1}{2}$
 - 18 $(\frac{141}{7}, \frac{123}{7})$, $\tan \theta = \frac{41}{47}$
 - 19 (3.76, 3.76)
 - 20 $\frac{13}{12}$

Answers of general exercises

- 1 c
- 2 B
- 3 A
- 4 d
- 5 A
- 6 d
- 7 A
- 8 $c_m = \frac{2}{3}$ cm $y_m = 1$ cm
- 9 3.2cm, 2.08cm
- 10 $\frac{5}{7}$ 13cm at point B
- 11 $5\sqrt{7}$ cm
- 12 $(\frac{7}{8}, \frac{9}{14})$, $\frac{\sqrt{337}}{14}$
- 13 7cm
the coordinate of the center of gravity
 $(7, 2.4)$, $\tan e = \frac{12}{35}$
- 14 $(\frac{130}{3}, 25)$
- 15 m = 200gm
- 16 (-2, -3)
- 17 $\tan \theta = \frac{2}{9}$
- 18 $\tan \theta = \frac{25}{31}$
- 19 $\tan \theta = \frac{24}{25}$
- 20 the center of gravity lies at point E
- 21 20 cm, 20 cm

- 22 $(\frac{710}{81}, 8)$, $\tan \theta = \frac{355}{324}$
- 23 $\tan^{-1} \frac{5}{8}$
- 24 24cm

Second: Dynamics

Unit 1: Collinear motion

Answers of exercises (1 - 1)

- 1 d
 - 2 a
 - 3 c
 - 4 c
 - 5 d
 - 6 d
 - 7 c
 - 8 (1) ← (5) (2) ← (6)
(3) ← (4)
 - 9 (1) negative - the body is decelerated
(2) positive - the body is accelerated
(3) positive - the body is decelerated
 - 10 (1) positive - the body is accelerated
(2) negative the body is decelerated
(3) positive - the body is accelerated
 - 11 a -1, -6
b 4, -3, -4
c $\therefore \approx S \in [-8, 2]$
 - 12 a $V = \pm K \sqrt{A^2 - X^2}$
b $\pm \frac{\sqrt{3}}{2} k$
c $t_1 = (\frac{-\pi}{4} + 2m\pi) \times \frac{1}{k}$
 $t_2 = (\frac{7\pi}{6} + 2m\pi) \times \frac{1}{k}$
 $c = \frac{AK^2}{2}$
 - 13 $\frac{15}{8}$ m/sec²
 - 14 ± 5 , zero
 - 15 a $\frac{-2}{x^3}$ b -64 acceleration units
 - 16 9x, 18 acceleration units
- #### Answers of exercises (1 - 2)
- 1 c
 - 2 d
 - 3 b
 - 4 d
 - 5 d
 - 6 b
 - 7 d
 - 8 a
 - 9 d
 - 10 26.1 meter
 - 11 $\therefore V = t^2 - 6t + 2$

$$\therefore x = \frac{1}{3}t^3 - 3t^2 + 2t, \quad x(8) = \frac{-16}{3}$$

12 $t = 2$, maximum velocity = $\frac{32}{3}$ w/m

13 $V = \pm \sqrt{2}$ w/m

$$x = \pm 4\sqrt{2} \text{ meter}$$

14 $V = \pm 7$ m/sec

$$x = \frac{-13}{3} \text{ m or } x = 4 \text{ m}$$

13 6 kg .m/sec

14 a -24 kg .m/sec

b -72 kg .m/sec

c -48 km .m/sec

15 a 162 kg .m/sec

b 54 gm .m/sec

16 a 576 gm .m/sec b zero

Answers of general exercises

1 d 2 c 3 d 4 c

5 a 6 c 7 a

8 -4 cm/sec, 4 km/sec²

9 $x(\frac{\pi}{2}) = -1$

10 a -6 m/sec, 6 m/sec²

c $\therefore V$ increases in the interval] 2, ∞ [

V decreases in the interval] 0, 2[

d 28 meter

11 -6m/sec, $\frac{26}{4}$ meter

12 c(0) = -6 m/sec²

c(2) = 6 m/sec², 10m/sec

13 $x(1) = \frac{11}{6}$ m

14 43.4m

Answers of exercises (2 - 2)

1 b 2 a 3 b 4 c

5 c

6 a $F = 40$ newtons $k = 120$ newtons

b $F = 80$ newtons

c $F = 25$ newtons, $k = 49$ newtons

d $F = 40$ newtons

7 a 84 newtons b 45 newtons

c 28 newtons, 22 newtons

d $F = 18$ newtons

8 48 kg.wt 9 $\frac{2}{3}$ 16 kg.wt

10 180 km/h 11 140 km/h

12 1.875 w.km 13 6 km/h

14 $\therefore m^2 = 40$ kg.wt 15 7 cars

16 1000 kg.wt

Unit 2: Newton's laws of motion

Answers of exercises (2-1)

1 b 2 c 3 b 4 d

5 b, d

6 20 m/sec 7 3042gm.m/sec

8 160000gm.cm/w

9 9.3m/sec 10 10.5 kg .m/sec

11 $10^5 \times 10^4$ kg .m/sec

12 a 29.4 kg .m/sec

b 39.2 kg .m/sec

c 39.2 kg .m/sec

Answers of exercises (2-3)

1 d 2 c 3 d 4 d

5 c 6 c

7 figure (27) $F = 118$ newtons

figure (28) $F = 10$ newtons

figure (29) 131 newtons figure (30) 20kg

figure (31) 8 kg figure (32) 4 kg

9 figure (33) - 5m/sec²,

figure (34) - $\frac{1}{5}$ m/sec²

figure (35) 4.8m/sec²

10 c = 30cm/sec²

- 11 980 newtons
 12 $62.5w$ kg 13 367.5cm/sec
 14 $800m = \text{newtons per ton of mass}$, $t = 25 w$
 15 $59w$. kg 16 1292.5 kg.wt
 17 $0.8 w$. ton 18 $\frac{3\sqrt{2}}{2}$ m/sec²
 19 $\frac{\sqrt{29}}{2}$ newtons
 20 $\frac{1}{2}(3\hat{i} + 2\hat{j} + 6\hat{V})$
 21 $a = 6$, $b = 3$, $c = 1$
 22 $a = 1$, $b = -9$

- 23 first: $1\hat{C}$ second: 47 newtons
 24 600 dyne 25 170 dyne
 26 $\vec{m} = (8t^2 + 10t + 2)\hat{C}$
 27 $V(2) = 2\text{m/sec}$ $B(2) = 1.5\text{m}$
 28 $c \leq \frac{4}{15} \cdot 3$ 29 410 kg.wt
 29 160 dyne
 30 402 kg.wt

Answers of exercises (2-4)

- 1 60 2 420, 112
 3 72 4 35, 42
 5 a 80 kg.wt, b 76.4 kg.wt
 c 77.6 kg.wt
 6 a 40 gm b 245 gm
 c 100 gm
 7 1.4m/sec^2 8 15 kg.wt
 9 108 kg.wt, zero
 10 $k = 14$ gm, $R = 15$ gm
 11 1.4 m/t upwards

Answers of exercises (2-5)

- 1 a 2.45 upwards b 9.8m/sec
 c $\sqrt{3}$
 2 a $\frac{49}{75}$ downwards b 2.94
 c 14.4
 3 a 4 c 5 c 6 c
 7 2.12m/sec^2 , 8w.kg

- 8 $c = 5.1\text{m/sec}^2$, $\frac{\sqrt{3}}{2}\text{kg.wt}$
 9 $-8.3 \times \frac{1}{\sqrt{2}}$ m/sec²
 $9.2 \sqrt{2}$ kg.wt
 10 $\frac{t_1}{t_2} = 1$

Answers of exercises (2-6)

- 1 a $\frac{2}{7}$ b $\frac{1}{2}$ c $\frac{1}{4}$
 d $7. \approx$ e $35. \approx$ f $33. \approx$
 g $27. \approx$ h 8.6 kg
 2 $m_r = 1$
 3 a 19.8 newtons b 11.8 newtons
 4 $\frac{1}{4}$ 5 1.08

Answers of exercises (2-7)

Complete:

- 1 a zero b 3 c 2
 2 a 4.9 b 9.8 c zero d 39.2
 3 a 140 b 480 c 960 d 360
 4 a $\frac{1}{3}d$ b $\frac{8}{3}k$ c 4.9 d 7.35
 e $\frac{1}{2}$ f 1.25
 5 a $\frac{7}{2}d$ b $\frac{10}{7}$ c $\frac{10}{7}\sqrt{2}$
 d $\frac{7}{2}d$ e $\frac{4}{7}d$
 6 a 3.5 b 25.2 c $25.2\sqrt{3}$
 7 b 280cm/sec
 8 2.45m/sec^2 , 73.5 newtons, 1.96 meter
 9 $\frac{3}{2}$ 10 $t = \frac{4}{7}$ s
 11 70 cm/t² 780 gm.wt
 12 4.2 m/sec, 19.6kg 13 420km
 14 22.68kg , 274.98 newtons
 15 180cm/sec^2
 72000 dyne, $72000\sqrt{2}$ dyne
 16 2sec
 17 $1.47\sqrt{2}$ newtons
 $70\sqrt{10}$ cm/sec, 50cm

- 11 $30\sqrt{481}$ joule
 12 $\vec{F} = 18\hat{j} + 24\hat{j}$, 3600 work measuring unit
 13 55125 joule 14 30kg
 15 $S = 98$ meter 16 30 meter
 17 49×10^4 joule
 18 (1) zero (2) 480.2 joule (3) -456.19 joule
 19 $F = m + 2000$ kg.wt 500 meter
 = 5 kg.wt
 20 582300 joule -44100 joule -88200 joule
 88200 joule
 21 a 12600 work unit
 b $\frac{1}{3}$ 28333 work unit
 22 .53 joule 23 k d r work unit

Answers of exercises (4 - 2)

- 1 15000 2 8 3 15 4 2.5×10^5
 5 6240 6 2.94
 7 $\frac{1}{4000}$ second 8 70 cm/sec
 9 $A = 60^\circ$ $B = 120^\circ$
 10 4.9 joule 11 -17.199 joule zero
 12 14 m/sec 13 $K = 16$ kg
 14 first: $A = \frac{3}{2}$, $B = -2$ second: 68 joule
 third: 72 joule
 15 $V = 75$ m/sec 16 $\frac{3}{2}$
 17 kinetic energy lost = $17.6 - 5.1 = 12.5$ joule
 18 1.6 meter
 19 0.06 kg 16.9 meter 1.68 kg.meter /sec
 20 first: $V = 2$ meter /sec
 second : 837.9 joule third: 34.2 kg.meter /sec
 21 first: 8m/sec
 second: 14336 joule third: $\frac{3}{7}$ 37691 kg.wt

Answers of exercises (4 - 3)

- 1 a 9.8 joule b zero
 c 9.8 joule
 2 686 3 3.4×10^6 4 19.6
 5 4.2 6 27 erg 7 72
 8 20.58 joule
 9 34.3 joule
 10 4900 joule 6 second 4774.56 joule
 11 4.9 joule 2.6 w . kg
 12 14112 joule 13 5 m/sec
 14 4 joule 0.9 joule 15 140 cm/sec
 16 13.5975 joule

Answers of exercises (4 - 4)

- 1 31 dyne.cm/sec 2 5 kg.wt
 3 36 kg.wt 4 126 horses
 5 126 km/h 6 30 km/h
 7 54 km/h 8 187.5 horses
 9 $m = 900$ kg.wt power 150 horses
 10 power = 400 horses
 11 43.2 m/sec 12 12 km/h
 13 80 horses
 14 80 horses $\frac{432}{7}$ km/h
 15 270 horses
 16 a 39 joule b 13 watt
 c 19 watt
 17 a 1122 joule b 374 watt
 c power = 647 watt
 18 a $18t\hat{i} + 6\hat{j}$
 b $54t^3 + 12t$ c 240 watt
 19 a 135 horses b 2250 kg.wt. m
 c 180 horses
 20 400 joule
 21 a $\vec{F} = -6(\cos 2t, \sin 2t)$
 b $\frac{9}{2} - 3(\cos 2t + \sin 2t)$
 c $-6(\cos 2t - \sin 2t)$

Answers of general exercises

unit 4

- 1 57600 erg
- 2 $m = 18 \text{ gm.wt}$ $m = 840 \text{ gm}$ $t = 6 \text{ sec}$
- 3 90 horses
- 4 $V = 7w/m$ 13720 joule 36400 kg.wt
- 5 $w = -1.96 \text{ joule}$
- 6 $11 \hat{i} + 23 \hat{j}$ 180 joule power = 120 watt
- 7 2 horses **8** 5292 joule
- 9 14.7 joule 24.99 joule
- 10 $c = \frac{1}{4} \text{ m/sec}^2$ $V = \frac{200}{13} \text{ m/sec}$
- 11 first: $\frac{100}{3}$ horses
second: $c = \frac{49}{200} \text{ m/sec}^2$
- 12 102.4 horses
- 13 **a** 10 joule **b** 2.4 joule
- 14 $V = 10 \text{ m/sec}$ 48 joule
- 15 $\vec{F} = \frac{2}{5} (6 \cos 2t \hat{i} + 20 e^{-4t} \hat{j})$
 $40 e^{2\pi} \pi \text{ watt}$ $(5 e^{2\pi} - \frac{107}{10}) \text{ joule}$

Test 1

Question 1

- 1 d 2 b 3 b 4 c
- 5 b 6 b

Question 2

- 1 175.4 newtons.m 2 $O \sin(\theta + L)$

Question 3

- 1 $25\vec{C}$ is distant 45cm from A
- 2 50 newtons 600 newtons.cm

Question 4

- 1 $x_1 = 5\sqrt{3}$ $x^2 = 5$ $\theta = 12.8^\circ$
- 2 $(\frac{35}{6}, \frac{5}{2}\sqrt{3})$

Question 5

- 1 -500 250 zero
- 2 $(\frac{407}{72}, \frac{245}{8})$

Test 2

Question 1

- 1 a 2 a 3 b 4 b
- 5 b 6 b

Question 2

- 1 $7\hat{i} + 8\hat{j} - 9\vec{V}$ $\frac{\sqrt{194}}{\sqrt{35}}$

Question 3

- 1 9cm 2 $46\sqrt{2} \text{ kg.wt}$

Question 4

- 2 B C is inclined at an angle of tangent $\frac{4}{5}$ to the horizontal

Question 5

- 1 first: $\frac{65}{7}$ $\frac{10\sqrt{3}}{7}$
second: $\frac{15}{20}$ $\frac{5}{3}\sqrt{3}$

Test 3

Question 1

- 1 75 2 350 newtons.cm
- 3 $\pm 4 (1, -2)$ 4 3000 newtons.cm
- 5 center of the sphere
- 6 intersection point of medians

Question 2

- 1 $36^\circ 52'$ 2 6392.3

Question 3

- 1 70cm $O = 2 \text{ newtons}$
- 2 $35\sqrt{3} \text{ newtons.cm}$

Question 4

- 1 $r_b = \frac{1}{2} O$ $r_a = \frac{3}{4} O$ $x = \frac{\sqrt{3}}{4} O$

Question 5

- 2 center of gravity is $(\frac{1}{3}, \frac{1}{6})$

Test 4

Question 1

- 1 c 2 b 3 b 4 d
- 5 d 6 a

Question 3

- ① zero
 ② first: $\hat{j} = 10 \bar{2}$ gm.wt second: $S = 332.5\text{cm}$

Question 4

- ① the distance required = $19.6 \times 2 + 40.4 = 79.6$ meter

Question 5

- ① 32 horses
 ② first: 36.75 joule second: 11.025 joule
 third: $70 \bar{30}$ cm/w

Test 8
Question 1

- ① $K = 8$ kg ② 600 dyne
 ③ downwards , upwards ④ 50cm/sec
 ⑤ 7 m/sec ⑥ 240.1 joule

Question 2

- ① The three speeds equal
 Perpendicular m does more work
 ② 137.2 joule , $\frac{1}{5}$ m/sec

Question 3

- ① 10w . gm 280cm/sec
 ② 20 m/sec

Question 4

- ① a 280 cm/sec S = 100 cm
 ② a 4 joule b zero
 c 20 joule d zero

Question 5

- ① 68 gm.cm/sec
 ② $E = 9.75\text{m/sec}^2$
 $c = 1.25\text{m/sec}^2$

Test 9
Question 1

- ① $A = 9$ B = 7
 ② 160 gm.wt ③ $m = 2000$ kg.wt
 ④ 24.5 kilowatt/h
 ⑤ 20 horses ⑥ $\frac{8}{15}$ horses

Question 2

- ① $m = 8$ kg.wt
 $X = 22 + 8 = 30w$. kg
 ② 280cm/sec^2 W = $225w$. kg
 $m = 140$ gm
 ascending $r_1 = 200w$. gm
 descending $r_2 = 180 w$. gm

Question 3

- ① first: 14m/sec second: $\frac{21}{10}$ kg.m/sec
 ② $V = 14\text{m/sec}$

Question 4

- ① $V = \frac{21}{5}$ m/sec $m = 4.2$ newtons
 $t = 5$ second
 ② $c = 1.4\text{m/sec}^2$ K = 70kg

Question 5

- ① $V = 27\text{km/h}$
 ② total work = 14.7 joule
 $V = 3.35\text{m/sec}$

Test 10**Question 1**

- ① $50\sqrt{3}$ kg.wt
- ② 3 m/w
- ③ $V = 280\text{cm/sec}$
- ④ 3600 joule
- ⑤ 4 horses
- ⑥ -120000gm.cm/sec

Question 2

- ① first: $B = 3$ $A = 1$
 second: 960 joule
- ② $c = 70\text{cm/sec}^2$, vertical distance = 35cm

Question 3

- ① 39km/h ②

Question 4

- ① 5.6m/sec , $m = 4600\text{w/kg}$
- ② 12.25m/sec

Question 5

- ① a $c = 7\text{cm/sec}^2$
 b $\frac{57}{340}$
- ② $K = 20\text{kg}$, $R = 16\text{w. kg}$

المواصفات الفنية:

رقم الكتاب:	
مقاس الكتاب:	$\frac{1}{8}$ (٨٢ × ٥٧) سم
طبع المتن:	٤ لون
طبع الغلاف:	٤ لون
ورق المتن:	٨٠ جم أبيض
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عدد الصفحات بالغلاف:	٣١٦ صفحة

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