Chapter 2

Wave theory

2.1 – Wave motion: Is defined as the mode of turbulence traveling from point to the other, through medium, the turbulence is mode or type of physical state, it genereted by moving source.

2.2 – Transverse waves:

If the wave trayeling in X-direction and (y) is a vertical, the contour of the wave in (x,y) plane is shown in Fig (1)

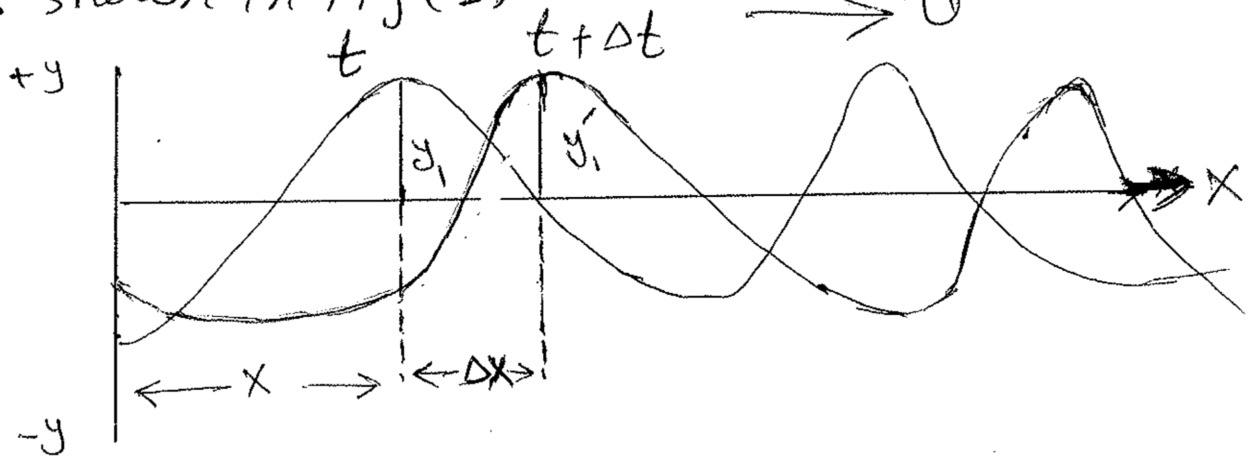


Fig (1) Wave transverse

Let this curve represented by

$$y = f(x) \quad -- \quad (1)$$

the contour of wave with constant velocity (v) therefore as (t) increases, a given Ordinate such as (y) after a time Δt has elapsed, be found at y_1' farther to the right by an amount $\Delta x = v\Delta t$

So that the equation(1) can be written as:

$$y = f(x - vt) \quad --- (2)$$

and when two time (t) and (Δt) we have.

$$\left.\begin{aligned} y_1 &= f(x - vt) \\ \text{and} \quad y_1' &= f[(x + \Delta x) - v(t + \Delta t)] \end{aligned}\right\} \quad --- (3)$$

if we substitute $\Delta x = v\Delta t$ we get:

$$y_1 = y_1'$$

The general equation for any transverse wave in aplane is:

$$y = f(x \mp vt) \quad --- (4)$$

the plus sign is to be used if the wave traveled to the left; i.e in the $(-x)$ direction, while the negative sign is to be used if the wave travel to the right $(+x)$ direction.

The partial differerential equation of wave equation traveling along x-axis may be written as:

$$\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2} \quad --- (5)$$

from eq (4) $y = f(x - vt)$ derivative with respect to (t).

$$\frac{\partial y}{\partial t} = -v f'(x - vt)$$

$$\frac{\partial^2 y}{\partial t^2} = v^2 f''(x - vt) \quad \text{-- (6)}$$

then when derivative with respect to (X)

$$\frac{\partial y}{\partial x} = f'(x - vt)$$

$$\frac{\partial^2 y}{\partial x^2} = f''(x - vt) \quad \text{--- (7)}$$

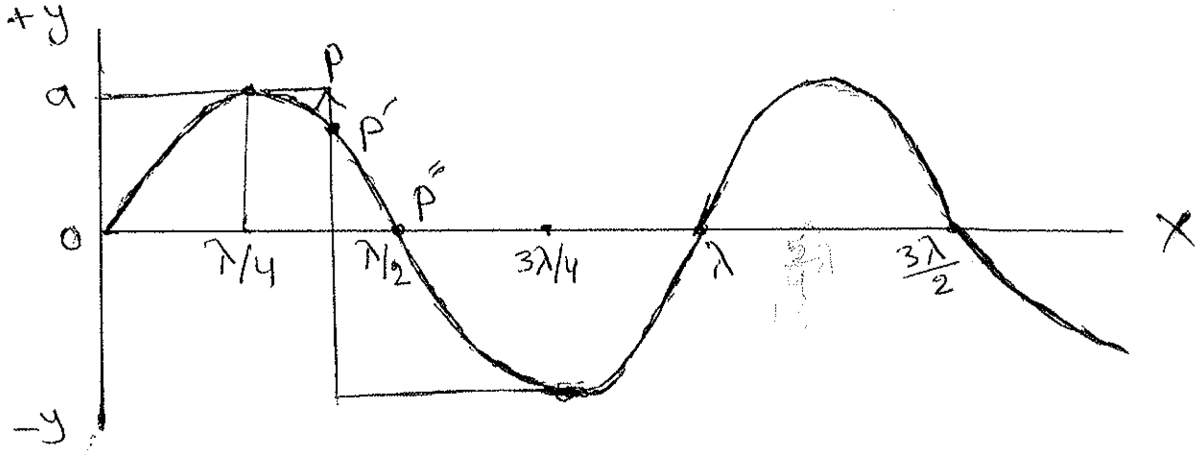
eqs(6) and (7) are verification for eq (4).

2.3 - Sine waves: The simplist type of wave is that for which the function in equation (4) is a sine and Cosine.

The instantaneous displacement (y) Can be represented by:

$$y = a \sin \frac{2\pi x}{\lambda} \quad \text{-- (8)}$$

The constant (a and λ) may be seen from the curve in Fig (2)



Fig(2) Contour of sine wave at $t=0$

as show in Fig (2). The contour of sine displaced toward (+x) axis with velocity (v) and any one particale such as (p) will carry out a simple Harmonic vibration occupying the successive position p, p', p'' ---; as the wave motion.

also: λ is the distance after the curve repeats;
Its wave length.

and a is the maximum displacement is called the amplitude of the wave.

To make the wave move we must introduce the time (t) so eq (8) become:

$$y = a\sin\frac{2\pi}{\lambda}(x - vt) \quad --- \quad (9)$$

The contour of wave will be displaced toward (+x) with velocity v.

The time for a whole vibration of any particle is the period (T) and its reciprocal of vibration per second is the Frequency (ν) we have.

$$\nu = \frac{1}{T} \quad \text{and} \quad v = \lambda \nu = \lambda \frac{1}{T}$$

therefore equation (9) become

$\because v = \lambda \nu$

$(x - vt)$

$(x - \lambda \nu t)$

$$y = a \sin \frac{2\pi}{\lambda}(x - \lambda \nu t)$$

$$= a \sin \left(\frac{2\pi x}{\lambda} - \frac{2\pi \lambda \nu t}{\lambda}\right)$$

$$= a \sin \left(\frac{2\pi}{\lambda} x - \underbrace{2\pi \nu}_{\omega} t\right)$$

where $(2\pi\nu)$ is the angular frequency (ω).

then:

$$y = a \sin \left(\frac{2\pi}{\lambda} x - \omega t\right)$$

where $2\pi/\lambda$ is the number of wave in a distance of 2π cm or called wave number (K) or propagation number.

then $$y = a \sin (Kx - \omega t) \quad \text{--- (10)}$$

where y is the stantaneous displacement of the particle and is measured by vertical distance. Now the addition of a constant to the quantity is of little physical significance, since such a constant may be eliminated by suitably adjusting the zero of the time scale.

Thus the equation (10) can be written:

$$y = a\sin(\omega t - Kx + \pi)$$
$$= a\cos(\omega t - Kx + \frac{\pi}{2})$$

$$\therefore \left.\begin{array}{l} y = a\sin(\omega t - Kx) \\ y = a\cos(\omega t - Kx) \end{array}\right\} \quad --- (11)$$

2.4- phase and phase difference:

The phase difference at any instant between two particles at position x_2 and x_1 is therefore:

$$\delta = K(x_2 - x_1) = \frac{2\pi}{\lambda}\Delta \quad --- (12)$$

where δ = phase difference

Δ = path difference between two particles in x- Coordinate.

This term refers to the quantity in brackets in equation (11).

In the physical optics we shall used the path defference for two separate waves not for two particles in the same waves or a single wave.

for different media. We use the optical path (d) when the wave passes through these media.
So the phase difference become:

$$\delta = \frac{2\pi}{\lambda}(\text{optical path difference})$$

$$\delta = \frac{2\pi}{\lambda}\left(\sum_i n_i d_i - \sum_j n_j d_j\right) \quad --- \quad (13)$$

2-5- phase velocity or wave velocity:

from equation (10) can be evaluating (y) as a function of (x) for several values of (t)

when $t = 0 \Rightarrow y = a \sin KX$ Fig (3-a)

when $t = \frac{1}{4}T$ (T is the time for one period)

Then $\omega t = 2\pi \nu \cdot \frac{1}{4}T = 2\pi \frac{1}{\cancel{T}} \cdot \frac{1}{4}\cancel{T} = \frac{\pi}{2}$ rad

as show in Fig (3-b)

when $t = \frac{1}{2}T \Rightarrow \omega t = 2\pi\nu \cdot \frac{1}{2}T = 2\pi \frac{1}{\cancel{T}} \cdot \frac{1}{2}\cancel{T}$

$= \pi$ rad Fig (3-c)

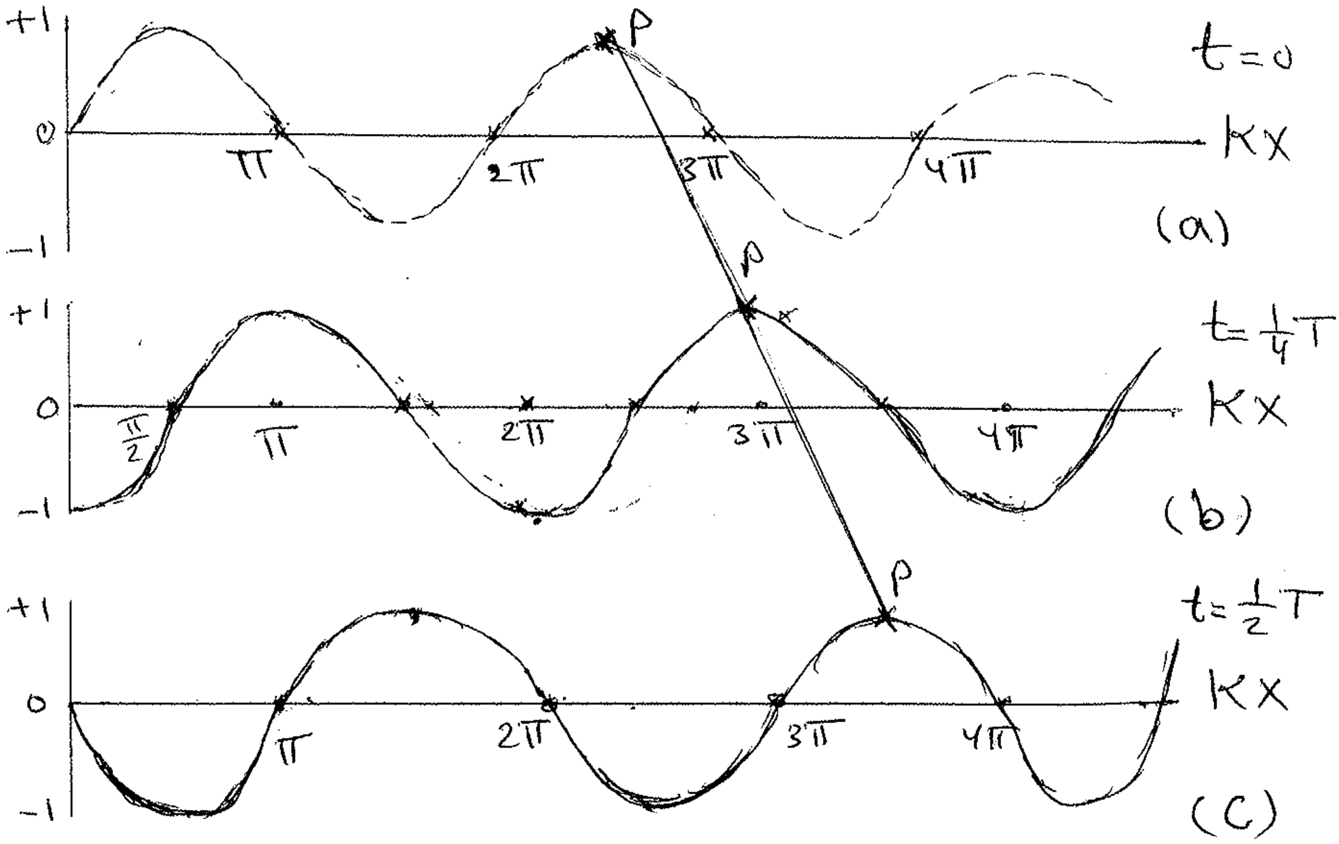


Fig (3) curves for $y = a\sin(KX - wt)$ at Three instant of time $t=0$, $t=\frac{1}{4}T$, $t=\frac{1}{2}T$

The point p is apoint of a constant phase is discription by Condition :

$$(wt - Kx) = \text{Constant} \quad \text{---} \quad (14)$$

by differential eq (14) with respect (t) get :

$$\frac{d}{dt}(wt - Kx) = \frac{d}{dt}(\text{constant})$$

$$w - K\frac{dx}{dt} = 0$$

$$\therefore \frac{dx}{dt} = \frac{w}{K} = \frac{2\pi\nu}{2\pi/\lambda} = \lambda\nu \quad \text{--- (15)}$$

Then the velocity of a constant (phase point) is called phase velocity:

$$V = \frac{W}{K} = \frac{dx}{dt} = \lambda\nu$$

For a wave traveling toward $(-x)$ the constant phase takes the form $(wt + Kx)$ and

$$V = -\frac{w}{K} = -\lambda\nu$$

The ratio (W/K) depends on the physical properties of the medium also on the frequency.

problem 1: The equation of wave motion is

$$y = 7 \sin 2\pi (20t - 0.05X)$$

Find the wave velocity and maximum particle Velocity; If X and t are in cm and sec.

Solution: The given equation of wave motion is

$$y = 7\sin 2\pi (20t - 0.05X)$$

Comparing above equation with equation of wave motion

$$y = a\sin(wt - KX)$$

$$a = 7 \text{ cm}, \quad W = 20 \text{ s}^{-1}$$

$$K = 0.05 \text{ cm}^{-1}$$

$$\therefore \quad V = \frac{W}{K} = \frac{20}{0.05} = 400 \text{ cm/sec}$$

The displacement of a particle at time t is

$$y = a\sin(wt - kx)$$

The velocity of a particle at time t is

$$\frac{\partial y}{\partial t} = aW\cos(wt - kx)$$

maximum particle velocity $= aW$

$$= (7)(20)(2\pi) = 880 \text{ cm/sec}$$

Thus the velocity of the given wave is (400) and maximum particle velocity is (880 cm/sec) problem 2: The equation of a wave travelling over a string is: $y = (0.20\,mm)\sin[(31.4\,m^{-1})X + (314\,s^{-1})t]$

a: In which direction does the wave travel?

b: Find the wave speed, Frequency and the wavelength of the wave.

c: what is the maximum displacement and maximum speed of a portion of the string.

Sol a - The equation of wave along positive X-direction

$y = a \sin(KX - \omega t)$

Then $a = 0.20 \times 10^{-3}\,m$

$K = 31.4\,m^{-1}$

$\omega = 314\,s^{-1}$

b - Wave speed $v = \frac{\omega}{K} = \frac{314}{31.4} = 10\,m/sec$

$\omega = 2\pi\nu \Rightarrow \nu = \frac{\omega}{2\pi} = \frac{314}{2 \times 3.14} = 50\,Hz$

$\because K = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{K} = \frac{2 \times 3.14}{31.4} = 0.2$ meter

$\lambda = 20\,cm$

c - Miximum displacement $= a = 0.20 \times 10^{-3}\,m$

velocity of the wave

$\frac{\partial y}{\partial t} = (0.20 \times 10^{-3} \times 314\,ms^{-1})\cos(31.4X + 314t)$

$= 62.8 \times 10^{-3}\,m/sec$

H-W ①/ A transverse wave travels along the positive X-direction with a speed of 20 m/sec. The amplitude of the wave is (0.30) cm and the wavelength is (2 cm)

a- Write a suitable wave equation which describes the wave b- what is the displacement of the particle at X = 2.0 cm and at t = 0 according to the wave equation written?

H-W 2/ Given the equations of transverse waves:

Find: amplitude, Frequency, wavelength, periodic time T, velocity of wave &

1- $y = 4 \sin 2\pi (0.2X - 3t)$

2- $y = 6 \sin (0.4t - 25X)$

3- $y = 10 \sin (6t - 0.5X)$

H.W (3), A wave is specified by $y = 6 \sin 2\pi (8t - 4x + \frac{3}{4})$

Find (a) the amplitude (b) the wavelength (c) Frequency (d) initial phase angle (e) initial displacement at $t = 0$ and $x = 0$

H.W (4) A wave is expressed by $y = 10 \sin (6t - 0.5x)$ where the time is in second and distance in Centemeters. Find the velocity and acceleration of a particle (3cm) from the origin at $t = 24$ sec.

H-W (5) Using the relation of equation (10) show that the phase of a sine wave may be variously expressed as:

$$\frac{2\pi}{T}\left(t - \frac{x}{v}\right), \quad 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right), \quad 2\pi\nu\left(t - \frac{x}{v}\right)$$

2-6: Amplitude and Intensity:

The waves transport energy, and the amount of it that flows per second across unit area perpendicular to the direction of travel is called the intensity of the wave. If the wave flows continuously with the velocity V, there is a definite (energy density) or total energy per unit volume.

According to equation $y = a\sin(\omega t - \alpha)$ where α is the value of (KX) the velocity of the particle is

$$\frac{dy}{dt} = \omega a \cos(\omega t - \alpha)$$

The maximum Kinetic Energy

$$\frac{1}{2}m\left[\frac{dy}{dt}\right]^2_{max} = \frac{1}{2}m\omega^2 a^2$$

Since this also the total energy of the particle and is proportional to the energy per unit volume:

$$\text{Energy density} \simeq \omega^2 a^2$$

also the intensity I is given

$$I = \omega^2 a^2 V^2 \quad \text{--- (16)}$$

From spherical wave the intensity decrease as the inverse squar of the distance from the source.

$$I \alpha \frac{1}{r^2} \quad \& \quad a \alpha \frac{1}{r}$$

and one may write the equation of a spherical wave as

$$y = \frac{a}{r} \sin(wt - kr) \quad --- \quad (17)$$

a: is the amplitude at unit distance from the source.

السعة على مسافة قدرها الوحدة من المصدر.

For plane waves the fraction dI/I of the intensity lost in traversing an infinitesimal thickness dx is proportional to dx so that

في الموجات المستوية يتناسب الكسر المفقود من الشدة $\frac{dI}{I}$ أثناء عبور سمك متناهي الصغر dx من الوسط مع قيمة dx.

$$\frac{dI}{I} = -\alpha \, dx$$

For above equation is integreted to give:

$$\int_{0}^{x} \frac{dI}{I} = -\alpha \int_{0}^{x} dx$$

للحصول على الشدة تكامل المعادلة.

we find

$$\boxed{I_x = I_0 e^{-\alpha x}} \quad --- \quad (18)$$

[illegible]

equation (18) is called lambert law.
We shall refer to as exponential law of opsorption.

أو نسميه القانون الأسي للامتصاص.

where:
I_o is the incident intensity
I_x is the intensity at thickness (x).
α is absorption Coefficient.

2.7: Frequency and wavelength:

Any wave motion is generated by vibrating source
The wave length in agiven medium is then determined by the velocity in that medium by dividing the velocity by the frequency $V = \lambda \nu \Rightarrow \lambda = \frac{V}{\nu}$

For since wavelengths proportional to velocities we have

$$\frac{\lambda}{\lambda_m} = \frac{c}{V} = n$$

The optical path Corresponding to adistance(d) in any medium is therefore:

$$nd = \frac{\lambda}{\lambda_m} d$$

Example: A given point is vibrating with a period of 5 sec and as amplitude of (3 cm) if the initial phase angle is ($\frac{\pi}{3}$) rad, Find ⓐ the initial displacement ⓑ the displacement after 12 sec.

Sol

$$y = a\sin(\omega t + \alpha)$$

since the angular speed ω is 2π in 5 sec or $\frac{2\pi}{5}\frac{rad}{sec}$

at time $t = 0$:

a- $y = 3\sin\left(\frac{2\pi}{5}(0) + \frac{\pi}{3}\right)$

b- after 12 sec gives:

$$y = 3\sin\left(\frac{2\pi}{5}(12) + \frac{\pi}{3}\right)$$

$$= 3\sin\left(4.8\pi + \frac{\pi}{3}\right)$$

The total phase angle of $\left(4.8\pi + \frac{\pi}{3}\right)$ is equivalent to

$864^\circ + 60^\circ = 924^\circ$

$\sin 924^\circ = -0.407$

$\therefore\ y = 3(-0.407) = -1.220$ cm

phase angles :

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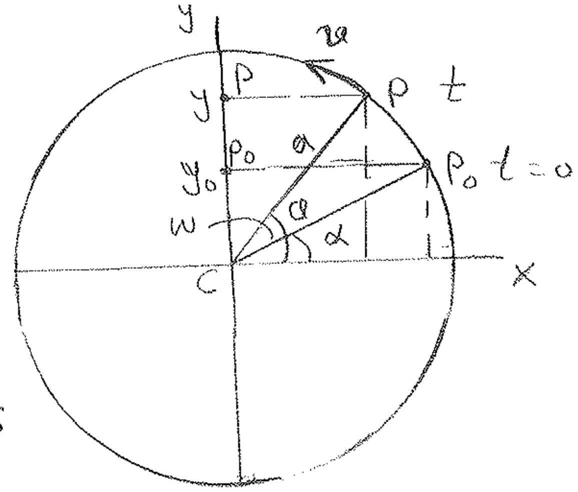
The angle θ measured counterclockwise from the X-axis specifying the position is called the phase angle.

As an example consider a point moving up and down along the y-axis as show in fig( ) the position of the mass point (p) is given by the projection of the graph point p_1 on the y-axis.

From the triangle PpC on the diagram.



$y = a\sin\theta$ --- (1)

the angular speed ω is constant we can write from any angle.

$\theta = \omega t$ -- (2)

subsititution in Eq. (1) gives

$y = a\sin\omega t$ --- (3)

At time t=0 the graph point is at P_o and the mass point is at P_o. at some later time t when the mass point is at P the graph point is at p and we must modify Eq(3) by adding the angle (α) as follows:

$y = a\sin(\omega t + \alpha)$ — (4)

The angle α is a constant and is called the initial phase angle. It is express all angles in radian.