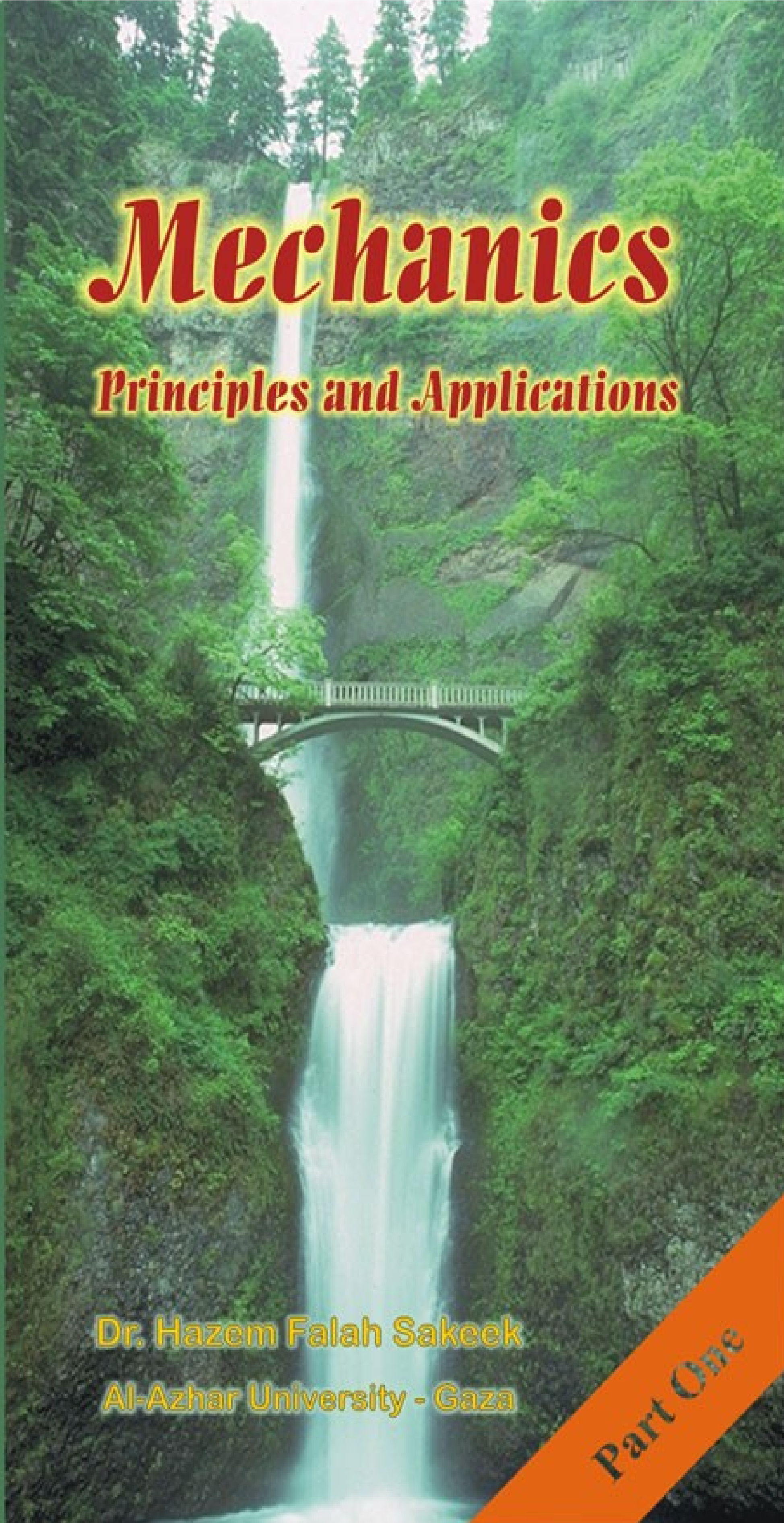
LECTURES IN GENERAL PHYSICS

Mechanics

Principles and Applications



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Part One

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GENERAL PHYSICS

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Mechanics

Principles and Applications

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الميكـــانيكـــــا
أساسيات وتطبيقات

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جامعـــة الأزهـــــر-غـــزة

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مقدمة

علم الـفيزيـــاء

منذ نشأة الخليقة على سطح الأرض شرع الإنسان يتساءل عن كيفية وجود الأشياء وعن سبب وجودها. هذا التساؤل كان دافعه الطبيعة الفضولية لدى البشر الذين متعهم الله بنعمة العقل والتفكير. ولكن بسبب جهل الإنسان القديم وخوفه فإنه كان يعــزو ظـــواهر الطبيعة إلى وجود قوى خارقة مجهولة، فعبدها ظاناً أنها المسئولة عن بقائه. ولكن مـــع مزيد من الملاحظات والاكتشافات ودافع الحاجة إلى الاختراع والابتكار أدرك أن الطبيعة تحكمها قوانين مترابطة تربط بين نشاطاته كإنسان وعلاقته بالعالم الجامد والعـــالم الحـــي على حد سواء.

إن العلم الذي يدرس الطبيعة التي خلقها الله سبحانه وتعالى في تكامـــل وتناســق إبداعي رائع هو "علم الفيزياء" الذي سمى قديما "علم الطبيعة". ولقد اســتطاع الإنســـان البدائي التوصل إلى بعض مفاهيم هذا العلم نتيجة لحاجته للحصول على الطعام فبابتكـــاره لعملية الصيد بالرمح كان عليه أن يكون مدركاً لكميتين فيزيائيتين أساسيتين هما المســـافة والزمن ومن ثم كان عليه معرفة سرعة الجسم النسبية ليتمكن من إصابة هدفه.

في الحقيقة يمكن أن نقول أن علم الفيزياء هو العلم الذي يهتم بالإجابـــة عـــن أي سؤال يبدأ بـــ "لماذا" فعلى سبيل المثال تساءل العالم نيوتن "لماذا ســـقطت التفاحـــة إلـــى أسفل؟" وبالإجابة عن هذا السؤال توصل نيوتن إلى وضع القانون العام للجاذبيـــة الـــذي يربط القوى التجاذبية بين الكواكب والأقمار. وبعد فهم طبيعة هذه القوى تمكن الإنســـان من وضع أقمار صناعية تدور حول الأرض في مدارات ثابتة أفادت البشرية في مجالات عديدة وعلى رأسها مجال الاتصالات.

أهمية علم الفيزياء

إن تطور علم الفيزياء هو نتيجة طبيعية لحاجة الإنسان إلى إيجاد تفسير للظواهر الطبيعية وفهم سلوكها والقوى المؤثرة عليها من خلال استنباط قوانين ترتبط ببعضها. إن التطور التكنولوجي الملحوظ في جميع المجالات سواء في الطب أو الهندسة أو الفضاء أو الاتصالات أو الكمبيوتر وغيرها ما هو إلا تطبيقات لنتائج أبحاث واكتشافات فيزيائية. فعلى سبيل المثال علم الفيزياء هو علم أساسي في مجال الطب يستخدم في تشخيص المرض سواء كان باستخدام أشعة اكس أو النظائر المشعة أو الرنين المغناطيسي أو الأمواج فوق الصوتية حيث تعتبر جميعها تطبيقات لأبحاث واكتشافات فيزيائية ولا يمكن أن يكون هناك علاج بدون تشخيص فكلما تطورت وسائل التشخيص أمكن القضاء على أمراض كانت فتاكة، أما الهندسة بجميع فروعها ومجالاتها فهي تطبيق عملي لعلم الفيزياء فمثلا تحويل الطاقة الحرارية إلى طاقة حركية أساسها قوانين فيزيائية استخدمها المهندسون الميكانيكيون في تصميم وسيلة النقل والمحرك منذ زمن بعيد. أما بالنسبة لمجال الاتصالات فقد شهد تطورا ملحوظا مع تطور الاكتشافات الفيزيائية فقد أدى اكتشاف الكهرباء وفهم قوانينها إلى استخدامها كوسيلة للاتصالات عن طريق إرسال المعلومات على شكل نبضات كهربائية خلال الأسلاك النحاسية. وبعد اكتشاف الفيزيائيين لأشعة الليزر والألياف الضوئية تحولت تكنولوجيا الاتصالات من استخدام الكهرباء إلى استخدام الضوء لما في ذلك من ميزات تفوق سابقتها بكثير. أما بالنسبة إلى علم الكمبيوتر فهو مثال واضح للتطبيقات الفيزيائية فبعد فهم طبيعة المواد وخواصها الكهربية ومن ثم اكتشاف أشباه الموصلات أصبحت هذه المواد البنية الأساسية للدوائر الإلكترونية للكمبيوتر، ولا شك أن التقدم الملحوظ في تكنولوجيا صناعة الكمبيوتر هو نتيجة للتقدم في الأبحاث والاكتشافات الفيزيائية فمثلا احتلت الشاشات التي تستخدم البلورات السائلة محل الشاشات التقليدية فأصبح الكمبيوتر بكل إمكاناته بحجم كتاب صغير.

من هذه الأمثلة ندرك أن علم الفيزياء هو علم أساسي لفهم باقي العلوم وتطويرها وقد أدركت الدول المتقدمة أهمية علم الفيزياء فشجعت على دراسته وأولته اهتماما كبيرا من حيث دعم الأبحاث العلمية وتشجيعها في مختلف المجالات الفيزيائية.

الشكوى من صعوبة دراسة الفيزياء

من الشائع بين الناس عامة والطلبة خاصة أن مادة الفيزياء صعبة ومعقـدة جـداً وهذا في حد ذاته غير صحيح فإن دراسة علوم الفيزياء تحتاج من الدارس إلى اسـتخدام مهارته في إمعان الفكر وربط المعلومات السابقة والحديثة مع بعضها ببعض والخـروج باستنتاج منطقي مقنع، ولأن علم الفيزياء هو علم تجريبي يعتمد علـى القيـاس وبالتـالي يحتاج إلى معادلات وقوانين رياضية تربط الكميات الفيزيائية، وغالبا ما يتحـول تركيـز الدارس لموضوع الفيزياء من الفهم الفيزيائي إلى التعامل مع أرقام مجردة فلا يستطيع فهم معناها ومن هنا يصبح التعامل مع كل مسألة على أنها موضـوع درس جديـد وقـوانين جديدة بالرغم من أن تلك المسألة ما هي إلا تطبيق آخر للقانون نفسه ولكن الشيء الجديد ما هو إلا مجهول آخر فيصاب الدارس بالإحباط لفشله في حل السؤال. ولكن إذا ما اتبع الأسلوب الصحيح في دراسة هذا العلم فستكون دراسته ميسرة وشيقة جدا.

الفيزياء ليست رياضيات فهناك فرق شاسع بـين الاثنتـين. الفيزيـاء تسـتعين بالمعادلات الرياضية فقط بعد تحديد الكميات الأساسية التي تؤثر في النمـوذج الفيزيـائي تحت الدراسة وباستخدام المفاهيم الفيزيائية يمكن إهمال تأثير بعض تلك الكميات وبعـدها يأتي دور الرياضيات لتحويل العلاقة الفيزيائية إلى معادلة رياضية تحل وتبسط صورتها.

ومن هنا جاءت فكرة تأليف هذه السلسلة من كتب "محاضرات في الفيزياء العامة" التي تتناول دراسة أجزاء أساسية من علم الفيزياء التي يدرسها الطالـب فـي المرحلـة الجامعية. يتناول الجزء الأول من كتاب محاضرات في الفيزياء العامة شرح مبادئ علـم الميكانيكا وتطبيقاتها. وقد راعيت في عرض الموضوعات سـهولة العبـارة ووضـوح المعنى. وتم التركيز على حل العديد من الأمثلة بعد كل موضوع لمزيد مـن التوضـيح على ذلك الموضوع، وفي نهاية كل فصل تم حل العديد من المسائل المتنوعة التي تغطي ذلك الفصل، هذا بالإضافة إلى المسائل في نهاية كل فصل للطالب ليحلها خلال دراسـته. تم الاعتماد على اللغة العربية في توضيح وشرح بعض المواضيع وكذلك في التعليق على حلول الأمثلة وللاستفادة من هذا الكتاب ينصح باتباع الخطوات التالية:

- حاول حل الأمثلة المحلولة في الكتاب دون الاستعانة بالنظر إلى الحل الموجود.
- اقرأ صيغة السؤال للمثال المحلول عدة مرات حتى تستطيع فهم السؤال جيداً.
- حدد المعطيات ومن ثم المطلوب من السؤال.

- حدد الطريقة التي ستوصلك إلى إيجاد ذلك المطلوب على ضوء المعطيات والقوانين.
- قارن حلك مع الحل الموجود في الكتاب مستفيداً من أخطائك.

يحتوي الكتاب على عشرة فصول، يركز الفصل الأول على الوحدات الفيزيائية وعلم المتجهات والتعامل معها لأنها تشكل الأساس الرياضي للعديد من المفاهيم الفيزيائية، والفصل الثاني والثالث يتناولان علم ميكانيكا الحركة الخطية من ناحية علم وصف الحركة "الكينماتيكا" ومن ناحية مؤثرات الحركة "الديناميكا"، كما يدرس الفصلان الرابع والخامس الطاقة الميكانيكية والشغل وعلاقتهما ببعض. يتناول الفصل السادس دراسة قانون الحفاظ على كمية الحركة والتصادمات بأنواعها المختلفة كتطبيق على علم الميكانيكا الخطية، أما الفصل السابع فيركز على مفهوم الحركة الدورانية وأساسياتها مع توضيح التشابه بينها وبين الحركة الخطية. أما الفصل الثامن فيدرس مفهوم عجلة الجاذبية وقانون الجذب العام لنيوتن وتطبيقاته. والفصل التاسع يدرس نوعاً آخراً من الحركة هو الحركة الاهتزازية وتطبيقاتها. والفصل العاشر يدرس ميكانيكا الموائع وأساسياتها الفيزيائية وتطبيقاتها العملية. كما تم في نهاية كل فصل الإجابة على بعض الأسئلة التي تدور حول موضوع الفصل وكذلك تم اختيار عدد من الأسئلة والتمارين للطالب ليتدرب على حلها، كما تم وضع ما يزيد عن مائة مسألة اختيار متعدد في نهاية هذا الكتاب لتغطي فصول الكتاب.

آمل أن أكون قد قدمت لأبنائنا الدارسين من خلال هذا العمل المتواضع ما يعينهم على فهم واستيعاب هذا الفرع من فروع المعرفة. كما أتقدم بالشكر لكل من يقدم نصيحة حول هذا الكتاب وموضوعاته.

والله من وراء القصد

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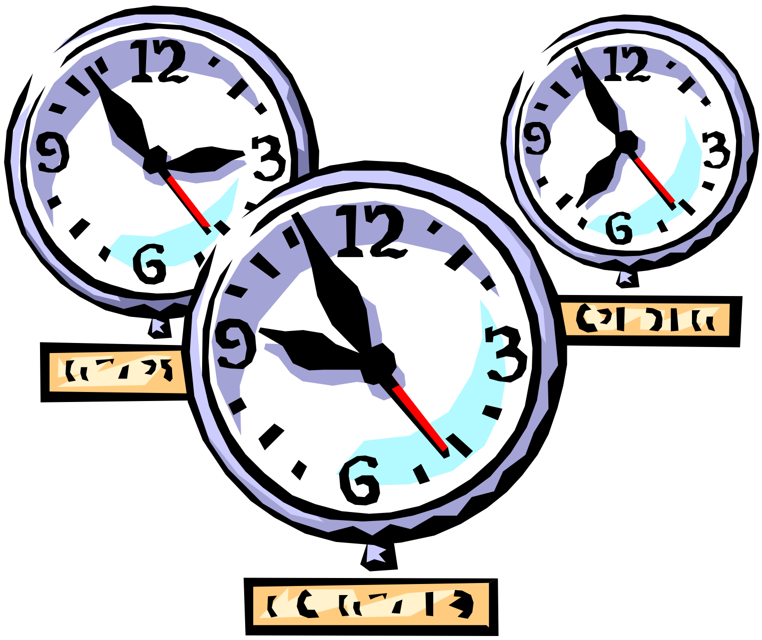
Chapter 1

Introduction: Physics and Measurements

مقدمة: الفيزياء والقياس



INTRODUCTION: PHYSICS AND MEASUREMENTS

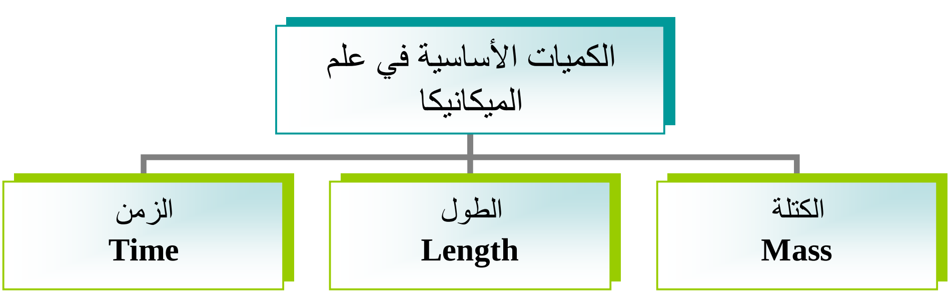
1.1 Physics and Measurements

علم الفيزياء هو علم تجريبي يهتم بكشف أسرار الطبيعة، فكل شيء نعرفه عن هذا الكون وعن القوانين التي تحكمه تم التوصل إليها عن طريق القياسات والملاحظات لأي ظاهرة طبيعية. ويعرف علم الفيزياء أيضاً بأنه علـــم القيـــاس ***Science of measurements*** يقول العالم الشهير كلفن "عندما تستطيع قياس ما تتكلم عنه وتعبر عنه بالأرقام فإنـــك إذاً تعرف شيئاً عنه، ولكنها عندما لا تستطيع التعبير عنه بالأرقام فإن معرفتك في هذه الحالة غير كافية ولكن تعتبر البداية".

1.2 Physical Quantity

لتعريف الكمية الفيزيائية *Physical Quantity* فإنه يجب أولا أن نعرف طريقة قيـــاس هذه الكمية أو طريقة حسابها رياضياً من كميات أخرى. فعلى سبيل المثال يمكن تعريـــف المسافة والزمن بواسطة وصف الطريقة التي يمكن أن نقيس كلاً منهما، وبالتـــالي يمكـــن تعريف سرعة جسم متحرك بواسطة حساب حاصل قسمة المسافة على الزمن. في هـــذه الحالة فإن كلاً من المسافة والزمن هما كميتان فيزيائيتان أساسيتان بينما السرعة فهي كمية فيزيائية مشتقة *Derived Physical Quantity*.

تسمى هذه الطريقة من التعريـــف بـــالتعريف الإجرائـــي *Operational Definition*. وبالتالي تعتمد على وصف طريقة القياس لأية كمية فيزيائية. هناك كميات فيزيائية كثيرة تعتمد على هذه الطريقة من التعريف وهذه هي الكميات الأساسية فمثلاً في علم الميكانيكا فإن الكميات الأساسية التي سنستخدمها هي الكتلة والطول والزمن.



1.3 Unit systems

Two systems of units are widely used in the world, the metric and the British systems. The metric system measures the length in meters whereas the British system makes use of the foot, inch, ….. The metric system is the most widely used. Therefore the metric system will be used in this book.

By international agreement the metric system was formalized in 1971 into the *International System of Units* (SI). There are seven basic units in the SI as shown in table 1.3. "*For this book only three units are used, the meter, kilogram, and second*".

Quantity	Name	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Temperature	kelvin	K
Electric current	ampere	A
Number of particles	mole	mol
Luminous intensity	candela	cd

Mass

The SI unit of mass is the *Kilogram*, which is defined as the mass of a specific platinum-iridium alloy cylinder.

Time

The SI unit of time is the *Second*, which is the time required for a cesium-133 atom to undergo 9192631770 vibrations.

Length

The SI unit of length is Meter, which is the distance traveled by light is vacuum during a time of 1/2999792458 second.

1.3.1 Units of Length

تعتبر وحدة قياس المسافة (الكيلومتر) كبيرة في بعض الأحيان فمثلاً لقياس طول غرفــة الدراسة أو قياس مسافة عرض الشارع فإنه يمكن استخدام وحدات مشتقة مثل المتــر أو

السنتمتر أو الميليمتر، أما في حالة قياس مسافات ذرية فإننا نستخدم وحدات أصغر مثـــل الأنجسترم. الجدول التالي يوضح قيمة وحدات المسافة المشتقة بالمتر.

1	kilometer	(km)	$=10^{3}$m
1	decimeter	(dm)	$=10^{-1}$m
1	centimeter	(cm)	$=10^{-2}$m
1	millimeter	(mm)	$=10^{-3}$m
1	micrometer	(μm)	$=10^{-6}$m
1	nanometer	(nm)	$=10^{-9}$m
1	angstrom	(Å)	$=10^{-10}$m
1	picometer	(pm)	$=10^{-12}$m
1	femtometer	(fm)	$=10^{-15}$m

1.3.2 Power of ten prefixes

كثيراً ما تكون الوحدات الأساسية (الكيلومتر والكيلوجرام والثانية) إما صغيرة أو كبيـــرة نسبة لما نقوم بقياسه من كميات فيزيائية لذا فقد تم تسمية وحدات عملية أخري موضـــحة في الجدول التالي:

number	prefix	Abbreviation
10^{18}	exa-	E
10^{15}	peta	P
10^{12}	tera-	T
10^{9}	giga-	G
10^{6}	mega-	M
10^{3}	kilo-	K
10^{-2}	centi-	C
10^{-3}	milli-	M
10^{-6}	micro-	μ
10^{-9}	nano-	N
10^{-12}	pico-	P
10^{-15}	femto-	F
10^{-18}	atto-	A

1.4 Derived quantities

All physical quantities measured by physicists can be expressed in terms of the three basic unit of length, mass, and time. For example,

speed is simply length divided by time, and the ***force*** is actually mass multiplied by length divided by time squared.

$$[\text{Speed}] = L/T = LT^{-1}$$

$$[\text{Force}] = ML/T^2 = MLT^{-2}$$

where [Speed] is meant to indicate the unit of speed, and M, L, and T represents mass, length, and time units.

1.5 Dimensional Analysis

The word dimension in physics indicates the physical nature of the quantity. For example the distance has a dimension of ***length***, and the speed has a dimension of ***length/time***.

The dimensional analysis is used to check the formula, since the dimension of the left hand side and the right hand side of the formula must be the same.

تستخدم تحليل الأبعاد Dimensional Analysis في التأكد من صحة المعادلات والعلاقات الرياضية المشتقة في الفيزياء حيث أن وحدة الطرف الأيمن للمعادلة يجب أن يساوي وحدة الطرف الأيسر للمعادلة، وإلا فإن المعادلة غير صحيحة.

Example 1.1

Using the dimensional analysis check that this equation $x = ½\, at^2$ is correct, where x is the distance, a is the acceleration and t is the time.

Solution

$$x = ½\, at^2$$

الطرف الأيسر للمعادلة له بعد طول، ولكي تكون المعادلة صحيحة فإن الطرف الأيمن يجب أن يكون له بعد طول أيضاً، وللتحقق من صحة المعادلة نستخدم تحليل الأبعاد لطرفي المعادلة.

$$L = \frac{L}{T^2} \times T^2 = L$$

This equation is correct because the dimension of the left and right side of the equation have the same dimensions.

Example 1.2

Show that the expression $v = v_o + at$ is dimensionally correct, where v and v_o are the velocities and a is the acceleration, and t is the time

Solution

The right hand side

$$[v] = \frac{L}{T}$$

The left hand side

$$[at] = \frac{L}{T^2} \times T = \frac{L}{T}$$

Therefore, the expression is dimensionally correct.

Example 1.3

Suppose that the acceleration of a particle moving in circle of radius r with uniform velocity v is proportional to the r^n and v^m. Use the dimensional analysis to determine the power n and m.

Solution

Let us assume a is represented in this expression

$$a = k\, r^n v^m$$

Where k is the proportionality constant of dimensionless unit.

The right hand side

$$[a] = \frac{L}{T^2}$$

The left hand side

$$[k\ r^n\ v^m] = L^n\left(\frac{L}{T}\right)^m = \frac{L^{n+m}}{T^m}$$

therefore

$$\frac{L}{T^2} = \frac{L^{n+m}}{T^m}$$

hence

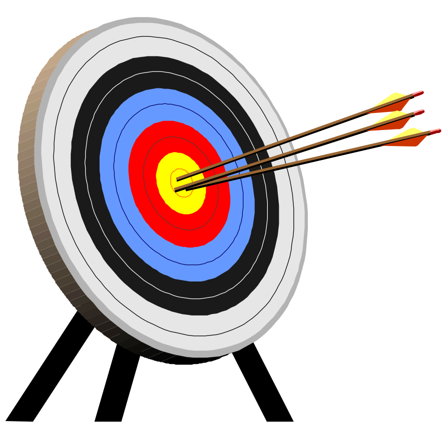
n+m=1 and m=2

Therefore. n =-1 and the acceleration a is

$$a = k\,r^{-1}\,v^2$$

k = 1

$$a = \frac{v^2}{r}$$



1.6 Vector and Scalar

جميع الكميات الفيزيائية (أساسية أو مشتقة) يمكن تقسيمها إلى نوعين، الأول الكمية القياسية ***scalar*** والثانية الكمية المتجهة ***vector***. **الكمية القياسية** يمكن تحديدها بمقدارها فقط، مثل أن تقول أن كتلة جسم 5kg. أما **الكمية المتجهة** تحتاج إلى أن تحدد اتجاهها بالإضافة إلى مقدارها، مثل سرعة الرياح 10km/h غرباً.

في الجدول التالي قائمة ببعض الكميات القياسية والكميات المتجهة.

Vector Quantity	**Scalar Quantity**
Displacement	Length
Velocity	Mass
Force	Speed
Acceleration	Power
Field	Energy
Momentum	Work

1.7 Coordinate system

نحتاج في حياتنا العملية إلى تحديد موقع جسم ما في الفراغ سواءً كان ساكناً أم متحركاً، ولتحديد موقع هذا الجسم فإننا نستعين بما يعرف بالإحداثيات ***Coordinates***، وهناك نوعان من الإحداثيات التي سوف نستخدمها في هذا الكتاب وهما ***Rectangular coordinates*** و ***polar coordinates***.

1.7.1 The rectangular coordinates

The rectangular coordinate system in two dimensions is shown in Figure 1.1. This coordinate system is consist of a fixed reference point (0,0) which called the origin. A set of axis with appropriate scale and label.

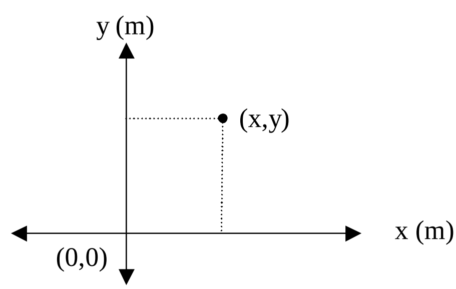


Figure 1.1

1.7.2 The polar coordinates

Sometimes it is more convenient to use the polar coordinate system (r,q), where r is the distance from the origin to the point of rectangular coordinate (x,y), and q is the angle between r and the x axis.

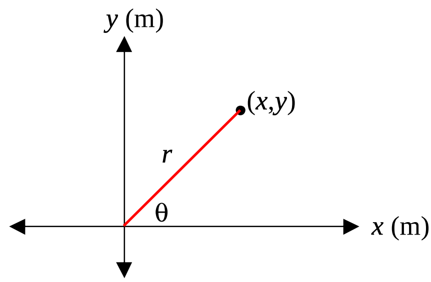


Figure 1.2

1.7.3 The relation between coordinates

The relation between the rectangular coordinates (x,y) and the polar coordinates (r,q) is shown in Figure 1.3, where,

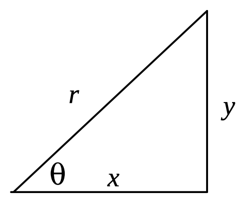


Figure 1.3

$$x = r\cos q \qquad (1.1)$$

And

$$y = r\sin q \qquad (1.2)$$

Squaring and adding equations (1.1) and (1.2) we get

$$r = \sqrt{x^2 + y^2} \qquad (1.3)$$

Dividing equation (1.1) and (1.2) we get

$$\tan q = \frac{x}{y} \qquad (1.4)$$

Example 1.4

The polar coordinates of a point are r = 5.5m and q =240°. What are the Cartesian coordinates of this point?

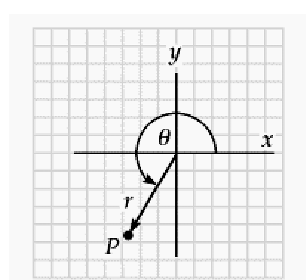


Figure 1.4

Solution

$x = r \cos\theta = 5.5\times\cos 240^{o} = -2.75$ m

$y = r \sin\theta = 5.5\times\sin 240^{o} = -4.76$ m

1.8 Properties of Vectors

1.8.1 Vector addition

Only vectors representing the same physical quantities can be added. To add vector $\vec{A}$ to vector $\vec{B}$ as shown in Figure 1.5, the resultant vector $\vec{R}$ is

$$\vec{R} = \vec{A} + \vec{B} \qquad (1.5)$$

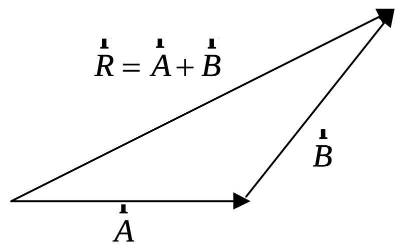


Figure 1.5

Notice that the vector addition obeys the commutative law, *i.e.*

$$\vec{A} + \vec{B} = \vec{B} + \vec{A} \qquad (1.6)$$

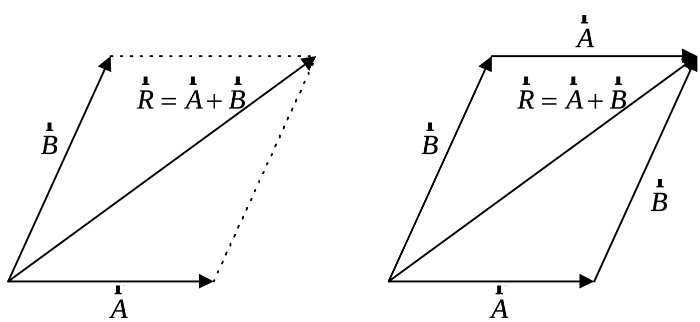


Figure 1.6

Notice that the vector addition obeys the associative law, *i.e.*

$$\vec{A} + (\vec{B} + \vec{C}) = (\vec{A} + \vec{B}) + \vec{C} \qquad (1.7)$$

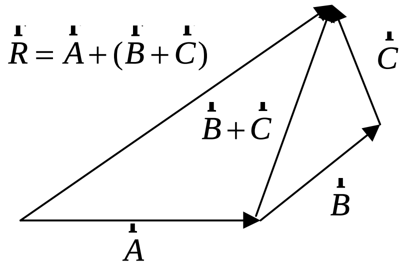


Figure 1.7

1.8.2 Vector subtraction

The vector subtraction $\vec{A} - \vec{B}$ is evaluated as the vector subtraction ***i.e.***

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B}) \qquad (1.8)$$

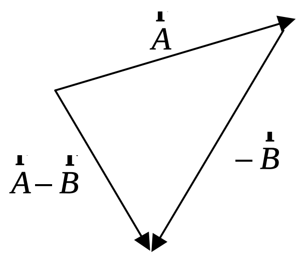


Figure 1.8

where the vector $-\vec{B}$ is the negative vector of $\vec{B}$

$$\vec{B} + (-\vec{B}) = 0 \qquad (1.9)$$

1.9 The unit vector

A unit vector is a vector having a magnitude of unity and its used to describe a direction in space.

المتجه $\vec{A}$ يمكن تمثيله بمقدار المتجه A ضرب متجه الوحدة $\boldsymbol{a}$ كالتالي

$$\vec{A} = \boldsymbol{a}\, A \qquad (1.10)$$

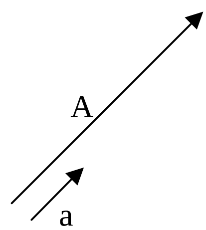


Figure 1.9

كذلك يمكن تمثيل متجهات وحدة (i, j, k) لمحـــاور الإســـناد المتعامـــدة rectangular (x, y, z) coordinate system كما في الشكل التالي:-

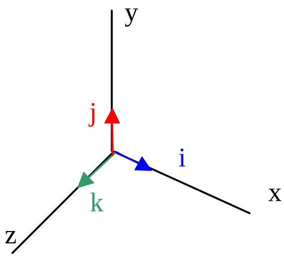


Figure 1.10

i ≡ a unit vector along the *x*-axis
j ≡ a unit vector along the *y*-axis
k ≡ a unit vector along the *z*-axis

1.10 Components of a vector

Any vector $\vec{A}$ lying in *xy* plane can be resolved into two components one in the *x*-direction and the other in the *y*-direction as shown in Figure 1.11

$$A_x = A \cos q \qquad (1.11)$$

$$A_y = A \sin q \qquad (1.12)$$

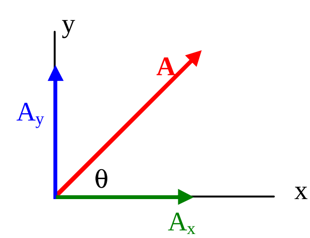


Figure 1.11

عند التعامل مع عدة متجهات فإننا نحتاج إلى تحليل كل متجه إلى مركباته بالنســـبة إلـــى محاور الإسناد (*x*,*y*) مما يسهل إيجاد المحصلة بدلاً من استخدام الطريقة البيانية لإيجـــاد المحصلة.

The magnitude of the vector $\vec{A}$

$$A = \sqrt{A_x^2 + A_y^2} \qquad (1.13)$$

The direction of the vector to the *x*-axis

$$q = \tan^{-1} \frac{A_y}{A_x} \qquad (1.14)$$

A vector $\vec{A}$ lying in the *xy* plane, having rectangular components A_x and A_y can be expressed in a unit vector notation

$$\vec{A} = A_x i + A_y j \qquad (1.15)$$

ملاحظة: يمكـن اسـتخدام طريقـة تحليـل المتجهات في جمع متجهين $\vec{A}$ و $\vec{B}$ كما في الشكل التالي:

$$\vec{A} = A_x i + A_y j$$
$$\vec{B} = B_x i + B_y j$$

$$\vec{R} = \vec{A} + \vec{B} = (A_x + B_x) i + (A_y + B_y) j$$

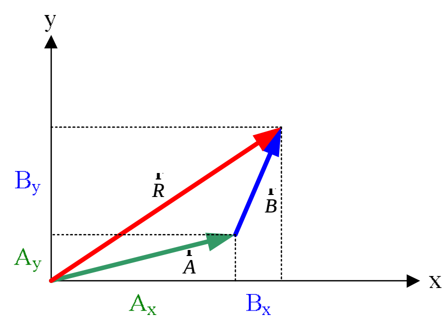


Figure 1.12

Example 1.5

Find the sum of two vectors $\vec{A}$ and $\vec{B}$ given by

$$\vec{A} = 3i + 4j \qquad \text{and} \qquad \vec{B} = 2i - 5j$$

Solution

Note that A_x=3, A_y=4, B_x=2, and B_y=-5

$$\vec{R} = \vec{A} + \vec{B} = (3+2)i + (4-5)j = 5i - j$$

The magnitude of vector $\vec{R}$ is

$$R = \sqrt{R_x^{\ 2} + R_y^{\ 2}} = \sqrt{25+1} = \sqrt{26} = 5.1$$

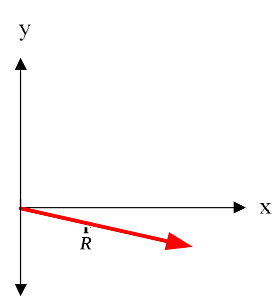


Figure 1.13

The direction of $\vec{R}$ with respect to *x*-axis is

$$q = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{-1}{5} = -11^o$$

Example 1.6

The polar coordinates of a point are *r*=5.5m and θ=240°. What are the rectangular coordinates of this point?

Solution

$x = r\cos\theta = 5.5 \times \cos 240 = -2.75$ m

$y = r\sin\theta = 5.5 \times \sin 240 = -4.76$ m

Example 1.7

Vector $\vec{A}$ is 3 units in length and points along the positive *x* axis. Vector $\vec{B}$ is 4 units in length and points along the negative *y* axis. Use graphical methods to find the magnitude and direction of the vector (a) $\vec{A}+\vec{B}$, (b) $\vec{A}-\vec{B}$

Solution

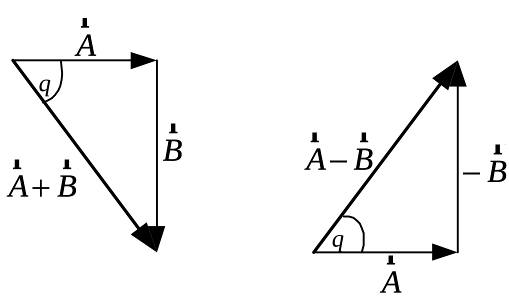


Figure 1.14

$|\vec{A}+\vec{B}| = 5$ $\qquad$ $|\vec{A}-\vec{B}| = 5$

$\theta = -53^{o}$ $\qquad$ $\theta = 53^{o}$

Example 1.8

Two vectors are given by $\vec{A} = 3i - 2j$ and $\vec{B} = -i - 4j$. Calculate (a) $\vec{A}+\vec{B}$, (b) $\vec{A}-\vec{B}$, (c) $|\vec{A}+\vec{B}|$, (d) $|\vec{A}-\vec{B}|$, and (e) the direction of $\vec{A}+\vec{B}$ and $|\vec{A}-\vec{B}|$.

Solution

(a) $\vec{A}+\vec{B} = (3i-2j)+(-i-4j) = 2i-6j$

(b) $\vec{A}-\vec{B} = (3i-2j)-(-i-4j) = 4i+2j$

(c) $|\vec{A}+\vec{B}| = \sqrt{2^2+(-6)^2} = 6.32$

(d) $|\vec{A}-\vec{B}| = \sqrt{4^2+2^2} = 4.47$

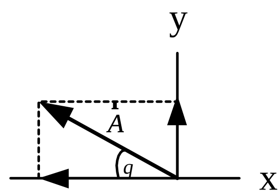
(e) For $\vec{A}+\vec{B}$, $\theta = \tan^{-1}(-6/2) = -71.6^{o} = 288^{o}$

For $\vec{A}-\vec{B}$, $\theta = \tan^{-1}(2/4) = 26.6^{o}$

Example 1.9

A vector $\vec{A}$ has a negative *x* component 3 units in length and positive *y* component 2 units in length. (a) Determine an expression for $\vec{A}$ in unit vector notation. (b) Determine the magnitude and direction of $\vec{A}$. (c) What vector $\vec{B}$ when added to $\vec{A}$ gives a resultant vector with no *x* component and negative *y* component 4 units in length?

Solution



A_x = -3 units & A_y = 2 units

(a) $\vec{A} = A_x i + A_y j = -3i + 2j$ units

(b) $|\vec{A}| = \sqrt{A_x^{\,2} + A_y^{\,2}} = \sqrt{(-3)^2 + (2)^2} = 3.61\ units$

$\theta = \tan^{-1}(2/-3) = 33.7^o$ (relative to the –x axis)

(c) $R_x = 0$ & $R_y = -4$; since $\vec{R} = \vec{A} + \vec{B}$, $\vec{B} = \vec{R} - \vec{A}$

$B_x = R_x - A_x = 0-(-3) = 3$

$B_y = R_y - A_y = -4-2 = -6$

Therefore,

$$\vec{B} = B_x i + B_y j = (3i - 6j)\ \ units$$

Example 1.10

A particle moves from a point in the *xy* plane having rectangular coordinates (-3,-5)m to a point with coordinates (-1,8)m. (a) Write vector expressions for the position vectors in unit vector form for these two points. (b) What is the displacement vector?

Solution

(a) $\vec{R}_1 = x_1 i + y_1 j = (-3i - 5j)m$

$\vec{R}_2 = x_2 i + y_2 j = (-i + 8j)m$

(b) Displacement = $\Delta\vec{R} = \vec{R}_2 - \vec{R}_1$

$$\Delta\vec{R} = (x_2 - x_1)i + (y_2 - y_1)j = -i - (-3i) + 8j - (-5j) = (2i + 13j)m$$

1.11 Product of a vector

There are two kinds of vector product the first one is called scalar product or dot product because the result of the product is a scalar quantity. The second is called vector product or cross product because the result is a vector perpendicular to the plane of the two vectors.

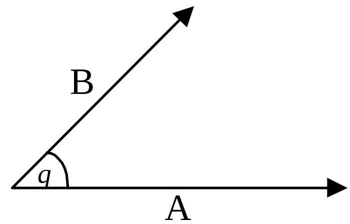


Figure 1.15

ينتج من الضرب القياسي كمية قياسية وينتج من الضرب الإتجاهي كمية متجهة

1.11.1 The scalar product

يعرف الضرب القياسي scalar product بالضرب النقطي dot product وتكون نتيجة الضرب القياسي لمتجهين كمية قياسية، وتكون هذه القيمة موجبة إذا كانت الزاوية المحصورة بين المتجهين بين 0 و 90 درجة وتكون النتيجة سالبة إذا كانت الزاوية المحصورة بين المتجهين بين 90 و 180 درجة وتساوي صفراً إذا كانت الزاوية 90.

$$\vec{A}.\vec{B} = \text{+ve when } 0 \geq q > 90^o$$

$$\vec{A}.\vec{B} = \text{-ve when } 90^o < q \leq 180^o$$

$$\vec{A}.\vec{B} = \text{zero when } q = 0$$

يعرف الضرب القياسي لمتجهين $\vec{A}, \vec{B}$ بحاصل ضرب مقدار المتجه الأول $\vec{A}$ في مقدار المتجه الثاني $\vec{B}$ في جيب تمام الزاوية المحصورة بينهما.

$$\vec{A}.\vec{B} = |A||B|\cos q \qquad (1.16)$$

يمكن إيجاد قيمة الضرب القياسي لمتجهين باستخدام مركبات كل متجه كما يلي:

$$\vec{A} = A_x i + A_y j + A_z k \qquad (1.17)$$

$$\vec{B} = B_x i + B_y j + B_z k \qquad (1.18)$$

The scalar product is

$$\vec{A}.\vec{B} = (A_x i + A_y j + A_z k).(B_x i + B_y j + B_z k) \qquad (1.19)$$

بضرب مركبات المتجه $\vec{A}$ في مركبات المتجه $\vec{B}$ ينتج التالي:

$$\begin{aligned}\vec{A}.\vec{B} = (&A_x i.B_x i + A_x i.B_y j + A_x i.B_z k \\ &+ A_y j.B_x i + A_y j.B_y j + A_y j.B_z k \\ &+ A_z k.B_x i + A_z k.B_y j + A_z k.B_z k)\end{aligned} \qquad (1.20)$$

Therefore

$$\therefore\ \vec{A}.\vec{B} = A_x B_x + A_y B_y + A_z B_z \qquad (1.21)$$

The angle between the two vectors is

$$\cos q = \frac{\vec{A}.\vec{B}}{|A||B|} = \frac{A_x B_x + A_y B_y + A_z B_z}{|A||B|} \qquad (1.22)$$

Example 1.11

Find the angle between the two vectors

$$\vec{A} = 2i + 3j + 4k\,, \quad \vec{B} = i - 2j + 3k$$

Solution

$$\cos q = \frac{A_x B_x + A_y B_y + A_z B_z}{|A||B|}$$

$$A_x B_x + A_y B_y + A_z B_z = (2)(1)+(3)(-2)+(4)(3)=8$$

$$|A| = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{29}$$

$$|B| = \sqrt{1^2 + (-2)^2 + 3^2} = \sqrt{14}$$

$$\cos q = \frac{18}{\sqrt{29}\sqrt{14}} 0.397 \Rightarrow q = 66.6^o$$

1.11.2 The vector product

يعرف الضرب الاتجاهي *vector product* بـ *cross product* وتكون نتيجة الضرب الاتجاهي لمتجهين كمية متجهة. قيمة هذا المتجه $\vec{C} = \vec{A} \times \vec{B}$ واتجاهه عمودي على كل من المتجهين $\vec{A}$ و $\vec{B}$ وفي اتجاه دوران بريمة من المتجه $\vec{A}$ إلى المتجه $\vec{B}$ كما في الشكل التالي:

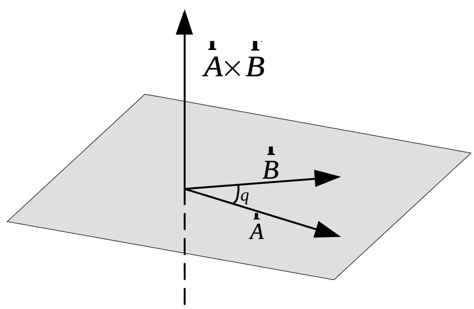


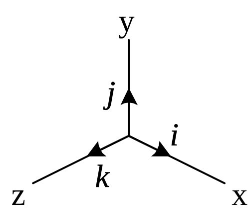
Figure 1.16

$$\vec{A} \times \vec{B} = AB \sin\theta \qquad (1.23)$$

$$\vec{A} \times \vec{B} = (A_x i + A_y j + A_z k) \times (B_x i + B_y j + B_z k) \qquad (1.24)$$

To evaluate this product we use the fact that the angle between the unit vectors *i*, *j* , *k* is 90°.

$$i \times i = 0 \qquad i \times j = k \qquad i \times k = -j$$
$$j \times j = 0 \qquad j \times k = i \qquad j \times i = -k$$
$$k \times k = 0 \qquad k \times i = j \qquad k \times j = -i$$



$$\vec{A} \times \vec{B} = (A_y B_z - A_z B_y) i + (A_z B_x - A_x B_z) j + (A_x B_y - A_y B_x) k \qquad (1.25)$$

If $\vec{C} = \vec{A} \times \vec{B}$, the components of $\vec{C}$ are given by

$$C_x = A_y B_z - A_z B_y$$
$$C_y = A_z B_x - A_x B_z$$
$$C_z = A_x B_y - A_y B_x$$

Example 1.12

If $\vec{C} = \vec{A} \times \vec{B}$, where $\vec{A} = 3i - 4j$, and $\vec{B} = -2i + 3k$, what is $\vec{C}$?

Solution

$$\vec{C} = \vec{A} \times \vec{B} = (3i - 4j) \times (-2i + 3k)$$

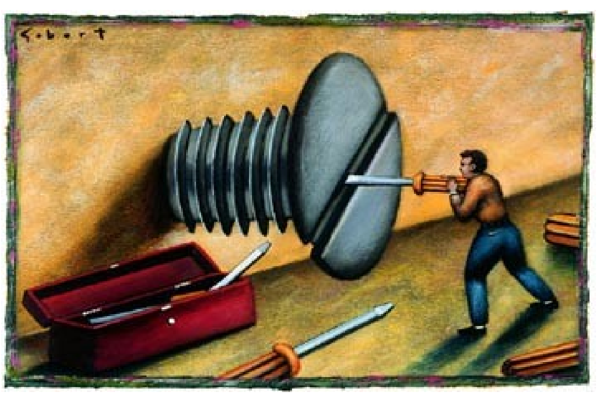
which, by distributive law, becomes

$$\vec{C} = -(3i \times 2i) + (3i \times 3k) + (4j \times 2i) - (4j \times 3k)$$

Using equation (123) to evaluate each term in the equation above we get

$$\vec{C} = 0 - 9j - 8k - 12i = -12i - 9j - 8k$$

The vector $\vec{C}$ is perpendicular to both vectors $\vec{A}$ and $\vec{B}$.



1.12 Problems

[1] Show that the expression $x=vt+1/2at^2$ is dimensionally correct, where x is a coordinate and has units of length, v is velocity, a is acceleration, and t is time.

[2] Which of the equations below are dimensionally correct?

(a) $v = v_o+at$

(b) $y = (2\text{m})\cos(kx)$, where $k = 2\ \text{m}^{-1}$.

[3] Show that the equation $v^2 = v_o^2 + 2at$ is dimensionally correct, where v and v_o represent velocities, a is acceleration and x is a distance.

[4] The period T of simple pendulum is measured in time units and given by

$$T = 2p\sqrt{\frac{l}{g}}$$

where l is the length of the pendulum and g is the acceleration due to gravity. Show that the equation is dimensionally correct.

[5] Suppose that the displacement of a particle is related to time according to the expression $s = ct^3$. What are the dimensions of the constant c?

[6] Two points in the xy plane have Cartesian coordinates (2, -4) and (-3, 3), where the units are in m. Determine (a) the distance between these points and (b) their polar coordinates.

[7] The polar coordinates of a point are r = 5.5m and q = 240°. What are the cartisian coordinates of this point?

[8] A point in the xy plane has cartesian coordinates (-3.00, 5.00) m. What are the polar coordinates of this point?

[9] Two points in a plane have polar coordinates (2.5m, 30°) and (3.8, 120°). Determine (a) the cartisian coordinates of these points and (b) the distance between them.

[10] A point is located in polar coordinate system by the coordinates r = 2.5m and q =35°. Find the x and y coordinates of this point, assuming the two coordinate system have the same origin.

[11] Vector $\vec{A}$ is 3.00 units in length and points along the

positive *x* axis. Vector $\vec{B}$ is 4.00 units in length and points along the negative *y* axis. Use graphical methods to find the magnitude and direction of the vectors (a) $\vec{A}+\vec{B}$, (b) $\vec{A}-\vec{B}$.

[12] A vector has x component of -25 units and a y component of 40 units. Find the magnitude and direction of this vector.

[13] Find the magnitude and direction of the resultant of three displacements having components (3,2) m, (-5, 3) m and (6, 1) m.

[14] Two vector are given by $\vec{A} = 6i - 4j$ and $\vec{B} = -2i+5j$. Calculate (a) $\vec{A}+\vec{B}$, (b) $\vec{A}-\vec{B}$, $|\vec{A}+\vec{B}|$, (d) $|\vec{A}-\vec{B}|$, (e) the direction of $\vec{A}+\vec{B}$ and $\vec{A}-\vec{B}$.

[15] Obtain expressions for the position vectors with polar coordinates (a) 12.8m, 150^o; (b) 3.3cm, 60^o; (c) 22cm, 215^o.

[16] Find the x and y components of the vector $\vec{A}$ and $\vec{B}$ shown in Figure 1.17. Derive an expression for the resultant vector $\vec{A}+\vec{B}$ in unit vector notation.

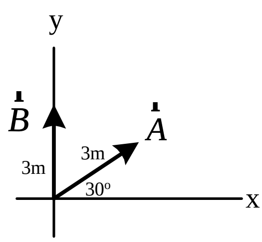


Figure 1.17

[17] A vector $\vec{A}$ has a magnitude of 35 units and makes an angle of 37^o with the positive *x* axis. Describe (a) a vector $\vec{B}$ that is in the direction opposite $\vec{A}$ and is one fifth the size of $\vec{A}$, and (b) a vector $\vec{C}$ that when added to $\vec{A}$ will produce a vector twice as long as $\vec{A}$ pointing in the negative y direction.

[18] Find the magnitude and direction of a displacement vector having *x* and *y* components of -5m and 3m, respectively.

[19] Three vectors are given by $\vec{A}$=6i, $\vec{B}=9j$, and $\vec{C}=(-3i+4j)$. (a) Find the magnitude and direction of the resultant vector. (b) What vector must be added to these three to make the resultant vector zero?

[20] A particle moves from a point in the *xy* plane having Cartesian coordinates (-3.00, -5.00) m to a point with coordinates (-1.00, 8.00) m. (a)

Write vector expressions for the position vectors in unit-vector form for these two points. (b) What is the displacement vector?

[21] Two vectors are given by $\vec{A}$= 4i+3j and $\vec{B}$= -i+3j. Find (a) $\vec{A}.\vec{B}$ and (b) the angle between $\vec{A}$ and $\vec{B}$.

[22] A vector is given by $\vec{A}$= -2i+3j. Find (a) the magnitude of $\vec{A}$ and (b) the angle that $\vec{A}$ makes with the positive *y* axis.

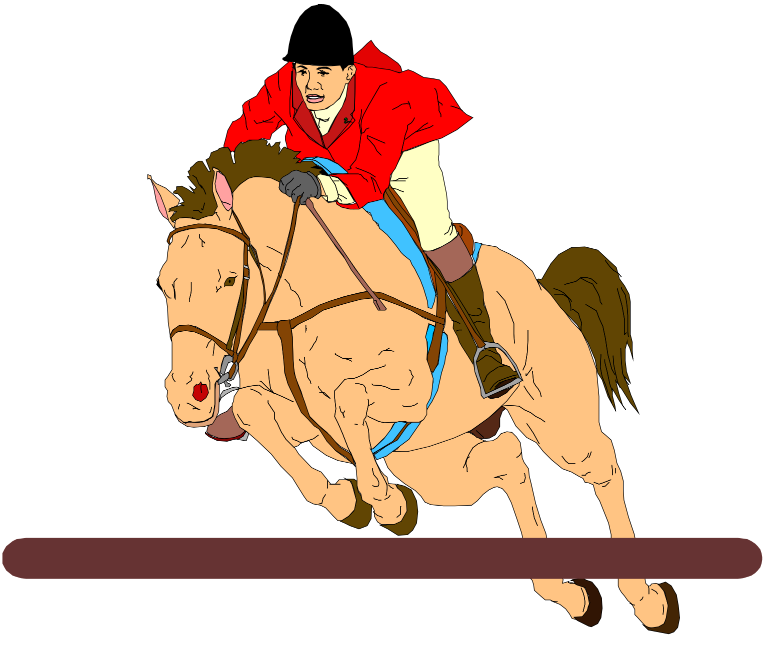
[23] Vector $\vec{A}$ has a magnitude of 5 units, and $\vec{B}$ has a magnitude of 9 units. The two vectors make an angle of 50^o with each other. Find $\vec{A}.\vec{B}$.

[24] For the three vectors $\vec{A}$=3i+j-k, $\vec{B}$= -i+2j+5k, and $\vec{C}$= 2j-3k, find $\vec{C}.(\vec{A}-\vec{B})$

[25] The scalar product of vectors $\vec{A}$ and $\vec{B}$ is 6 units. The magnitude of each vector is 4 units. Find the angle between the vectors.

Chapter 2

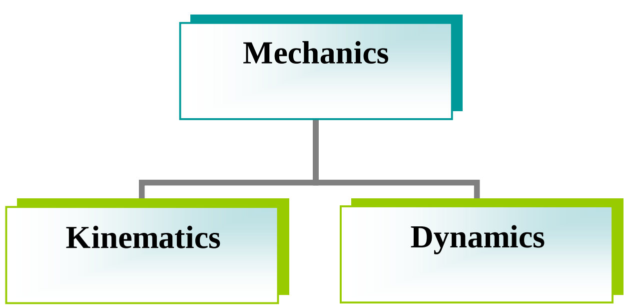
Mechanics



علم الميكانيكا

علم الميكانيكا

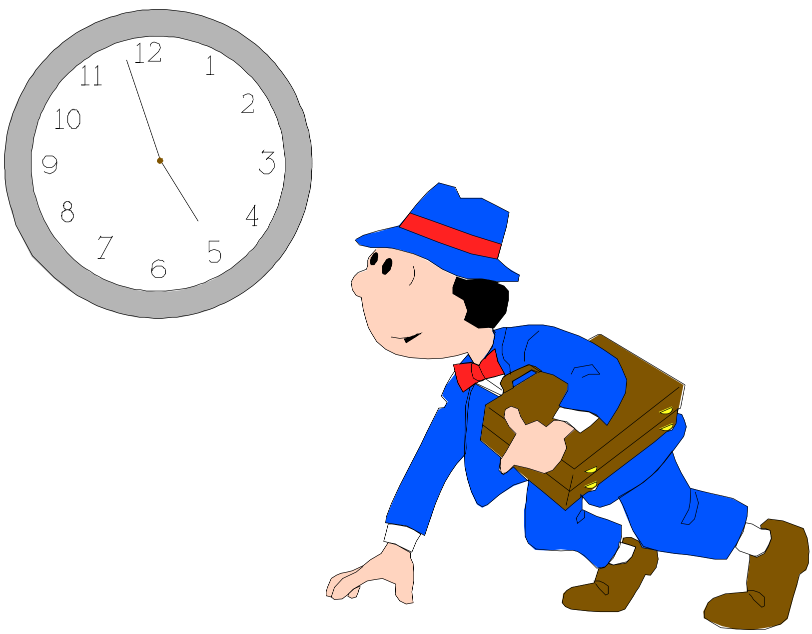
علم الميكانيكا من العلوم الواسعة التي تهتم بحركة الأجسام ومسبباتها، ويتفرع من هذا العلم فروع أخرى مثل الكينماتيكا ***Kinematics*** و الديناميكا ***Dynamics***. وعلم الكينماتيكا يهتم بوصف حركة الأجسام دون النظر إلى مسبباتها، أما علم الديناميكا ***Dynamics*** فهو يدرس حركة الأجسام ومسبباتها مثل القوة والكتلة. وفي هذا الفصل سنقوم بدراسة حركة الأجسام وعلاقتها بكل من الإحداثيات المكانية والزمنية. ثم سندرس الفرع الثاني وهو علم الديناميكا.



Chapter 2

Mechanics: Kinematics
Description of Motion

الكيناماتيكا: علم وصف الحركة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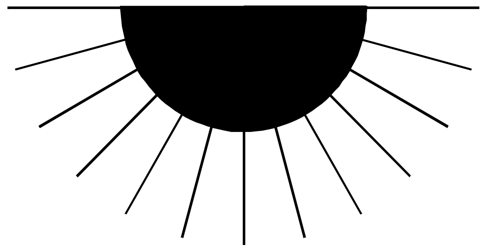


KINEMATICS DESCRIPTION OF MOTION

2.1 The position vector and the displacement vector

من أساسيات دراسة علم وصف الحركة الكينماتيكا ***Kinematics*** للأجسام المادية هو دراسة كل من الإزاحة ***Displacement*** و السرعة ***Velocity*** و العجلة ***Acceleration***. ونحتاج هنا إلى اعتماد محاور إسناد لتحديد موضع الجسم المتحرك عند أزمنة مختلفة ومن المناسب اعتماد محاور الإسناد الكارتيزية أو ما سميت بـ ***rectangular coordinate (x,y,z)***، فمثلاً نحتاج إلى تحديد موقع جسم ما إلى إسناده إلى مرجعية محددة فمثلاً يمكن اعتبار متجه الموضع ***Position vector*** هو المتجه الواصل من مركز إسناد معين إلى مكان الجسم الذي يراد تحديده. كما في الشكل 2.1 حيث تم اعتبار مركز الإسناد في بعدين فقط هو مركز المحاور x, y

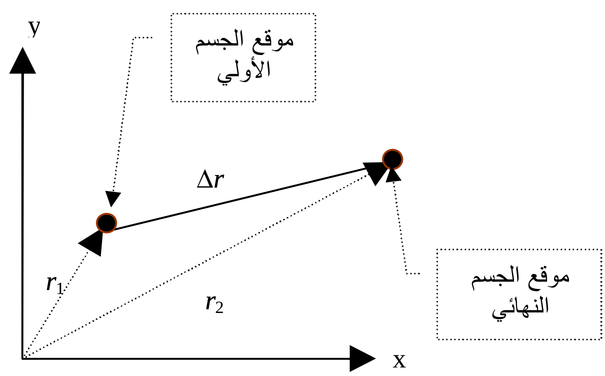


Figure 2.1

في الشكل 2.1 متجه الموضع r_1 يحدد موضع الجسم عند بداية الحركة ومتجه الموضع r_2 يحدد موقع الجسم النهائي بعد زمن وقدره $\Delta t = t_2 - t_1$ و هنا فإن الإزاحة للجسم تعطى بالمعادلة (2.3)

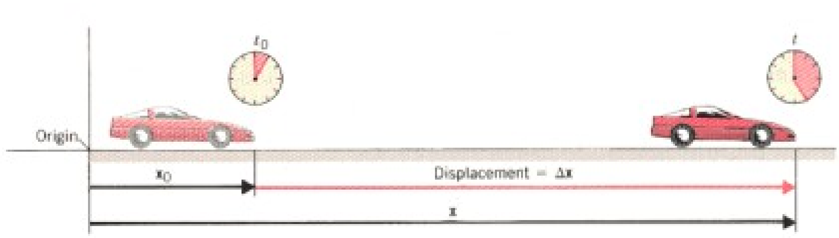
$$\vec{r}_1 = x_1 i + y_2 j \qquad (2.1)$$

$$\vec{r}_2 = x_2 i + y_2 j \qquad (2.2)$$

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 \qquad (2.3)$$

Δr is called the displacement vector which represent the change in the position vector.

لاحظ أن الإزاحة displacement $\Delta\vec{r}$ تعتمد على المسافة بين نقطتي البدايـــة والنهايـــة فقط ولا تعتمد على المسار الذي يسلكه الجسم.



Example 2.1

Write the position vector for a particle in the rectangular coordinate (*x*, *y*, *z*) for the points (5, -6, 0), (5, -4), and (-1, 3, 6).

Solution

For the point (5, -6, 0) the position vector is $\vec{r} = 5i - 6j$

For the point (5, -4) the position vector is $\vec{r} = 5i - 4j$

For the point (-1, 3, 6) the position vector is $\vec{r} = -i + 3j + 6k$

Example 2.2

Calculate the displacement vector for a particle moved from the point (4, 3, 2) to a point (8, 3, 6).

Solution

The position vector for the first point is $\vec{r}_1 = 4i + 3j + 2k$

The position vector for the second point is $\vec{r}_2 = 8i + 3j + 6k$

The displacement vector $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$

$$\therefore \Delta\vec{r} = 4i + 4k$$

Example 2.3

If the position of a particle is given as a function of time according to the equation

$$\vec{r}(t) = 3t^2 i + (3t - 2)j$$

where t in seconds. Find the displacement vector for t_1=1 and t_2=8

Solution

First we must find the position vector for the time t_1 and t_2

For t_1 $\quad \vec{r}_1(t_1) = 3i + j$

For t_2 $\quad \vec{r}_2(t_2) = 192i + 22j$

The displacement vector

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = 192i + 22j - 3i + j$$

$$\Delta\vec{r} = 189i + 21j$$

2.2 The average velocity and Instantaneous velocity

عند انتقال الجسم من موضع البداية عند الزمن t_1 إلى موضع النهاية عند الزمن t_2 فإن حاصل قسمة الإزاحة على فرق الزمن Δt (t_2-t_2) يعرف بالسرعة ***Velocity*** وحيث أن الجسم يقطع المسافة بسرعات مختلفة فإن السرعة المحسوبة تسمى بمتوسط السرعة ***Average velocity***. ويمكن تعريف السرعة عند أية لحظة بالسرعة اللحظية ***Instantaneous velocity***.

The ***average velocity*** of a particle is defined as the ratio of the displacement to the time interval.

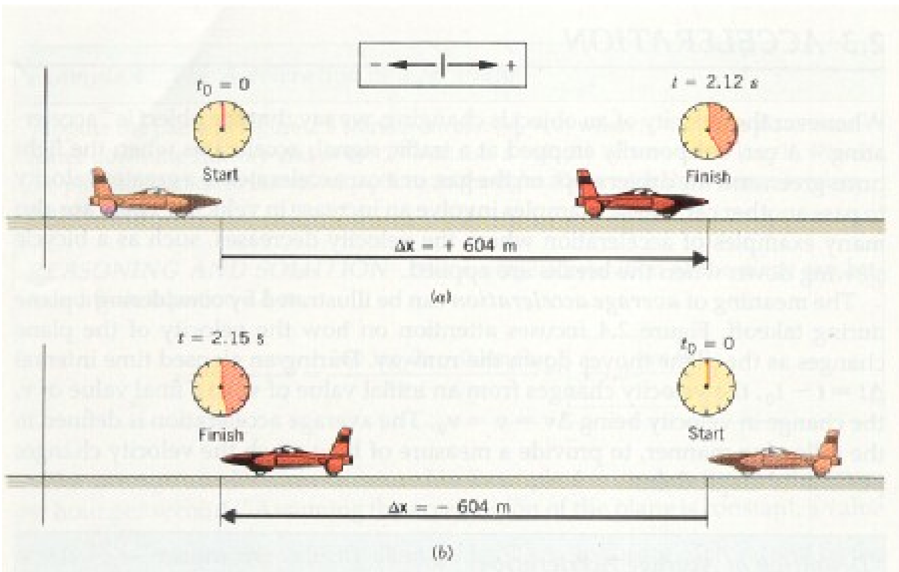
$$\vec{v}_{ave} = \frac{\Delta\vec{r}}{\Delta t} \qquad (2.4)$$

The ***instantaneous velocity*** of a particle is defined as the limit of the average velocity as the time interval approaches zero.

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} \qquad (2.5)$$

$$\therefore \vec{v} = \frac{d\vec{r}}{dt} \qquad (2.6)$$

The unit of the velocity is (m/s)



2.3 The average acceleration and Instantaneous acceleration

عند انتقال الجسم من موضع البداية عند الزمن t_1 إلى موضع النهاية t_2 بسرعة ابتدائية v_1 وعند النهاية كانت السرعة v_2 فإن معدل تغير السرعة بالنسبة إلى الزمن يعـرف باسـم التسارع ***Acceleration*** أو متوسـط التسـارع ***Average Acceleration***، ويكـون التسارع اللحظي ***Instantaneous acceleration*** هو السرعة اللحظية على الزمن.

The ***average acceleration*** of a particle is defined as the ratio of the change in the instantaneous velocity to the time interval.

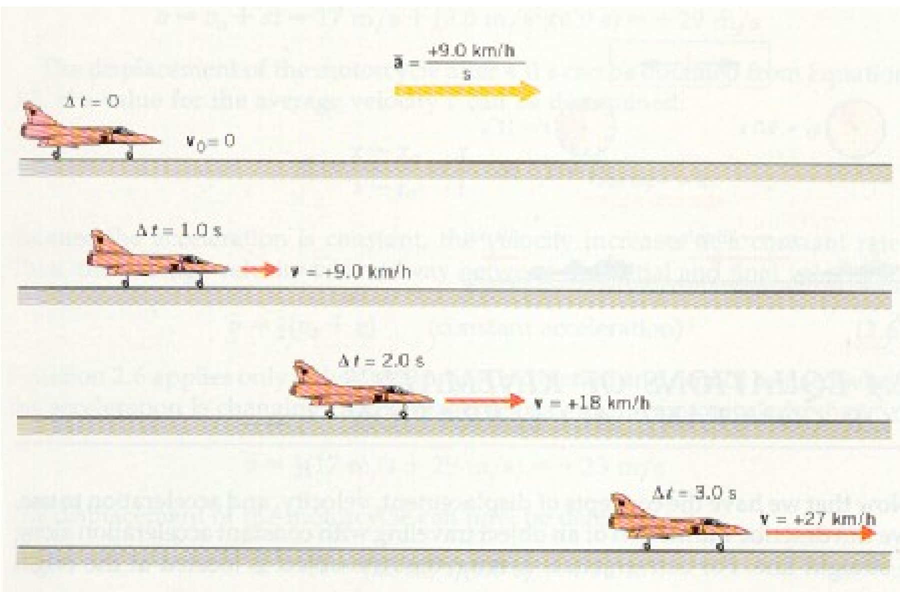
$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} \qquad (2.7)$$

The ***instantaneous acceleration*** is defined as the limiting value of the ratio of the average velocity to the time interval as the time approaches zero.

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} \qquad (2.8)$$

The unit of the acceleration is (m/s^2)

لنفترض طائرة تبدأ الحركة من السكون أي v_o=0 عند زمن t_o=0 كما في الشكل أدنـــاه. وبعد فترة زمنية قدرها 29s تصل الطائرة إلى سرعة 260k/h فإن العجلـــة المتوســطة للطائرة هي 9km/h/s



يوضح الشكل أعلاه تأثير العجلة على زيادة سرعة الطائرة للأربع ثوان الأولى من انطلاقها حيث تكون السرعة بعد زمن قدره ثانية يساوي 9km/h وبعد زمن ثانيتين تصل السرعة إلى 18km/h وهكذا ....

Example 2.4

The coordinate of a particle moving along the x-axis depends on time according to the expression

$$x = 5t^2 - 2t^3$$

where x is in meters and t is in seconds.

1. Find the velocity and acceleration of the particle as a function of time.
2. Find the displacement during the first 2 seconds.
3. Find the velocity and acceleration of the particle after 2 seconds.

Solution

(a) The velocity and acceleration can be obtained as follow

$$v = \frac{dx}{dt} = 10t - 6t^2$$

$$a = \frac{dv}{dt} = 10 - 12t$$

(b) using the equation $x = 5t^2 - 2t^3$ substitute for t=2s

$x = 4\text{m}$

(c) using the result in part (a)

$v = -4$ m/s

$a = -14$ m/s^2

Example 2.5

A man swims the length of a 50m pool in 20s and makes the return trip to the starting position in 22s. Determine his average velocity in (a) the first half of the swim, (b) the second half of the swim, and (c) the round trip.

Solution

(a) $v_1 = \dfrac{d}{t_1} = \dfrac{50}{20} = 2.5$ m/s

(b) $v_2 = \dfrac{d}{t_2} = \dfrac{-50}{20} = -2.27$ m/s

(c) Since the displacement is zero for the round trip, $v_{ave} = 0$

Example 2.6

A car makes a 200km trip at an average speed of 40 km/h. A second car starting 1h later arrives at their mutual destination at the same time. What was the average speed of the second car?

Solution

$t_1 = \dfrac{d}{v_1} = \dfrac{200}{40} = 5\text{h}$ for car 1

$t_2 = t_1 - 1 = 4\text{h}$ for car 2

$v_2 = \dfrac{d}{t_2} = \dfrac{200}{4} = 50\text{km/h}$

Example 2.7

A particle moves along the x-axis according to the equation $x=2t+3t^2$, where x is in m and t is in second. Calculate the instantaneous velocity and instantaneous acceleration at t=3s.

Solution

$v(t) = \dfrac{dx}{dt} = 2+6\,(3) = 20\text{m/s}$

$a(t) = \dfrac{dv}{dt} = 6\text{m/s}^2$

Therefore at $t = 3$s

$v = 20$m/s

$a = 6\text{m/s}^2$

Motion in One Dimension

2.4 One-dimensional motion with constant acceleration

سندرس الآن الحركة في بعد واحد وذلك فقط عندما تكــون العجلــة ثابتــة ***constant acceleration***. وفــي هــذه الحالــة تكــون العجلــة اللحظيــة ***Instantaneous acceleration*** تساوى متوسط العجلة ***Average acceleration***. ونتيجة لذلك فــإن السرعة إما أن تتزايد أو تتناقص بمعدلات متساوية خلال الحركة.

ويعبر عن ذلك رياضياً على النحو التالي:-

Instantaneous acceleration = Average acceleration

$$a = a_{ave} = \frac{v - v_o}{t - t_o} \qquad (2.9)$$

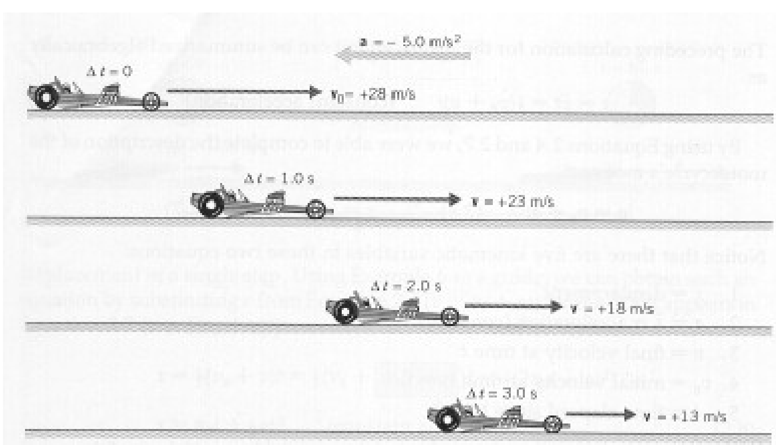
Let $t_o = 0$ then the acceleration

$$a = \frac{v - v_o}{t} \qquad (2.10)$$

or

$$v = v_o + at \qquad (2.11)$$

من المعادلة (2.11) يمكن إيجاد السرعة v عند أي زمن t إذا عرفنا السرعة الابتدائية v_o والعجلة الثابتة a التي يتحرك بها الجسم. وإذا كانت العجلة تساوي صفراً فإن السرعة لا تعتمد على الزمن، وهذا يعني أن السرعة النهائية تساوي السرعة الابتدائية. لاحظ أيضاً أن كل حد من حدود المعادلة السابقة له بعد سرعة (m/s).



يوضح الشكل أعلاه تأثير عجلة ثابتة مقدارها $-5 m/s^2$ في تقليل السرعة بمقدار 5m/s كل ثانية.

Since the velocity varies linearly (خطـــي) with time we can express the average velocity as

$$v_{ave} = \frac{v + v_o}{2} \qquad (2.12)$$

To find the displacement Δx $(x\text{-}x_o)$ as a function of time

$$\Delta x = v_{ave}\,\Delta t = \left(\frac{v + v_o}{2}\right)t \qquad (2.13)$$

or

$$x = x_o + \frac{1}{2}\,(v+v_o)\,t \qquad (2.14)$$

Also we can obtain the following equations

$$x = x_o + v_o\,t + \frac{1}{2}\,a\,t^2 \qquad (2.15)$$

$$v^2 = v_o^2 + 2a(x\text{-}x_o) \qquad (2.16)$$

من المعادلة (2.15) نلاحظ أن المسافة المقطوعة $(x\text{-}x_o)$ تساوي المسافة المقطوعة نتيجة السرعة الابتدائية وهو الحد $v_o t$ بالإضافة إلى المسافة نتيجة للعجلة الثابتة، وهذا يظهر في الحد الأخير من المعادلة $1/2at^2$، وإن كل حد من حدود المعادلة له بعد مسافة (m).

لاحظ أيضاً أنه إذا كانت العجلة تساوي صفراً فإن المسافة المقطوعة تساوي السرعة فـــي الزمن.

$$x - x_o = v_o t \qquad (2.17)$$

إذا كانت السرعة الابتدائية تساوي صفراً تكون المسافة المقطوعة تساوي

$$x - x_o = \frac{1}{2}\,a\,t^2 \qquad (2.18)$$

Example 2.8

A body moving with uniform acceleration has a velocity of 12cm/s when its x coordinate is 3cm. If its x coordinate 2s later is -5cm, what is the magnitude of its acceleration?

Solution

$$x = x_o + v_o\, t + \frac{1}{2}\, a\, t^2$$

$$-5 = 3 + 12\times 2 + 0.5\, a\, (2)^2$$

$$a = -16 \text{ cm/s}^2$$

Example 2.9

A car moving at constant speed of 30m/s suddenly stalls at the bottom of a hill. The car undergoes a constant acceleration of -2m/s^2 while ascending the hill.

1. Write equations for the position and the velocity as a function of time, taking x=0 at the bottom of the hill where v_o = 30m/s.
2. Determine the maximum distance traveled by the car up the hill after stalling.

Solution

1.

$$x = x_o + v_o\, t + \frac{1}{2}\, a\, t^2$$

$$x = 0 + 30\, t - t^2$$

$$x = 30\, t - t^2 \text{ m}$$

$$v = v_o + at$$

$$v = 30 - 2t \text{ m/s}$$

2. x reaches a maximum when $v = 0$ then,

$v = 30 - 2t = 0$ therefore $t = 15$ s

$x_{max} = 30\, t - t^2$

$x = 30\, t - t^2 = 30\,(15) - (15)^2 = 225$m

2.5 Application of one-dimensional motion with constant acceleration

2.5.1 Free Fall

من التطبيقات الهامة على العجلة الثابتة constant acceleration السقوط الحــر Free fall تحت تأثير عجلة الجاذبية الأرضية g حيث أن عجلة الجاذبية الأرضية ثابتة نســبياً على ارتفاعات محدودة من سطح الأرض واتجاهها دائما في اتجاه مركز الأرض، وبالتالي يمكن استخدام المعادلات الأربع السابقة مع تغيير الرمز x بالرمز y وكذلك التعويض عن العجلة a بعجلة الجاذبية الأرضية بإشارة سالبة $-g$ وذلك لأن عجلة الجاذبية الأرضية دائماً في اتجاه مركز الأرض وهذا يعبر عنه من خلال المحور y السالب كما في الشكل 2.2.

$$v = v_o - g\,t \qquad (2.19)$$

$$y = y_o + \frac{1}{2}(v+v_o)t \qquad (2.20)$$

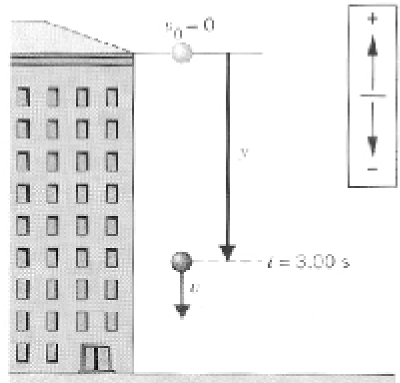
$$y = y_o + v_o\,t - \frac{1}{2}\,g\,t^2 \qquad (2.21)$$

$$v^2 = v_o^2 - 2g\,(y-y_o) \qquad (2.22)$$

Figure 2.2

Example 2.10

A stone is dropped from rest from the top of a building, as shown in Figure 2.4. After 3s of free fall, what is the displacement y of the stone?



Solution

From equation (2.21)

$$y = y_o + v_o\, t - \frac{1}{2}\, g\, t^2$$

$$y = 0 + 0 - \frac{1}{2}\,(9.8) \times (3)^2 = -44.1\text{m}$$

Example 2.11

A stone is thrown upwards from the edge of a cliff 18m high as shown in Figure 2.5. It just misses the cliff on the way down and hits the ground below with a speed of 18.8m/s.

(a) With what velocity was it released?

(b) What is its maximum distance from the ground during its flight?

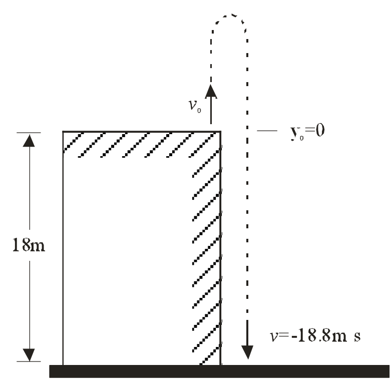


Figure 2.5

Solution

Let y_o = 0 at the top of the cliff.

(a) From equation

$$v^2 = v_o^2 - 2g\,(y - y_o)$$

$$(18.8)^2 = v_o^2 - 2\times9.8\times18$$

$$v_o^2 = 0.8 \text{ m/s}$$

(b) The maximum height reached by the stone is h

$$h = \frac{v^2}{2g} = \frac{18}{2\times9.8} = 18 \text{ m}$$

Example 2.12

A student throws a set of keys vertically upward to another student in a window 4m above as shown in Figure 2.6. The keys are caught 1.5s later by the student.

(a) With what initial velocity were the keys thrown?

(b) What was the velocity of the keys just before they were caught?

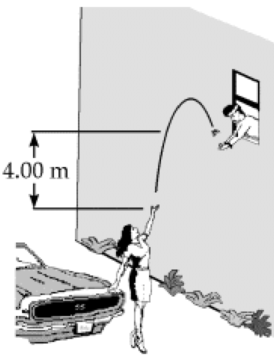


Figure 2.6

Solution

(a) Let y_o=0 and y=4m at t=1.5s then we find

$$y = y_o + v_o\, t - \frac{1}{2}\, g\, t^2$$

$$4 = 0 + 1.5\, v_o - 4.9\, (1.5)^2$$

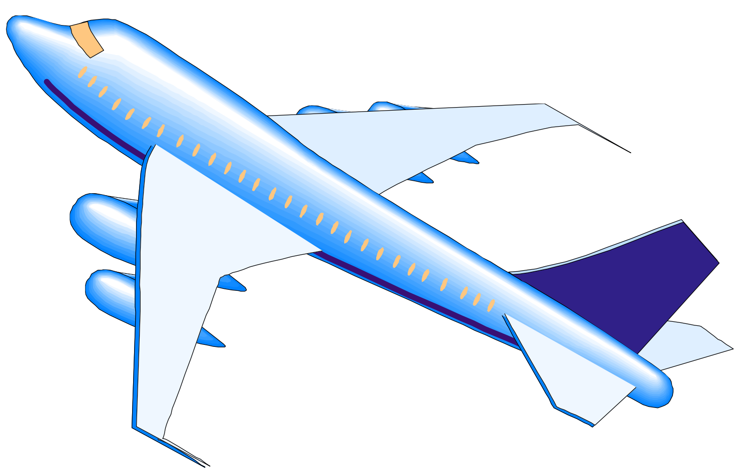
$$v_o = 10 \text{ m/s}$$

(b) The velocity at any time $t > 0$ is given by

$$v = v_o + at$$

$$v = 10 - 9.8\,(1.5) = -4.68 \text{ m/s}$$

Motion in two dimensions



2.6 Motion in two dimensions

Motion in two dimensions like the motion of projectiles and satellites and the motion of charged particles in electric fields. Here we shall treat the motion in plane with constant acceleration and uniform circular motion.

درسنا في الفصل السابق الحركة في بعد واحد أي عندما يتحرك الجسم في خط مستقيم على محور x أو أن يسقط الجسم سقوطاً حراً في محور y، سندرس الآن حركة جسم في بعدين أي في كل من x,y مثل حركة المقذوفات حيث يكون للإزاحة والسرعة مركبتان في اتجاه المحور x والمحور y.

2.7 Motion in two dimension with constant acceleration

Assume that the magnitude and direction of the acceleration remain unchanged during the motion.

The position vector for a particle moving in two dimensions (*xy* plane) can be written as

$$\vec{r} = x_i + y_j \qquad (2.23)$$

where *x*, *y*, and *r* change with time as the particle moves
The velocity of the particle is given by

$$v = \frac{dr}{dt} = \frac{dx}{dt}i + \frac{dy}{dt}j \qquad (2.24)$$

$$\vec{v} = v_x i + v_y j \qquad (2.25)$$

Since the acceleration is constant then we can substitute

$$v_x = v_{xo} + a_x t \qquad v_y = v_{yo} + a_y t$$

this give

$$v = (v_{xo} + a_x t)i + (v_{yo} + a_y t)j$$

$$= (v_{xo}\, i + v_{yo}\, j) + (a_x\, i + a_y j)\, t$$

then

$$v = v_o + a\, t \qquad (2.26)$$

من المعادلة (2.26) نستنتج أن سرعة جسم عند زمن محدد t يساوى الجمع الاتجاهى للسرعة الابتدائية والسرعة الناتجة من العجلة المنتظمة.

Since our particle moves in two dimension x and y with constant acceleration then

$$x = x_o + v_{xo}\, t + \frac{1}{2}\, a_x\, t^2 \quad \& \qquad y = y_o + v_{yo}\, t - \frac{1}{2}\, a_y\, t^2$$

but

$$r = xi + yj$$

$$r = (x_o + v_{xo}\, t + \frac{1}{2}\, a\, t^2)i + (y_o + v_{yo}\, t - \frac{1}{2}\, g\, t^2)j$$

$$= (x_o\, i + y_o\, j) + (v_{xo} i + v_{yo} j)t + \frac{1}{2}\, (a_x i + a_y j)t^2$$

$$r = r_o + v_o t + \frac{1}{2} a\, t^2 \qquad (2.27)$$

من المعادلة (2.27) نستنتج أن متجه الإزاحة r-r_o هو عبارة عن الجمع الإتجاهى لمتجه الإزاحة الناتج عن السرعة الابتدائية $v_o t$ والإزاحة الناتجة عن العجلة المنتظمة $\frac{1}{2} a\, t^2$.

2.8 Projectile motion

تعتبر حركة المقذوفات ***Projectile motion*** من الأمثلة على الحركة في بعدين، وسوف نقوم بإيجاد معادلات الحركة للمقذوفات لتحديد الإزاحة الأفقية والرأسية والسرعة والعجلة من خلال العديد من الأمثلة.

Example 2.13

A good example of the motion in two dimension it the motion of projectile. To analyze this motion lets assume that at time t=0 the projectile start at the point x_o=y_o=0 with initial velocity v_o which makes an angle q_o, as shown in Figure 2.5.

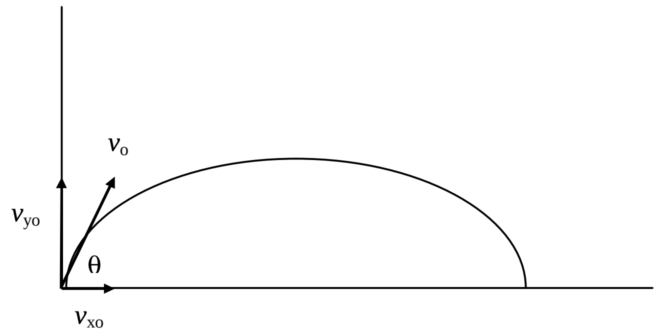


Figure 2.5

then

$$v_x = v_{xo} = v_o\cos\theta_o = \text{constant}$$

$$v_y = v_{yo} - gt = v_o\sin\theta_o - gt$$

$$x = v_{xo}\, t = (v_o\cos\theta_o)t \qquad (2.28)$$

$$y = v_{yo}\, t - \frac{1}{2}\, g\, t^2 = (v_o\sin\theta_o)t - \frac{1}{2}\, g\, t^2 \qquad (2.29)$$

2.8.1 Horizontal range and maximum height of a projectile

It is very important to work out the range (R) and the maximum height (h) of the projectile motion.

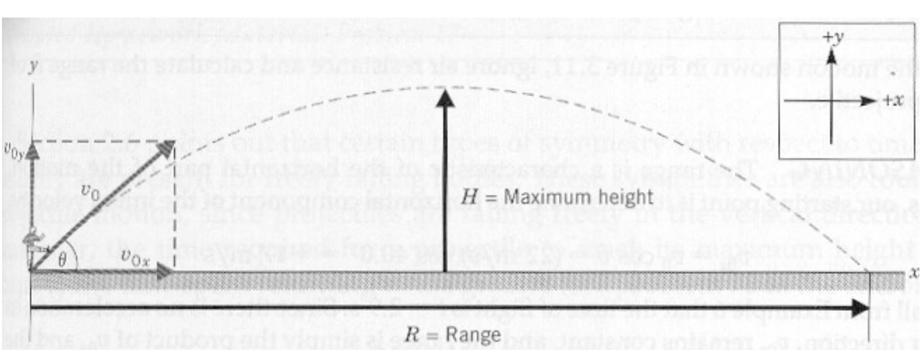


Figure 2.6

To find the maximum height h we use the fact that at the maximum height the vertical velocity $v_y=0$

by substituting in equation

$$v_y = v_o\sin\theta_o - gt \qquad (2.30)$$

$$t_1 = \frac{v_o \sin\theta_o}{g} \qquad (2.31)$$

To find the maximum height h we use the equation

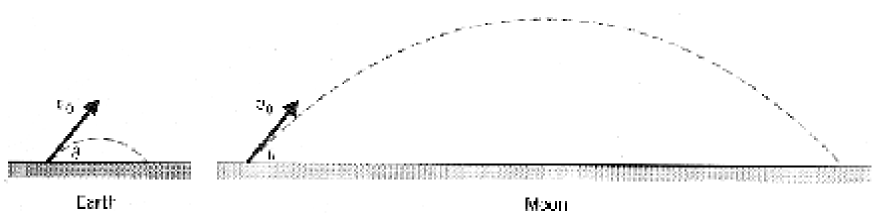
$$y = (v_o\sin\theta_o)t - \frac{1}{2} g t^2 \qquad (2.32)$$

by substituting for the time t_1 in the above equation

$$h = (v_o\sin\theta_o)\frac{v_o \sin\theta_o}{g} - \frac{1}{2} g \left(\frac{v_o \sin\theta_o}{g}\right)^2 \qquad (2.33)$$

$$h = \frac{v_o^2 \sin^2\theta_o}{2g} \qquad (2.34)$$

من المعادلة (2.34) نلاحظ أقصى ارتفاع يصل إليه الجسم المتحرك فـــي بعـــدين كحركـــة المقذوفات على عجلة الجاذبية، وعليه فإن المقذوفات على سطح القمر تأخذ مساراً ذا مـــدى وارتفاع أكبر منه على سطح الأرض كما في الشكل أدناه.



Example 2.14

A long-jumper leaves the ground at an angle of 20^o to the horizontal and at a speed of 11 m/s. (a) How far does he jump? (b) The maximum height reached?

Solution

(a) $x = (v_o \cos q_o)t = (11 \times \cos 20)\ t$

x can be found if t is known, from the equation

$$v_y = v_o \sin\theta_o - gt$$

$$0 = 11 \sin 20 - 9.8\ t_1$$

$t_1 = 0.384$ s where t_1 is the time required to reach the top then $t = 2t_1$

$$t = 0.768 \text{ s}$$

therefore

$$x = 7.94 \text{ m}$$

(b) The maximum height reached is found using the value of t_1=0.384s

$$y_{max} = (v_o \sin\theta_o)t_1 - \frac{1}{2} g\, t_1^2$$

$$y_{max} = 0.722 \text{ m}$$

Example 2.15

A ball is projected horizontally with a velocity v_o of magnitude 5m/s. Find its position and velocity after 0.25s

Solution

Here we should note that the initial angle is 0. The initial vertical velocity component is therefore 0. The horizontal velocity component equals the initial velocity and is constant.

$$x = v_o t = 5 \times 0.25 = 1.25 \text{ m}$$

$$y = - 1/2\ g\ t^2 = -0.306 \text{ m}$$

the distance of the projectile is given by

$$r = \sqrt{x^2 + y^2} = 1.29 \text{ m}$$

The component of velocity are

$$v_x = v_o = 5 \text{ m/s}$$

$$v_y = -g\ t = -2.45 \text{ m/s}^2$$

The resultant velocity is given by

$$v = \sqrt{v_x^2 + v_y^2} = 5.57 \text{ m/s}$$

The angle θ is given by

$$\theta = \tan^{-1} \frac{v_y}{v_x} = -26.1^o$$

Example 2.16

An object is thrown horizontally with a velocity of 10m/s from the top of a 20m high building as shown in Figure 2.7. Where does the object strike the ground?

Solution

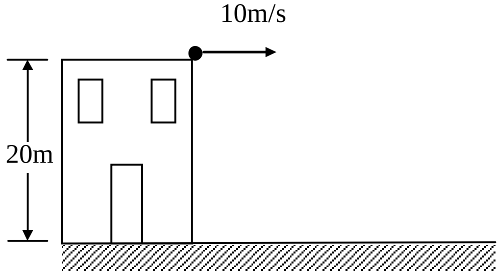


Figure 2.7

Consider the vertical motion

$$v_{yo} = 0$$

$$a_y = 9.8 \text{ m/s}^2$$

$$y = 20 \text{ m}$$

then

$$y = v_{oy} + 1/2\ g\ t^2$$

$$t = 2.02 \text{ s}$$

Consider the horizontal motion

$v_{xo} = v_x = 10$ m/s

$x = v_x\, t$

$x = 20.2$ m

Example 2.17

Suppose that in the example above the object had been thrown upward at an angle of 37° to the horizontal with a velocity of 10m/s. Where would it land?

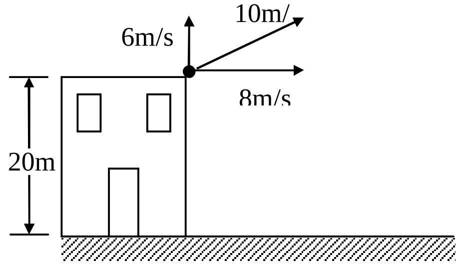


Figure 2.8

Solution

Consider the vertical motion

$v_{oy} = 6$ m/s

$a_y = -9.8$m/s2

$y = 20$m

To find the time of flight we can use

$$y = v_{yo}\, t - \frac{1}{2}\, g\, t^2$$

since we take the top of the building is the origin the we substitute for $y = -20$m

$$-20 = 6\, t - \frac{1}{2}\, 9.8\, t^2$$

$t = 2.73$s

Consider the horizontal motion

$v_x = v_{xo} = 8$m/s

then the value of x is given by

$x = v_x\, t = 22$m

Example 2.18

In Figure 2.9 shown below where will the ball hit the wall

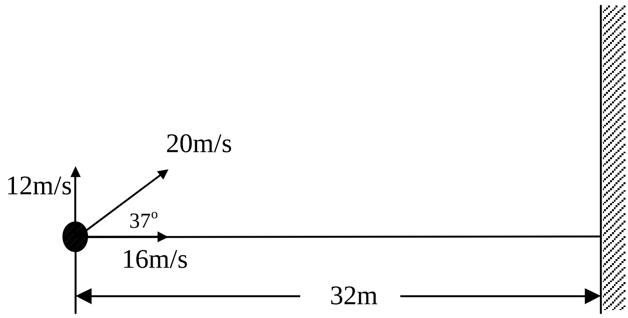


Figure 2.9

Solution

$$v_x = v_{xo} = 16\text{m/s}$$

$$x = 32\text{m}$$

Then the time of flight is given by

$$x = vt$$

$$t = 2\text{s}$$

To find the vertical height after 2s we use the relation

$$y = v_{yo}\, t - \frac{1}{2}\, g\, t^2$$

Where v_{yo} = 12m/s, t =2s

$$y = 4.4\text{m}$$

Since y is positive value, therefore the ball hit the wall at 4.4m from the ground

To determine whether the ball is going up of down we estimate the velocity and from its direction we can know

$$v_y = v_{yo} - gt$$

$$v_y = -7.6\text{m/s}$$

Since the final velocity is negative then the ball must be going down.

Example 2.19

A fish swimming horizontally has velocity $v_o = (4i + j)$ m/s at a point in the ocean whose distance from a certain rock is $r_o = (10i - 4j)$ m. After swimming with constant acceleration for 20.0 s, its velocity is $v = (20i - 5j)$ m/s. (a) what are the components of the acceleration? (b) what is the direction of the acceleration with respect to unit vector *i*? (c) where is the fish at $t = 25$ s and in what direction is it moving?

Solution

At $t = 0$, $v_o = (4i + j)$ m/s, and $r_o = (10i - 4j)$ m,

At $t = 20$s, $v = (20i - 5j)$ m/s

$$\text{(a) } a_x = \frac{\Delta v_x}{\Delta t} = \frac{20\text{m/s} - 4\text{m/s}}{20\text{s}} = 0.800 \text{ m/s}^2$$

$$a_y = \frac{\Delta v_y}{\Delta t} = \frac{-5m/s - 1m/s}{20s} = -0.300 \text{ m/s}^2$$

$$\text{(b) } \theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{-0.300 \text{ m/s}^2}{0.800 \text{ m/s}^2}\right)$$

$$= -20.6° \text{ or } 339° \text{ from the } +x \text{ axis}$$

(c) At $t = 25$ s, its coordinates are:-

$$x = x_o + v_{xo}\, t + 1/2\, a_x t^2$$

$$= 10 \text{ m} + (4\text{m/s})(25 \text{ s}) + 1/2\,(0.800 \text{ m/s}^2)(25 \text{ s})^2 = 360 \text{ m}$$

$$y = y_o + v_{yo}\, t + 1/2\, a_y\, t^2$$

$$= -4 \text{ m} + (1 \text{ m/s})(25 \text{ s}) + 1/2\,(-0.300 \text{ m/s}^2)\,(25 \text{ s})^2 = -72.8 \text{ m}$$

$$\theta = \tan^{-1}\left(\frac{v_y}{va_x}\right) = \tan^{-1}\left(\frac{-6.5m/s}{24m/s}\right) = -15°$$

(where v_x and v_y were evaluated at $t = 25$ s)

Example 2.20

A particle initially located at the origin has an acceleration of $\boldsymbol{a} = 3\boldsymbol{j}$ m/s^2 and an initial velocity of $\boldsymbol{v}_o = 5\boldsymbol{i}$ m/s. Find (a) the vector position and velocity at any time *t* and (b) the coordinates and speed of the particle at $t = 2$ s.

Solution

Given $\quad \boldsymbol{a} = 3\boldsymbol{j}$ m/s^2 , $\qquad \boldsymbol{v}_o = 5\boldsymbol{i}$ m/s $\qquad \boldsymbol{r}_o = 0\boldsymbol{i} + 0\boldsymbol{j}$

(a) $\boldsymbol{r} = \boldsymbol{r}_o + \boldsymbol{v}_o\, t + \frac{1}{2}\, \boldsymbol{a}\, t^2 = (5t\boldsymbol{i} + 1.5\, t^2\boldsymbol{j})$ m

$\boldsymbol{v} = \boldsymbol{v}_o + \boldsymbol{a}t = (5\boldsymbol{i} + 3t\boldsymbol{j})$ m/s

(b) At $t = 2$ s, we find

$$\boldsymbol{r} = 5(2)\boldsymbol{i} + 1.5(2)2\boldsymbol{j} = (10\boldsymbol{i} + 6\boldsymbol{j}) \text{ m}$$

That is,

$$(x,y) = (10 \text{ m}, 6 \text{ m})$$

$$\boldsymbol{v} = 5\boldsymbol{i} + 3(2)\boldsymbol{j} = (5\boldsymbol{i} + 6\boldsymbol{j})\text{m/s}$$

so

$$v = |\boldsymbol{v}| = \sqrt{v_x^{\ 2} + v_y^{\ 2}} = 7.81 \text{ m/s}$$

Example 2.21

A ball is thrown horizontally from the top of a building 35 m high. The ball strikes the ground at a point 80 m from the base of the building. Find (a) the time the ball is in flight, (b) its initial velocity, and (c) the *x* and *y* components velocity just before the ball strikes the ground.

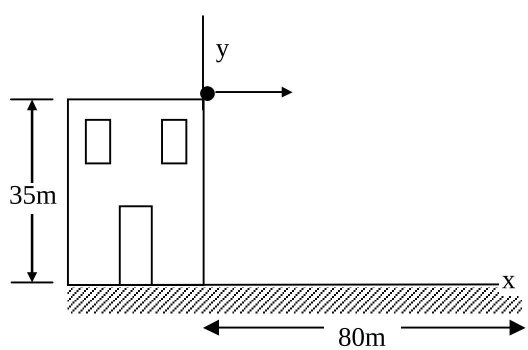


Figure 2.10

Solution

$x_o=0$ $y_o=35m.$

$v_{xo}=v_o$ $a_x=0$

$v_{yo}=0$ $a_y=9.8m/s^2$

(a) when the ball reaches the ground, x = 80m and y = 0

To find the time it takes to reach

The ground,

$$y = y_o + v_{yo}\, t - \frac{1}{2}\, a_y\, t^2 = 35 - 4.9t^2 = 0$$

thus

t = 2.67s

(b) Using $x = x_o+v_{xo}t = v_o t$ with t = 2.67s

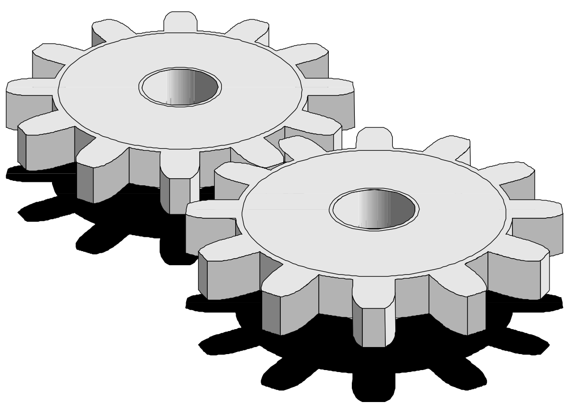
$80 = v_o(2.67)$

$v_o=29.9m/s$

(c) $v_x=v_{xo}=29.9m/s$

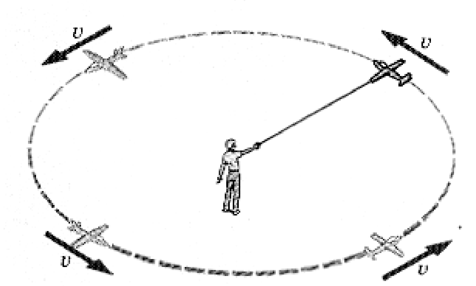
$v_y=v_{yo} - gt = 0-9.8\ (2.67) = -26.2m/s$

Motion in Uniform Circle



2.9 Motion in Uniform Circular Motion

من الممكن أن يتحرك جسم على مسار دائري بسرعة خطية ثابتة ***linear constant speed***. قد يخطر لنا الآن أن العجلة في هذه الحالة تساوى صفراً، وذلك لأن السرعة ثابتة، وهذا غير صحيح لأن الجسم يتحرك على مسار دائري لذا توجد عجلة. ولشرح ذلك نحن نعلم أن السرعة كمية متجه، والعجلة هي عبارة عن كمية متجه لأنها تساوى معدل التغير في السرعة بالنسبة للزمن، والتغير في السرعة قد يكون في المقدار أو في الاتجاه. وفي حالة حركة الجسم على مسار دائري فإن العجلة لا تؤثر على مقدار السرعة إنما تغير من اتجاه السرعة، ولهذا فإن الجسم يتحرك على مسار دائري وبسرعة ثابتة. يكون متجه السرعة دائما عمودياً على نصف القطر وفى اتجاه المماس عند أية نقطة على المسار الدائري كما في الشكل 2.11.



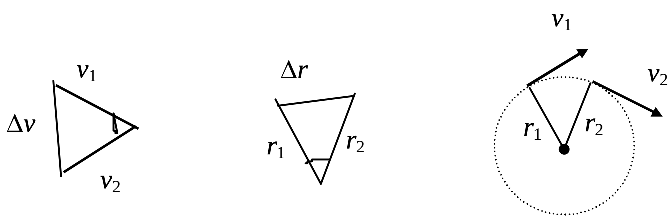


Figure 2.11

$$\frac{\Delta v}{\Delta r} = \frac{v}{r} \qquad (2.35)$$

$$\Delta v = \frac{v}{r}\Delta r \qquad (2.36)$$

Divide both sides by Δt

$$\frac{\Delta v}{\Delta t} = \frac{v}{r}\frac{\Delta r}{\Delta t} \qquad (2.37)$$

$$a = \frac{v}{r}v = \frac{v^2}{r} \qquad (2.38)$$

$$a_{\perp} = \frac{v^2}{r} \qquad (2.39)$$

Example 2.22

A particle moves in a circular path 0.4m in radius with constant speed. If the particle makes five revolution in each second of its motion, find (a) the speed of the particle and (b) its acceleration.

Solution

(a) Since r=0.4m, the particle travels a distance 0f $2\pi r$ = 2.51m in each revolution. Therefore, it travels a distance of 12.57m in each second (since it makes 5 rev. in the second).

$$v = 12.57\text{m}/1\text{sec} = 12.6 \text{ m/s}$$

(b) $a_{\perp} = \frac{v^2}{r}$ = 12.6/0.4 = 395m/s^2

Example 2.23

A train slows down as it rounds a sharp horizontal turn, slowing from 90km/h to 50km/h in the 15s that it takes to round the bend. The radius of the curve is 150m. Compute the acceleration at the train.

Solution

يجب تحويل السرعة من وحدة km/h إلى وحدة m/s كالتالي:-

$$50km/h = \left(50\frac{km}{h}\right)\left(10^3\frac{m}{km}\right)\left(\frac{1h}{3600s}\right) = 13.89m/s$$

$$90km/h = \left(90\frac{km}{h}\right)\left(10^3\frac{m}{km}\right)\left(\frac{1h}{3600s}\right) = 25m/s$$

when v=13.89m/s

$$a_{\perp} = \frac{v^2}{r} = \frac{13.89}{150} = 1.29\text{m/s}^2$$

$$a = \frac{v^2}{r} = \frac{13.89-25}{15} = -0.741\text{m/s}^2$$

$$a = \sqrt{{a_r}^2 + {a_t}^2} = \sqrt{(1.29)^2 + (-0.741)^2} = 1.48m/s^2$$

2.10 Questions with solutions

(1) Can the average velocity and the instantaneous velocity be equal? Explain.

Answer: *Yes, when a body moves with constant velocity* $v_{ave}=v$

(2) If the average velocity is nonzero for some time interval, does this mean that the instantaneous velocity is never zero during this interval? Explain.

Answer: *No, the average velocity may be nonzero, but the particle may come to rest at some instant during this interval. This happens, for example, when the particle reaches a turning point in its motion.*

(3) A ball is thrown vertically upward. What are its velocity and acceleration when it reaches its maximum altitude? What its acceleration just before it strikes the ground?

Answer: *At the peak of motion* $v=0$ *and* $a=-g$*. during its entire flight.*

(4) A child throws a marble into the air with an initial velocity v_o. Another child drops a ball at the same instant. Compare the acceleration of the two objects while they are in flight.

Answer: *Both have an acceleration of* $-g$*, since both are freely falling.*

(5) A student at the top of a building of height h throws one ball upward with an initial speed v_o and then throws a second ball downward with the same initial speed. How do the final velocities of the balls compare when they reach the ground?

Answer: *They are the same.*

(6) Describe a situation in which the velocity of a particle is perpendicular to the position vector.

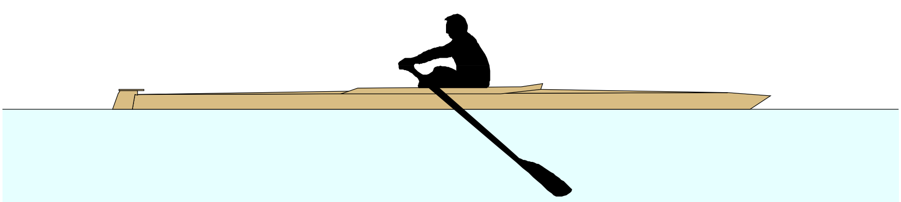
Answer: *A particle moving in a circular path, where the origin of r is at the center of the circle.*

(7) Can a particle accelerate if its speed is constant? Can it accelerate if its velocity is constant?

Answer: *Yes, Its speed may be constant, but the direction of v may change causing an acceleration. However a particle has zero acceleration when its velocity is constant.*
Note that constant velocity means that both the direction and magnitude of v remain constant.

(8) Explain whether or not the following particles have an acceleration: (a) a particle moving in a straight line with constant speed and (b) a particle moving around a curv with constant speed.

Answer: *(a) The acceleration is zero, since v and its direction remains constant.*
(b) The particle has an acceleration since the direction of v changes.



2.11 Problems

(1) An athlete swims the length of a 50-m pool in 20 s and makes the return trip to the starting position in 22 s. Determine his average velocity in (a) the first half of the swim, (b) the second half of the swim, and (c) the round trip.

(2) A particle moves along the *x* axis according to the equation $x = 2t + 3t^2$, where *x* is in meters and *t* is in seconds. Calculate the instantaneous velocity and instantaneous acceleration at *t* = 3.0 s.

(3) When struck by a club, a golf ball initially at rest acquires a speed of 31.0 m/s. If the ball is in contact with the club for 1.17 ms, what is the magnitude of the average acceleration of the ball?

(4) A railroad car is released from a locomotive on an incline. When the car reaches the bottom of the incline, it has a speed of 30 mi/h, at which point it passes through a retarder track that slows it down. If the retarder track is 30 ft long, what negative acceleration must it produce to stop the car?

(5) An astronaut standing on the moon drops a hammer, letting it fall 1.00 m to the surface. The lunar gravity produces a constant acceleration of magnitude 1.62 m/s^2. Upon returning to Earth, the astronaut again drops the hammer, letting it fall to the ground from a height of 1.00 m with an acceleration of 9.80 m/s^2. Compare the times of fall in the two situations.

(6) The Position of a particle along the x-axis is given by $x=3t^3-7t$ where x in meters and t in seconds. What is the average velocity of the particle during the interval from t=2sec to t=5sec?

(7) A car makes a 200km trip at an average speed of 40km/h. A second car starting 1h later arrives at their mutual destination at the same time. What was the average speed of the second car?

(8) A particle is moving with a velocity v_o=60m/s at *t*=0. Between *t*=0 and *t*=15s the velocity decreases uniformly to zero. What was the average acceleration during this 15s interval? What is the significance of the sign of your answer?

(9) A car traveling in a straight line has a velocity of 30m/s at some instant. Two seconds

later its velocity is 25m/s. What is its average acceleration in this time interval?

(10)A particle moving in a straight line has a velocity of 8m/s at t=0. Its velocity at t=20s is 20m/s. What is its average acceleration in this time interval?

(11)A car traveling initially at a speed of 60m/s is accelerated uniformaly to a speed 85m/s in 12s. How far does the car travel during the 12s interval?

(12)A body moving with uniform acceleration has a velocity of 12cm/s when its coordinate is 3cm. If its x coordinate 2s later is -5cm, what is the magnitude of its acceleration?

(13)The initial speed of a body is 5.2m/s. What is its speed after 2.5s if it (a) accelerates uniformaly at 3m/s^2 and (b) accelerates uniformaly at -3m/s^2.

(14)

(15)Two trains started 5 minutes apart. Starting from rest, each capable of maximum speed of 160km/h after uniformly accelerating over a distance of 2km. (a) What is the acceleration of each train? (b) How far ahead is the first train when the second one starts? (c) How far apart are they when they are both traveling at maximum speed?

(16)A ball is thrown directly downward with an initial velocity of 8m/s from a height of 30m. When does the ball strike the ground?

(17)A hot air balloon is traveling vertically upward at a constant speed of 5m/s. When it is 21m above the ground, a package is released from the balloon. (a) How long after being released is the package in the air? (b) What is the velocity of the package just before impact with the ground? (c) Repeat (a) and (b) for the case of the balloon descending at 5m/s.

(18)A ball thrown vertically upward is caught by the thrower after 20s. Find (a) the initial velocity of the ball and (b) the maximum height it reaches.

(19)A stone falls from rest from the top of a high cliff. A second stone is thrown downward from the same height 2s later with an initial speed of 30m/s. If both stones hit the ground below simultaneously, how high is the cliff?

(20)At t=0, a particle moving in the xy plane with constant acceleration has a velocity of

$v_o=3i - 2j$ at the origin. At t=3s, its velocity is $v=9i+7j$. Find (a) the acceleration of the particle and (b) its coordinates at any time t.

(21)The position vector of a particles varies in time according to the expressions $r=(3i-6t^2j)$ m. (a) Find expressions for the velocity and acceleration as a function of time. (b) Determine the particle's position and velocity at t=1s.

(22)A student stands at the edge of a cliff and throws a stone horizontally over the edge with speed of 18m/s. The cliff is 50m above the ground. How long after being released does the stone strike the beach below the cliff? With what speed and angle of impact does it land?

(23)A football kicked at an angle of 50o to the horizontal, travels a horizontal distance of 20m before hitting the ground. Find (a) the initial speed of the football, (b) the time it is in the air, and (c) the maximum height it reaches.

(24)A projectile is fired in such a way that its horizontal range is equal to three times its maximum height. What is the angle of projection?

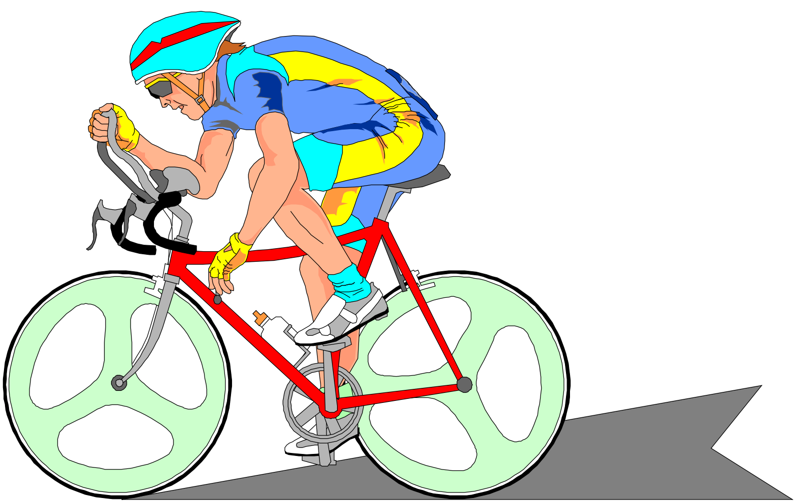
(25)Find the acceleration of a particle moving with a constant speed of 8m/s in a circle 2m in radius.

(26)The speed of a particle moving in a circle 2m in radius increases at the constant rate of $3m/s^2$. At some instant, the magnitude of the total acceleration is $5m/s^2$. At this instant find (a) the centripetal acceleration of the particle and (b) its speed.

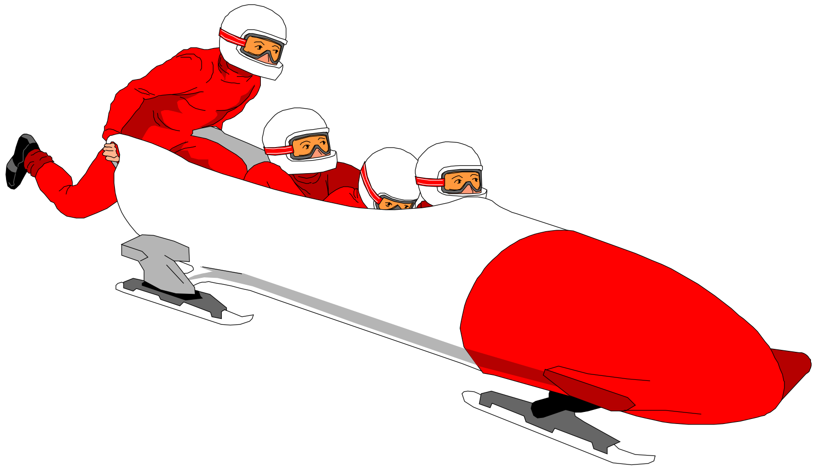
(27)A student swings a ball attached to the end of a string 0.6m in length in a vertical circle. The speed of the ball is 4.3m/s at its highest point and 6.5m/s at its lowest point. Find the acceleration of the ball at (a) its highest point and (b) its lowest point.

Chapter 3

Mechanics: Dynamics
The Law of Motion



الديناميكا: قوانين الحركة



MECHANICS: DYNAMICS

THE LAW OF MOTION

3.1 The law of motion

في الجزء السابق ركزنا على علم وصف الحركة من إزاحة وسرعة وعجلة دون النظــر إلى مسبباتها وهذا العلم يسمى علم الكينماتيكا ***Kinematics***، وفى هذا الجزء من المقرر سوف ندرس مسبب الحركة وهو كمية فيزيائية هامة تدعى القوة ***Force*** والتـــي وضـــع العالم نيوتن ثلاث قوانين أساسية تعتمد على الملاحظات التجريبية التي أجراها منذ أكثــر من ثلاث قرون. والعلم الذي يدرس العلاقة بين حركة الجسم والقوة المؤثرة عليه هو من علوم الميكانيكا الكلاسيكية ***Classical mechanics*** والتـــي تعــرف باســم ديناميكـــا ***Dynamics***، وكلمة كلاسيك هنا تدل على أننا نتعامل فقط مع سرعات اقل بكثير مـــن سرعة الضوء وأجسام أكبر بكثير من الذرة.

3.2 The concept of force

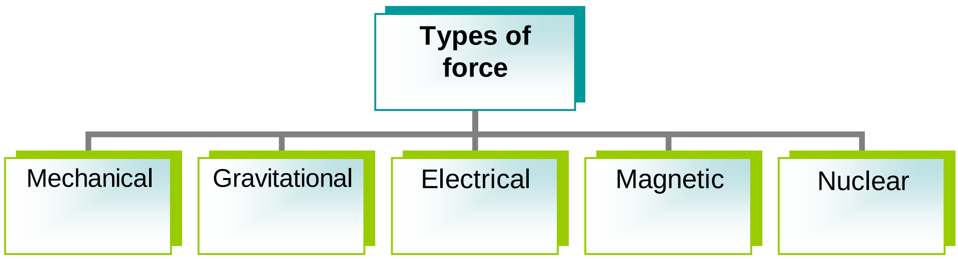
نتعامل في حياتنا اليومية مع العديد من أنواع القوى المختلفة التي قد تؤثر على الأجســـام المتحركة فتغير من سرعتها مثل شخص يدفع عربة أو يسحبها أو أن تؤثر القـــوة علـــى الأجسام الساكنة لتبقيها ساكنة مثل الكتاب على الطاولة أو الصور المعلقة علـــى الحـــائط. ويكون تأثير القوة مباشر ***Contact force*** مثل سحب زنبرك أو دفع صندوق ويمكن أن يكون تأثير القوة عن بعد ***Action-at-a-distance*** مثل تنافر أو تجاذب قطبي مغناطيس.

It is not always force needed to move object from one place to another but force are also exist when object do not move, *for example* when you read a book you exert force holding the book against the force of gravitation.

يعرف الجسم الساكن بأنه في حالة اتزان equilibrium عندما تكـــون محصـــلة القـــوى المؤثرة عليه تساوي صفراً.

It is very important to know that when a body is at rest or when moving at constant speed we say that the net force on the body is zero *i.e.* the body in ***equilibrium***.

يوجد العديد من أنواع القوة الموجودة في الطبيعة وهي أما أن تكون ميكانيكية أو جاذبيـــة أو كهربية أو مغناطيسية أو نووية. وسندرس في هذا المقرر من الكتـــاب النـــوع الأول والثاني.



ولدراسة القوى الميكانيكية سنبدأ بدراسة قوانين نيوتن للحركة.

3.3 Newton's laws of motion

Newton's first law, *the law of equilibrium* states that an object at rest will remain at rest and an object in motion will remain in motion with a constant velocity unless acted on by a *net external force.*

Newton's second law, *the law of acceleration*, states that the acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass.

Newton's third law, *the law of action-reaction*, states that when two bodies interact, the force which body "A" exerts on body "B" (the action force) is equal in magnitude and opposite in direction to the force which body "B" exerts on body "A" (the reaction force). A consequence of the third law is that forces occur in pairs. Remember that the action force and the reaction force act on different objects.

3.3.1 Newton's first and second law

يشرح القانون الأول لنيوتن حالة الأجسام التي تؤثر عليها مجموعة قوى محصلتها تساوي صفراً، حيث يبقى الجسم الساكن ساكناً والجسم المتحرك يبقى متحركاً بسرعة ثابتة. أما قانون نيوتن الثاني فيختص بالأجسام التي تؤثر عليها قوة خارجية تؤدي إلى تحريكها بعجلة a أو أن تغير من سرعتها إذا كانت الأجسام متحركة. وهنا يجدر الإشارة إلى أن القانون الثاني يحتوي القانون الأول بتطبيق أن العجلة تساوي صفراً $a = 0$.

$$\sum \vec{F} = m\vec{a} \qquad (3.1)$$

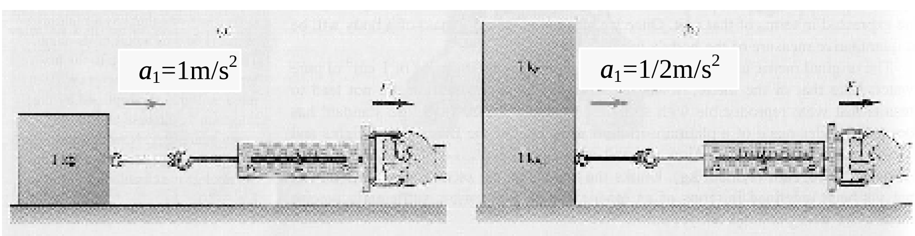
where m is the mass of the body and a is the acceleration of the body

Then the unit of the force is (Kg.m/s^2) which is called Newton (N)

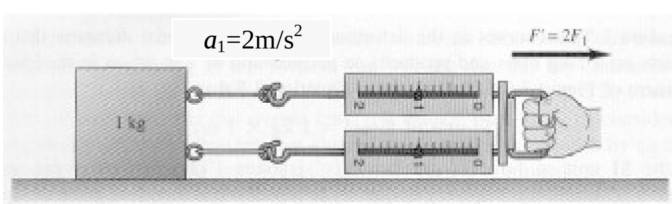
وقد سميت وحدة القوة بنيوتن تكريماً للعالم نيوتن.

$\sum \vec{F} = 0$ *Newton's first law*

$\sum \vec{F} = m\vec{a}$ *Newton's second law*



في الشكل أعلاه إذا زادت الكتلة بمقدار الضعف مع ثبوت قوة الشد فإن العجلة تقل بمقدار النصف.



في الشكل أعلاه إذا تضاعفت قوة الشد فإن العجلة تزداد بمقدار الضعف.

Example 3.1

Two forces, F_1 and F_2, act on a 5-kg mass. If F_1 =20 N and F_2 =15 N, find the acceleration in (a) and (b) of the Figure 3.1

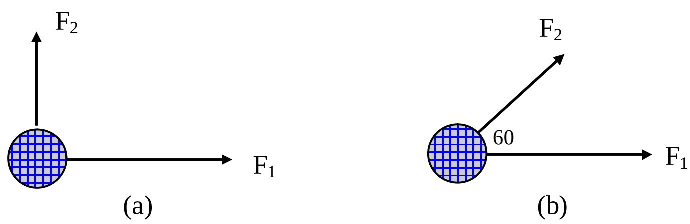


Figure 3.1

Solution

(a) $\sum F = F_1 + F_2 = (20\boldsymbol{i} + 15\boldsymbol{j})$ N

$\sum F = ma \quad \therefore 20\boldsymbol{i} + 15\boldsymbol{j} = 5\,\boldsymbol{a}$

$\boldsymbol{a} = (4\boldsymbol{i} + 3\boldsymbol{j})$ m/s^2 or $\quad \boldsymbol{a} = 5$m/s^2

(b) $F_{2x} = 15 \cos 60 = 7.5$ N

$F_{2y} = 15 \sin 60 = 13$ N

$F_2 = (7.5\boldsymbol{i} + 13\boldsymbol{j})$ N

$\sum F = F_1 + F_2 = (27.5\boldsymbol{i} + 13\boldsymbol{j}) = ma = 5\,\boldsymbol{a}$

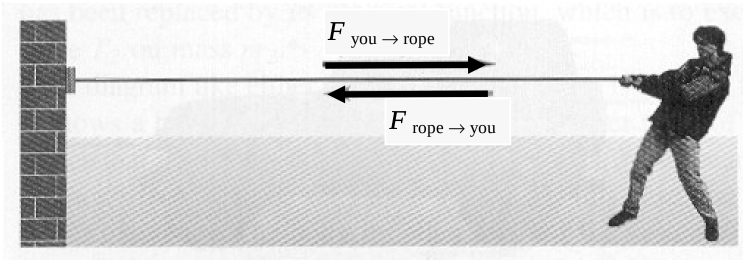
$\boldsymbol{a} = (5.5\boldsymbol{i} + 2.6\boldsymbol{j})$ m/s^2 or $\quad \boldsymbol{a} = 6.08$m/s^2

3.3.2 Newton's third law

يختص القانون الثالث لنيوتن على القوة المتبادلة بين الأجسام حيث أنه إذا أثرت بقوة على جسم ما وليكن كتاب ترفعه بيدك فإن الكتاب بالمقابل يؤثر بنفس مقدار القوة علـــى يـــدك وفي الاتجاه المعاكس.

$$\vec{F}_{12} = -\vec{F}_{21} \qquad (3.2)$$

والرمز F_{12} يعني القوة التي يتأثر بها الجسم الأول نتيجة للجسم الثاني.



يتضح من الشكل أعلاه مفهوم قانون نيوتن الثالث للفعل ورد الفعل، حيث يشد الشـــخص الجدار بواسطة الحبل وبالمقابل فإن الحبل يشد الشخص كرد فعل.

Example 3.2

A ball is held in a person's hand. (a) Identify all the external forces acting on the ball and the reaction to each of these forces. (b) If the ball is dropped, what force is exerted on it while it is in "flight"? Identify the reaction force in this case.

Solution

(a) The external forces acting on the ball are

(1) F_H, the force which the hand exerts on the ball.
(2) W, the force of gravity exerted on the ball by the earth.

The reaction forces are

(1) To F_H: The force which the ball exerts on the hand.

(2) To W: The gravitational force which the ball exerts on the earth.

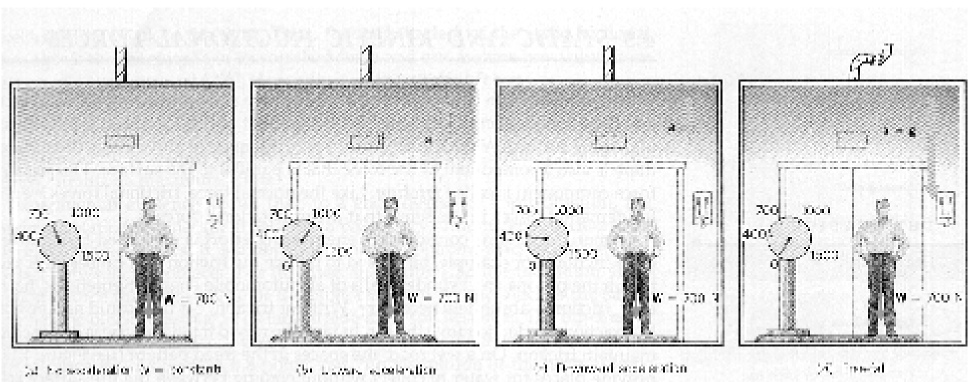
(b) When the ball is in free fall, the only force exerted on it is its weight, W, which is exerted by the earth. The reaction force is the gravitational force which the ball exerts on the earth.

3.4 Weight and tension

3.4.1 Weight

نعلم جميعا أن الوزن ***Weight*** هو كمية فيزيائية لها وحدة القوة (N) وهى ناتجة من تأثير عجلة الجاذبية الأرضية g على كتلة الجسم m، وبتطبيق قانون نيوتن الثاني على جسم موجود على بعد قريب من سطح الأرض حيث يتأثر بقوة الجاذبية الأرضية ومقدارها كتلة الجسم في عجلة الجاذبية الأرضية، وبالتالي فإن الوزن

$$\vec{W} = m\vec{g} \qquad (3.3)$$



في الشكل أعلاه يوضح تأثير تغيير العجلة على وزن الشخص في مصعد كهربي حيث يتغير وزن الشخص في حالة صعود أو هبوط المصعد.

(1) عندما يتحرك المصعد بدون عجلة (سرعة ثابتة) فإن وزن الشخص W=700N.

(2) عندما يتحرك المصعد إلى الأعلى فإن وزن الشخص يصبح W=1000N.

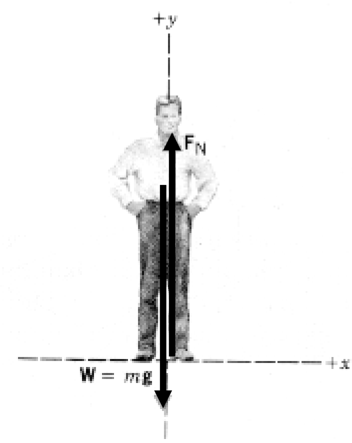
(3) عندما يتحرك المصعد إلى الأسفل فإن وزن الشخص يصبح W=400N.

(4) عندما يسقط المصعد سقوطاً حراً فإن الوزن يصبح صفراً (حالة انعدام الوزن).

في الحالة الأولى عندما تكون العجلة تساوي صفراً يكون الوزن المقاس هو الوزن الحقيقي للشخص، بينما الوزن المقاس في الحالات الثلاث الأخرى فيدعى الوزن الظاهري. ولتوضيح التغير في الوزن الظاهري بالنسبة إلى الوزن الحقيقي سنستخدم قانون نيوتن الثاني:

بتحليل القوى المؤثرة على الشخص في المصـــعد نجد أن هنالك قوتين الأولى هـــي وزن الشـــخص $W=mg$ والقوة الأخرى هي قوة رد فعل المصعد على الشخص F_N. بتطبيق قانون نيـــوتن الثـــاني نجد أن

$$\sum \vec{F} = F_N - mg = ma$$



where a is the acceleration of the elevator and the person.

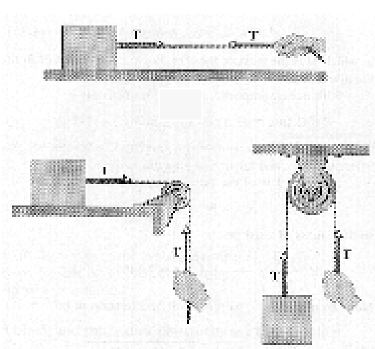
$$\underbrace{F_N}_{\text{Apparent weight}} = \underbrace{mg}_{\text{True Weight}} + ma$$

عندما يتحرك المصعد إلى الأعلى تكون العجلة a موجبة. أما عندما يتحرك المصعد للأسفل فإن a تكون سالبة.

$F_N = mg + ma$ when the elevator moves upward

$F_N = mg - ma$ when the elevator moves downward

3.4.2 Tension



عند سحب جسم بواسطة حبل فإن القوة المـــؤثرة على الجسم من خلال الحبل تـــدعى قـــوة الشـــد ***Tension*** ويرمز لها بـــالرمز T ووحدتـــه N. ويظهر في الشكل صور مختلفة مـــن قـــوة الشـــد وكيفية تحديدها على الشكل.

Example 3.3

An electron of mass 9.1×10^{-31} kg has an initial speed of 3.0×10^{5} m/s. It travels in a straight line, and its speed increases to 7.0×10^{5} m/s in a distance of 5.0cm. Assuming its acceleration is constant, (a) determine the force on the electron and (b) compare this force with the weight of the electron, which we neglected.

Solution

$$F = ma \qquad \text{and} \qquad v^2 = v_0^2 + 2ax \quad \text{or} \qquad a = \frac{(v^2 - v_o^2)}{2x}$$

$$F = \frac{m(v^2 - v_o^2)}{2x} \qquad = \qquad 3.6\times10^{-18}\text{N}$$

(b) The weight of the electron is

$$W = m\,g = (9.1 \times 10^{-31}\text{ kg})\,(9.8\text{ rn/s}^2) = 8.9 \times 10^{-30}\text{ N}$$

The accelerating force is approximately 10^{11} times the weight of the electron.

Example 3.4

Two blocks having masses of 2 kg and 3 kg are in contact on a fixed smooth inclined plane as in Figure 3.2.

(a) Treating the two blocks as a composite system, calculate the force ***F*** that will accelerate the blocks up the incline with acceleration of 2m/s^2,

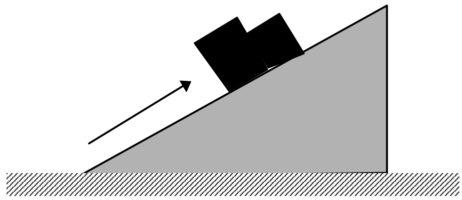


Figure 3.2

Solution

We can replace the two blocks by an equivalent 5 kg block as shown in Figure 3.3. Letting the *x* axis be along the incline, the resultant force on the system (the two blocks) in the *x* direction gives

$$\sum F_x = F - W \sin(37^o) = m\, a_x$$

$$F - 5\,(0.6) = 5(2)$$

$$F = 39.4 \text{ N}$$

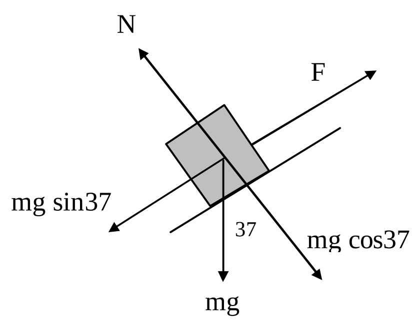
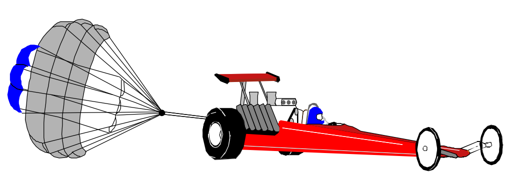


Figure 3.3

Example 3.5

The parachute on a race car of weight 8820N opens at the end of a quarter-mile run when the car is travelling at 55 m/s. What is the total retarding force required to stop the car in a distance of 1000 m in the event of a brake failure?



Solution

W = 8820 N, g = 9.8 m/s^2 , v_o = 55 m/s, v_f = 0, $x_f - x_o$ = 1000 m

$$m = \frac{W}{g} = 900 \text{ kg}$$

$$v_f^{\,2} = v_o^{\,2} + 2a(x - x_o),$$

$$0 = 55^2 + 2a(1000), \qquad \text{giving} \qquad a = -1.51 \text{ m/s}^2$$

$$\sum F = ma = (900 \text{ kg})(-1.51 \text{ m/s}^2) = -1.36 \times 10^3 \text{ N}$$

The minus sign means that the force is a retarding force.

Example 3.6

Two masses of 3 kg and 5 kg are connected by a light string that passes over a smooth pulley as shown in the Figure. Determine (a) the tension in the string, (b) the acceleration of each mass, and (c) the distance each mass moves in the first second of motion if they start from rest.

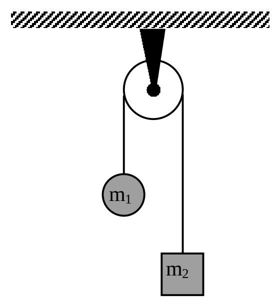


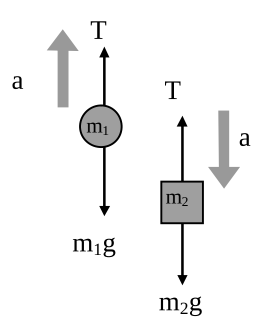
Figure 3.4

Solution

(a)

$$m_1a = T - m_1g \qquad (1)$$

$$m_2a = m_2g - T \qquad (2)$$



Add (1) and (2)

$$(m_1 + m_2)\, a = (m_2 - m_1)\, g$$

$$a = \frac{m_2 - m_1}{(m_2 + m_1)g} = \frac{5-3}{(5+3)(9.8)} = 2.45 \text{m/s}^2$$

(b)

$$T = m_2\,(g - a) = 5(9.80 - 2.45) = 36.6 \text{ N}$$

(c) Substitute a into (1)

$$T = m_1\,(a + g) = \frac{2m_1 m_2 g}{m_1 + m_2}$$

$$s = \frac{at^2}{2} \quad (v_o = 0),$$

At t = 1s, $$s = \frac{(2.45)(1^2)}{2} = 1.23\text{m}$$

Example 3.7

Two blocks connected by a light rope are being dragged by a horizontal force F as shown in the Figure 3.5. Suppose that F = 50 N, m_1 = 10 kg, m_2 = 20 kg,

(a) Draw a free-body diagram for each block.

(b) Determine the tension, T, and the acceleration of the system.

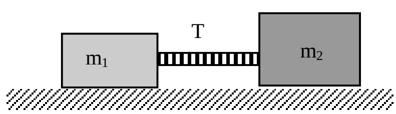
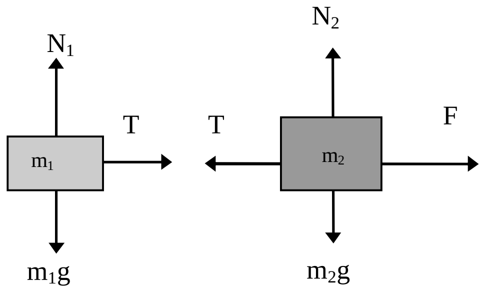


Figure 3.5

Solution



(b)

$\sum F_x(m_1) = T = m_1 a$ $\sum F_x(m_2) = 50 - T = m_2 a$

$\sum F_y(m_1) = N_1 - m_1 g = 0$ $\sum F_y(m_2) = N_2 - m_2 g = 0$

$T = 10\, a,$ $50 - T = 20\, a$

Adding the expression above gives

$50 = 30\, a,$

$a = 1.66 \text{ m/s}^2$

$T = 16.6\text{N}$

Example 3.8

Three blocks are in contact with each other on a frictionless, horizontal surface as shown in Figure 3.6. A horizontal force F is applied to m_1=2kg, m_2=3kg, m_3=4kg, and F=18N, find (a) the acceleration of the blocks, (b) the resultant force on each block, and (c) the magnitude of the contact forces between the blocks.

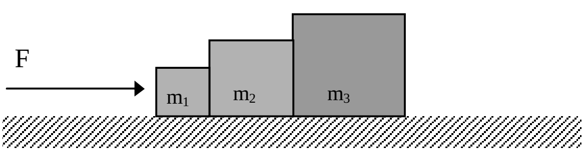


Figure 3.6

Solution

(a) $F = ma$; $18 = (2+3+4)\, a$; $a = 2\text{m/s}^2$

(b) The force on each block can be found by knowing mass and acceleration:

$F_1 = m_1 a = 2\times 2 = 4\text{N}$

$F_2 = m_2 a = 3\times 2 = 6\text{N}$

$F_3 = m_3 a = 4\times 2 = 8\text{N}$

(c) The force on each block is the resultant of all contact forces. Therefore,

$$F_1 = 4\text{N} = F - P,$$

where P is the contact force between m_1 and m_2

$$P = 14\text{N}$$

$$F_2 = 6\text{N} = P - Q,$$

where Q is the contact force between m_2 and m_3

$$Q = 8\text{N}$$

Example 3.9

What horizontal force must be applied to the cart shown in Figure 3.7 in order that the blocks remain stationary relative to the cart? Assume all surfaces, wheels, and pulley is frictionless. (Hint: Note that the tension in the string accelerates m_1)

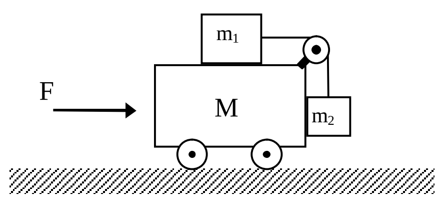


Figure 3.7

Solution

Note that m_2 should be in contact with the cart.

$$\sum F = ma$$

For m_1:

$$T = m_1 a$$

$$a = \frac{m_2 a}{m_1}$$

For m_2: $\qquad T - m_2 g = 0$

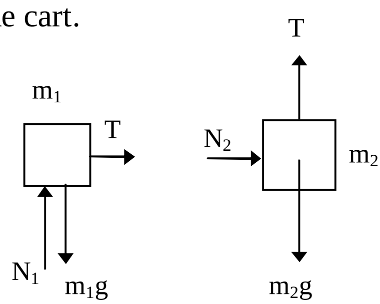


Figure 3.8

For all 3 blocks: $F = (M + m_1 + m_2)a$

$$F = (M + m_1 + m_2)\,\frac{m_2 g}{m_1}$$

Example 3.10

Two blocks of mass 2kg and 7kg are connected by a light string that passes over a frictionless pulley as shown in Figure 3.9. The inclines are smooth. Find (a) the acceleration of each block and (b) the tension in the string.

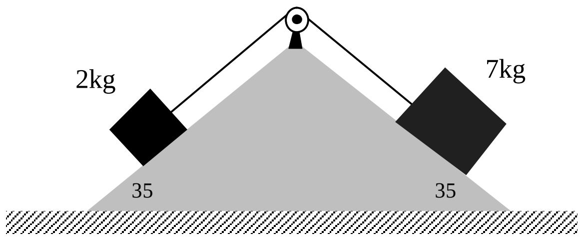


Figure 3.9

Solution

Since it has a larger mass, we expect the 7-kg block to move down the plane. The acceleration for both blocks should have the same magnitude since they are joined together by a nonstretching string.

$\sum F_1 = m_1 a_1$ $\qquad\qquad$ $-m_1 g \sin 35^{o} + T = m_1 a$

$\sum F_2 = m_2 a_2$ $\qquad\qquad$ $-m_2 g \sin 35^{o} + T = -m_2 a$

and

$-(2)(9.80) \sin 35^{o} + T = 2a$

$-(7)(9.80) \sin 35^{o} + T = -7a$

$T = 17.5\text{N}$ $\qquad\qquad$ $a = 3.12\text{m/s}^2$

Example 3.11

Find the acceleration of the 20kg block shown in Figure 3.10

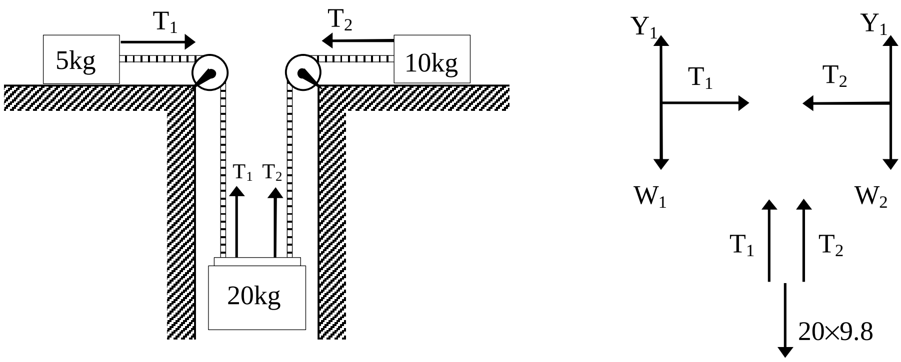


Figure 3.10

Solution

لحل هذا السؤال سنقوم بعزل الكتلة الأولى 5kg ونعتبر اتجاه حركتهـــا ناحيـــة اليمـــين موجبة. وحيث أن الحركة في اتجاه T وبالتالي فإن القوة Y_1 تلغى قوة الوزن W_1.

$F = m\,a$ $\qquad\qquad T_1 = 5\,a \qquad\qquad (1)$

وبالمثل للكتلة الأخرى 10kg

$F = m\,a$ $\qquad\qquad T_2 = 10\,a \qquad\qquad (2)$

وحيث أن كلا الكتلتين سوف تتحركان بنفس العجلة.

$196\text{N} - T_1 - T_2 = 20\,a \qquad\qquad (3)$

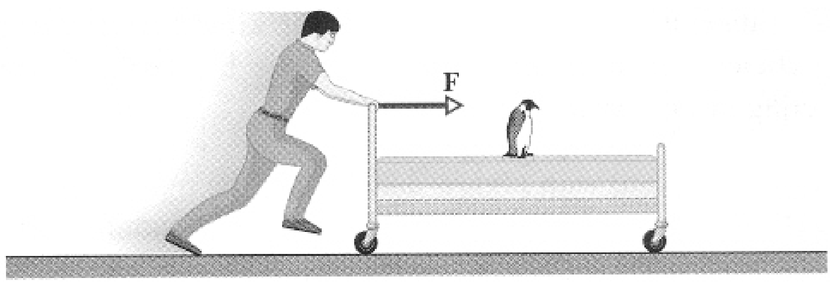
وبحل المعادلات (1) و (2) و (3) نجد أن

$a = 5.6\ \text{m/s}^2$

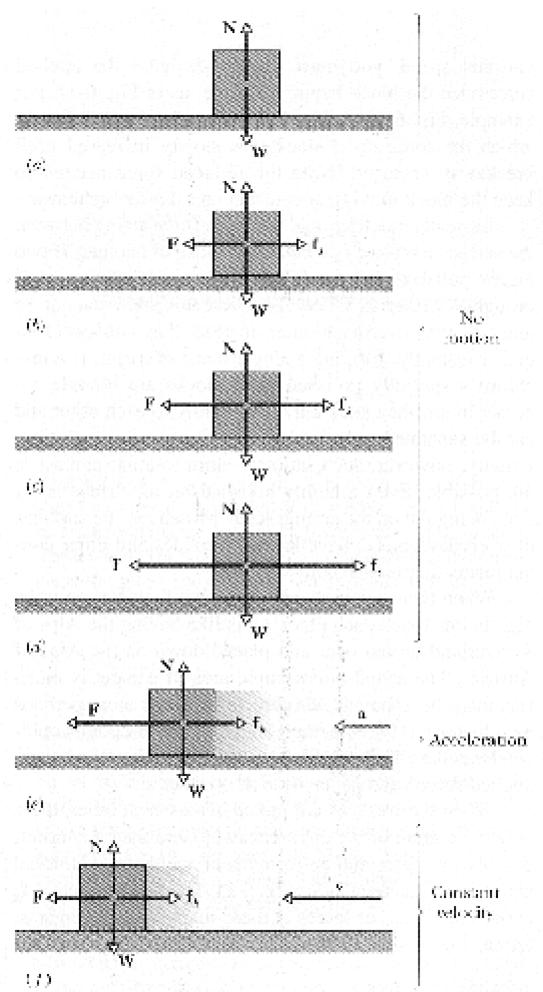
$T_1 = 28$ N

$T_2 = 56$ N

FORCE OF FRICTION



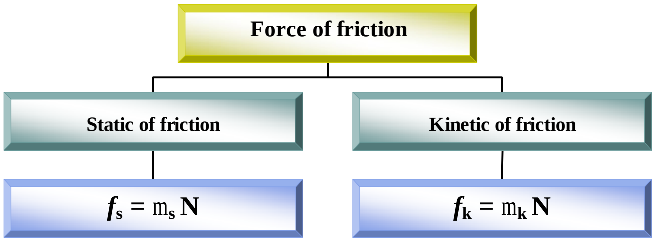
3.5 Force of friction



لقد أهملنا سابقاً القوة الناتجة عن الاحتكاك وذلك بفرض أن الأجسام تتحرك على أسطح ناعمة smooth surfaces وذلك حتى لا نزيد عدد المعادلات الرياضية المصاحبة لحل مسائل الميكانيكا، ولكن وبعد أن قطعنا شوطاً في التعامل مع متجهات القوة بمختلف أنواعها مثل الوزن W والشد T ورد الفعل N والقوة الخارجية المؤثرة على الحركة F، سندخل نوع آخر من القوة المؤثرة على الحركة وهي قوة الاحتكاك ***force of friction*** ويرمز لها بالرمز f واتجاه هذه القوة دائماً عكس اتجاه الحركة وهي ناتجة عن خشونة الأسطح المتحركة.

من التجارب العملية لوحظ أن قوة الاحتكاك للأجسام الساكنة أكبر من قوة الاحتكاك للأجسام المتحركة. وهذا شيء نلاحظه في حياتنا العملية حيث يحتاج الشخص إلى قوة كبيرة في بداية الأمر لتحريك صندوق خشبي على الأرض ولكن بعد أن يتحرك الجسم نلاحظ أن القوة اللازمة أصبحت أقل من ذي قبل وهذا لأن الجسم أصبح متحركاً وبالتالي فإن قوة الاحتكاك تصبح أقل.

لهذا السبب يمكن تقسيم الاحتكاك إلى نوعين هما الاحتكاك السكوني ***static friction*** والاحتكاك الحركي ***kinetic friction***.



ولقد وجد عمليا أن قوة الاحتكاك تتناسب طردياً مع قوة رد الفعل لهذا فإن الاحتكاك يمكن أن يكتب كالتالي:

$$f = \mu N \qquad (3.5)$$

حيث μ تسمى معامل الاحتكاك، وفى حالة الاحتكاك السكوني تسمى ***Coefficient of static friction***، μ_s أما في حالة الاحتكاك الحركي تسمى ***Coefficient of kinetic friction***، μ_k.

وعند تمثيل العلاقة بين القوة المؤثرة على جسم وقوة الاحتكاك بيانياً ينتج الشكل التالي:

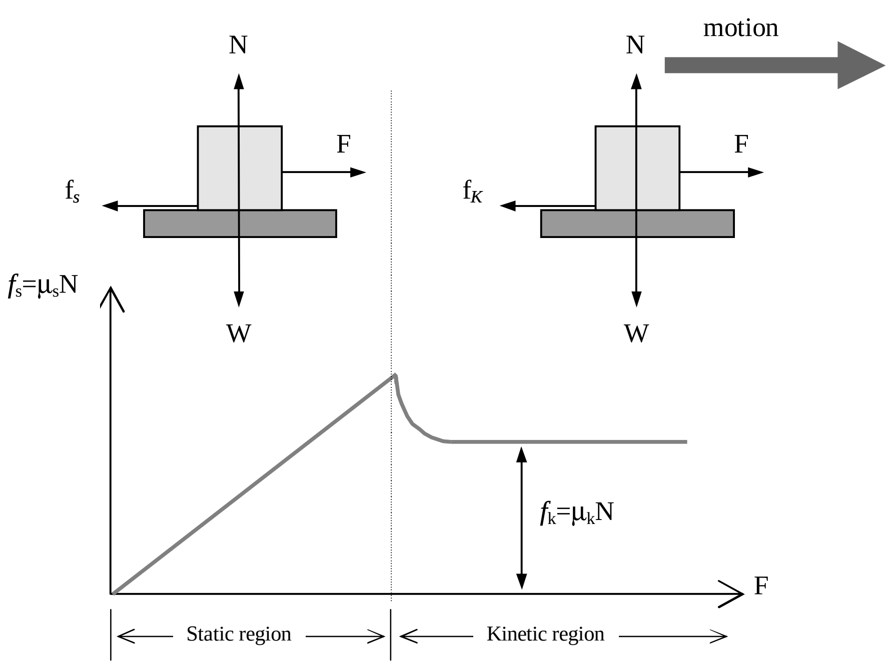


Figure 3.11

معامل الاحتكاك الحركي يكون دائما أكبر من معامل الاحتكاك السكوني ومعامل الاحتكاك ليس له وحدة.

3.5.1 Evaluation of the force of friction

Case (1) when a body slides on a horizontal surface

$$f_k = \mu_k N$$

since $N = mg$ **(كما هو في الشكل المقابل)**

$$f_k = \mu_k mg$$

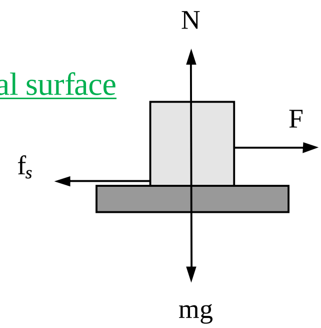


Figure 3.12

Case (2) when a body slides on an inclined surface

$$f_k = \mu_k N$$

since $N = mg\cos\theta$ **(كما هو في الشكل المقابل)**

$$f_k = \mu_k mg\cos\theta$$

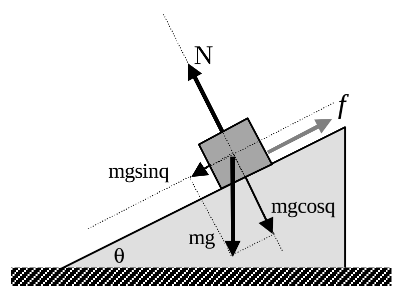


Figure 3.13

Example 3.12

Two blocks are connected by a light string over a frictionless pulley as shown in Figure 3.14. The coefficient of sliding friction between m_1 and the surface is μ. Find the acceleration of the two blocks and the tension in the string.

Solution

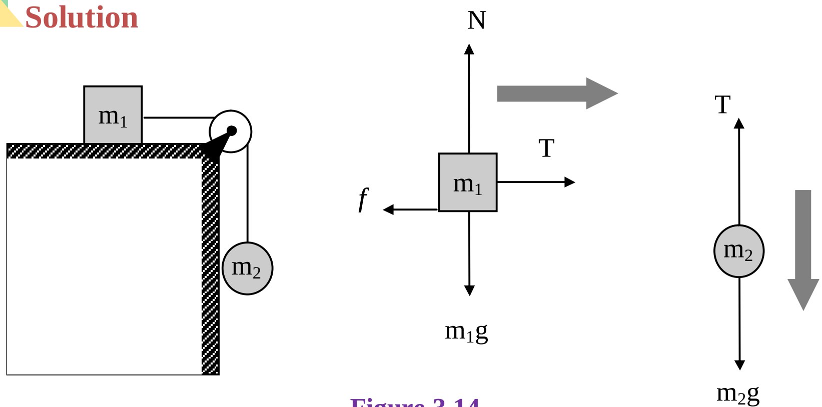


Figure 3.14

Consider the motion of m_1. Since its motion to the right, then $T>f$. If T were less than f, the blocks would remain stationary.

$$\sum F_x \text{ (on } m_1) = T - f = m_1 a$$

$$\sum F_y \text{ (on } m_1) = N - m_1 g = 0$$

since $f = \mu N = m_1 g$, then

$$T = m_1(a+\mu g)$$

For m_2, the motion is downward, therefore $m_2 g>T$. Note that T is uniform through the rope. That is the force which acts on the right is also the force which keeps m_2 from free falling. The equation of motion for m_2 is:

$$\sum F_y \text{ (on } m_2) = T - m_2 g = - m_2 a \qquad \Rightarrow \qquad T = m_2(g-a)$$

Solving the above equation

$$m_2(a+\mu g) - m_2(g-a) = 0$$

$$a = \left(\frac{m_2 - m m_1}{m_1 + m_2}\right) g$$

The tension T is

$$T = m_2\left(1 - \frac{m_2 - m m_1}{m_1 + m_2}\right)g = \frac{m_1 m_2 (1 + m) g}{m_1 + m_2}$$

Example 3.13

A 3kg block starts from rest at the top of 30^{o} incline and slides a distance of 2m down the incline in 1.5s. Find (a) the acceleration of the block, (b) the coefficient of kinetic friction between the block and the plane, (c) the friction force acting on the block, and (d) the speed of the block after it has slid 2m.

Solution

Given m = 3kg, $\theta = 30^{o}$, x = 2m, t = 1.5s

$$x = \frac{1}{2}at^2 \quad \Rightarrow \quad 2 = \frac{1}{2}a\,(1.5)^2 \quad \Rightarrow \quad a = 1.78\text{m/s}^2$$

$$mg\sin30 - f = ma \Rightarrow \quad f = m(g\sin30 - a)\; f = 9.37\text{N}$$

$$N - mg\cos30 = 0 \Rightarrow \quad N = mg\cos30$$

f = 9.37N

$$\mu_k = \frac{f}{N} = 0.368$$

$$v^2 = v_o^2 + 2a\,(x - x_o)$$

$$v^2 = 0 + 2(1.78)(2) = 7.11$$

then

v = 2.67m/s

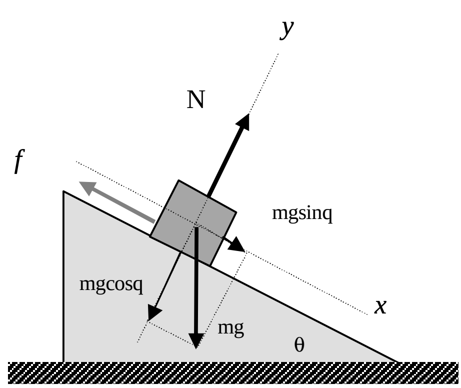


Figure 3.15

Example 3.14

A 2kg block is placed on top of a 5kg block as shown in figure 3.16. The coefficient of kinetic friction between the 5kg block and the surface is 0.2. A horizontal force *F* is applied to the 5kg block? (b) Calculate the force necessary to pull both blocks to the right with an acceleration of 3m/s^2. (c) Find the minimum coefficient of static friction between the blocks such that the 2kg block does not slip under an acceleration of 3m/s^2.

Solution

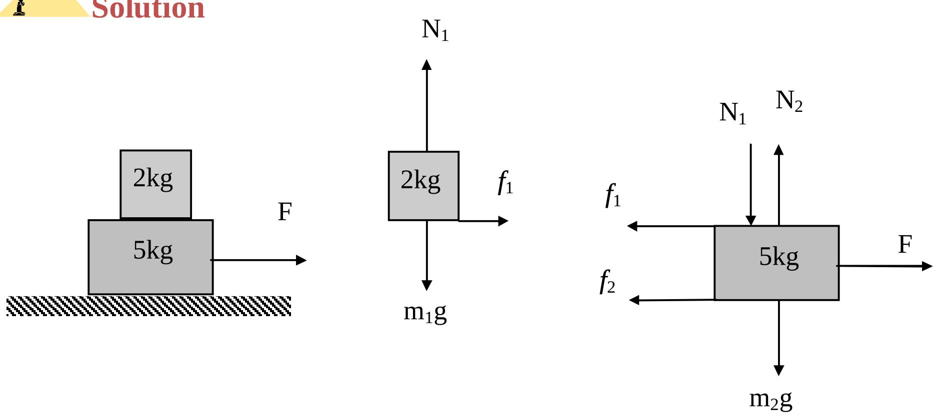


Figure 3.16

(a) The force of static friction between the blocks accelerates the 2kg block.

(b) $F - f_2 = ma \Rightarrow \quad F - \mu N_2 = ma$

$$F - (0.2)\,[(5+2)\times 9.8] = (5+2)\times 3 \quad \Rightarrow \; F = 4.7\text{N}$$

(c) $f = \mu N_1 = m_1 a$

$$\mu\,(2\times 9.8) = 2\times 3$$

$$\mu = 0.3$$

Example 3.15

A 5kg block is placed on top of a 10kg block. A horizontal force of 45N is applied to the 10kg block, while the 5kg block tied to the wall. The coefficient of kinetic friction between the moving surfaces is 0.2. (a) Draw a free-body diagram for each block. (b) Determine the tension in the string and the acceleration of the 10kg block.

Solution

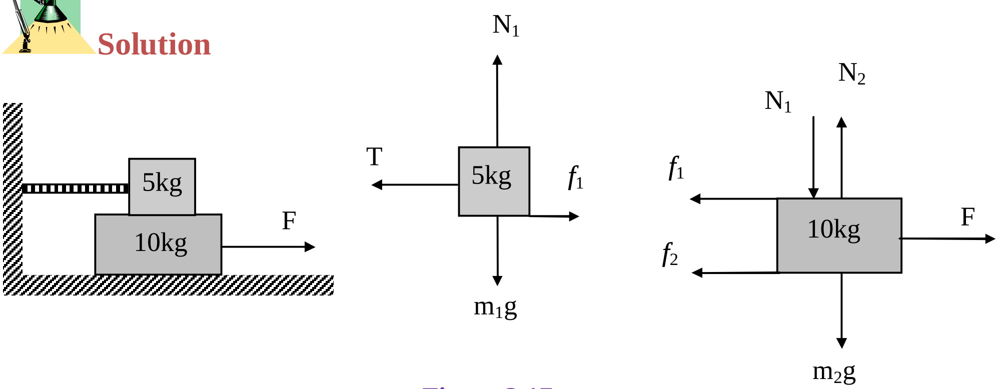


Figure 3.17

Consider the 5kg block first

$$f_1 - T = 0 \quad \Rightarrow \qquad T = f_1 = \mu mg = 0.2 \times 5 \times 9.8 = 9.8\text{N}$$

Consider the 10kg block

$$\Sigma F_x = ma \qquad 45 - f_1 - f_2 = 10a \qquad (1)$$

$$\Sigma F_y = 0 \qquad N_2 - N_1 - 10\text{g} = 0 \qquad (2)$$

$$f_2 = \mu N_2$$

but $N_2 = N_1 + 10g$ from equation (2)

then $f_2 = \mu(N_1 + 10g) = 29.4\text{N}$

from equation (1)

$$45 - 9.8 - 29.4 = 10\,a$$

$$a = 0.58\text{m/s}^2$$

Example 3.16

A coin is placed 30cm from the centre of a rotating, horizontal turntable. The coin is observed to slip when its speed is 50cm/s. What is the coefficient of static friction between the coin and the turntable?

Solution

$$N = mg \qquad (1)$$

$$f = m\frac{v^2}{r} \qquad (2)$$

Since $f = \mu_s N = \mu_s mg$

substitute in equation (2)

$$\mu_s mg = m\frac{v^2}{r}$$

$$\mu_s = \frac{v^2}{rg} = 0.085$$

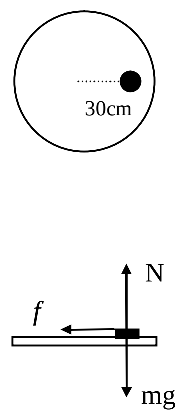


Figure 3.18

Example 3.17

A cart is loaded with bricks has a total mass of 18kg and is pulled at constant speed by a rope. The rope is inclined at 20^o above the horizontal and the cart moves on a horizontal plane. The coefficient of kinetic friction between the ground and the cart is 0.5. (a) What is the tension in the rope? When the cart is moved 20m, (b) How much work is done on the cart by the rope? (c) How much work is done by the friction force?

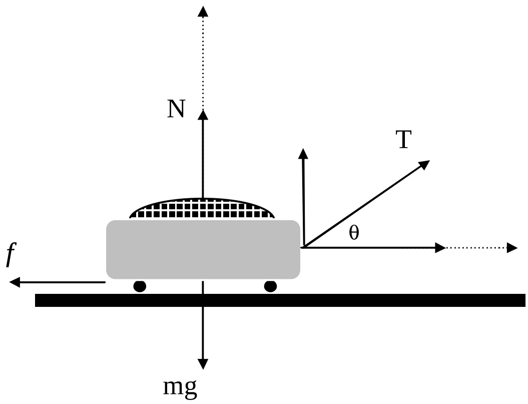


Figure 3.19

Solution

$$T\cos q - f = 0 \qquad (1)$$

$$N + T\sin q - mg = 0 \qquad (2)$$

$$f = \mu N = \mu(mg - T\sin q) \qquad (3)$$

Substitute (3) in (1)

$$T\cos q - \mu(mg - T\sin q) = 0$$

$$\therefore T = \frac{mmg}{\cos q + m\sin q} = 79.4\text{N}$$

(b) $W_T = T\cos q \times s = 1.49\text{kJ}$

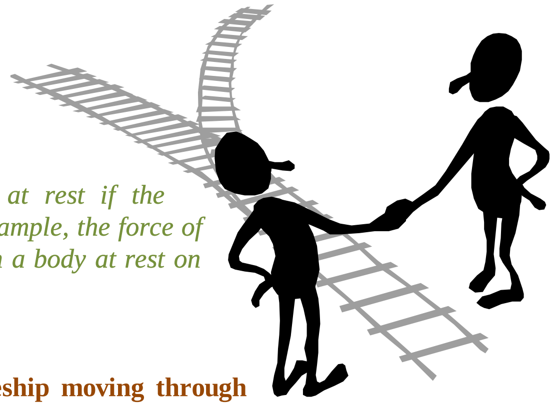
لان السرعة ثابتة وبالتالي العجلة تساوى صفر وهذا يؤدى إلى أن محصلة القوة تساوى صفر، إذا الشغل الكلى سيساوى صفر.

$$W_{net} = W_T + W_f = 0$$

then

$$W_f = - W_T = -1.49\text{kJ}$$

3.6 Questions with solution



1. If an object is at rest, can we conclude that there are no external forces acting on it?

Answer: No. The body may be at rest if the resultant force on it is zero. For example, the force of gravity and the normal force act on a body at rest on a table.

2. A space explorer is in a spaceship moving through space far from any planet or star. She notices a large rock, taken as a specimen from an alien planet, floating around the cabin of the spaceship. Should she push it gently toward a storage compartment or kick it toward the compartment? Why?

Answer: Regardless of the location of the rock, it still has mass, and a large force is necessary to move it. Newton's third law says that if he kicks it hard enough to provide the large force, the force back on his leg will be very unpleasant.

3. How much does an astronaut weigh out in space, far from any planets?

Answer: Zero. Since $w = mg$, and $g = 0$ in space, then $w = 0$.

4. Identify the action-reaction pairs in the following situations: (a) a man takes a step; (b) a snowball hits a girl in the back; (c) a baseball player catches a ball; (d) a gust of wind strikes a window.

Answer: (a) As a man takes a step, the action is the force his foot exerts on the earth; the reaction is the force of the earth on his foot. (b) The action is the force exerted on the girl's back by the snowball; the reaction is the force exerted on the snowball by the girl's back. (c) The action is the force of the glove on the ball; the reaction is the force of the ball on the glove. (d) The action is the force exened on the window by the air molecules; the reaction is the force on the air molecules exerted by the window.

5. While a football is in flight, what forces act on it? What are the action-reaction pairs while the football is being kicked, and while it is in flight?

Answer: When a football is in flight, the only force acting on it is its weight, assuming that we neglect air resistance. While it is being kicked, the forces acting on it are its weight and the force exerted on it by the kicker's foot. The reaction to the weight is the gravitational force exerted on the earth by the football. The reaction to the force exerted by the kicker's foot is the force exerted on the foot by the football.

6. A ball is held in a person's hand. (a) Identify all the external forces acting on the ball and the reaction to each. (b) If the ball is dropped, what force is exerted on it while it is falling? Identify the reaction force in this case. (Neglect air resistance.)

Answer: (a) The external forces on a ball held in a person's hand are its weight and the force of the hand upward on the ball. The reaction to the weight is the upward pull of the ball on the earth because of gravitational attraction. The reaction to the force on the ball by the hand is the downward force on the hand exerted by the ball. (b) When the ball is falling, the only force on it is its weight. The reaction force is the upward force on the earth exerted by the ball because of gravitational attraction.

7. If a car is travelling westward with a constant speed of 20 m/s, what is the resultant force acting on it?

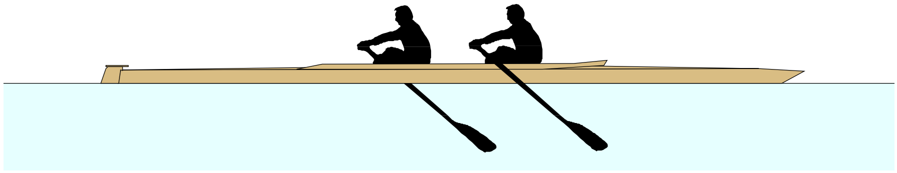
Answer: If an object moves with constant velocity, the net force on it is zero.

8. A rubber ball is dropped onto the floor. What force causes the ball to bounce back into the air?

Answer: When the ball hits the earth, it is compressed. As the ball returns to its original shape, it exerts a force on the earth, and the reaction to this thrusts it back into the air.

9. What is wrong with the statement, "Since the car is at rest, there are no forces acting on it."? How would you correct this sentence?

Answer: Just because an object is at rest does not mean that no forces act on it. For example, its weight always acts on it. The correct sentence would read, "Since the car is at rest, there is no net force acting on it."



3.7 Problems

1. A force, ***F***, applied to an object of mass m1 produces an acceleration of $3 m/s^2$. The same force applied to a second object of mass m_2 produces an acceleration of $1 m/s^2$. (a) What is the value of the ratio m_1/m_2? (b) If m_1 and m_2 are combined, find their acceleration under the action of the force ***F***.

2. A 6-kg object undergoes an acceleration of $2 m/s^2$. (a) What is the magnitude of the resultant force acting on it? (b) If this same force is applied to a 4-kg object, what acceleration will it produce?

3. A force of 10N acts on a body of mass 2 kg. What is (a) the acceleration of the body, (b) its weight in N. and (c) its acceleration if the force is doubled?

4. A 3-kg particle starts from rest and moves a distance of 4m in 2s under the action of a single, constant force. Find the magnitude of the force.

5. A 5.0-g bullet leaves the muzzle of a rifle with a speed of 320 m/s. What average force is exerted on the bullet while it is travelling down the 0.82m-long barrel of the rifle?

6. Two forces F_1 and F_2 act on a 5-kg mass. If F_1 = 20 N and F_2 = 15 N, find the acceleration in (a) and (b) of the Figure 3.20.

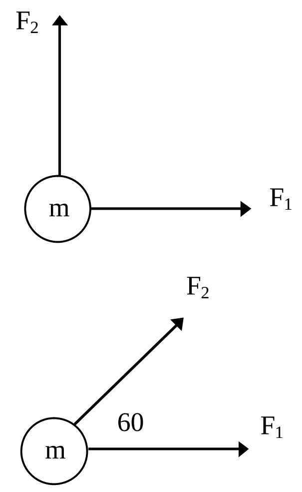


Figure 3.20

7. An electron of mass 9.1×10^{-31}kg has an initial speed of 3.0×105m/s. It travels in a straight line, and its speed increases to 7.0×10^5 m/s in a distance of 5.0 cm. Assuming its acceleration is constant, (a) determine the force on the electron and (b) compare this

force with the weight of the electron, which we neglected.

8. A 25kg block is initially at rest on a rough, horizontal surface. A horizontal force of 75N is required to set the block in motion. After it is in motion, a horizontal force of 60N is required to keep the block moving with constant speed. Find the coefficient of static and kinetic friction from this information.

9. The coefficient of static friction between a 5kg block and horizontal surface is 0.4. What is the maximum horizontal force that can be applied to the block before it slips?

10. A racing car accelerates uniformly from 0 to 80 km/h in 8s. The external force that accelerates the car is the friction force between the tires and the road. If the tires do not spin, determine the minimum coefficient of friction between the tires and the road.

11. Two blocks connected by a light rope are being dragged by a horizontal force F as shown in the Figure 3.5. Suppose that F = 50 N, m_1 = 10 kg, m_2 = 20 kg, and the coefficient of kinetic friction between each block and the surface is 0.1.

(a) Draw a free-body diagram for each block.
(b) Determine the tension, T, and the acceleration of the system.

12. The parachute on a race car of weight 8820N opens at the end of a quarter mile run when the car travelling at 55m/s. What is the total retarding force required to stop the car in a distance 1000m in the event of a brake failure?

13. Find the tension in each cord for the systems described in the Figure 3.21.

14. A 72-kg man stands on a spring scale in an elevator starting from rest, the elevator ascends, attaining it maximum velocity of 1.2 m/s in 0.8 s. It travels wit this constant velocity for the next 5.0 s. The elevator then undergoes a uniform negative acceleration for 1.5s and comes to rest. What does the spring scale

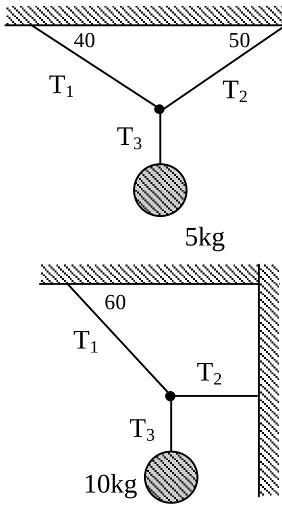


Figure 3.21

register (a) before the elevator starts to move (b) during the first 0.8s? (c) while the elevator is travelling at constant velocity? (d) during the negative acceleration period?

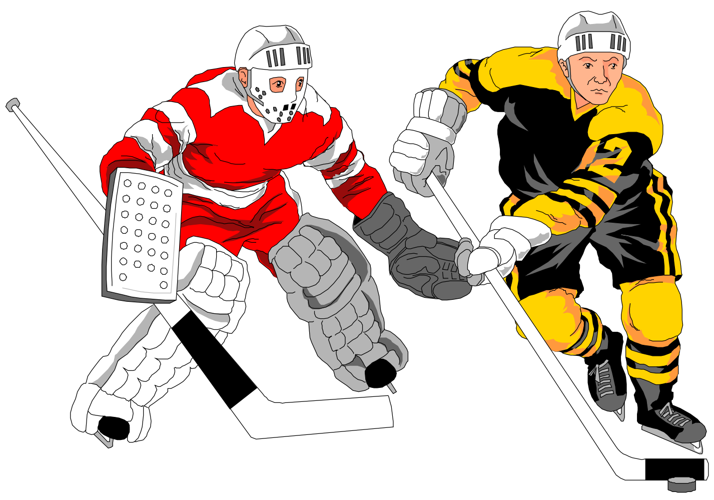
15.A toy car completes one lap around a circular track (a distance of 200m) in 25s. (a) What is the average speed? (b) If the mass of the car is 1.5kg, what is the magnitude of the centripetal force that keep it in a circle?

16.What centripetal force is required to keep 1.5kg mass moving in a circle of radius 0.4m at speed of 4m/s?

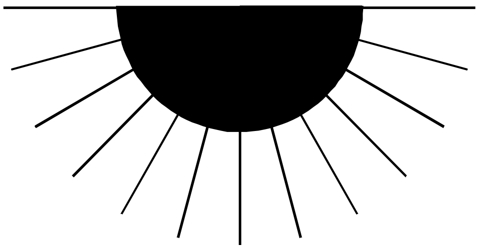
17.A 3kg mass attached to a light string rotates in circular motion on a horizontal, frictionless table. The radius of the circle is 0.8m, and the string can support a mass of 25kg before breaking. What range of speeds can the mass have before the string breaks?

Chapter 4

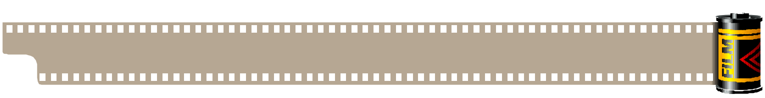
Work and Energy



الشغل والطاقة

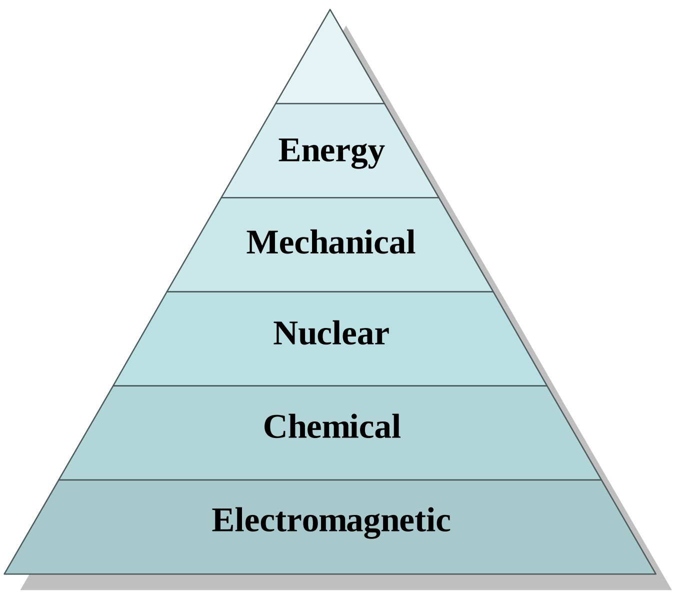


WORK AND ENERGY



4.1 Work and Energy

إن مفهوم الشغل والطاقة مهم جداً في علم الفيزياء، حيث توجد الطاقة في الطبيعة في صور مختلفة مثل الطاقة الميكانيكية ***Mechanical energy***، والطاقة الكهرومغناطيسية ***Electromagnetic energy***، والطاقة الكيميائية ***Chemical energy***، والطاقة الحرارية ***Thermal energy***، والطاقة النووية ***Nuclear energy***. إن الطاقة بصورها المختلفة تتحول من شكل إلى آخر ولكن في النهاية الطاقة الكلية ثابتة. فمثلا الطاقة الكيميائية المختزنة في بطارية تتحول إلى طاقة كهربية لتتحول بدورها إلى طاقة حركية. ودراسة تحولات الطاقة مهم جداً لجميع العلوم.



وفى هذا الجزء من المقرر سوف نركز على ***Mechanical energy***. وذلك لأنه يعتمد على مفاهيم القوة التي وضعها نيوتن في القوانين الثلاثة، ويجدر الذكر هنا أن الشغل والطاقة كميات قياسية وبالتالي فإن التعامل معها سيكون أسهل من استخدام قوانين نيوتن للحركة، وذلك لأننا كنا نتعامل وبشكل مباشر مع القوة وهى كمية متجهة. وحيث أننا لم نجد أية صعوبة في تطبيق قوانين نيوتن وذلك لأن مقدار القوة المؤثرة على حركة

الأجسام ثابت، ولكن إذا ما أصبحت القوة متغيرة وبالتالي فإن العجلة ستكون متغيرة وهنا يكون التعامل مع مفهوم الشغل والطاقة اسهل بكثير في مثل هذه الحالات.

ولكن قبل أن نتناول موضوع الطاقة فإننا سوف نوضح مفهوم الشغل الـــذي هـــو حلقـــة الوصل ما بين القوة والطاقة.

والشغل قد يكون ناتجاً من قوة ثابتة ***constant force*** أو من قـــوة متغيـــرة ***varying force***. وسوف ندرس كلا النوعين في هذا الفصل.

4.2 Work done by a *constant force*

اعتبر وجود جسم يتحرك إزاحة مقدارها *s* تحت تأثير قوة *F*، وهنا سوف نأخذ حالة بسيطة عندما تكون الزاوية بين متجه القوة ومتجه الإزاحة يساوي صفراً وفي الحالة الثانية عندما تكون هناك زاوية بين متجه الإزاحة ومتجه القوة وذلك للتوصل إلى القانون العام للشغل.

- **قوة منتظمة في اتجاه الحركة**

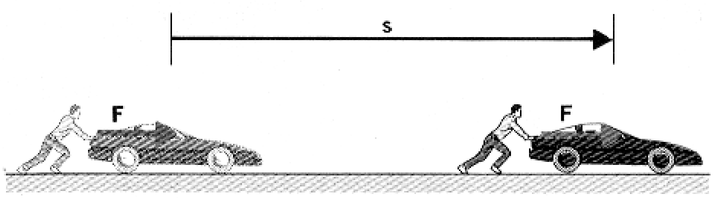


Figure 4.1

The work in this case is given by the equation

$$W = F\,s \qquad (4.1)$$

- **قوة منتظمة تعمل زاوية مقدارها q مع اتجاه الحركة**

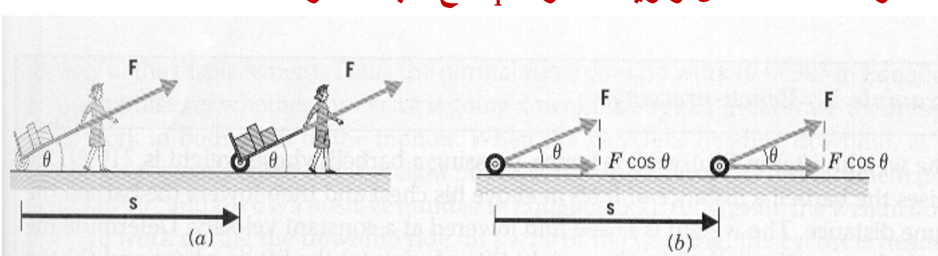


Figure 4.2

The work in this case is done by the horizontal component of the force

$$W = F\cos\theta\, s \qquad (4.2)$$

The above equation can be written in the directional form as dot product

$$W = \vec{F}\,.\,\vec{s} \qquad (4.3)$$

The unit of the work is N.m which is called Joule (J).

Example 4.1

Find the work done by a 45N force in pulling the luggage carrier shown in Figure 4.2 at an angle $q = 50^o$ for a distance $s = 75$m.

Solution

According to equation 4.2, the work done on the luggage carrier is

$$W = (F\cos q)\ s = 45 \cos 50^o \times 75 = 2170\text{J}$$

Work can be positive or negative

Important Notes

- ***The object must undergo a displacement s.***
- ***F must have a non-zero component in the direction of s.***
- ***Work is zero when there is no displacement.***
- ***Work is zero when the force is perpendicular to the displacement.***
- ***Work is positive when F is indirection of displacement or when 0£q<90 as in Figure 4.3(a).***
- ***Work is negative when F is in opposite direction of displacement or when 90<q£180 as in Figure4.3(b).***

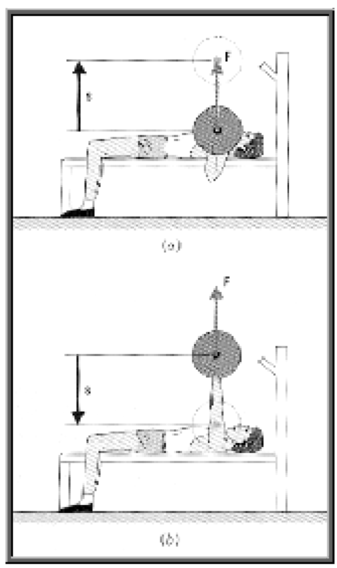


Figure 4.3

Example 4.2

The weight lifter shown in Figure 4.3 is bench-pressing a barbell whose weight is 710N. He raises the barbell a distance 0.65m above his chest and then lowers the barbell the same distance. Determine the work done on the barbell by the weight lifter during (a) the lifting phase and (b) the lowering phase.

Solution

(a) The work done by the force *F* during the lifting phase is

$$W = (F\cos q\ )\ s = 710 \cos 0^{o} \times 0.65 = 460\text{J} \quad [\textbf{\textit{Positive work}}]$$

(a) The work done by the force *F* during the lowering phase is

$$W = (F\cos q\ )\ s = 710 \cos 180^{o} \times 0.65 = -460\text{J} \ [\textbf{\textit{Negative work}}]$$

Example 4.3

A force $F = (6i - 2j)$ N acts on a particle that undergoes a displacement $s = (3i + j)$ m. Find (a) the work done by the force on the particle and (b) the angle between *F* and *s*.

Solution

(a) $W = \vec{F}.\vec{s} = (6i - 2j).(3i + j) = (6)(3) + (-2)(1) = 18 - 2 = 16\text{J}$

(b) $$F = \sqrt{F_x^2 + F_y^2} = \sqrt{6^2 + (-2)^2} = 6.32\text{N}$$

$$s = \sqrt{s_x^2 + s_y^2} = \sqrt{3^2 + 1^2} = 3.16\text{m}$$

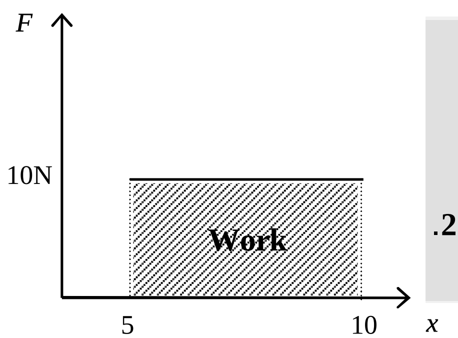
$W = F\ s \cos q$

$$\cos q = \frac{W}{Fs} = \frac{16}{6.32 \times 3.16} = 0.8012$$

$q = \cos^{-1}(0.8012) = 36.8^{o}$

4.3 Work done by a varying force

ذكرنا سابقا أن استخدام مفهوم الشغل سوف يساعدنا في التعامل مع الحركة عندما تكون القوة غير منتظمة، ولتوضيح ذلك دعنا نفترض أن قوة منتظمة قدرها 10N تؤثر على جسم ليتحرك مسافة من x_i=5m إلى x_f=25m وبالتالي فإن الإزاحة مقدارها 20m، ولتمثيل ذلك بيانياً نرسم محور القوة ومحور الإزاحة كما في الشكل، وبالتالي تكون القوة هي خط مستقيم يوازي محور x.

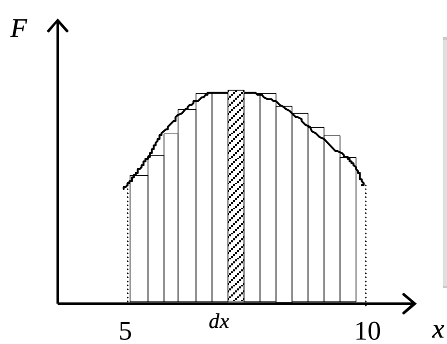


Work = $F\,s$ = 10×(25-5) = 200J

وهذا عبارة عن المساحة تحت المنحنى وهى مساحة المستطيل الذي عرضه 10N وطوله 20m.

Figure 4.4

أما في حالة كون القوة متغيرة خلال الإزاحة كما هو مبين في الشكل التالي:



$$W = \int_{x_i}^{x_f} F_x dx$$

وهذا عبارة عن المساحة تحت المنحنى أيضا.

Figure 4.5

في هذه الحالة نأخذ إزاحة صغيرة قدرها Δx حتى تكون القوة المؤثرة لهذه الإزاحة منتظمة وهنا يكون الشغل المبذول يعطى بالعلاقة التالية:

$$\Delta W = F_x \Delta x \qquad (4.4)$$

وإذا قمنا بتقسيم منحنى القوة إلى أجزاء صغيرة وحسبنا الشغل المبذول خلال كــل جــزء وجمعناهم، فإنه يمكن التعبير عن ذلك بالعلاقة الرياضية التالية:

$$W = \sum_{x_i}^{x_f} F_x \Delta x \qquad (4.5)$$

وعند جعل الإزاحة Δx أصغر ما يمكن أي أنها تؤول إلى الصفر لكي نحصل على قــيم أدق فإن المعادلة السابقة تتحول إلى

$$W = \int_{x_i}^{x_f} F_x dx \qquad (4.6)$$

وهذه هي الصورة العامة للشغل (لاحظ أن $F_x = F \cos q$).

$$W = \int_{x_i}^{x_f} F . dx \qquad (4.7)$$

Example 4.4

If an applied force varies with position according to $F_x = 3x^3 - 5$, where x is in m, how much work is done by this force on an object that moves from x=4m to x=7m?

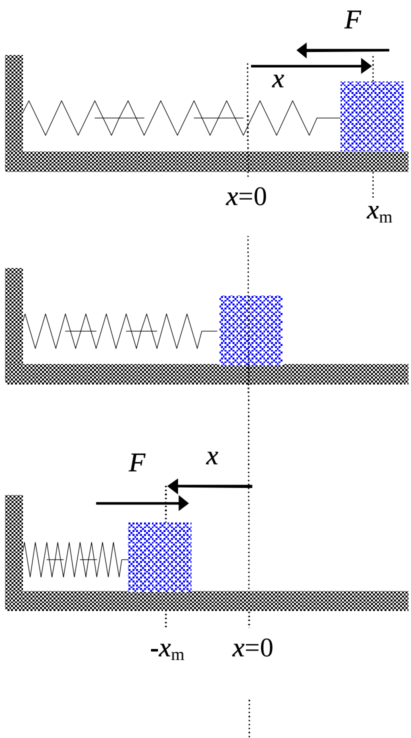
Solution

$$F = 3x^3 - 5$$

$$W = \int_{x1}^{x2} F dx \qquad \int_4^7 (3x^3 - 5)dx \qquad \left[\frac{3}{4}x^4 - 5x\right]_4^7$$

$$W = 1.59\text{kJ}$$

4.4 Work done by a spring



يعتبر الزنبرك Spring تطبيقاً عملياً على قوة متغيرة مع الإزاحة حيث أن القوة في حالة الزنبرك تعطى بالقانون التالي وهو قانون هوك Hooke's law.

$$F_s = -k\,x$$

حيث k هو ثابت الزنبرك، والإشارة السالبة تدل على أن قوة شد الزنبرك في عكس اتجاه الإزاحة x.

Work done by a spring:

$$W_s = W_{-xm\to 0} + W_{0\to xm} = \text{zero}$$

وذلك لأن الشغل المبذول لتحريك الجسم بواسطة قوة الزنبرك من $x_i=-x_m$ إلى $x_f=0$ يساوي الشغل المبذول لتحريك الجسم بواسطة قوة الزنبرك من $xi=0$ إلى $x_f=x_m$ ولكن بالسالب.

Work done by an external agent:

الشغل المبذول بواسطة مؤثر خارجي لتحريك الجسم المتصل بزنبرك ببطء من $x_i=0$ إلى $x_f=x_m$

$$W_{Fapp} = \frac{1}{2} k x_m^2$$

Figure 4.6

الشكل السابق 4.5 يوضح مراحل إزاحة جسم مرتبط بزنبرك كمثال على القوة المتغيرة حيث أن القوة الاسترجاعية للزنبرك تتغير مع تغير الإزاحة. ولحساب الشغل المبذول بواسطة شخص يشد ببطء الزنبرك من $x_i=-x_m$ إلى $x_f=0$ نعتبر أن القوة الخارجية F_{app} تساوي قوة الزنبرك F_s أي أن

$$F_{app} = -(-kx) = kx \qquad (4.8)$$

The work done by the external agent is

$$W_{F_{app}} = \int_0^{x_m} F_{app}\,dx = \int_0^{x_m} kx\,dx = \frac{1}{2} k x_m^{\;2} \qquad (4.9)$$

لاحظ أن الشغل المبذول بواسطة قوة خارجية تساوي سالب الشغل المبذول بواسطة قوة شد الزنبرك.

4.5 Work and kinetic energy

تعلمنا في أجزاء سابقة أن الجسم يتسارع إذا أثرت عليه قوة خارجية. فإذا فرضنا هنا أن جسم كتلته m يتعرض إلى قوة منتظمة مقدارها F في اتجاه محور x. وبتطبيق قانون نيوتن الثاني نجد أن

$$F_x = m\,a \qquad (4.10)$$

فإذا كانت الإزاحة الكلية التي تحركها الجسم هي s فإن الشغل المبذول في هذه الحالة يعطى بالمعادلة

$$W = F_x\, s = (m\,a)\, s \qquad (4.11)$$

ومن معلومات سابقة عن جسم يتحرك تحت تأثير عجلة ثابتة

$$s = \frac{1}{2}(v_i+v_f)\,t \quad \& \quad a = \frac{v_f - v_i}{t}$$

وبالتعويض في معادلة الشغل نحصل على

$$W = m\left(\frac{v_f - v_i}{t}\right)\frac{1}{2}\left(v_i + v_f\right)t \qquad (4.12)$$

$$W = \frac{1}{2}\,m v_f^2 - \frac{1}{2}\,m v_i^2 \qquad (4.13)$$

The product of one half the mass and the square of the speed is defined as the ***kinetic energy*** of the particle and has a unit of J

$$K = \frac{1}{2}\,m v^2 \qquad (4.14)$$

$$W = K_f - K_i \qquad (4.15)$$

This means that the work is the change of the kinetic energy of a particle.

$$W = \Delta K \qquad (4.15)$$

لاحظ أن طاقة الحركة K دائما موجبة ولكن التغير في طاقة الحركة ΔK يمكن أن يكون سالباً أو موجباً أو صفراً.

Example 4.5

A fighter-jet of mass 5×10^4kg is travelling at a speed of $v_i=1.1\times10^4$m/s as showing in Figure 4.7. The engine exerts a constant force of 4×10^5N for a displacement of 2.5×10^6m. Determine the final speed of the jet.

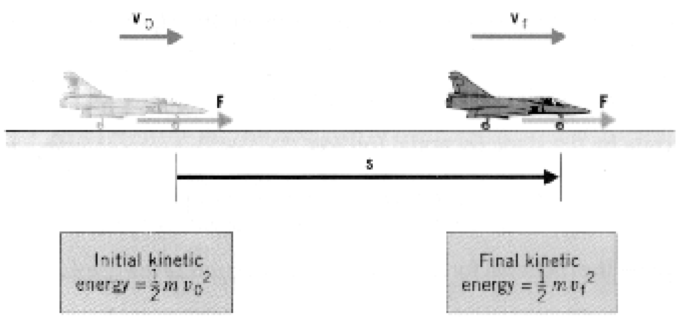


Figure 4.7

Solution

According to equation 4.7, the work done on the engine is

$$W = (F\cos q)\; s = 4\times10^5 \cos 0^o \times 2.5\times10^6 = 1\times10^{12}\text{J}$$

The work is positive, because the force and displacement are in the same direction as shown in Figure 4.7. Since $W = K_f - K_i$ the final kinetic energy of the fighter jet is

$$K_f = W + K_i$$

$$= (1\times10^{12}\text{J}) + ½\,(5\times10^4\text{kg})\,(1\times10^4\text{m/s})^2 = 4.031\times10^{12}\text{J}$$

The final kinetic energy is $K_f = ½\, m v_f^2$, so the final speed is

$$v_f = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2(4.03\times10^{12})}{5\times10^4}} = 1.27\times10^4 \text{ m/s}$$

حيث أن المحرك يبذل شغلاً موجباً لذا كانت السرعة النهائية أكبر من السرعة الابتدائية.

4.6 Power

***The power** is defined as the time rate of energy transfer*. If an external force is applied to an object, and if the work done by this force is ΔW it the time interval Δt, then the average power is:

$$P_{\text{ave}} = \frac{\Delta W}{\Delta t} \qquad (4.16)$$

The instantaneous power is given by

$$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt} \qquad (4.17)$$

$$P = \frac{dW}{dt} = F.\frac{ds}{dt} \qquad (4.18)$$

$$\therefore P = F.v \qquad (4.19)$$

The unit of the power is J/s which is called watt (W).

Example 4.6

A 65-kg athlete runs a distance of 600 m up a mountain inclined at 20^{o} to the horizontal. He performs this feat in 80s. Assuming that air resistance is negligible, (a) how much work does he perform and (b) what is his power output during the run?

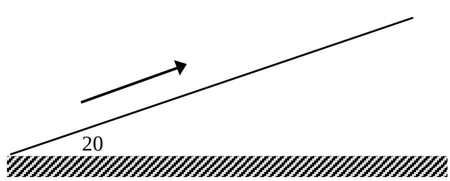


Figure 4.8

Solution

Assuming the athlete runs at constant speed, we have

$$W_A + W_g = 0$$

where W_A is the work done by the athlete and W_g is the work done by gravity. In this case,

$$W_g = -mgs(\sin\theta)$$

So

$$W_A = -W_g = +mgs(\sin\theta)$$

$$= (65\text{kg})(9.80\text{m/s}^2)(600\text{m})\sin 20^\circ$$

(b) His power output is given by

$$P_A = \frac{W_A}{\Delta t} = \frac{1.31\times 10^5\,J}{80\,s} = 1.63\text{kW}$$

Example 4.7

A 4-kg particle moves along the x-axis. Its position varies with time according to $x = t + 2t^3$, where x is in m and t is in s. Find (a) the kinetic energy at any time t, (b) the acceleration of the particle and the force acting on it at time t, (c) the power being delivered to the particle at time t, and (d) the work done on the particle in the interval $t = 0$ to $t = 2$ s.

Solution

Given $m = 4$ kg and $x = t + 2t^3$, we find

$$v = \frac{dx}{dt} = \frac{d}{dt}(t + 2t^3) = 1 + 6t^2$$

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(4)(1 + 6t^2)^2 = (2 + 24t^2 + 72t^4)\text{ J}$$

(b) $a = \frac{dv}{dt} = \frac{d}{dt}(1 + 6t^2) = 12t \text{ m/s}^2$

$F = m\,a = 4(12t) = 48t$ N

(c) $P = \frac{dW}{dt} = \frac{dK}{dt} = \frac{d}{dt}(2 + 24t^2 + 72t^4) = (48t + 288t^3)$W

[or use $P = Fv = 48t\,(1 + 6t^2)$]

(d) $W = K_f - K_i$ where $t_i = 0$ and $t_f = 2$s.

At $t_i = 0$,

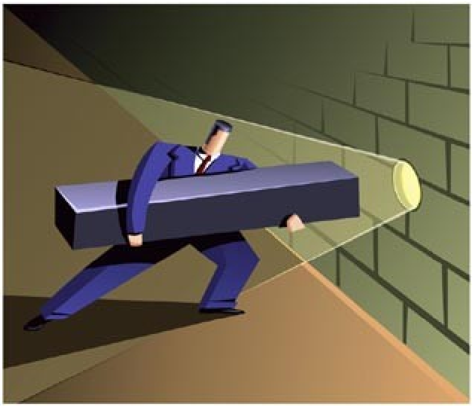
$K_i = 2$J

At $t_f = 2$ s,

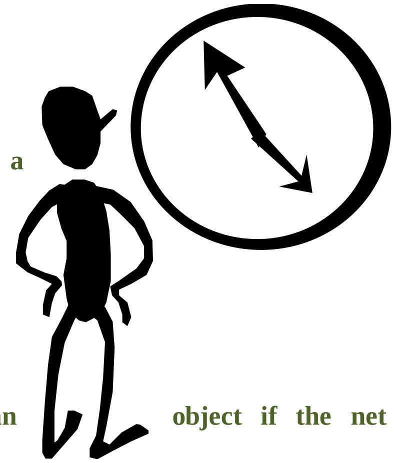
$K_f = [2 + 24(2)^2 + 72(2)^4] = 1250$ J

Therefore,

W = 1.25 × 103 J



4.7 Questions with solution



1 Can the kinetic energy of an object have a negative value?

Answer: No. Kinetic energy = $\frac{1}{2}mv^2$*. Since* v^2 *is always positive, K is always positive.*

2 What can be said about the speed of an object if the net work done on that object is zero?

Answer: Its speed remains unchanged. This can be seen from the work-energy theorem. Since $W = \Delta K = 0$*, it follows that* $v_f = v_i$*.*

3 Using the work-energy theorem, explain why the force of kinetic friction always has the effect of reducing the kinetic energy of a particle,

Answer: The work done by the force of sliding friction is always negative. Since the work done is equal to the change in kinetic energy, it follows that the final kinetic energy is less than the initial kinetic energy.

4 One bullet has twice the mass of a second bullet. If both are fired such that they have the same velocity, which has more kinetic energy?

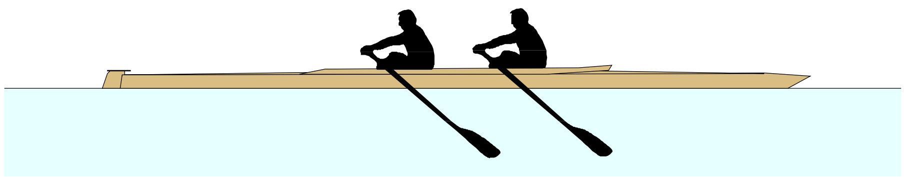
Answer: The kinetic energy of the more massive bullet is twice that of the lower mass bullet.

5 When a punter kicks a football, is he doing any work on the ball while his toe is in contact with the ball? Is he doing any work on the ball after it loses contact with his toe? Are there any forces doing work on the ball while it is in flight?

Answer: As the punter kicks the football, he exerts a force on the ball and also moves the ball. Thus, he is doing work on the ball. After his toe loses contact with the ball, the punter no longer exerts a force on it, and thus, he can no longer do work on it. While in flight, the only force doing any work on the ball, in the absence of air resistance, is the weight of the ball.

6 Do frictional forces always reduce the kinetic energy of a body? If your answer is no, give examples which illustrate the effect.

Answer: No. For example, if you place a crate on the bed of a truck and the truck accelerates, the friction force acting on the crate is what causes it to accelerate, assuming it doesn't slip. Another example is a car which gets its acceleration because of the friction force between the road and its tires. This force is in the direction of motion of the car and produces an increase in the car's kinetic energy. Of course, the source of the energy is the combustion of fuel in the car's engine.



4.8 Problems

(1) If a man lifts a 20-kg bucket from a well and does 6 kJ of work, how deep is the well? Assume the speed o the bucket remains constant as it is lifted.

(2) A 65kg woman climbs a flight of 20 stairs, each 23 cm high. How much work was done against the force of gravity in the process?

(3) A horizontal force of 150 N is used to push a 40-kg box on a rough horizontal surface through a distance of 6m. If the box moves at constant speed, find (a) the work done by the 150-N force, (h) the work done by friction.

(4) When a 4-kg mass is hung vertically on a certain light spring that obeys Hooke's law, the spring stretches 2.5cm. If the 4-kg mass is removed, (a) how far will the spring stretch if a 1.5-kg mass is hung on it, and (b) how much work must an external agent do to stretch the same spring 4.0 em from its unstretched position?

(5) If an applied force varies with position according to $F_x = 3x^3 - 5$, where x is in m, how much work is done by this force on an object that moves from $x = 4$ m to $x = 7$ m?

(6) A 0.6-kg particle has a speed of 2 m/s at point A and kinetic energy of 7.5J at point B. What is (a) its kinetic energy at point A? (b) its velocity at point B? (c) the total work done on the particle as it moves from A to B?

(7) A 0.3-kg ball has a speed of 15 m/s. (a) What is its kinetic energy? (b) If its speed is doubled, what is its kinetic energy?

(8) Calculate the kinetic energy of a 1000kg satellite orbiting the earth at speed of 7×10^3m/s.

(9) A mechanic pushes a 2500kg car from rest to a speed *v* doing 5000J of work in the process. During this time, the car moves 25m. Neglecting friction between the car and the road, (a) What is the final speed, *v*, of the car? (b) What is the horizontal force exerted on the car?

(10)A 3-kg mass has an initial velocity $v_o = (6i - 2j)$ m/s. (a) What is its kinetic energy at this time? (b) Find the change in its kinetic energy if its velocity changes to (8*i* + 4*j*) m/s.

(11)A 700-N marine in basic training climbs a 10-m vertical rope at uniform speed in 8 s. What is his power output?

(12)A weightlifter lifts 250kg through 2m in 1.5s. What is his power output?

(13)A 200kg cart is pulled along a level surface by an engine. The coefficient of friction between the carte and surface is 0.4. (a) How much power must the enginge deliver to move the carte at constant speed of 5m/s? (b) How much work is done by the engine in 3min?

(14)A 1500-kg car accelerates uniformly from rest to a speed of 10 m/s in 3s. Find (a) the work done on the car in this time, (b) the average power delivered the engine in the first 3s, and (c) the instantaneous power delivered by the engine at t = 2s.

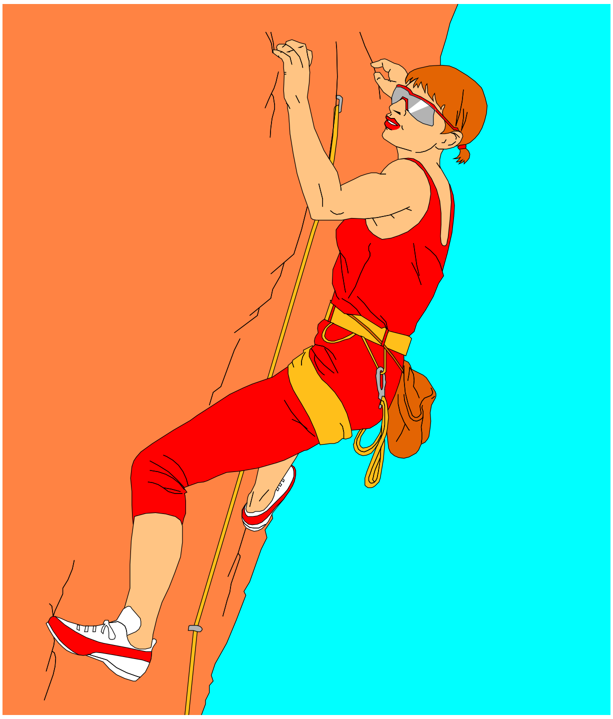
(15)A woman raises a 10-kg flag front the ground to the top of a 10-m flagpole at constant velocity, 0.25 m/s. (a)Find the work done by the woman while raising the flag. (b) Find the work done by gravity. (c) What is the power output of the woman while raising the flag?

(16)A 4-kg particle moves along the x-axis. Its position varies with time according to $x = t + 2t^3$, where x is in m and t is in s. Find (a) the kinetic energy at any time t, (b) the acceleration of the particle and the force acting on it at time t, (c) the power being delivered to the particle at time t, and (d) the work done on the particle in the interval t = 0 to t = 2 s.

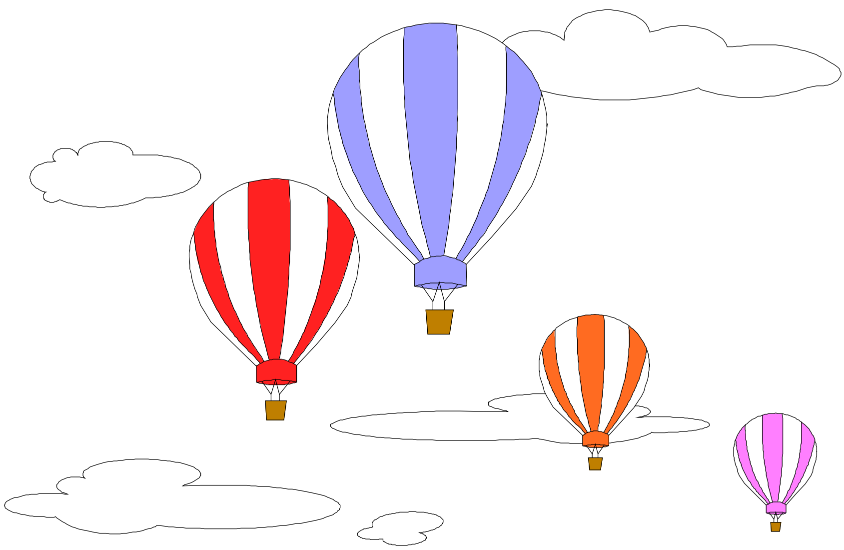
(17)The resultant force acting on a 2kg particle moving along the x axis varies as $F = 3x^2 - 4x + 5$, where x is in m and F, is in N. (a) Find the net work done on the particle as it moves from x = 1m to x = 3m. (b) If the speed of the particle is 5 m/s at x = 1m, what is its speed at x = 3 m?

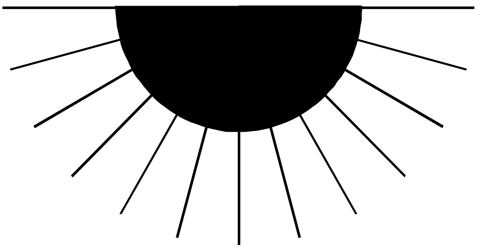
Chapter 5

Potential energy and conservation energy

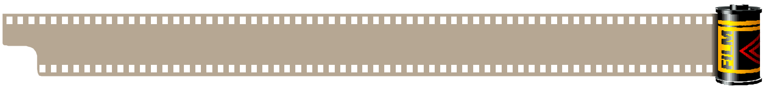


طاقة الوضع وقانون الحفاظ على الطاقة





POTENTIAL ENERGY AND CONSERVATION ENERGY



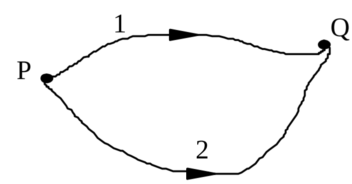
5.1 Potential energy and conservation energy

درسنا في الفصل السابق مفهوم طاقة الحركة ***Kinetic energy*** لجسم متحرك ووجدنا أن طاقة حركة الجسم تتغير عندما يبذل شغل على الجسم. سندرس في هذا الفصل نوعاً آخر من أنواع الطاقة الميكانيكية وهو طاقة الوضع ***Potential energy***. ويمكن لطاقة الوضع أن تتحول إلى طاقة حركة أو إلى بذل شغل. وتجدر الإشارة هنا إلى أن أنواع القوى التي درسناها هي إما قوة عجلة الجاذبية الأرضية (F_g) أوقوة الاحتكاك (f) أوقوة الشد (T) أوالقوة المؤثرة الخارجية (F_{app})، هذه القوى تقسم إلى نوعين، إما قوى محافظة ***conservative forces*** أو قوى غير محافظة ***non-conservative***. فإذا كان **الشغل** الناتج عن قوة ما لا يعتمد على المسار فإن هذه القوة تكون **محافظة**، أما إذا كان **الشغل** يعتمد على المسار فإن هذه القوة تكون **غير محافظة**.

5.2 Conservative forces

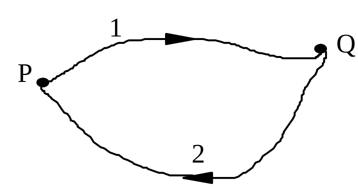
A force is conservative when the *work* done by that *force* acting on a particle moving between two points is *independents* of the path the particle takes between the points.

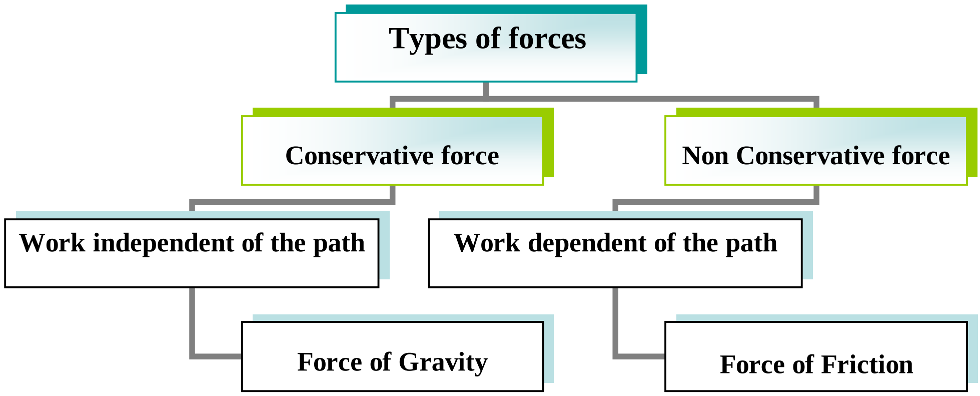
$$W_{PQ}(\text{along } 1) = W_{PQ}(\text{along } 2)$$



The total work done by a conservative force on a particle is zero when the particle moves around any closed path and returns to its initial position.

$$W_{PQ}(\text{along } 1) = -\,W_{PQ}(\text{along } 2)$$





تعتبر قوة الجاذبية الأرضية مثالاً على القوة المحافظة، فعند نقل جسم من موضع إلى آخر فإن الشغل المبذول يعتمد على القوة *mg* وعلى الإزاحة بين نقطتي البداية والنهايــة، ولا يعتمد الشغل على المسار فإذا كانت نقطة البداية والنهاية لها نفس الارتفاع عــن ســطح الأرض فإن الشغل يكون صفراً.

الشغل لا يعتمد على المسار عند نقل جسم من موضع آخر لأن قوة الجاذبية الأرضية قوة محافظة.

$$W_g = - mg (y_f - y_i)$$

كما وأن القوة الاسترجاعية للزنبرك قوة محافظة حيث أن الشغل يعتمد على نقطتي البداية والنهاية فقط ولا يعتمد على المسار، وقد لاحظنا في الفصل السابق أن الشــغل المبــذول بواسطة الزنبرك يساوي صفراً في حركة الزنبرك دورة كاملة حيث يكــون فيهــا نقطــة النهاية هي العودة إلى نقطة البداية.

5.3 Potential energy

When the work done by conservative force we found that the work does not depend on the path taken by the particle. Therefore we can define a new physical quantity called the change in potential energy ΔU.

The Change potential energy is defined as

$$\Delta U = (-W) = U_f - U_i = -\int_{x_i}^{x_f} F_x \, dx \qquad (5.1)$$

علمنا سابقاً أن الشغل يساوى التغير في طاقة الحركة، ولكن إذا تحرك جسم تحت تأثير قوة محافظة مثل قوة عجلة الجاذبية الأرضية إزاحة محددة فإن الشغل هنا يعتمد على نقطتي البداية والنهاية ولا يعتمد على المسار. وهنا لا نستطيع القول أن الشغل يساوى التغير في طاقة الحركة. فمثلاً إذا حاول شخص رفع كتلة ما من سطح الأرض إلى ارتفاع معين قدره h فإن هذا الشخص سيبذل شغلاً موجباً مساوياً لـ mgh لان القوة التي بذلها في اتجاه الحركة، ولكن من وجهة نظر الجسم فإنه بذل شغلاً سالباً قدره $-mgh$ وذلك لأن قوته (وزنه) في عكس اتجاه الإزاحة، هذا الشغل السالب يدعى طاقة الوضع التي اكتسبها الجسم عند تحريكه من نقطة إلى أخرى تحت تأثير قوة محافظة (قوة عجلة الجاذبية الأرضية).

5.4 Conservation of mechanical energy

لنفترض وجود جسم يتحرك في بعد واحد x تحت تأثير قوة محافظة F_x, فإن الشغل المبذول بواسطة القوة يساوي التغير في طاقة حركة الجسم.

$$W = \Delta K = -\Delta U \qquad (5.2)$$

$$\Delta K = -\Delta U \qquad (5.3)$$

$$\Delta K + \Delta U = \Delta(K + U) = 0 \qquad (5.4)$$

This is the law of conservation of mechanical energy, which can be written as

$$K_i + U_i = K_f + K_f \qquad (5.5)$$

Law of conservation mechanical energy

5.5 Total mechanical energy

لنعرف الطاقة الميكانيكية الكلية ***Total mechanical energy*** بحاصل جمع طاقة الحركة وطاقة الوضع للجسم.

$$\boldsymbol{E = K + U}$$

الطاقة الميكانيكية الكلية ← E

طاقة الحركة ← K

طاقة الوضع ← U

ومن هنا يمكن كتابة قانون الحفاظ على الطاقة الميكانيكية على النحو التالي:

$$\boldsymbol{E_i = E_f}$$

Law of conservation mechanical energy

The law of conservation of mechanical energy states that the total mechanical energy of a system remains constant for conservative force only. This means that when the kinetic energy increased the potential energy decrease.

Examples

Example 5.1

A 0.2 kg bead is forced to slide o a frictionless wire as in the Figure. The bead starts from rest at A and ends up at B after colliding with a light spring of force constant *k*. If the spring compresses a distance of 0.1 m, what is the force constant of the spring?

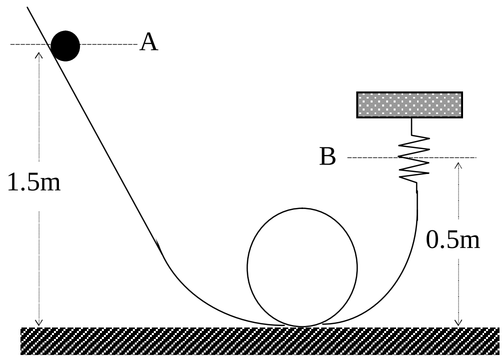


Figure 5.1

Solution

The gravitational potential energy of the bead at A -with respect to the lowest point is

$$U_i = mgh_i = (0.2\text{ kg})\,(9.8\text{ rn/s}^2)\,(1.5\text{ m}) = 2.94\text{ J}$$

The kinetic energy of the bead at A is zero since it starts from rest. The gravitational potential energy of the bead at B is

$$U_f = mgh_f = (0.2\text{ kg})\,(9.8\text{ m/s}^2)\,(0.5\text{ m}) = 0.98\text{ J}$$

Since the spring is part of the system, we must also take into account the energy stored in the spring at B. Since the spring compresses a distance $x_m = 0.1\text{m}$, we have

$$U_S = \frac{1}{2}\,k\,x_m^2 = \frac{1}{2}\,k\,(0.1)^2$$

Using the principle of energy conservation gives

$$U_i = U_f + U_s$$

$$2.94\text{ J} = 0.98\text{ J} + \frac{1}{2}\,k\,(0.1)^2$$

$$k = 392\text{ N/m}$$

Example 5.2

A block of mass 0.2 kg is given an initial speed v_o = 5 m/s on a horizontal, rough surface of length 2m as in Figure 5.2. The coefficient of kinetic friction on the horizontal surface is 0.30. If the curved part of the track is frictionless, how high does the block rise before coming to rest at B?

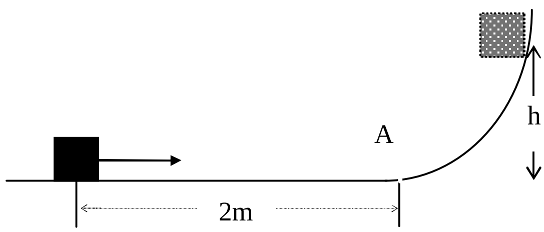


Figure 5.2

Solution

The initial kinetic energy of the block is

$$K_o = \frac{1}{2}mv^2 = \frac{1}{2}(0.2\text{kg})(5\text{m/s})^2$$

$$= 2.50 \text{ J}$$

The work done by friction along the horizontal track is

$$W_f = -fd = -\mu mgd = -(0.30)(0.2)(9.8)(2) = -1.18 \text{ J}$$

Using the work-energy theorem, we can find the kinetic energy at A

$$W_f = k_A - K_o = K_A - 2.50$$

$$K_A = 2.50 + W_f = 2.50 - 1.18 = 1.32 \text{ J}$$

Since the curved track is frictionless, we can equate the kinetic energy of the block at A to its gravitational potential energy at B.

$$mgh = K_A = 1.32 \text{ J}$$

$$h = \frac{1.32\text{J}}{0.2\text{kg} \times 9.8\text{m/s}^2} = 0.673 \text{ m}$$

Example 5.3

A single conservative force $F_x = (2x + 4)$ N acts on a 5-kg particle, where x is in m. As the particle moves along the x axis from $x = 1$ m to $x = 5$ m, calculate (a) the work done by this force, (b) the change in the potential energy of the particle, and (c) its kinetic energy at $x = 5$ m if its speed at $x = 1$ m is 3 m/s.

Solution

$F_x = (2x + 4)$ N $\qquad x_i = 1$ m $\qquad x_f = 5$ m $\qquad v_i = 3$m/s

(a) $W_F = \int_{xi}^{xf} F_x dx = \int_{xi}^{xf} (2x+4)dx = \left[x^2 + 4x\right]_1^5 = 5^2 + 4(5) - [1^2 + 4(1)] = 40J$

(b) $\Delta U = -W_F = -40J$

(c) $\Delta K + \Delta U = 0 \quad \Rightarrow \quad \Delta K = -\Delta U = -40.0$ J

$K_f - \frac{1}{2} m v_i^2 = 40J$

$K_f = 40.0\ J + 22.5\ J = 62.5J$

Example 5.4

A bead slides without friction around a loop-the-loop. If the bead is released from a $h = 3.5R$, what is its speed at point A? How large is the normal force on it if its mass is 5.0?

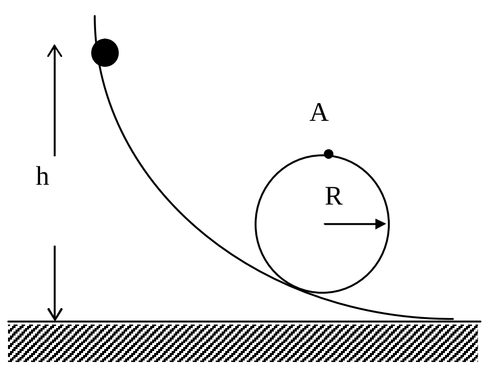


Figure 5.3

Solution

It is convenient to choose the reference point of potential energy to be at the lowest point of the bead's motion. Since $v_i = 0$ at the start,

$$E_i = K_i + U_i = 0 + mgh = mg(3.5R)$$

The total energy of the bead at point A can be written as

$$E_A = K_A + U_A = \frac{1}{2} m v_A^2 + mg(2R)$$

Since mechanical energy is conserved, $E_i = E_A$, and we get

$$\frac{1}{2} m v_A^2 + mg(2R) = mg(3.5R)$$

$$v_A^2 = 3gR \quad \text{or } v_A = \sqrt{3gR}$$

To find the normal force at the top, it is use to construct a free-body diagram as shown, where both N and mg are downward Newton's second law gives

$$N + mg = \frac{m v_A^2}{R} = \frac{m(3gR)}{R} = 3mg \Rightarrow \qquad N = 3mg - mg = 2mg$$

> وذلك لأن عند النقطة A الجسم يتحرك على مسار دائري.

$$N = 2(5 \times 10^{-3}\ \text{kg}\ (9.80\text{m/s}^2) = 0.098\text{N}$$

Example 5.5

A 25-kg child on a swing 2 m long is released from rest when the swing supports make an angle of 30° with the vertical. (a) Neglecting friction, find the child's speed at the lowest position. (b) If the speed of the child at the lowest position is 2 m/s, what is the energy loss due to friction?

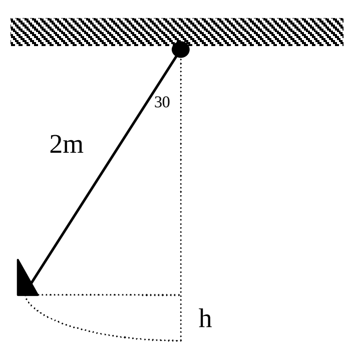


Figure 5.4

Solution

(a) First, note that the child falls through a vertical distance of h = (2 m) - (2 m) cos 30^{o} = 0.268 m

Taking U = 0 at the bottom, and using conservation of energy gives

$$K_i + U_i = K_f + U_f$$

$$0 + mgh = \frac{1}{2}\, mv^2 + 0$$

$$v = \sqrt{2gh} = 29 \text{ m/s}$$

(b) If v_f = 2 m/s, and friction is present, then

$$W_f = \Delta K + \Delta U = \left(\frac{1}{2}\, mv_f^2 - \frac{1}{2}\, mv_i^2\right) + 0 - mgh$$

$$W_f = -15.6\text{J}$$

Example 5.6

A block of mass 0.25kg is placed on a vertical spring of constant *k*=5000N/m, and is pushed downward compressing the spring a distance of 0.1 m. As the block is released, it leaves the spring and continues to travel upward. To what maximum height above the point of release does the block rise?

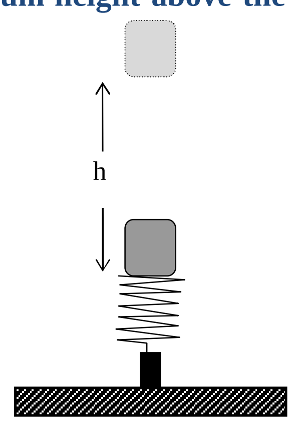


Figure 5.5

Solution

Taking U_g = 0 to be at the point of release, and noting that = v_i = 0, gives

$$E_i = K_i + U_i = 0 + (U_s + U_g)_i$$

$$E_i = \frac{1}{2}\, k\, x^2 + 0 = 25\text{J}$$

When the mass reaches its maximum height h, v_f = 0, and the spring is unstretched, so U_S = 0.

$$E_f = K_f + U_f = 0 + mgh = (0.25 \text{ kg})(9.80 \text{ m/s}^2)\ h$$

Since mechanical energy is conserved, we have $E_f = E_i$, or

$$(0.25\ \text{kg})(9.80\ \text{m/s}^2)h = 25\text{J}$$

$$h = 10.2\text{m}$$

Example 5.7

A ball whirls around in a vertical circle at the end of a string. If the ball's total energy remains constant, show that the tension in the string at the bottom is greater than the tension at the top by six times the weight of the ball.

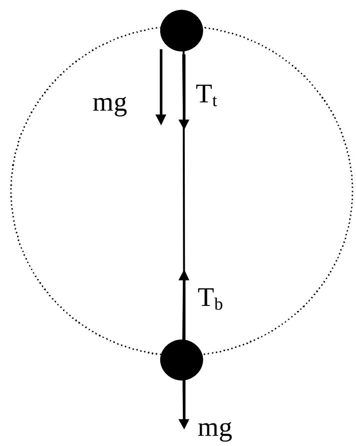


Figure 5.6

Solution

Applying Newton's second law at the bottom (b) and top (t) of the circular path gives

$$T_b - mg = \frac{mv_b^2}{R} \qquad (1)$$

$$T_t + mg = \frac{mv_t^2}{R} \qquad (2)$$

Subtracting (1) and (2) gives

$$T_b = T_t + 2mg + \frac{m(v_b^2 - v_t^2)}{R} \qquad (3)$$

Also, energy must be conserved; that is,

$$\Delta K + \Delta U = 0. \text{ So,}$$

$$\frac{1}{2}mv_b^2 - \frac{1}{2}mv_t^2 + (0 - 2mgR) = 0$$

$$m\frac{(v_b^2 - v_t^2)}{R} = 4mg \qquad (4)$$

Substituting (4) into (3) gives

$$\boldsymbol{T_b = T_t + 6mg.}$$

Example 5.8

A 20kg block is connected to a 30kg block by a light string that passes over a frictionless pulley. The 30kg block is connected to a light spring of force constant 250N/m, as in shown in the Figure. The spring is unstretched when the system is as shown in the figure, and the incline is smooth. The 20kg block is pulled a distance of 20cm down the incline (so that the 30kg block is 40 cm above the floor) and is released from rest. Find the speed of each block when the 30kg block is 20cm above the floor (that is, when the spring is unstretched).

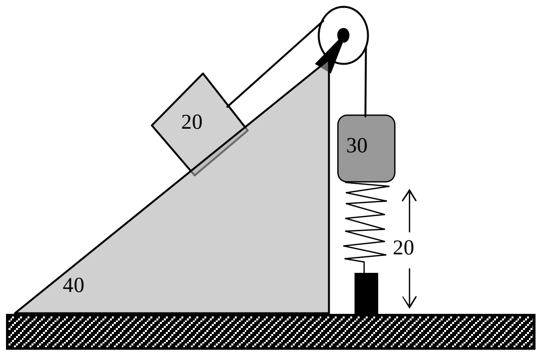


Figure 5.7

Solution

لحل هذا السؤال نفرض أن x هي المسافة التي استطال بها الزنبرك نتيجة لسحب الكتلة 20kg مسافة محددة وبالتالي فإن x=0.2m. وكذلك نفرض أن طاقة الوضع U_g=0 مقاسه عند أدنى قيمة للكتلة 20kg قبل تركها. فإذا كانت v هي سرعة الكتلتين عند مرورهما بموضع الاتزان قبل استطالة الزنبرك.

$$\Delta K + \Delta Us + \Delta Ug = 0$$

$$(K_f - K_i) + (U_{sf} - U_{si}) + (U_{gf} - U_{gi}) = 0$$

$$\left[\frac{1}{2}(m_1 + m_2)v^2 - 0\right] + (0 - \frac{1}{2}kx^2) + (m_2 gx\sin\theta - m_1 gx) = 0$$

نعوض في المعادلة السابقة بالقيم ونحل المعادلة لحساب قيمة السرعة.

$$v = 1.24\text{m/s}$$

5.6 Non-conservative forces and the work-energy theorem

في حالة التعامل مع قوة غير محافظة مثل قوة الاحتكاك بالإضافة إلى قوى محافظة، فإننا لا نستطيع أن نستخدم القانون السابق والذي ينص على أن التغير في الطاقة الميكانيكية الكلية يساوى صفراً لأن هناك جزءً من الطاقة يضيع على شكل حرارة بواسطة الشغل المبذول نتيجة لقوة الاحتكاك. لذلك نحتاج إلى قانون أشمل وأعم ليشمل جميع أنواع القوى.

نعلم سابقا أن الشغل يساوى التغير في طاقة الحركة

$$W = \Delta K \qquad (5.6)$$

وحيث أن الشغل قد يكون مبذولاً بواسطة قوى محافظة W_c وأحياناً يكون الشغل مبذولاً بواسطة قوى غير محافظة يرمز له بالرمز W_{nc}.

$$W_{nc} + W_c = \Delta K \qquad (5.7)$$

وحيث أن الشغل بواسطة قوة محافظة W_c يساوى سالب التغير في طاقة الوضع.

$$W_{nc} + -\Delta U = \Delta K \qquad \Rightarrow \qquad W_{nc} = \Delta K + \Delta U$$

وهذا يعنى أن الشغل المبذول بواسطة قوة غير محافظة يساوى التغير طاقة الحركة بالإضافة إلى التغير في طاقة الوضع.

$$W_{nc} = (K_f+U_f) - (K_i + U_i) \qquad (5.8)$$

$$\boldsymbol{W_{nc} = E_f - E_i} \qquad (5.9)$$

وهذا يمثل القانون العام للعلاقة بين الشغل والطاقة والذي ينص على أن الشغل المبذول بواسطة قوة غير محافظة يساوى التغير الكلى في الطاقة الميكانيكية.

Example 5.9

A 3kg block slides down a rough incline 1m in length as shown in the figure. The block starts from rest at the top and experience a constant force of friction of 5N. the angle of inclination is 30°. (a) Use energy methods to determine the speed of the block when it reach the bottom of the incline.

Solution

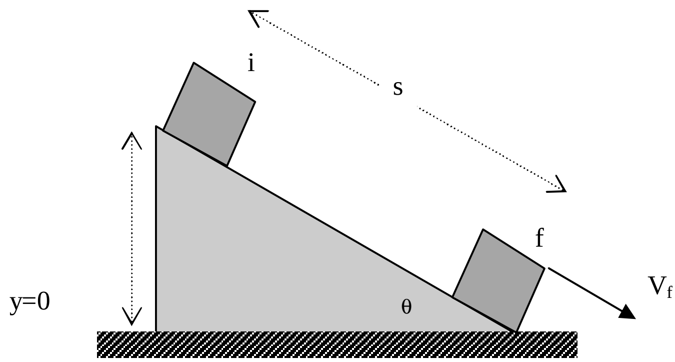


Figure 5.8

$$W_{nc} = E_f - E_i$$

$$W_{nc} = (K_f + U_f) - (K_i + U_i)$$

$$-f\,s = (1/2\ mv^2 + 0) - (0 + mgh)$$

ومن هذه المعادلة يمكن إيجاد السرعة النهائية للجسم المنزلق. كذلك لاحظ يمكن إيجاد السرعة النهائية باستخدام قانون نيوتن الثاني.

5.7 Questions with solution

1 A bowling ball is suspended from the ceiling of a lecture hall by a strong cord. The bowling ball is drawn from its equilibrium position and released from rest at the tip of the demonstrator's nose. If the demonstrator remains stationary, explain why he will not be struck by the ball on its return swing. Would the demonstrator be safe if the ball were given a push from this position?

Answer: *The total energy of the system (bowling ball) must be conserved. Since the ball initially has a potential energy mgh, and no kinetic energy, it cannot have any kinetic energy when returning to its initial position. Of course, air resistance will cause the ball to return to a point slightly below its initial position. On the other hand, if the ball is given a push, the demonstrator's nose will be in big trouble.*

2 Can the gravitational potential energy of an object ever have a negative value? Explain.

Answer: *The potential energy mgy of an object depends on the position of the reference frame. If the below the object, we call the potential energy positive (positive y value). If the object is below the origin, the potential energy is negative (negative y value). This is the convention used in the text.*

3 A ball is dropped by a person from the top of a building, while another person at the bottom observes its motion. Will these two people agree on the value of the ball's potential energy? On the change in potential energy of the ball? On the kinetic energy of the ball?

Answer: *The two will not necessarily agree on the potential energy, since this depends on the origin--which may be chosen differently for the two observers. However, the two must agree on the value of the change in potential energy, which is independent of the choice of the reference frames. The two will also agree on the kinetic energy of the ball, assuming both observers are at rest with respect to each other, and hence measure the same v.*

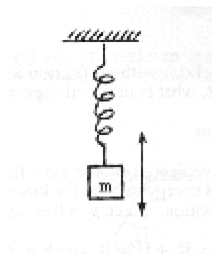
4 When nonconservative forces act on a system, does the total mechanical energy remain constant?

Answer: *No. Nonconservative forces such as friction will either decrease or increase the total mechanical energy of a system. The mechanical energy remains constant when conservative forces only act on the system, or when nonconservative forces do zero work.*

5 If three different conservative forces and one nonconservative force act on a system, how many potential energy terms will appear in the work-energy theorem?

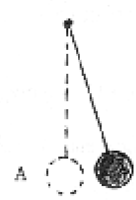
Answer: Three. One for each conservative force.

6 A block is connected to a spring that is suspended from the ceiling. If the block is set in motion and air resistance is neglected, will the total energy of the system be conserved? How many forms of potential energy are there for this situation?

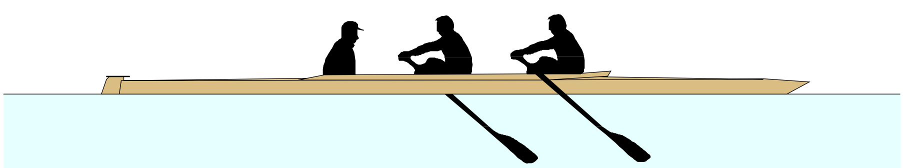


Answer*: Yes, the total mechanical energy is served since there are only conservative forces ting: gravity and the spring force. Hence, there two forms of potential energy: gravitational potential energy (mgy) and spring potential energy (½ky*^{2}*) .*

7 Consider a ball fixed to one end of a rigid rod with the other end pivoted on a horizontal axis so that the rod can rotate in a vertical plane. What are the positions of stable and unstable equilibrium?



Answer: *Only one position (A), the lowest point, stable. All other positions are ones of unstable equilibrium.*



5.8 Problems

1) A single conservative force acting on a particle varies as $\boldsymbol{F} = (-Ax + Bx^2)\boldsymbol{i}$ N, where A and B are constants and x is in m. (a) Calculate the potential energy associated with this force, taking $U = 0$ at $x = 0$. (b) Find the change in potential energy and change in kinetic energy as the particle moves from $x = 2$ m to $x = 3$ m.

2) A 4-kg particle moves along the x-axis under the influence of a single conservative force. If the work done on the particle is 80J as the particle moves from x=2m to x=5m, find (a) the change in the particle's kinetic energy, (b) the change in its potential energy, and (c) its speed at x=5m if it starts at rest at x=2m.

3) A single conservative force $F_x = (2x + 4)$ N acts on a 5-kg particle, where x is in m. As the particle moves along the x axis from $x = 1$ m to $x = 5$ m, calculate (a) the work done by this force, (b) the change in the potential energy of the particle, and (c) its kinetic energy at x = 5 m if its speed at x =1m is 3 m/s.

4) Use conservation of energy to determine the final speed of a mass of 5.0kg attached to a light cord over a massless, frictionless pulley and attached to another mass of 3.5 kg when the 5.0 kg mass has fallen (starting from rest) a distance of 2.5 m as shown in Figure 5.9

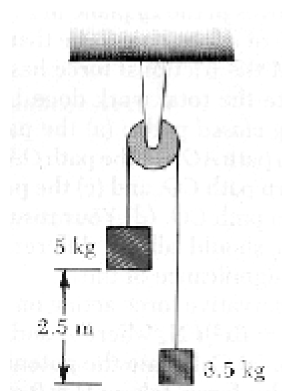


Figure 5.9

5) A 5-kg mass is attached to a light string of length 2m to form a pendulum as shown in Figure 5.10. The mass is given an initial speed of 4m/s at its lowest position. When the string makes an angle of 37° with the vertical, find (a) the change in the potential energy of the mass, (b) the speed of the mass, and (c) the tension in the string. (d) What is the maximum height reached by the mass above its lowest position?

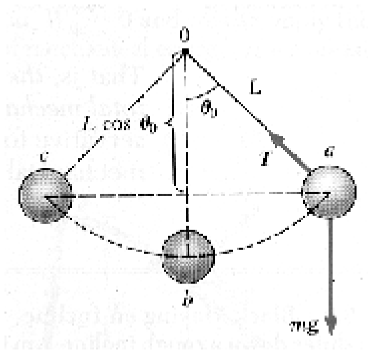


Figure 5.10

6) A 0.5-kg ball is thrown vertically upward with an initial speed of 16 m/s. Assuming its initial potential energy is zero, find its kinetic energy, potential energy, and total mechanical energy (a) at its initial position, (b) when its height is 5m, and (c) when it reaches the top of its flight. (d) Find its maximum height using the law of conservation of energy.

7) Two masses are connected by a light string passing over a light frictionless pulley as shown in Figure 5.11. The 5-kg mass is released from rest. Using the law of conservation of energy, (a) determine the velocity of the 3-kg mass just as the 5-kg mass hits the ground. (b) Find the maximum height to which the 3kg mass will rise.

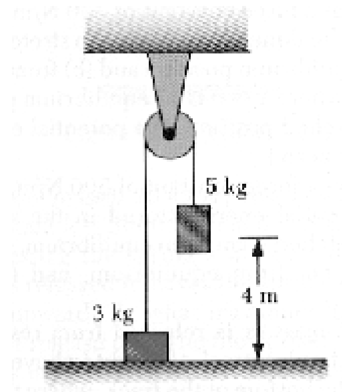


Figure 5.11

8) A 5-kg block is set into motion up an inclined plane as in Figure 5.12 with an initial speed of 8 m/s. The block comes to rest after travelling 3 m along the plane, as shown in the diagram. The plane is inclined at an angle of 30' to the horizontal. (a) Determine the change in kinetic energy. (b) Determine the change in potential energy. (c) Determine the frictional force on the block (assumed to be constant). (d) What is the coefficient of kinetic friction?

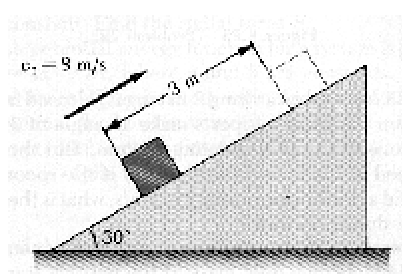


Figure 5.12

9) A block with a mass of 3 kg starts at a height h = 60 em on a plane with an inclination angle of 30', as shown in Figure 5.13. Upon reaching the bottom of the ramp, the block slides along a horizontal surface. If the coefficient of friction on both surfaces is ,U_k = 0.20, how far will the block slide on the horizontal surface before coming to rest? [Hint: Divide the path into two straight-line parts].

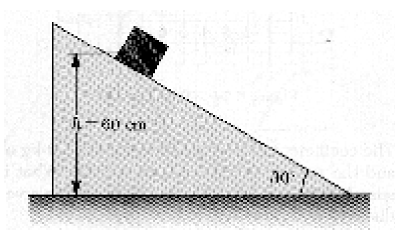


Figure 5.13

10)The coefficient of friction between the 3.0-kg object and the surface in Figure 5.14 is 0.40. What is the speed of the 5.0-kg mass when it has fallen a vertical distance of 1.5 m?

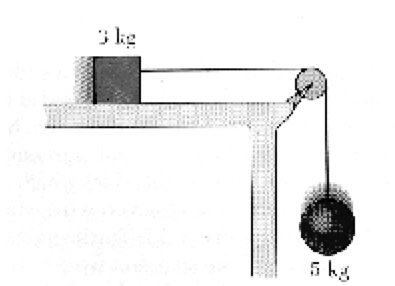


Figure 5.14

11)A mass of 2.5 kg is attached to a light spring with k = 65 N/m. The spring is stretched and allowed to oscillate freely on a frictionless horizontal surface, When the spring is stretched 1 0 cm, the kinetic energy of the attached mass and the elastic potential energy are equal. What is the maximum speed of the mass?

12)A block of mass 0.25 kg is placed on a vertical spring of constant k = 5000 N/m and is pushed downward, compressing the spring a distance of 0.1 m. As the block is released it leaves the spring and continues to travel upward. To what maximum height above the point of release does the block rise?

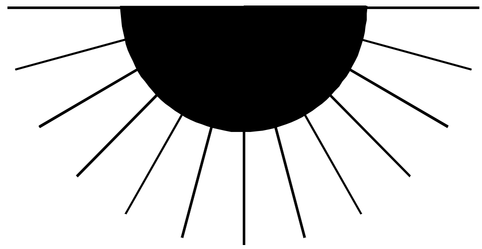
13)A block of mass 2 kg is kept at rest by compressing a horizontal massless spring having a spring constant k = 100 N/m by 10 cm. As the block is released it travels on a rough horizontal surface a distance of 0.25 m before it stops. Calculate the coefficient of kinetic friction between the horizontal surface and the block.

14)A 6-kg mass, when attached to a light vertical spring of length 10 cm, stretches the spring by 0.3 cm. The mass is now pulled downward, stretching the spring to a length of 10.7 cm, and released. Find the speed of the oscillating mass when the spring's length is 10.4cm.

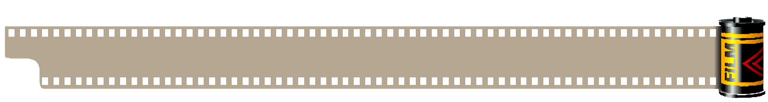
Chapter 6

The Linear Moment and Collisions

كمية الحركة والتصادم



THE LINEAR MOMENT AND COLLISIONS



عندما يتصادم جسمان فإن حركتهما يمكن أن توصف من خلال قانون الحفاظ على الطاقة والحفاظ على كمية الحركة. في هذا الباب سندرس كيف نستخدم مفهوم الطاقــة وكميــة الحركة لوصف التصادم بين الأجسام.

6.1 The Linear Moment

***The linear moment (p)** of a particle is defined as the mass of the particle multiplied by its velocity.*

$$\vec{p} = m\vec{v} \qquad (6.1)$$

The linear moment is a vector quantity and has a unit of kg.m/s.

تدعى كمية الحركة الخطية (***Linear moment***) في بعض الأحيان باسم العزم الخطــي ويرتبط بمفهوم القوة المؤثرة على الجسم من خلال قانون نيوتن الثاني، حيث تعرف القوة بأنها معدل التغير في كمية الحركة الخطية للجسم. فإذا كانت القوة المؤثرة تساوي صفراً فإن كمية الحركة الخطية تكون ثابتة.

From Newton's second law of motion we have

$$\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} \qquad (6.2)$$

$$\therefore \vec{F} = \frac{d\vec{p}}{dt} \qquad (6.3)$$

or we can write the equation as

$$d\vec{p} = \vec{F}dt \qquad (6.4)$$

أي أن التغير في كمية الحركة الخطية هو القوة في الفترة الزمنية لتأثير القــوة. لإيجــاد التغير في كمية الحركة الخطية لجسم من حالة ابتدائية p_i عند زمن t_i إلى حالة نهائية p_f عند زمن t_f.

$$\Delta\vec{p} = \vec{p}_f - \vec{p}_i = \int_{t_i}^{t_f} \vec{F}dt \qquad (6.5)$$

الطرف الأيمن للمعادلة يعبر عن كمية فيزيائية جديدة تدعى الصــدمة ***Impulse*** والتــي تعرف بالقوة المؤثرة خلال فترة زمنية قصيرة.

***Impulse** ($\vec{I}$) is a vector quantity defined as the force acting in short time and its equal to the change in momentum of the particle.* The impulse has a unit of N.s.

$$\vec{I} = \Delta\vec{p} = \int_{t_i}^{t_f} \vec{F}dt \qquad (6.6)$$

The impulse equation is equivalent to the Newton's second law of motion.

Example 6.1

A car travelling at speed of 300m/s strikes a stone of mass 0.5kg and 20cm in size. Estimate the force exerted by the stone on the car.

Solution

كمية الحركة للحجر تساوي صفر لأنه كان ثابت قبل اصطدام السيارة به. بعد التصـــادم يتحرك الحجر بسرعة السيارة وبالتالي يمكن حساب التغير في كمية الحركة من العلاقـــة التالية:

$$\vec{I} = \Delta\vec{p} = \int_{t_i}^{t_f} \vec{F}dt$$

therefore,

$$\int_{t_i}^{t_f} \vec{F}dt = \Delta\vec{p} = \Delta(m\vec{v})$$

$$= 0.5\times300 - 0 = 150\text{N.s}$$

To find the force we should estimate the time

$$t = \frac{x}{v} = \frac{0.2}{300} = 7\times10^{-4}\,s$$

زمن التصـادم يحسب من الزمن المستغرق للسـيارة لقطـع مسـافة الحجــر والتـــي هـــي 20سنتمتر.

The force F is equal to 2×10^{5}N

Example 6.2

A ball of mass 0.4kg is thrown against a brick wall. When it strikes the wall it is moving horizontally to the left at 30m/s, and it rebounds horizontally to the right at 20m/s. Find the impulse of the force exerted on the wall.

Solution

The impulse of the force exerted on the wall is equal to the change in momentum,

$$\vec{I} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i$$

therefore, the initial momentum p_i of the ball

$$p_i = mv = 0.4\times(-30) = -12 \text{ kg.m/s}$$

therefore, the final momentum p_f of the ball

$$p_f = mv = 0.4\times(20) = 8 \text{ kg.m/s}$$

the change in momentum is

$$\Delta\vec{p} = \vec{p}_f - \vec{p}_i = 8-(-12) = 20 \text{ kg.m/s}$$

Hence, the impulse of the force exerted on the ball is 20 N.s. Since the impulse is positive, the force must be toward the right.

Example 6.3

A ball of mass 0.1kg is dropped from height *h*=2m above the floor as shown in Figure 6.1. It rebound vertically to height h'=1.5m after colliding with the floor. (a) Find the momentum of the ball before and after the ball

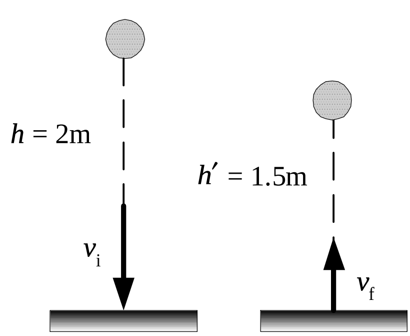


Figure 6.1

colliding with the floor. (b) Determine the average force exerted by the floor on the ball. Assume the collision time is 10^{-2}s.

Solution

(a) From the energy conservation, we can find the velocity of the ball before and after the collision,

$$\frac{1}{2}mv_i^{\,2} = mgh$$

$$\frac{1}{2}mv_f^{\,2} = mgh'$$

hence,

$$v_i = \sqrt{2gh} = \sqrt{2\times 9.8\times 2} = 6.26 m/s$$

$$v_f = \sqrt{2gh'} = \sqrt{2\times 9.8\times 1.5} = 5.42 m/s$$

The initial and final momentum is,

$$\vec{p}_i = m\vec{v}_i = -0.626\, j\, kgm/s$$

$$\vec{p}_f = m\vec{v}_f = 0.542\, j\, kgm/s$$

(b) The force exerted by the floor on the ball is,

$$\Delta\vec{p} = \vec{p}_f - \vec{p}_i = \vec{F}\Delta t$$

$$\vec{F} = \frac{[0.542\, j - (-0.626\, j)]}{10^{-2}} = 1.17\times 10^{2}\, jN$$

نلاحظ أن قوة التصادم أكبر بكثير من قوة عجلة الجاذبية الأرضية، وعدم رجوع الكـــرة إلى نفس الارتفاع يعود إلى الطاقة المفقودة نتيجة الحرارة الناتجة من التصادم بين الكـــرة والأرض.

6.2 Conservation of linear momentum

عندما يتصادم جسمان مع بعضهما البعض فإن كل جسم سيغير كمية حركة الجسم الآخر لأن كل جسم سيؤثر بقوة على الجسم الآخر. وطبقاً لقانون نيوتن الثالث فإن القوتين متساويتان في المقدار ومتعاكستان في الاتجاه.

وهذا يؤدي إلى أن قوة الصدم ***Impulse*** خلال فترة التصادم متساويتان في المقدار متعاكستان في الاتجاه، وبالتالي فإن التغير في كمية الحركة الكلي للجسمين يبقى ثابتاً، وهذا ما يعرف بقانون الحفاظ على كمية الحركة ***Conservation of linear momentum***.

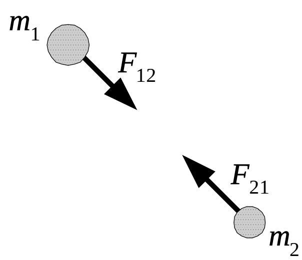


Figure 6.2

Suppose that at time t, two particles collide with each other, the momentum of particle 1 is p_1 and the momentum of particle 2 is p_2. In collision the particle exerts a force on each other as follow,

$$\vec{F}_{12} = \frac{d\vec{p}_1}{dt} \qquad \& \qquad \vec{F}_{21} = \frac{d\vec{p}_2}{dt}$$

where $\vec{F}_{12}$ is the force on particle 1 due to particle 2, and $\vec{F}_{21}$ is the force on particle 2 due to particle 1. From Newton's third law of motion, then,

$$\vec{F}_{12} = -\vec{F}_{21} \qquad (6.7)$$

hence,

$$\vec{F}_{12} + \vec{F}_{21} = 0 \qquad (6.8)$$

therefore,

$$\frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt} = \frac{d}{dt}(\vec{p}_1 + \vec{p}_2) = 0 \qquad (6.9)$$

Since the time derivative of the momentum is zero, therefore the total momentum ($\vec{P}$) remains constant, *i.e.*

$$\vec{P} = \vec{p}_1 + \vec{p}_2 = const. \qquad [\textit{Conservation of momentum}] \qquad (6.10)$$

If the initial velocity of the particles 1 and 2 is v_{1i} and v_{2i} and the final velocity of the particles 1 and 2 is v_{1f} and v_{2f} we get,

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f} \qquad (6.11)$$

or

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f} \qquad (6.12)$$

This equation represents the law of conservation of momentum.

Example 6.4

A cannon of mass 5000kg rest on frictionless surface as shown in figure 6.3. The cannon fired horizontally a 50kg cannonball. If the cannon recoil to the right with velocity 2m/s, what is the velocity of the cannonball just after it leaves the cannon?

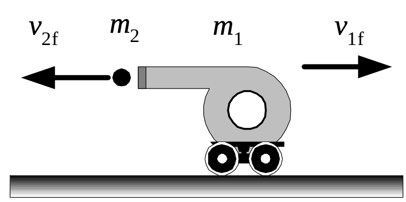


Figure 6.3

Solution

Using the conservation law of momentum

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$$

since the total momentum before firing is zero, therefore the total momentum after firing is zero as well.

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = 0 \quad \& \qquad m_1\vec{v}_{1f} + m_2\vec{v}_{2f} = 0$$

the velocity of the cannonball just after it leaves the cannon is

$$v_{2f} = \frac{-m_1 v_{1f}}{m_2} = \frac{-5000}{50} \times 2 = -200 \text{ m/s}$$

الإشارة السالبة تشير إلى أن السرعة النهائية للقذيفة تتحرك إلى اليسار عكس ارتداد الدبابة.

6.3 Collisions

التصادم بين جسمين ***collisions*** يعتبر من التطبيقات الهامة على قانون الحفاظ على كمية الحركة حيث يؤثر كل جسم على الآخر بقوة صدم ***impulse*** لفترة زمنية قصيرة وقد يكون التصادم ناتجاً عن تلامس الجسمين مع بعضهما البعض مثل التصادم الناتج عن كرات البلياردو أو تصادم كرة التنس مع المضرب أو أن يكون التصادم عن بعد مثل تصادم الأجسام المشحونة كتصادم بروتون مع أيون موجب.

يمكن تقسيم التصادمات بين الأجسام إلى ثلاثة أنواع معتمدين على مبدأ الحفاظ على الطاقة وكمية الحركة وأنواع التصادمات هي موضحة في الشكل 6.4.

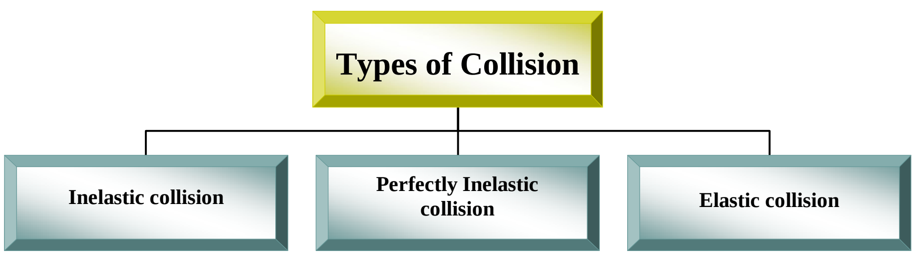


Figure 6.4

Inelastic collision: *is one in which the momentum is conserved, but the kinetic energy is not conserved.*

Perfectly Inelastic collision: *when the two objects stick together after collision and they move with the same velocity.*

Elastic collision: *is one in which the momentum and the kinetic energy are conserved.*

When two particles collide as shown in Figure 6.5 the impulse force $\vec{F}_{12}$ will change the momentum of particle 1 and the impulse force $\vec{F}_{21}$ will change the momentum of particle

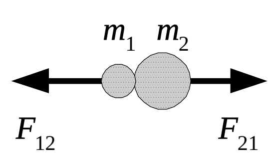


Figure 6.5

2, therefore,

$$\Delta \vec{p}_1 = \int_{t_i}^{t_f} \vec{F}_{12}\, dt \qquad \textit{the change momentum of } m_1$$

$$\Delta \vec{p}_2 = \int_{t_i}^{t_f} \vec{F}_{21}\, dt \qquad \textit{the change momentum of } m_2$$

By applying Newton's third law of motion we get,

$$\vec{F}_{12} = -\vec{F}_{21}$$

hence

$$\Delta \vec{p}_1 + \Delta \vec{p}_2 = 0 \qquad (6.13)$$

Therefore, the total momentum $\vec{P}$ is constant

السؤال الآن ماذا عن السرعة النهائية لجسم بعد التصادم؟ وللإجابة على هـذا التسـاؤل سندرس نوعين من أنواع التصادمات التي تكون فيها كميـة الحركـة محفوظـة وهمـا التصادمات غير المرنة كلياً والتصادمات المرنة.

6.3.1 Perfectly Inelastic collisions

في هذا النوع من التصادمات يكون لكل جسم سرعة ابتدائيـة وبعـد التصـادم يتحـرك الجسمان بسرعة نهائية واحدة لكليهما. وفي هذه الحالة تكون كمية الحركة محفوظـة أي أن كمية الحركة قبل التصادم تساوي كمية الحركة بعد التصادم. ولكن طاقة الحركة غير محفوظة لهذا نطبق قانون الحفاظ على كمية الحركة فقط في هذه الحالة.

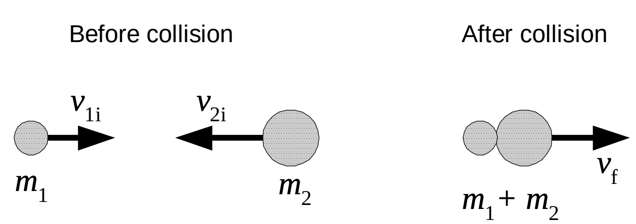


Figure 6.6

$$\vec{p}_{1i} + \vec{p}_{2i} \qquad = \qquad \vec{p}_{(1+2)f}$$

Before collision ***After collision***

By applying the law of conservation of momentum, therefore,

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = (m_1 + m_2)\vec{v}_f \qquad (6.14)$$

Hence, the final velocity of the two colliding particles is,

$$\vec{v}_f = \frac{m_1\vec{v}_{1i} + m_2\vec{v}_{2i}}{m_1 + m_2} \qquad (6.15)$$

Example 6.5

A car of mass 1000kg moving with velocity of 20cm/s hit another car of mass 2000kg at rest. The two cars moves with the same velocity due to collision. (a) What is the velocity of the two cars after the collision? (b) How much kinetic energy is lost in the collision?

Solution

(a) Since the two cars moves as one object therefore, the collision is inelastic.

Before collision the momentum is

$$p_i = m_1 v_{1i} = 1000 \times 20 = 2 \times 10^4 \text{ kg.m/s}$$

$$p_f = (m_1 + m_2)v_f = (1000 + 2000)v_f = 3000 v_f$$

The momentum before collision = the momentum after collision

therefore the final velocity is,

$$v_f = \frac{p_i}{m_1 + m_2} = \frac{2 \times 10^4}{3000} = 6.66 \text{ m/s.}$$

(b) The kinetic energy before collision (K_i) = the kinetic energy after collision (K_f)

$$K_i = K_f$$

$$K_i = \frac{1}{2} m_1 v_{1i}^{\ 2} + \frac{1}{2} m_2 v_{2i}^{\ 2} = \frac{1}{2} \times 1000 \times (20)^2 + 0 = 2 \times 10^5 \,\text{J}$$

$$K_f = \frac{1}{2}(m_1 + m_2) v_f^{\ 2} = \frac{1}{2} \times (1000 + 2000) \times (6.66)^2 + 0 = 0.66 \times 10^5 \,\text{J}$$

Hence, the kinetic energy is lost in the collision is

$$K_i - K_f = 1.34 \times 10^5 \,\text{J}$$

Example 6.6

A bullet of mass *m* is fired to a large wood block suspended by string. The bullet is stopped by the block of mass *M*, and the entire system swing through a height *h*. Find the initial velocity of the bullet before collision. Assume *m*=10g, *M*=2kg, and *h*=12cm ?

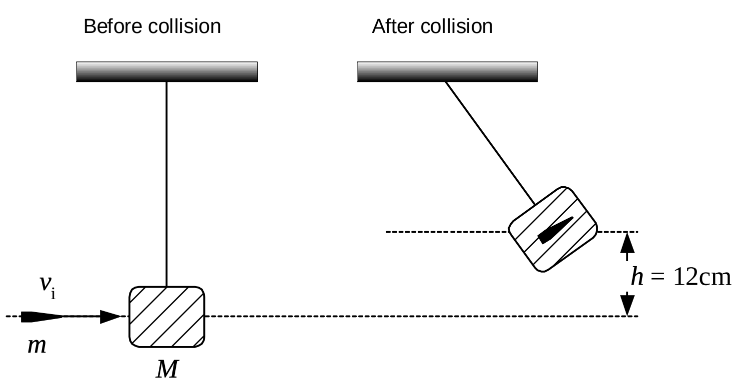


Figure 6.7

Solution

يستخدم هذا المثال كتجربة لإيجاد سرعة الأجسام المتحركة بسرعات عالية مثل سرعة انطلاق رصاصة. وسوف نقوم في الخطوة الأولى بتحديد السرعة النهائية من قانون الحفاظ على الطاقة الميكانيكية وفي الخطوة الثانية سنقوم بحساب السرعة الابتدائية للرصاصة قبل التصادم باستخدام قانون الحفاظ على الكتلة.

Since the collision is inelastic and the total momentum is conserved, then, from equation (6.15).

$$v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}$$ where $v_{2i} = 0$, $m_1 = m$ and $m_2 = M$

$$v_f = \frac{m v_{1i}}{m + M}$$

The kinetic energy after the collision is given by

$$K_f = \frac{1}{2}(m + M){v_f}^2$$

Substitute for v_f we get

$$K_f = \frac{m^2 {v_i}^2}{2(m + M)}$$ where v_i is the initial velocity of the bullet

This kinetic energy is transformed to potential energy *i.e.*

$$K_f = (m+M)\,g\,h$$

or

$$\frac{m^2 {v_i}^2}{2(m + M)} = (m + M)gh$$

$$v_i = \left(\frac{m + M}{m}\right)\sqrt{2gh}$$

ومن هذه المعادلة يمكن إيجاد السرعة الابتدائية للرصاصة من قياس ارتفاع المجموعة h بعد التصادم.

for m=10g, M=2kg, and h=12cm the velocity of the bullet is

$$v_i = 301.5 \text{ m/s}$$

6.3.2 Elastic collisions

في هذا النوع من التصادمات يكون لكل جسم سرعة ابتدائية وبعد التصادم يصبح لكل جسم سرعة نهائية. وفي هذه الحالة تكون كل من كمية الحركة وطاقة الحركة محفوظة أي أن كمية الحركة قبل التصادم تساوي كمية الحركة بعد التصادم وكذلك طاقة الحركة

قبل التصادم تساوي طاقة الحركة بعد التصادم. وهذان القانونان يؤديان إلـــى معـــادلتين لإيجاد مجهولين هما السرعة النهائية للجسمين المتصادمين.

For two particles m_1 and m_2 moving with initial velocities v_{1i} and v_{2i} undergo elastic collision. Find the final velocities v_{1f} and v_{2f} of the two particles after collision.

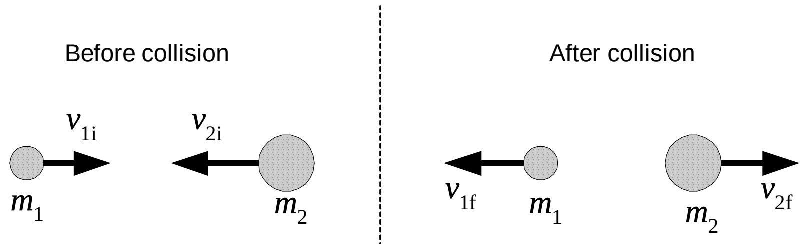


Figure 6.8

Before collision		***After collision***
$\vec{p}_{1i} + \vec{p}_{2i}$	=	$\vec{p}_{1f} + \vec{p}_{2f}$
$K_{1i} + K_{2i}$	=	$K_{1f} + K_{2f}$

To find the final velocities v_{1f} and v_{2f} we need two equations, since the collision is elastic then, the first equation is found using the law of conservation of momentum and the second equation is found the law of conservation of kinetic energy.

By applying the law of conservation of momentum, therefore,

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f} \qquad (6.16)$$

By applying the law of conservation of kinetic energy, therefore,

$$\frac{1}{2}m_1\vec{v}_{1i}^{\,2} + \frac{1}{2}m_2\vec{v}_{2i}^{\,2} = \frac{1}{2}m_1\vec{v}_{1f}^{\,2} + \frac{1}{2}m_2\vec{v}_{2f}^{\,2} \qquad (6.17)$$

حيث أن السرعة كمية متجهة فإننا نعتبر اتجاه السرعة موجباً إذا كان الجسم متحركاً إلـــى اليمين وتكون السرعة سالبة إذا كان الجسم متحركاً إلى اليسار.

rearranging equation (6.17) we get,

$$m_1(\vec{v}_{1i}^{\,2} - \vec{v}_{1f}^{\,2}) = m_2(\vec{v}_{2f}^{\,2} - \vec{v}_{2i}^{\,2}) \qquad (6.18)$$

If we take the terms of m_1 on one side and the terms of m_2 on the other side we get,

$$m_1(v_{1i}-v_{1f})(v_{1i}+v_{1f})=m_2(v_{2f}-v_{2i})(v_{2f}+v_{2i}) \quad (6.19)$$

rearranging equation (6.16) we get,

$$m_1(v_{1i}-v_{1f})=m_2(v_{2f}-v_{2i}) \quad (6.20)$$

by dividing the equations (6.16) & (6.19) to eliminate m_1 & m_2 we get

$$v_{1i}+v_{1f}=v_{2f}+v_{2i} \quad (6.21)$$

or

$$v_{1i}-v_{2i}=-(v_{1f}-v_{2f}) \quad (6.22)$$

المعادلة (6.22) تشير إلى أن فرق السرعة الابتدائية للجسمين تساوي سالب فرق السرعة النهائية للجسمين في حالة التصادم المرن.

يمكن استخدام المعادلة الأخيرة مع معادلة الحفاظ على كمية الحركة لإيجاد السرعات النهائية للجسمين المتصادمين تصادماً مرناً وتكون نتيجة حل المعادلتين جبرياً كما يلي:

$$v_{1f}=\left(\frac{m_1-m_2}{m_1+m_2}\right)v_{1i}+\left(\frac{2m_2}{m_1+m_2}\right)v_{2i} \quad (6.23)$$

$$v_{2f}=\left(\frac{2m_1}{m_1+m_2}\right)v_{1i}+\left(\frac{m_2-m_1}{m_1+m_2}\right)v_{2i} \quad (6.24)$$

عند التعويض في المعادلتين السابقتين يجب أن نأخذ في الحسبان إشارة اتجاه السرعة حيث تكون موجبة إذا كان الجسم متحركاً إلى اليمين وتكون السرعة سالبة إذا كان الجسم متحركاً إلى اليسار.

6.3.3 Special cases

Case 1: When $m_1 = m_2$, then $v_{1f} = v_{2i}$ and $v_{2f} = v_{1i}$

وهذا يعني أن الجسمين المتصادمين يتبادلان السرعة مع بعضهما البعض نتيجة للتصادم المرن كما يحدث في تصادم كرات البلياردو.

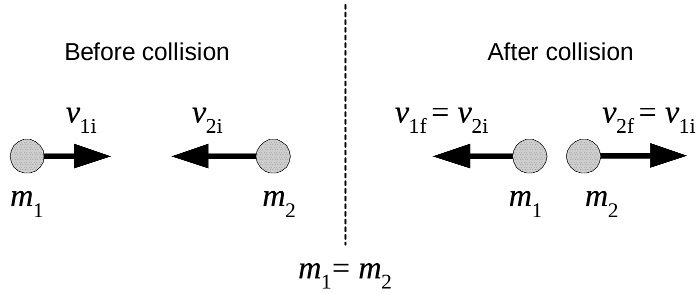


Figure 6.9

Case 2: when mass m_2 is initially at rest *i.e.* $v_{2i} = 0$, the final velocity is given by,

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{1i} \qquad (6.25)$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right)v_{1i} \qquad (6.26)$$

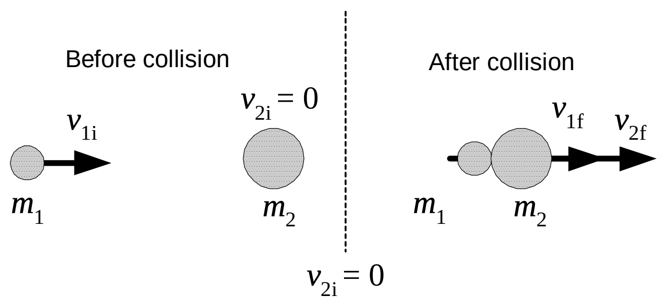


Figure 6.10

Case 3: when m_1 is very large compare with m_2, then $v_{1f} \approx v_{1i}$ and $v_{2f} \approx 2v_{1i}$.

وهذا يعني أنه في حالة صدم جسم ثقيل لجسم خفيف ساكن تصادماً مرناً فإن الجسم الثقيل سيتحرك بنفس سرعته قبل التصادم بينما الجسم الخفيف فسيتحرك بسرعة ضعف سـرعة الجسم الثقيل بعد التصادم.

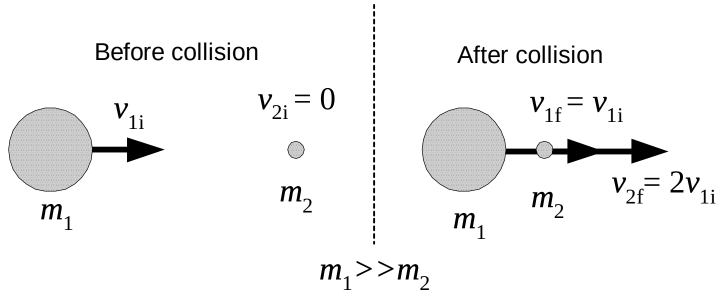


Figure 6.11

Example 6.7

A block of mass m_1=2kg moving to the right with a speed of 5m/s on frictionless horizontal track collides with a spring attached to a second block of mass m_2=3kg moving to the left with speed of 4m/s, as shown in figure 6.12. The spring has a spring constant of 500N/m. at the instant when the speed of 3m/s, determine the velocity of m_2 and (b) the distance x that the spring is compressed.

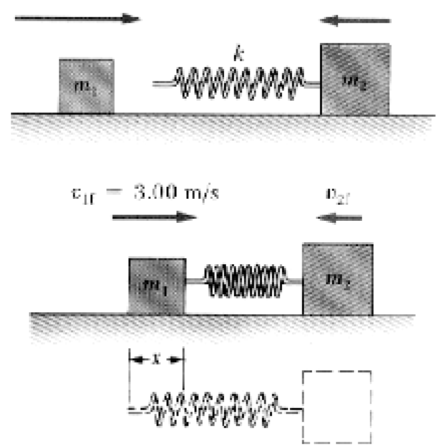


Figure 6.12

Solution

(a) Since the collision is elastic, the momentum and kinetic energy is conserved, we have

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

v_{2i} is negative because the direction of the velocity is to the left

$$2 \times 5 + 3 \times (-4) = 2 \times 3 + (3) v_{2f}$$

$$\therefore v_{2f} = -2.66\,\text{m/s}$$

The negative sign indicates that the final velocity of m_2 still moving towards the left.

(b) the distance x that the spring is compressed could be found using the law of conservation of kinetic energy.

$$\frac{1}{2}m_1 v_{1i}^{\;2} + \frac{1}{2}m_2 v_{2i}^{\;2} = \frac{1}{2}m_1 v_{1f}^{\;2} + \frac{1}{2}m_2 v_{2f}^{\;2} + \frac{1}{2}kx^2$$

The term ½ kx^2 is the energy of the spring after collision, if we substitute for the values we can get the value of x

$x = 0.34$ m

Example 6.8

A 12 g bullet is fired into 100g wooden block initially at rest on a horizontal surface. After impact, the block slides 7.5m before coming to rest. If the coefficient of friction between the block and the surface is 0.65, what was the speed of the bullet immediately before impact?

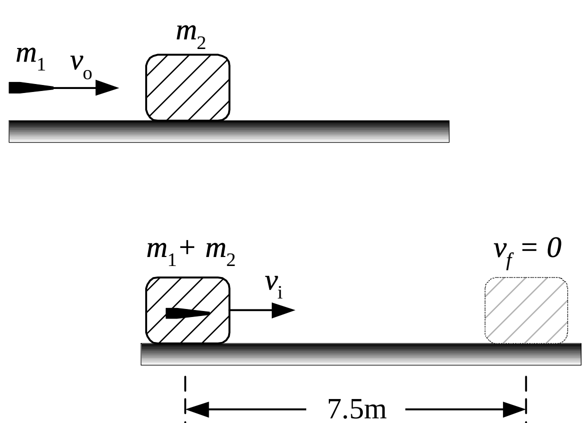


Figure 6.13

Solution

لنفترض أن السرعة التي انطلقت بها الرصاصة هي v_o وسرعة المجموعة بعد التصـــادم مباشرة هي v_i والسرعة النهائية للمجموعة هـــي صـــفر $v_f = 0$ لأنهـــا توقفـــت نتيجـــة الاحتكاك.

Since the collision is totally inelastic, and the momentum is conserved,

$$m_1 v_o = (m_1 + m_2) v_i$$

hence,

$$v_i = \left(\frac{12}{12+100} \right) v_o = (0.107) v_o \qquad (\text{ß})$$

The initial kinetic energy is lost due to the work done by the force of friction. Since,

$$W_f = \Delta K = -\frac{1}{2}(m_1 + m_2) v_i^{\,2}$$

and

$$W_f = -fs = -\mathrm{m} mgs$$

hence,

$$\frac{1}{2}(m_1 + m_2) v_i^{\,2} = \mathrm{m}(m_1 + m_2) gs$$

or

$$v_i^{\,2} = 2\mathrm{m} gs$$

substitute for v_i from equation (ß) we get,

$$(0.107)^2 v_o^{\,2} = 2 \times 0.65 \times 9.8 \times 7.5$$

$$v_o = 91.2\text{m/s}$$

Example 6.9

Consider a frictionless track ABC as shown in Figure 6.14. A block of mass m_1=5kg is released from A. It takes a head-on elastic collision with a block of mass m_2=10kg at b, initially at rest. Calculate the maximum height to which m1 will rise after the collision.

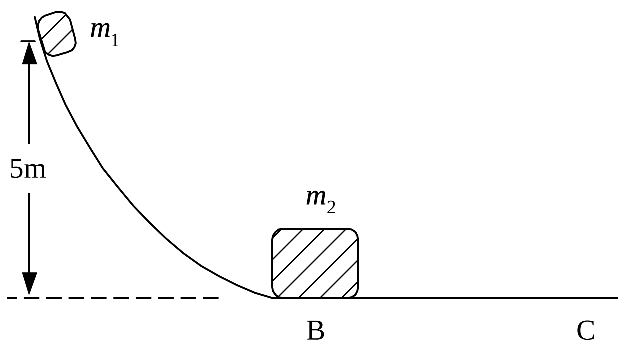


Figure 6.14

Solution

لنبدأ أولاً بإيجاد سرعة الكتلة m_1 عند النقطة B قبل التصادم مباشرة باستخدام قانون الحفاظ على الطاقة الميكانيكية، علماً بأن $v_A = 0$، نحصل على:

$$K_A + U_A = K_B + U_B$$

$$0 + m_1 gh = \frac{1}{2} m_1 v_B{}^2 + 0$$

$$v_B = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 5} = 9.9 \text{ m/s}$$

حيث أن الكتلة m_2 ثابتة قبل التصادم لذا نستخدم المعادلتين (6.25) (6.26) (الحالة الخاصة 2)

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) v_{1i} = \left(\frac{5 - 10}{5 + 10} \right) \times 9.9 = -3.3 \text{ m/s}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right)v_{1i} = \left(\frac{2\times5}{5+10}\right)\times9.9 = 6.6\,\text{m/s}$$

الإشارة السالبة للسرعة النهائية للكتلة m_1 تعني أن الكتلة m_1 ستتحرك إلى اليسار بينمـــا الكتلة m_2 ستتحرك إلى اليمين. ولإيجاد أقصى ارتفاع يمكن أن تصل إليه الكتلة m_1 بعد التصادم نستخدم قانون الحفاظ على الطاقة الميكانيكية مرة أخرى.

$$m_1 g h' = \frac{1}{2} m_1 v_{1f}^{\,2}$$

$$h' = \frac{v_{1f}^{\,2}}{2g} = \frac{(-3.3)^2}{2\times9.8} = 0.556\,\text{m}$$

Example 6.10

A block of mass m slides down a smooth, curved track and collides head-on with identical block at the bottom of the track as shown in Figure 6.15. (a) If the collision is assumed to be perfectly elastic, find the speed of the each block after the collision, and the speed of block B when it reaches point C. (b) If the collision is perfectly inelastic (the two blocks stick together), find the speed of the blocks right after the collision and the maximum distance they move above point B.

Solution

(a) the collision is perfectly elastic

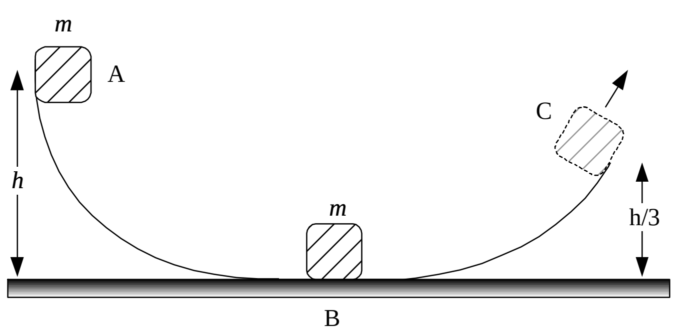


Figure 6.15

لنبدأ أولاً بإيجاد سرعة الكتلة عند النقطة A قبل التصادم مباشرة باستخدام قانون الحفــاظ على الطاقة الميكانيكية، علماً بطاقة الوضع عند النقطة A تتحول إلى طاقة حركــة عنــد النقطة B.

$$K_A + U_A = K_B + U_B$$

$$0 + mgh = \frac{1}{2}mv^2 + 0$$

$$v = \sqrt{2gh}$$

حيث أن الكتلتين متساويتان فإنه عند التصادم ستتبادل الكتلتان السرعات كما هو الحال في الحالة الخاصة 1.

When $m_1 = m_2$, then $v_{1i} = v$ and $v_{2i} = 0$, therefore, the velocity of block B after collision is $\sqrt{2gh}$, while block A comes to rest.

لإيجاد سرعة الكتلة B عند النقطة C نستخدم قانون الحفاظ على الطاقة الميكانيكية، علمــاً بأن طاقة الحركة عند النقطة B تتحول إلى طاقة حركة وطاقة وضع عند النقطة C.

$$\frac{1}{2}mv^2 + 0 = \frac{1}{2}mv_c^{\ 2} + mg\frac{h}{3}$$

$$\because v = \sqrt{2gh}$$

$$\frac{1}{2}m(2gh) = \frac{1}{2}mv_c^{\ 2} + mg\frac{h}{3}$$

$$v_c^{\ 2} = 2gh - \frac{2}{3}gh = \frac{4}{3}gh$$

$$v_c = 2\sqrt{\frac{gh}{3}}$$

(b) The collision is perfectly inelastic

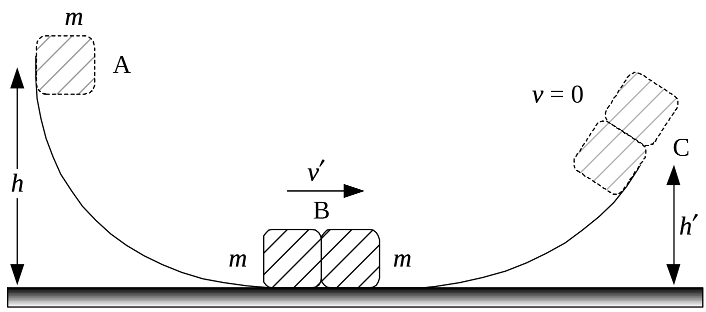


Figure 6.16

في هذه الحالة تكون الطاقة غير محفوظة نتيجة للتصادم غير المرن ولكن كمية الحركة محفوظة بعد التصادم مباشرة لذا فإن.

$$mv = (m+m)v'$$

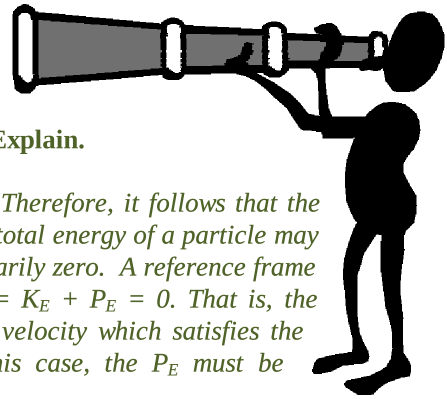
$$v' = \frac{1}{2}v = \frac{1}{2}\sqrt{2gh}$$

لإيجاد أقصى ارتفاع h' يمكن أن تصل إليه الكتلتان المتلاصقتان بعد التصادم نستخدم قانون الحفاظ على الطاقة الميكانيكية مرة أخرى. علماً بأن الطاقة الحركية عند النقطة B تساوي طاقة الوضع عند ارتفاع h'.

$$0 + (m+m)gh' = \frac{1}{2}(m+m)v'^2 + 0$$

$$h' = \frac{1}{2g}v'^2 = \frac{1}{2g}\left(\frac{1}{2}\sqrt{2gh}\right)^2 = \frac{1}{4}h$$

6.4 Questions with solutions



1. If the kinetic energy of a particle is zero, what is its linear momentum? If the total energy of a particle is zero, is its linear momentum necessarily zero? Explain.

Answer: *If $K_E = ½ mv^2 = 0$, then $v = 0$. Therefore, it follows that the linear momentum = mv = 0. Although the total energy of a particle may be zero, its linear momentum is not necessarily zero. A reference frame can be chosen such as the total energy = $K_E + P_E = 0$. That is, the particle has kinetic energy, and hence a velocity which satisfies the condition that $½ mv^2 + P_E = 0$. (In this case, the P_E must be negative.)*

2. If the velocity of a particle is doubled, by what factor is its momentum changed? What happens to its kinetic energy?

Answer: *Since p = mv, doubling v will double the momentum. On the other hand, since $KE = ½ mv^2$, doubling v would quadruple the kinetic energy.*

3. Does a large force always produce a larger impulse on a body than a smaller force? Explain.

Answer: *No, not necessarily. The impulse of a force depends on the (average) force and the time over which the force acts. The statement is only true if the times over which the forces, act are equal.*

4. In a perfectly elastic collision between two particles, does the kinetic energy of each particle change as a result of the collision?

Answer: *No, not necessary. The kinetic energies after the collision depend on the masses of the particles and their initial velocities. If the particles have equal mass, they exchange velocities.*

5. Is it possible to have a collision in which all of the kinetic energy is lost? If so, cite an example.

Yes. If two equal masses moving in opposite directions with equal speeds collide in elastically (they stick together), they are at rest after the collision. For example, two carts on a frictionless surface can be made to stick together with sticky paper.

6.5 Problems

1. A 3kg particle has a velocity of (3*i*-4*j*) m/s. Find its x and y component of momentum and the magnitude of its total momentum.

2. The momentum of a 1250kg car is equal to the momentum of 5000kg truck traveling at a speed of 10m/s. What is the speed of the car?

3. A 1500kg automobile travels eastward at speed of 8m/s. It makes a 90^o turn to the north in a time of 3s and continues with the same speed. Find (a) the impulse delivered to the car as a result of the turn and (b) the average force exerted on the car during the turn.

4. A 0.3kg ball moving along a straight line has a velocity of 5*i*m/s. It collides with the wall and rebounds with a velocity of -4*i*m/s. Find (a) the change in its momentum and (b) the average force exerted on the wall if the ball is in contact with the wall for 5×10^{-3}s.

5. If the momentum of an object is doubled in magnitude, what happens to its kinetic energy? (b) If the kinetic energy of an object is tripled, what happens to its momentum?

6. A 0.5kg football is thrown with a speed of 15m/s. A stationary receiver catches the ball and brings it to rest in 0.02s. (a) What is the impulse delivered to the ball? (b) What is the average force exerted on the receiver?

7. A single constant force of 60N accelerates a 5kg object from a speed of 2m/s to a speed of 8m/s. Find (a) the impulse acting on the object in this interval and (b) the time interval over which this impulse is delivered.

8. A 3kg steel ball strikes a massive wall with speed of 10m/s at an angle of 60^o with the surface. It bounces off with the same speed and angle (see Figure 6.17). If the ball is in contact with the wall for 0.2s, what is the average force exerted on the ball by the wall?

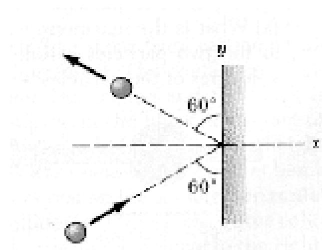


Figure 6.17

9. A 40kg child standing on frozen pond throws a 2kg stone to the east with a speed of 8m/s. Neglecting friction between the child and ice, find the recoil velocity of the child.

10. A 60kg boy and 40kg girl, both wearing skates, face each other at rest. The boy pushes the girl, sending her eastward with a speed of 4m/s. Describe the subsequent motion of the boy. (Neglect friction.)

11. A 2.5kg mass moving initially with a speed of 10m/s makes a perfectly inelastic head-on collision with a 5kg mass initially at rest. (a) Find the final velocity the composite particle. (b) How much energy is lost in the collision?

12. A 10g bullet is fired into a 2.5kg ballistic pendulum and becomes embedded in it. If the pendulum raises a vertical distance of 8cm, calculate the initial speed of the bullet.

13. A 1200kg car traveling initially with a speed of 25m/s in an easterly direction crashes into the rear end of a 9000kg truck moving in the same direction at 20m/s (Figure 6.18). The velocity of the car right after the collision is 18m/s to the rest. (a) What is the velocity of the truck right after the collision? (b) How much mechanical energy is lost in the collision? How do you account for this lost in energy?

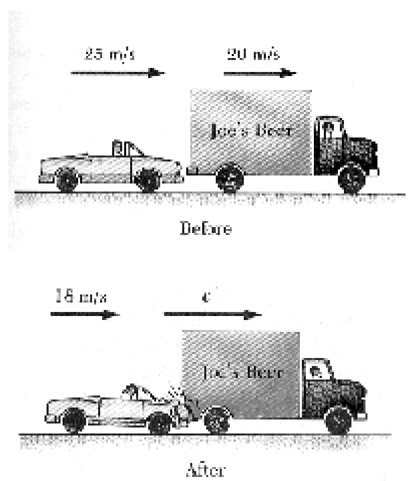
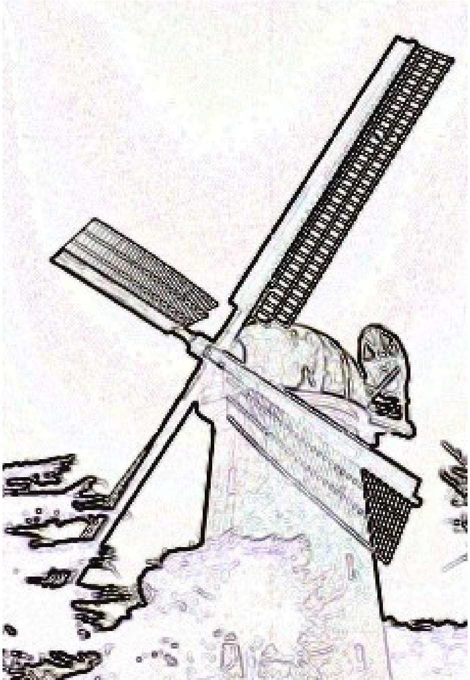


Figure 6.18

Chapter 7

Rotational motion



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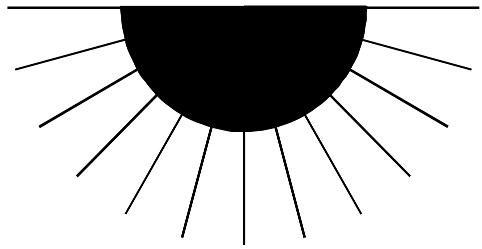


ROTATIONAL MOTION

Rotational Kinematics

درسنا في الفصول السابقة ميكانيكا الحركة الخطيــة مــن حيــث الإزاحــة والسرعة والعجلة والقوى المؤثرة على الحركة والشغل والطاقة. وفي هــذا الفصل سوف نطبق المفاهيم الأساسية للحركة الخطية على نوع جديــد مــن الحركة يعرف باسم الحركة الدورانية ***Rotational motion***. أي عنــدما يدور جسم حول محور ثابت كيف يمكن حساب الإزاحة (الإزاحة الزاويــة) والسرعة (السرعة الزاوية) والعجلة (العجلة الزاوية) والشغل والطاقة، كمــا سنتعرف على مفاهيم فيزيائية جديدة مثل عزم الازدواج.

7.1 Angular displacement

An arbitrary shape rigid body rotating about a fixed axis through point O as shown in Figure 7.1. Line OP is a line fixed with respect to the body and rotating with it. The position of the entire body is specified by the point O and the angle q which the line OP makes with x-axis. It is convenient to use the polar coordinate (r, q) (see Chapter 1) in describing the position of point P, where the only coordinate changing with time is the angle q ,while remains constant. In rectangular coordinate both x and y are changing with time.

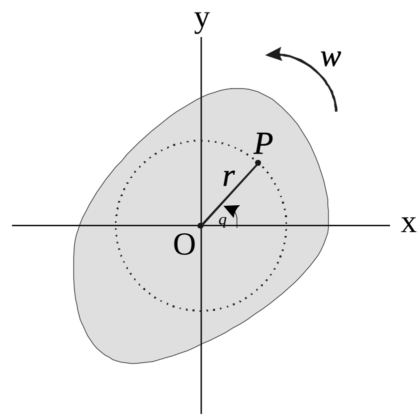


Figure 7.1

Due to the rotational motion the point P moves through an arc of length Δs where the Δs is related to the angular displacement Δq

$$\Delta s = r\Delta q \qquad (7.1)$$

$$\Delta q = \frac{\Delta s}{r} \qquad (7.2)$$

The unit of displacement Δs is in meter and the radius r is in meter as well, hence the angular displacement Δq has no unit, but it commonly the angle is measured by degrees or in radian (rad).

***Definition of radian**: the ratio of arc length to the radius of the circle.*

For one cycle the point P move and angle $\Delta q = 360^{\circ}$ and Δs is the circumference of the circle $2p\,r$, substitute in equation 7.2 we get

$$360^{\circ} \leftrightarrow 2p \ \ \text{rad}$$

therefore,

$$1\,\text{rad} = \frac{360}{2p} = 57.3^{\circ} \qquad (7.3)$$

hence

$$q = \frac{p}{180^{o}} q\,(\text{deg}) \qquad (7.4)$$

For example, 60^{o} equals $p/3$ rad and 45^{o} equals $p/4$ rad

The point P moves in time Δt to point Q then the angular displacement Δq is given by

$$\Delta q = q_2 - q_1 \qquad (7.5)$$

In time

$$\Delta t = t_2 - t_1 \qquad (7.6)$$

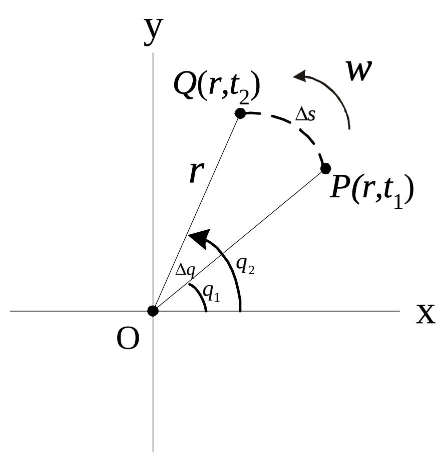


Figure 7.2

7.2 Angular velocity

We found in time $\Delta t = t_2 - t_1$ the angular displacement of the point P changed by $\Delta q = q_2 - q_1$, the *average angular velocity* of the point P ($\overline{w}$) is defined as the ratio the angular displacement to the time interval.

$$\overline{w} = \frac{q_2 - q_1}{t_2 - t_1} = \frac{\Delta q}{\Delta t} \qquad (7.7)$$

The *instantaneous angular velocity* (w) is defined as the limit of the average angular velocity $\overline{w}$ as the time Δt approaches zero

$$w = \lim_{\Delta t \to 0} \frac{\Delta q}{\Delta t} = \frac{dq}{dt} \qquad (7.8)$$

The angular velocity has a unit of rad/s. The angular velocity is positive when q increases (الحركة مع عقارب الساعة) and negative when q decreases (الحركة عكس عقارب الساعة).

7.3 Angular acceleration

If the angular velocity of a body changes, then the body will have an angular acceleration. If w_2 and w_1 are the instantaneous angular velocities at time t_1 and t_2, the average angular acceleration ($\overline{a}$) is defined as

$$\overline{a} = \frac{w_2 - w_1}{t_2 - t_1} = \frac{\Delta w}{\Delta t} \qquad (7.9)$$

The *instantaneous angular acceleration* (a) is defined as the limit of the average angular acceleration $\overline{a}$ as the time Δt approaches zero

$$a = \lim_{\Delta t \to 0} \frac{\Delta w}{\Delta t} = \frac{dw}{dt} \qquad (7.10)$$

The angular acceleration has a unit of rad/s^2.

نلاحظ أن كلاً من الإزاحة الزاوية (q) والسرعة الزاوية (w) والعجلة الزاوية (a) تناظر مفهوم الإزاحة (x) والسرعة (v) والعجلة (a) في الحركة الخطية.

لتحديد إشارة السرعة الزاوية (w) والعجلة الزاوية (a) فإنها تأخذ إشارة موجبة إذا كانت الحركة مع عقارب الساعة وسالبة إذا كانت في عكس عقارب الساعة.

واصطلاحاً يمكن تحديد اتجاه السرعة الزاوية باستخدام قبضة اليد اليمنى حيث تشير أصابع اليد إلى اتجاه الدوران ويشير إصبع الإبهام إلى اتجاه السرعة الزاوية كما في الشكل التوضيحي المقابل (داخل على الصفحة إذا كانت الحركة مع عقارب الساعة وخارج من الصفحة

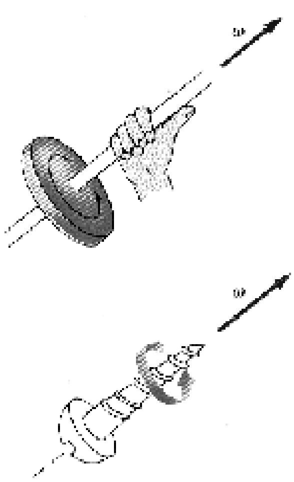


Figure 7.3

إذا كانت الحركة عكس عقارب الساعة). وحيث أن العجلة الزاوية تساوي التغيـــر فـــي السرعة الزاوية بالنسبة للزمن فإن العجلة الزاوية تكون في اتجاه السرعة الزاوية عنـــدما تكون السرعة متزايدة وتكون العجلة الزاوية في عكس اتجاه السرعة الزاويـــة إذا كانـــت السرعة الزاوية تتناقص.

Comparison of kinematics symbol of linear and rotational motion

	Linear motion	Rotational motion
Displacement	x	q
Velocity	v	w
Acceleration	a	a

7.4 Rotational motion with constant angular acceleration

Similar to the equation derived for the linear motion under constant acceleration we can get the same equation for the rotational motion under the constant acceleration. Let's $w = w_o$ and $q = q_o$ at t_o=0, we get,

$$w = w_o + at \qquad (7.11)$$

$$q = q_o + w_o t + \frac{1}{2} a t^2 \qquad (7.12)$$

$$w^2 = w_o{}^2 + 2a(q - q_o) \qquad (7.13)$$

Table 7.1 gives a comparison of kinematics equations for the rotational and linear motion.

Rotational motion at constant angular acceleration	Linear motion at constant acceleration
$w = w_o + at$	$v = v_o + at$
$q = q_o + w_o t + \frac{1}{2} a t^2$	$x = x_o + v_o t + \frac{1}{2} a t^2$
$w^2 = w_o{}^2 + 2a(q - q_o)$	$v^2 = v_o{}^2 + 2a(x - x_o)$

Example 7.1

A wheel rotates with angular velocity of 4rad/s at time *t*=0, and the angular acceleration is constant and equal to 2rad/s^2. A line OP in the wheel is horizontal at time *t*=0.

(a) What angle does this line make with horizontal at time t=3s?

(b) What is the angular velocity at this time?

Solution

(a) $q = q_o + w_o t + \frac{1}{2} a t^2$

$= 0+4\times3+1/2\times2\times3^2$

$= 21$ rad $= 21/2\pi$ rev $= 3.34$ rev

The wheel turns through three complete revolutions plus 0.34 revolution = 0.34 rev × 2π rad/rev = 2.15 rad = 123^o. Therefore the line OP turns through 123^o and makes an angle of 57^o with the horizontal.

(b) $w = w_o + at$

$=4+2\times3=10$ rad/s

Example 7.2

A grinding wheel, initially at rest, is rotated with constant angular acceleration α =5 rad/s^2 for 8s. The wheel is then brought to rest with uniform negative acceleration in 10 revolutions. Determine the negative acceleration required and the time needed bring the wheel to rest.

Solution

$$\omega = \omega_o + \alpha t = 0 + 5\times 8 = 40 \text{ rad/s}$$

Since the wheel is brought to rest after 10 revolutions, its angular displacement during this interval is

$$\theta = 10 \text{ rev} \times 2\pi \text{ rad/rev} = 20\pi \text{ rad}$$

In this second interval, ω = 0, and ω_o = 40rad/s, therefore

$$\omega^2 = \omega_o^2 + 2\alpha(\theta - \theta_o) = 0$$

$$\alpha = -\frac{\omega_o^2}{2\theta} = -\frac{(40)^2}{2(20\pi)} = -12.7 \text{ rad/s}^2$$

Since $\omega = \omega_o + \alpha t$, and ω = 0 then,

$$t = -\frac{\omega_o}{\alpha} = -\left(\frac{40}{-12.7}\right) = 3.14 \text{ s}$$

7.5 Relationship between angular and linear quantities

7.5.1 Angular velocity and linear velocity

When a rigid body rotates about a fixed axis, every particle of the body moves in a circle. The centre of the circle is the axis of rotation. In the Figure 7.4 the point P moves in a circle, the linear velocity vector is always tangent to the circular path, the velocity is known as tangential velocity and we can define it as,

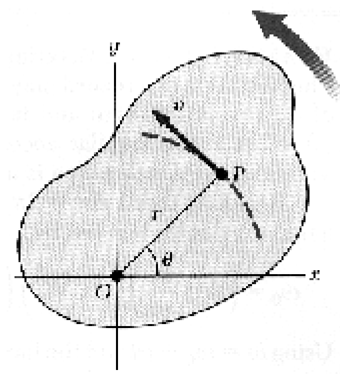


Figure 7.4

$$v = \frac{ds}{dt} \qquad (7.14)$$

$$\mathbf{Q}\, ds = rdq$$

$$\therefore v = r\frac{dq}{dt}$$

hence,

$$v = rw \qquad (7.15)$$

Therefore the tangential velocity of a point P rotating in a circle is equal to the distance from the axis of rotation multiply by the angular velocity. We can conclude that every point in the rigid body have the same angular velocity but have different tangential velocity, and the velocity increases as the point moves outward from the centre of rotation.

Example 7.3

Two wheels connected with string as shown in Figure 7.5. The small wheel rotates with angular velocity of 6 rad/s, what is the angular velocity of the other wheel. Assume that the string does not stretch.

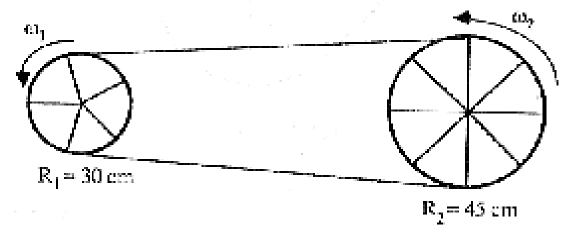


Figure 7.5

Solution

Since the string does not stretch then we use the equation (7.15) for both wheels and the linear velocity is the same for them.

$$v_1 = R_1 w_1 \qquad\qquad v_2 = R_2 w_2$$

But $v_1 = v_2$

$$R_1 w_1 = R_2 w_2 \quad \rightarrow \quad w_2 = \frac{R_1 w_1}{R_2} \rightarrow$$

$$w_2 = \frac{6 \times 30}{45} = 4 \text{ rad/s}$$

7.5.2 Angular acceleration and linear acceleration

If the angular velocity about a given axis change in magnitude only by Δw, the linear velocity in the direction tangent to the circle of radius r will change by Δv, where

$$\Delta v = r\Delta w$$

Divide both sides of the equation by Δt

$$\frac{\Delta v}{\Delta t} = r\frac{\Delta w}{\Delta t} \qquad (7.16)$$

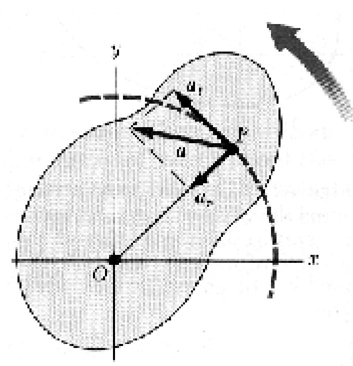


Figure 7.6

The limit as Δt approach to zero, then,

$$a_t = \frac{dv}{dt} = r\frac{dw}{dt} \qquad (7.17)$$

$$a_t = ra \qquad (7.18)$$

where a_t is the tangential component of the linear acceleration of a point on a rotating rigid body equals the distance of the point from the axis of rotation multiplied by the angular acceleration.

The radial components of acceleration of a point P in a rotating body as shown in Figure 7.6.

$$a_r = \frac{v^2}{r} = rw^2 \qquad (7.19)$$

Their vector sum is the acceleration ***a*** is the total linear acceleration of the body,

$$a = \sqrt{a_t^{\,2} + a_r^{\,2}} \qquad (7.20)$$

$$a = \sqrt{r^2a^2 + r^2w^2} = r\sqrt{a^2 + w^2} \qquad (7.21)$$

Example 7.4

A wheel 2m in diameter rotates with a constant angular acceleration of 4 rad/s^2. The wheel starts at rest at *t*=0, and the radius vector at point P on the rim makes an angle of 57.3^o with the horizontal at this time. At *t*=2s, find (a) the angular speed of the wheel, (b) the linear velocity and acceleration of the point P, and (c) the position of the point P.

Solution

(a) $w = w_o + at = 0 + at$ at t=2s

$w = 4 \times 2 = 8$ rad/s

(b) $v = rw$ =1×8=8 rad/s

$a_r = rw^2$ =1×8^2=64m/s^2 $\qquad a_t = ra$ =1×4=4m/s^2

(c) $q = q_o + w_o t + \frac{1}{2}at^2$ =1+[(1/2)×4×2^2]=9 rad

Example 7.5

A disk 8cm in radius rotates at a constant rate of 1200rev/min about its axis. Determine (a) the angular speed of the disk, (b) the linear speed at a point 3cm from its centre, (c) the radial acceleration of a point on the rim, and (d) the total distance a point on the rim moves in 2s.

Solution

(a) $w = 2pf = 2p\dfrac{1200}{60} = 40p = 126$ rad/s

(b) $v = rw = 40p \times 0.03 = 3.77$ m/s

(c) $a_r = rw^2 == (40p)^2 \times 0.08 = 1.26$ km/s^2

(d) $s = qr = wtr = 40p \times 2 \times 0.08 = 20.1$ m

Example 7.6

A 6kg block is released from A on a frictionless track as shown in Figure 7.7. Determine the radial and tangential components of acceleration for the block at P.

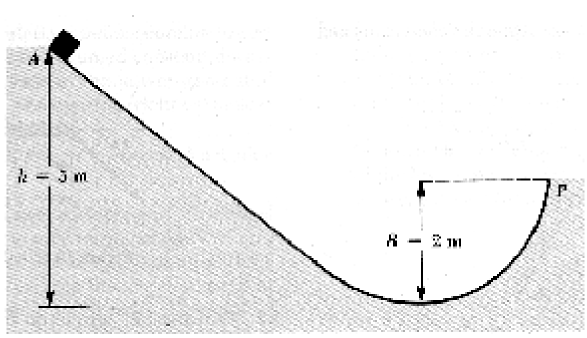


Figure 7.7

Solution

At the point P the block has two types of acceleration one is the gravitational acceleration (tangential) and the other is the centripetal acceleration (radial).

The gravitational acceleration is a_t = 9.8 m/s^2

The centripetal acceleration a_r is given by

$$a_r = \frac{{v_P}^2}{R}$$

To find the velocity v_p we use the law of conservation of energy

$$E_A = E_P$$

$$mgh + 0 = mgR + \frac{1}{2} m{v_P}^2$$

$${v_P}^2 = 2g(h - R)$$

$${v_P}^2 = 20(5-2) = 60\,(\text{m/s})^2$$

therefore

$$a_r = \frac{{v_P}^2}{R} = \frac{60}{2} = 30\,\text{m/s}^2$$

The resultant acceleration is

$$a = \sqrt{{a_t}^2 + {a_r}^2} = 31.5\ \text{m/s}^2$$

7.6 Rotational kinetic energy

For a rigid body consist of small particles rotating with angular velocity w. The kinetic energy of a of a particle of mass m_i in a rotating body is given by

$$K_i = \frac{1}{2} m_i v_i^2 \qquad (7.22)$$

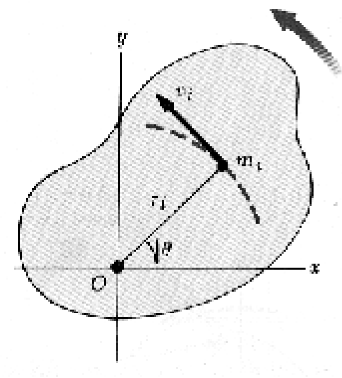


Figure 7.8

The total kinetic energy for all particles of the rotating body is the sum of the kinetic energies of the individual particles,

$$K = \sum K_i = \sum \frac{1}{2} m_i v_i^2 = \frac{1}{2} \sum m_i r_i^2 w^2 \qquad (7.23)$$

$$K = \frac{1}{2}\left(\sum m_i r_i^2\right) w^2 \qquad (7.24)$$

The quantity $\left(\sum m_i r_i^2\right)$ called the moment of inertia, *I*, and has a unit of kg.m^2

$$I = \sum m_i r_i^2 \qquad (7.25)$$

Therefore, the total energy of rotating body

$$K = \frac{1}{2} I w^2 \qquad (7.26)$$

where *I* is analogous to the mass *m* and w analogous to the *v*.

Example 7.7

The four particles in Figure 7.9 are connected by light rigid rods. If the system rotates in the *xy* plane about the z-axis with an angular velocity of 6rad/s.. (a) Calculate the moment of inertia of the system abut the z-axis (b) Calculate the kinetic energy of the system.

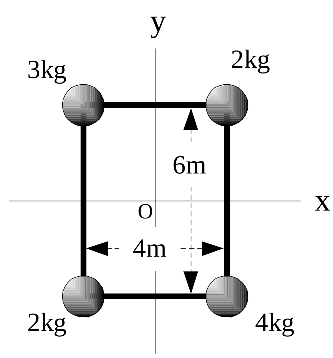


Figure 7.9

Solution

(a) The distance between the four particles and the z-axis is r.

$r^2 = 3^2 + 2^2 = 13\text{m}^2$

Therefore,

$$I = \sum m_i r_i^2 = (3\times13)+(2\times13)+(4\times13)+(2\times13)$$

$$= 11\times13 = 143\text{ kg.m}^2$$

(b) $$K = \frac{1}{2} I w^2 = \frac{1}{2}\times143\times6^2 = 2.57\text{ kJ}$$

Example 7.8

Three particles are connected by rigid rod of negligible mass lying along y axis as shown in Figure 7.10. If the system rotates about the x axis with an angular speed of 2rad/s, find (a) the kinetic energy of the particles and (b) the linear speed for each particle.

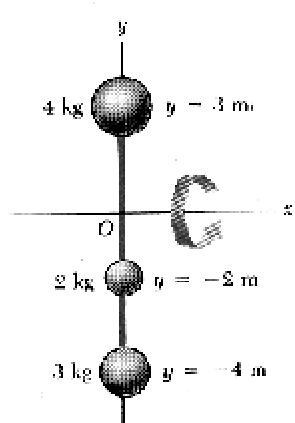


Figure 7.10

Solution

(a) The total kinetic energy is given by

$$K = \frac{1}{2} I w^2 \qquad \text{where } w = 2 \text{ rad/s}$$

The moment of inertia *I* is given by

$$I = \sum m_i r_i^2 = 4(3)^2 + 2(-2)^2 + 3(-4)^2 = 92 \text{ kg.m}^2$$

$$K = \frac{1}{2} I w^2 = \frac{1}{2} \times 92 \times 2^2 = 184 \text{ J}$$

(b) The linear velocity for each particle is given by

$$v = rw$$

Then

$$v_1 = r_1 w = 2 \times 3 = 6 \text{ m/s}$$

$$v_2 = r_2 w = 2 \times 4 = 8 \text{ m/s}$$

$$v_3 = r_3 w = 2 \times 2 = 4 \text{ m/s}$$

Moment of inertia of rigid bodies with different shapes

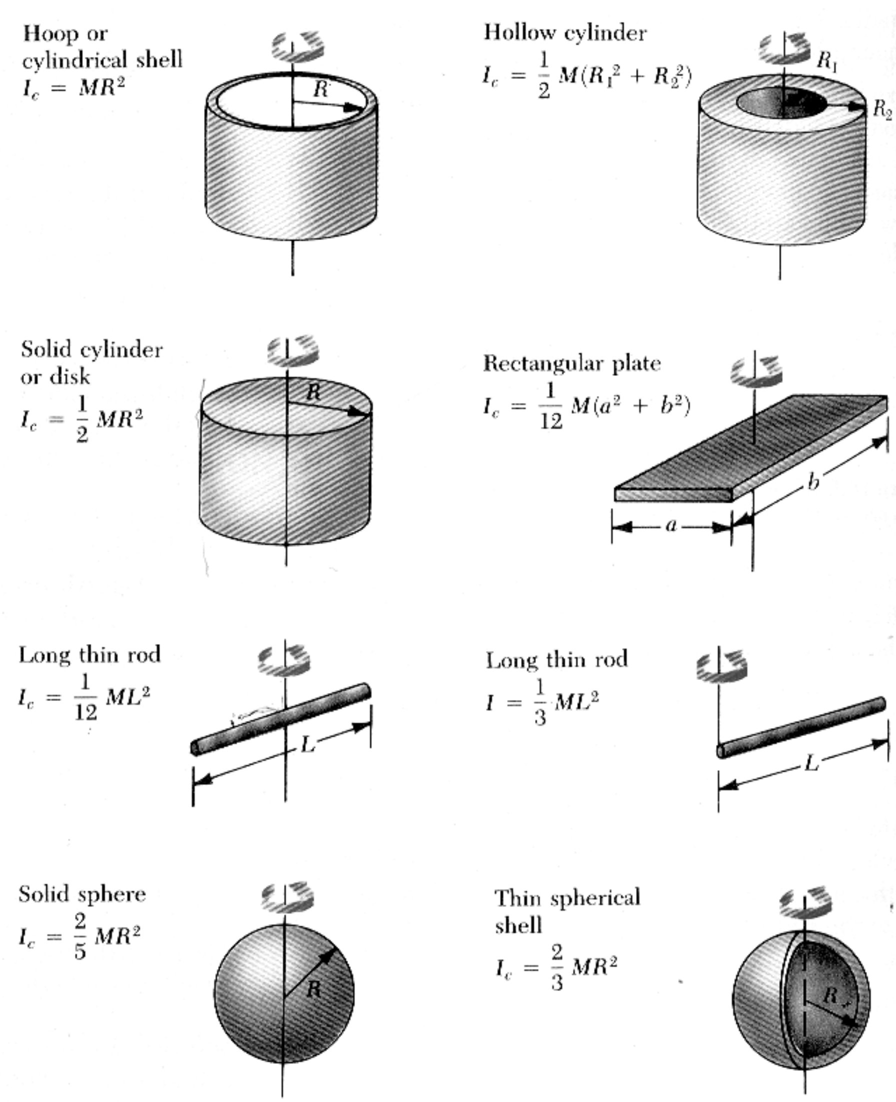


Figure 7.11

Rotational Dynamics

درسنا في الفصول الأولى من هذا الكتاب مفهوم القوة وقوانين نيوتن للحركـــة وســـوف ندرس في هذا الفصل كمية فيزيائية مناظرة للقوة في الحركة الدائرية وهي عزم الازدواج ***Torque*** (t) وسنركز أيضا على علاقة عزم الازدواج بقانون نيوتن الثـــاني للحركـــة الدورانية .

7.7 Torque

The torque in the rotational motion is equivalent to the force in the linear motion, the torque is defined as the force exerted on a rigid body pivoted about some axis to rotate it about that axis.

$$t = rF \sin f \qquad (7.27)$$

we can write the equation as

$$\vec{t} = \vec{r} \times \vec{F} \qquad (7.28)$$

The torque has a unit of N.m.

The quantity $r \sin f$ called the moment arm of the force. it represent the perpendicular distance from the line of action of the force to the axis of the rotation.

The torque is a vector quantity and its direction is determine by the right hand rule, if the rotation of the rigid body in the xy plane then the torque will be in the direction of the positive z axis if the rotation is counterclockwise and the torque in the negative z axis if the rotation is clockwise.

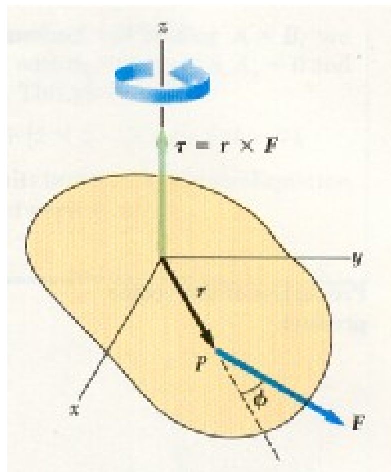


Figure 7.12

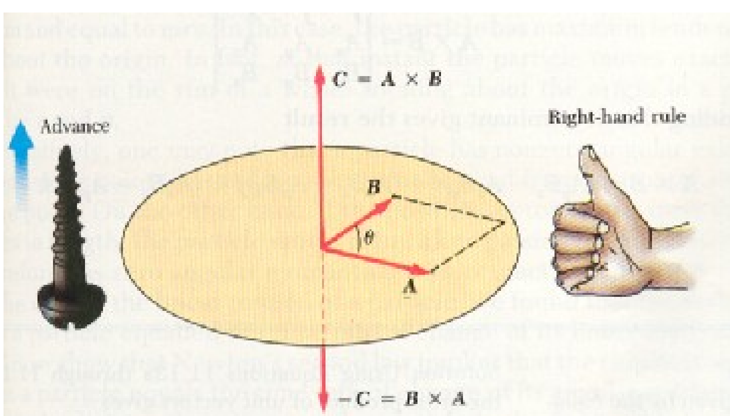


Figure 7.13

Example 7.9

A wheel 1m in diameter rotates on a fixed frictionless horizontal axle. Its moment of inertia about this axis is 5kg.m^2. A constant tension of 20N is maintained on a rope wrapped around the rim of the wheel, so as to cause the wheel to accelerate. If the wheel starts from rest at *t*=0, find (a) the angular acceleration of the wheel, (b) the wheel's angular speed at *t*=3s, (c) the kinetic energy of the wheel at *t*=3s, and (d) the length of rope unwound in the first 3s.

Solution

(a) $t = rF = 20\times0.5 = 10$ N.m

$$a = \frac{t}{I} = \frac{10}{5} = 2\ \text{rad/s}^2$$

(b) $w = w_o + at = 0 + at = 0 + 2\times3 = 6$ rad/s

(c) $K = \frac{1}{2}Iw^2 = \frac{1}{2}\times5\times6^2 = 90$ kJ

(d) $q = q_o + w_o t + \frac{1}{2}at^2 = 0+0+[(1/2)\times2\times3^2] = 9$ rad

$$s = rq = 0.5\times9 = 4.5\ \text{m}$$

7.8 Work and energy of rotational motion

If a force F is applied at a distance r from the axis of rotation as shown in Figure 7.14. The work done by the force to rotate the body through small distance ds in time dt is given by,

$$dW = \vec{F}.d\vec{s} = (F\sin\phi)rd\theta \qquad (7.29)$$

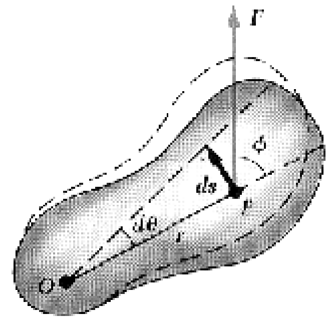


Figure 7.14

The quantity $F\sin\phi$ is the component of the force witch does work, and called the tangential component of the force. By definition the torque $\tau = rF\sin\phi$, then we can write the equation as,

$$dW = \tau\, d\theta \qquad (7.30)$$

Divide both sides of the equation by dt we get,

$$\frac{dW}{dt} = \tau\,\frac{d\theta}{dt} \qquad (7.31)$$

The left side of the equation is known as the power P delivered by the force, hence

$$P = \frac{dW}{dt} = \tau\omega \qquad (7.32)$$

7.9 Angular momentum

The angular momentum L of the particle relative to the origin O is defined by the cross product of its position vector and the linear momentum p *i.e.*

$$\vec{L} = \vec{r}\times\vec{p} \qquad (7.33)$$

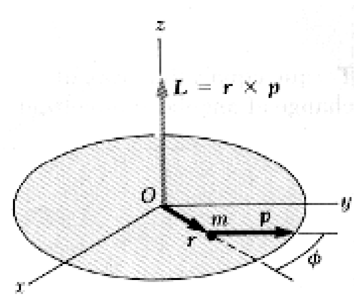


Figure 7.15

The angular momentum has unit kg.m^2/s. The direction of L is determined by the right-hand rule.

The magnitude of L is given by,

$$L = mvr\sin\phi \qquad (7.34)$$

where ϕ is the angle between *r* and *p*. The maximum value of *L* when ϕ = 90° and equal to *mvr*, in this case the particle rotates about the origin.

From equation (7.10)

$$v = r\omega$$

therefore

$$L_{max} = mvr = mr^2\omega \qquad (7.35)$$

$$L_{max} = I\omega \qquad (7.36)$$

This is equivalent to *p* = *mv* in the linear motion.

7.10 Relation between The torque and the angular momentum

In previous chapter we found that the force on a particle equal to the time rate of change of its linear momentum.

$$\vec{F} = \frac{d\vec{p}}{dt}$$

The torque is given by,

$$\vec{\tau} = \vec{r} \times \vec{F}$$

therefore,

$$\vec{\tau} = \vec{r} \times \frac{d\vec{p}}{dt}$$

By differentiate equation (7.33) we get

$$\frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r} \times \vec{p}) = \vec{r} \times \frac{d\vec{p}}{dt} + \frac{d\vec{r}}{dt} \times \vec{p} \qquad (7.37)$$

The result of the product of the last term in the right hand side is zero, since *v=dr/dt* is parallel to *p*, hence,

$$\frac{d\vec{L}}{dt} = \vec{r} \times \frac{d\vec{p}}{dt} \qquad (7.38)$$

$$\vec{\tau} = \frac{d\vec{L}}{dt} \qquad (7.39)$$

This shows that the torque acting on a particle is equal to the time rate of change of the particle's angular momentum.

Comparison between the equations of rotational and linear motion

	Linear motion	Rotational motion
Displacement	x	θ
Velocity	v	ω
Acceleration	a	α
Mass	m	I
Force	F	τ
Work	$W = \int F dx$	$W = \int \tau \, d\theta$
Kinetic energy	$K.E = 1/2\, mv^2$	$K.E = 1/2\, I\omega^2$
Power	$p=Fv$	$p = \tau\omega$
Momentum	$L=mv$	$L = I\omega$

7.11 Questions with solutions

1. A wheel rotates counterclockwise in the xy plane. What is the direction of ω? What is the direction of α the angular velocity is decreasing in time?

Answer: *From the right-hand rule (Figure 7.13), we see that ω is in the +z direction or out of the paper. Since ω is decreasing in time, α is into the paper (opposite ω).*

2. Are the kinematic expressions for θ, ω, and α valid when the angular displacement is measured in degree instead of in radians?

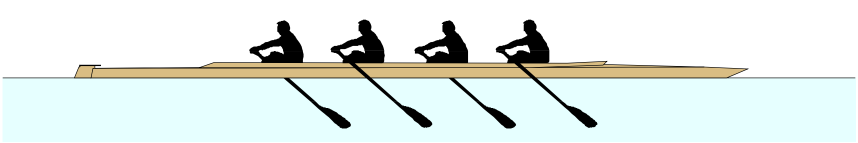
Answer: *Yes. However, it is conventional to use radians.*

3. A turntable rotates at a constant rate of 45 rev/min. What is the magnitude of its angular velocity in rad/s? What is its angular acceleration?

Answer: *The frequency of rotation is 45 rotations/min = 45/60 rotations/s. Since 1 rotation corresponds to an angular displacement of 2π rads, the angular frequency is $\omega = 2\pi f = 2\pi (45/60) = 4.71$ rad/s. Since ω is constant, the angular acceleration is zero.*

4. When a wheel of radius *R* rotates about a fixed axis, do all points on the wheel have the same angular velocity? Do they all have the same linear velocity? If the angular velocity is constant and equal to ω_o, describe the linear velocities and linear accelerations of the points at $r = 0$, $r = R/2$ and $r = R$.

Answer: *Yes. All points have the same angular velocity. This, in fact, is what makes angular quantities so useful in describing rotational motion. Not all points have the same linear velocities. point at r = 0 has zero linear velocity a acceleration; the point at r=R/2 has a linear velocity $v=(R/2)\omega_o$, and a linear acceleration equal to the centripetal acceleration $v^2/R/2 = R(\omega_o{}^2/2$. (The tangential acceleration is zero since ω_o is constant.) The point at r=R has a linear velocity $v = R\omega_o$ and a linear acceleration equal to $R\omega_o{}^2$.*



7.12 Problems

1. A wheel starts from rest and rotates with constant angular acceleration to an angular velocity of 12 rad/s in a time of 3s. Find (a) the angular acceleration of the wheel and (b) the angle in radians through which it rotates in this time.

2. The turntable of a record player rotates at the rate of 33 rev/min and takes 60s to come to rest when switched off. Calculate (a) its angular acceleration and (b) the number of revolutions it makes before coming to rest.

3. What is the angular speed in rad/s of (a) the earth in its orbit about the sun and (b) the moon in its orbit about the earth?

4. A wheel rotates in such away that its angular displacement in a time t is given by $q = at^2 + bt^3$, where a and b are constants. Determine equations for (a) the angular speed and (b) the angular acceleration, both as functions of time.

5. An electric motor rotating a workshop grinding wheel at a rate of 100 rev/min is switched off. Assuming constant negative acceleration of magnitude 2 rad/s^2 (a) how long will it take for the grinding wheel to stop? (b) through how many radians has the wheel turned during the time found in (a)?

6. The angular position of a point on a wheel can be described by $q = 5+10t+2t^2$ rad. Determine the angular position, speed, and acceleration of the point at $t = 0$ and $t = 3$ s.

7. A wheel, starting from rest, rotates with an angular acceleration a =(10+6t) rad/s^2, where t is in seconds. Determine the angle in radians through which the wheel has turned in the first four seconds.

8. A racing car travels on a circular track of radius 250m. If the car moves with a constant speed of 45m/s, find (a) the angular speed of the car and (b) magnitude and direction of the car's acceleration

9. The racing car described in Problem 8 starts from rest and accelerates uniformly to a speed of 45m/s in 15s Find (a) the average angular speed of the car in this interval, (b) the angular acceleration of the car, (c) magnitude of the car's linear acceleration at t =10s and (d) the total distance travelled in the first 30s.

10. A wheel 2m in diameter rotates with a constant angular acceleration of 4rad/s^2. The

wheel starts at rest at $t = 0$, and the radius vector at point P on the rim makes an angle of 57.3^o with the horizontal at this time. At $t = 2s$, find (a) the angular speed of the wheel, (b) the linear velocity and acceleration of the point P, and (c) the position of the point P.

11. A cylinder of radius 0.1m starts from rest and rotates about its axis with a constant angular acceleration 5 rad/s^2. At $t = 3s$, what is (a) its angular velocity (b) the linear speed of a point on its rim, and (c) the radial and tangential components of acceleration of a point on its rim?

12. A car is travelling at 36 km/h on a straight road. The radius of the tires is 25cm. Find the angular speed of one of the tires with its axle taken as the axis of rotation.

13. The system of particles described in Figure 7.9 rotates about the y axis. Calculate (a) the moment of inertia about the y axis and (b) the work required to take the system from rest to an angular speed of 6 rad/s.

14. A light rigid rod 1m in length rotates in the *xy* plane about a pivot through the rod's center. Two particles of mass 4kg and 3kg are connected to its ends (Figure 7.16). Determine the angular momentum of the system about the origin at the instant the speed of each particle is 5m/s.

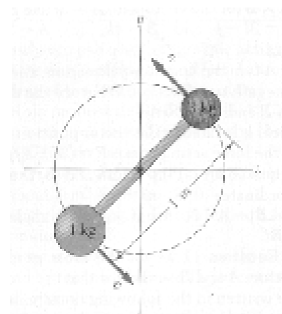


Figure 7.16

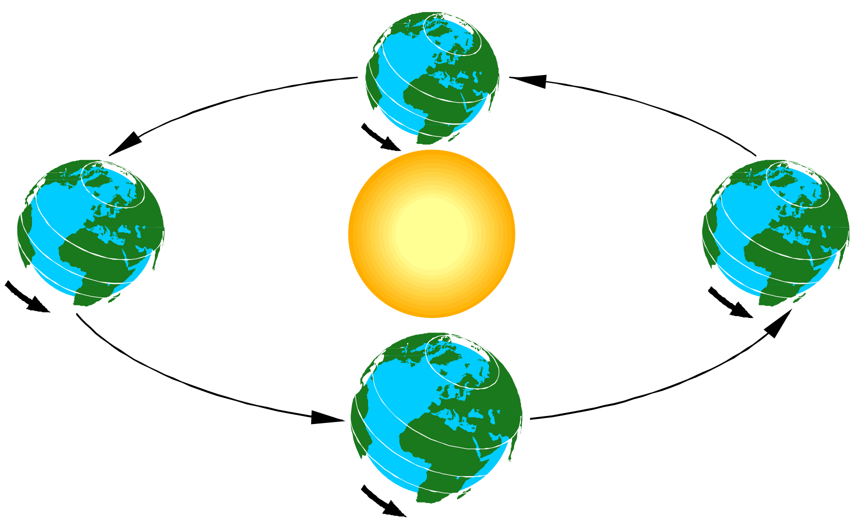
15. The position vector of a particle of mass 2kg is given as a function of time by r=(6i+5tj) m. Determine the angular momentum of the particle as a function of time.

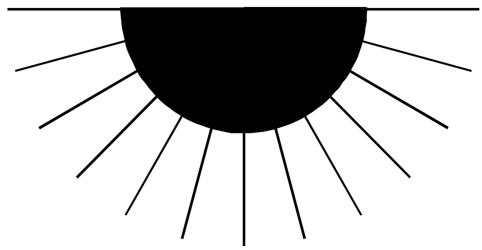
16. (a) Calculate the angular momentum of the earth due to its spinning motion about its axis. (b) Calculate the angular momentum of the earth due to its orbital motion about the sun and compare this with (a). (Take the earth-sun distance to be 1.49×10^{11} m.

Chapter 8

The law of universal gravitation

قانــــون الجــذب العــام





THE LAW OF UNIVERSAL GRAVITATION

8.1 The law of universal gravitation

وضع العالم نيوتن قانون الجاذبية العام بعد الرواية المشهورة عنه وهي سقوط التفاحة على رأسه بينما كان نائماً تحت شجرة، فتوصل إلى أن القوة التي أثرت على التفاحة لتسقط على الأرض هي نفس القوة التي تجذب القمر إلى الأرض. وتبين أيضاً أن قانون الجذب العام لنيوتن ينطبق على القوة المتبادلة بين الكواكب والأجسام المادية على حد سواء.

8.2 Newton's universal law of gravity

Newton's law of gravitational state that *every particle in the universe attract every another particle with a force proportional to the product of their masses and inversely proportional to the square of the distance between them.*

therefore,

$$F = G\frac{m_1 m_2}{r^2} \qquad (8.1)$$

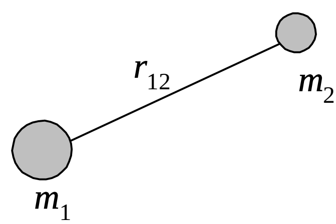


Figure 8.1

where G is the gravitational constant, and it is equal,

$$G = 6.67\times10^{-11}\,\frac{N.m^2}{kg^2} \qquad (8.2)$$

To write the force of gravitation equation in the vector form we make use of the unit vector $\hat{r}_{12}$ which has the magnitude of unity and directed from the mass m_1 to m_2, the force on m_2 due to m_1 is given by

$$\vec{F}_{21} = -G\frac{m_1 m_2}{r_{12}^2}\hat{r}_{12} \qquad (8.3)$$

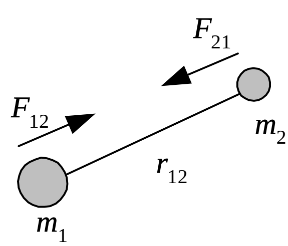


Figure 8.2

القوة المتبادلة بين كتلتين m_1 و m_2 هي ناتجة عن التأثير المتبادل بينهما وعليه فإن F_{21} هي قوة الجذب على الكتلة الثانية من تأثير الكتلة الأولى. كذلك فإن القوة F_{12} هي قوة الجذب على الكتلة الأولى من تأثير الكتلة الثانية وفي كلا الحالتين فإن القوتين متساويتان في المقدار ومتعاكستان في الاتجاه. ويعبر عن ذلك بالمعادلة التالية:

$$\vec{F}_{21} = -\vec{F}_{12} \qquad (8.4)$$

يمكن استخدام قانون الجذب العام لنيوتن لإيجاد القوة المتبادلة بين جسم كتلته m والكرة الأرضية، وهنا يتم التعامل مع كتلة الكرة الأرضية على أنها مركزة في المركز وتحسب المسافة من مركز الأرض إلى الجسم ويكون قانون الجذب العام هو

$$F = G\frac{M_e m}{R_e^{2}} \qquad (8.5)$$

where M_e is the mass of the earth and R_e is the radius of the earth.

Example 8.1

Three uniform spheres of mass 2kg, 4kg, and 6kg are placed at the corners of a right triangle as shown in Figure 8.3. Calculate the resultant gravitational force on the 4kg mass.

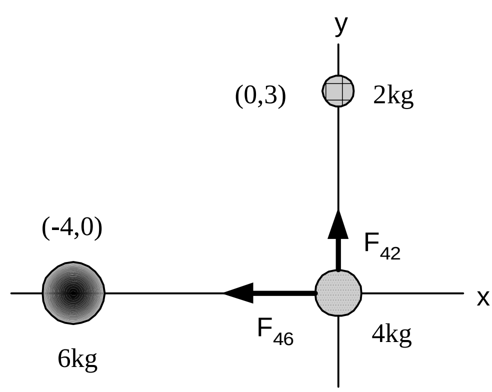


Figure 8.3

Solution

$$\vec{F}_4 = \vec{F}_{42} + \vec{F}_{46}$$

The force on the 4kg mass due to the 2kg mass is

$$\vec{F}_{42} = G\frac{m_4 m_2}{r_{42}^{2}}\, j$$

$$\vec{F}_{42} = (6.67\times10^{-11})\frac{4\times2}{3^2}\,j$$

$$\vec{F}_{42} = 5.93\times10^{-11}\,j\ \text{N}$$

The force on the 4kg mass due to the 6kg mass is

$$\vec{F}_{46} = G\frac{m_4 m_6}{r_{46}^{\ 2}}(-i)$$

$$\vec{F}_{46} = -(6.67\times10^{-11})\frac{4\times6}{4^2}\,i$$

$$\vec{F}_{46} = -10\times10^{-11}\,i\ \text{N}$$

hence,

$$\vec{F}_4 = (-10i + 5.93j)\times10^{-11}\ \text{N}$$

$$F_4 = 11.6\times10^{-11}\ \text{N} \qquad \& \qquad q = 149^o$$

Example 8.2

Two stars of masses *M* and 4*M* are separated by distance *d*. Determine the location of a point measured from *M* at which the net force on a third mass would be zero.

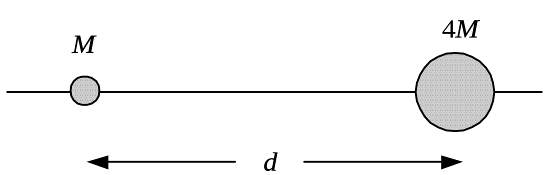


Figure 8.4

Solution

حتى تكون القوى المؤثرة على الكتلة الثالثة m فإن القوتين المؤثرتين على الكتلـــة الثالثـــة يجب أن تكونا متساويتين في المقدار ومتعاكستين في الاتجاه. وهذا يتحقق عندما يكـــون

موضـع الكتلة الثالثة بين الكتلتين M و $4M$ وبالقرب من الكتلة الأصغر كما فـــي الشـــكل 8.5.

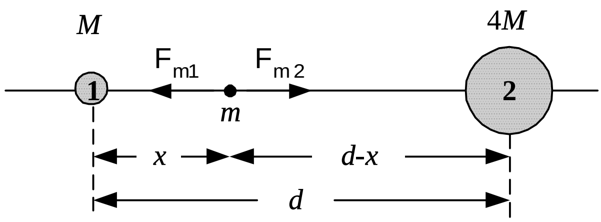


Figure 8.5

$$\vec{F}_{m2} = -\vec{F}_{m1}$$

$$G\frac{m4M}{(d-x)^2} = G\frac{mM}{(x)^2}$$

$$\frac{4}{(d-x)^2} = \frac{1}{(x)^2}$$

Solving for x then,

$$x = \frac{d}{3}$$

8.3 Weight and gravitational force

From Newton's second law we define the weight as a kind of force equal to mg where m is the mass of the particle and g the acceleration due to gravity, we can define the weight using the Newton's universal law of gravity as follow

$$W = mg = G\frac{M_e m}{R_e^{\ 2}} \qquad (8.6)$$

Therefore the acceleration due to gravity can be found as

$$g = G\frac{M_e}{R_e^{\ 2}} \qquad (8.7)$$

Substitute for the mass of earth $M_e = 5.98\times10^{24}$kg and the radius of the earth $R_e = 6.38\times10^6$m

$$\therefore g = G\frac{M_e}{R_e^{\ 2}} = 6.67\times10^{-11}\frac{5.98\times10^{24}}{6.38\times10^{6}} = 9.8 m/s^2 \qquad (8.8)$$

هنا يجب أن نذكر أن قوة الجاذبية بين كتلتين m_1 و m_2 هي من القوى ذات التأثير عن بعد action-at-a-distance وبالتالي يمكن أن نعتبر عجلة الجاذبية الأرضية على أنها مجال الجاذبية ***gravitational field*** ويمكن تعريف مجال الجاذبية الأرضية بأنها القوة المؤثرة على كتلة الجسم الموجود في مجال الجاذبية.

$$\vec{g} = \frac{\vec{F}}{m} \quad \rightarrow \quad \vec{g} = -\frac{GM_e}{r^2}\hat{r} \qquad (8.9)$$

والإشارة السالبة تدل على أن مجال الجاذبية الأرضية في مركز الأرض دائماً.

For a body of mass m a distance h above the earth then the distance r in the equation of the law of gravity is $r=R_e+h$

$$F = G\frac{M_e m}{r^2} = G\frac{M_e m}{(R_e+h)^2} \qquad (8.10)$$

and the acceleration due to gravity at altitude (ارتفاع) h, is given by

$$g' = G\frac{M_e}{r^2} = G\frac{M_e}{(R_e+h)^2} \qquad (8.11)$$

نستنتج من ذلك أن عجلة الجاذبية الأرضية تقل مع زيادة الارتفـاع عـن سـطح الأرض وتكون صفراً عندما تكون r في اللانهاية.

Example 8.3

Determine the magnitude of the acceleration of gravity at an altitude of 500km.

Solution

$$g' = G\frac{M_e}{(R_e + h)^2}$$

$$g' = 6.67\times10^{-11}\frac{5.98\times10^{24}}{(6.38\times10^{6} + 0.5\times10^{6})^2}$$

$$= 8.43 m/s^2$$

8.4 Gravitational potential energy

في الفصل السابق درسنا أن طاقة الوضع لجسم على سطح الأرض أو على ارتفاع h من سطح الأرض تساوي mgh وهذا عندما تكون h على مسافات قريبة من سطح الأرض أو عندما تكون h أصغر بكثير من نصف قطر الأرض.

سندرس الآن طاقة الوضع في مجال الجاذبية الأرضية عندما يتغير موضع الجسـم مـن مكان إلى آخر بالنسبة لمركز الأرض كما في الشكل التالي.

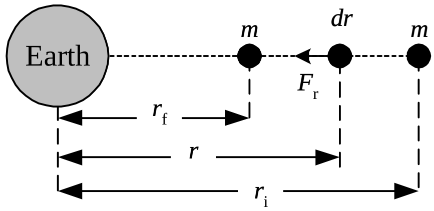


Figure 8.6

To move the particle of mass m from r_i to r_f in the gravitational field g a negative work W is done by an external agent since the external force F_{ex} is in opposite direction of the displacement. Therefore the change in gravitational potential energy associated with a given displacement dr is defined as the negative work done by the gravitational force during the displacement,

$$\Delta U = U_f - U_i = -\int_{r_i}^{r_f} F(r)dr \qquad (8.12)$$

When the particle move from r_i to r_f, it will be subjected to gravitational force given by

$$\vec{F} = -\frac{GM_e m}{r^2}\hat{r} \qquad (8.13)$$

Substitute in equation 8.12 we get

$$U_f - U_i = -GM_e m\int_{r_i}^{r_f} \frac{dr}{r^2} = GM_e m\left[-\frac{1}{r}\right]_{r_f}^{r_i} \qquad (8.14)$$

Hence

$$U_f - U_i = -GM_e m\left(\frac{1}{r_f} - \frac{1}{r_i}\right) \qquad (8.15)$$

Take U_i=0 at r_i=∞ we obtain the potential energy as a function of r from the centre of the earth

$$U(r) = -\frac{GM_e m}{r} \qquad (8.16)$$

The potential energy between any two particles m_1 and m_2 is given by

$$U = -G\frac{m_1 m_2}{r} \qquad (8.17)$$

نستنتج من المعادلة الأخيرة أن طاقة الوضع المتبادلة بين جسمين تتناسب عكسيا مع المسافة الفاصلة بينهما في حين أن قوة الجاذبية تتناسب عكسياً مع مربع المسافة بينهما.

تكون طاقة الوضع بين جسمين سالبة لأن القوة المتبادلة بينهما دائماً قوى تجاذبية، ويمكن أن نطلق على طاقة الوضع بين جسمين بطاقة الترابط ***Binding energy***.

For more than two particles the potential energy can be evaluated by the algebraic sum of the potential energy between any two particles.

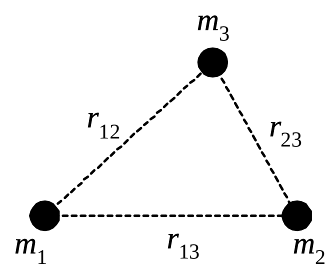


Figure 8.7

$$U_{total} = U_{12} + U_{13} + U_{23} \qquad (8.18)$$

$$U_{total} = -G\left(\frac{m_1 m_2}{r_{12}} + \frac{m_1 m_3}{r_{13}} + \frac{m_2 m_3}{r_{23}}\right) \qquad (8.19)$$

Example 8.4

A system consists of three particles, each of mass 5g, located at the corner of an equilateral triangle with sides of 30cm. (a) Calculate the potential energy of the system.

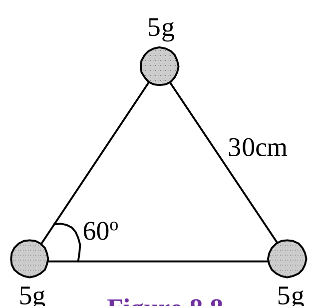


Figure 8.8

Solution

$$U_{total} = U_{12} + U_{13} + U_{23}$$

$$U_{total} = -G\left(\frac{m^2}{r} + \frac{m^2}{r} + \frac{m^2}{r}\right) = -\frac{3GM^2}{r}$$

$$U_{total} = -\frac{3\times 6.67\times 10^{-11}\times (0.005)^2}{0.3} = -1.67\times 10^{-14}\ J$$

8.5 Total Energy for circular orbital motion

When a body of mass *m* moving with speed *v* in circular orbit around another body of mass *M* where *M*>>*m* as the earth around the sun or satellite around the earth, the body of mass *M* is at rest with respect to the frame of reference. The total energy of the two body system is the sum of the kinetic energy and the potential energy.

$$E = K + U \qquad (8.20)$$

Hence

$$E = \frac{1}{2}mv^2 - \frac{GMm}{r} \qquad (8.21)$$

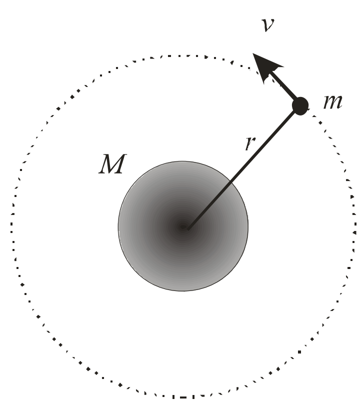


Figure 8.9

As the mass *m* moves from initial point *i* to a final point *f*, the total energy remains constant, therefore the total energy equation become,

$$E = \frac{1}{2}mv_i^2 - \frac{GMm}{r_i} = \frac{1}{2}mv_f^2 - \frac{GMm}{r_f} \qquad (8.22)$$

From Newton's second law $F = ma$ where *a* is the is the radial acceleration therefore,

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \qquad (8.23)$$

Multiply both sides by *r*/2

$$\frac{1}{2}mv^2 = \frac{GMm}{2r} \qquad (8.24)$$

Substitute from equation 8.24 into equation 8.22, we get

$$E = \frac{GMm}{2r} - \frac{GMm}{r} \qquad (8.25)$$

The total energy for circular orbit

$$E = -G\frac{Mm}{2r} \qquad (8.26)$$

Note that the total energy is negative in a circular orbit. And the kinetic energy is positive and equal to one half the magnitude of the potential energy. The total energy called the binding energy for the system.

8.6 Escape velocity

باستخدام مفهوم الطاقة الكليـــة ســـنقوم بحســـاب ســـرعة الإفلات ***velocity escape*** مـــن الجاذبيـــة الأرضـــية. وسرعة الإفلات هي أقل سرعة ابتدائية لجسم يقذف رأسياً ليتمكن الجسم من الإفلات من مجال الجاذبية الأرضية.

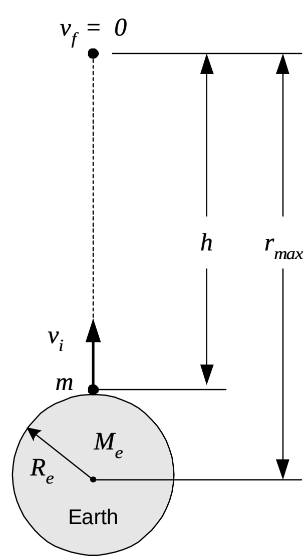


Figure 8.10

Suppose an object of mass m is projected vertically upward from the earth with initial speed $v_i = v$ and $r_i = R_e$. When the object is at maximum altitude, $v_f = 0$ and $r_f = r_{max}$.

In this case the total energy of the system (Earth & object) is conserved, we can use the equation 8.21

$$\frac{1}{2}mv_i^2 - \frac{GM_e m}{R_e} = -\frac{GM_e m}{r_{max}} \qquad (8.27)$$

solving for v_i^2 we get,

$$v_i^{\ 2} = 2GM_e\left(\frac{1}{R_e} - \frac{1}{r_{\max}}\right) \tag{8.28}$$

من هذه المعادلة إذا علمنا قيمة السرعة الابتدائية لانطلاق الجسم v_i يمكن حساب أقصى ارتفاع يمكن أن يصل إليه الجسم h حيث أن $h = r_{max}-R_e$.

لحساب سرعة الإفلات للجسم من مجال الجاذبية الأرضية مثل ما هو الحال عند إطلاق صاروخ فضائي أو مكوك من سطح الأرض إلى الفضاء الخارجي فإن سرعة الانطلاق الابتدائية التي يجب أن ينطلق بها المكوك يجب أن لا تقل عن سرعة الإفلات وإلا فإن المكوك سوف لن يصل إلى هدفه نتيجة لتأثير قوة الجاذبية. ولإيجاد سرعة الإفلات المطلوبة فإن ......

For the escape velocity the object will reach a final speed of $v_f = 0$ when $r_{max} = \infty$, therefore we substitute for $v_i = v_{esc}$ and we get

$$v_{esc} = \sqrt{\frac{2GM_e}{R_e}} \tag{8.29}$$

Note that the escape velocity does not depends on the mass of the object projected from the earth.

This equation can be used to evaluate the escape velocity from any planet in the universe if the mass and the radius of the planet are known.

Escape velocities for the planets	
Planet	v_{esc} **(km/s)**
Mercury	4.3
Venus	10.3
Earth	11.2
Moon	2.3
Mars	5.0
Jupiter	60
Saturn	36
Uranus	22
Neptune	24
Pluto	1.1
Sun	618

Table 8.1: Escape velocities for the planets

Example 8.5

(a) Calculate the minimum energy required to send a 3000kg spacecraft from the earth to a distance point in space where earth's gravity is negligible. (b) If the journey is to take three weeks, what average power would the engine have to supply?

Solution

(a) $$v_{esc} = \sqrt{\frac{2GM_e}{R_e}} = 1.12\times10^4\text{m/s}$$

$$K = \frac{1}{2}mv_{esc}^2 = \frac{1}{2}\times3000\times(1.12\times10^4)^2$$

$$= 1.88\times10^{11}\,J$$

(b) $$P_{av} = \frac{K}{\Delta t} = \frac{1.88\times10^{11}}{21\text{days}\times8.64\times10^4\,s/\text{day}} = 103kW$$

Example 8.6

A spaceship is fired from the Earth's surface with an initial speed of 2×10^4 m/s. What will its speed when it is very far from the Earth? (Neglect friction.)

Solution

Energy is conserved between the surface and the distant point

$$(K+U_g)_i + W_{nc} = (K+U_g)_f$$

$$\frac{1}{2}mv_i^2 - \frac{GM_E m}{R_E} + 0 = +\frac{1}{2}mv_f^2 - \frac{GM_E m}{\infty}$$

$$v_f^2 = v_i^2 - \frac{2GM_E}{R_E}$$

$$v_f^2 = v_i^2 - v_{esc}^2$$

$$v_f^2 = (2\times10^4)^2 - \frac{2(6.67\times10^{-11})^2(5.98\times10^{24})}{6.37\times10^6}$$

Thus,

$$v_f = 1.66\times10^4 \text{ m/s}$$

Example 8.7

Two planets of masses m_1 and m_2 and radii r_1 and r_2, respectively, are nearly at rest when they are an infinite distance apart. Because of their gravitational attraction, they head toward each other on a collision course. (a) When their center-to-center separation is d, find expressions for the speed of each planet and their relative velocity. (b) Find the kinetic energy of each planet just before they collide, if $m_1=2\times10^{24}$ kg, $m_2=8\times10^{24}$ kg, $r_1=3\times10^6$ m, and $r_2=2\times10^6$ m

Solution

(a) At infinite separation, U=0; and at rest, K=0. Since energy is conserved, we have

$$0=\frac{1}{2}m_1v_1^{\,2}+\frac{1}{2}m_2v_2^{\,2}-\frac{Gm_1m_2}{d} \qquad (1)$$

The initial momentum is zero and momentum is conserved. Therefore

$$0=m_1v_1-m_2v_2 \qquad (2)$$

Combine equations (1) and (2) to find v_1 and v_2

$$v_1=m_2\sqrt{\frac{2G}{d(m_1+m_2)}} \quad \text{and} \quad v_1=m_1\sqrt{\frac{2G}{d(m_1+m_2)}}$$

The relative velocity is

$$v_r=v_1-(v_2)=\sqrt{\frac{2G(m_1+m_2)}{d(m_1+m_2)}}$$

(b) Substitute for the given value for v_1 and v_2 we find that

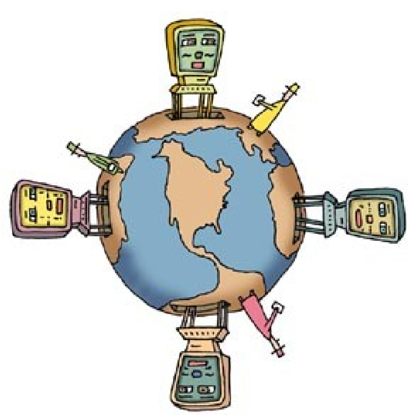
v_1 = 1.03×10^4 m/s and v_2= 2.58×10^3 m/s.

Therefore,

$$K_1=\frac{1}{2}m_1v_1^{\,2}=1.07\times10^{32}\text{ J}$$

and

$$K_2=\frac{1}{2}m_2v_2^{\,2}=2.67\times10^{31}\text{ J}$$



8.7 Problems

1. Two identical, isolated particles, each of mass 2 kg, are separated by a distance of 30 cm. What is the magnitude of the gravitational force of one particle on the other?

2. A 200-kg mass and a 500-kg mass are separated by a distance of 0.40 m. (a) Find the net gravitational force due to these masses acting on a 50-kg mass placed midway between them. (b) At what position (other than infinitely remote ones) would the 50-kg mass experience a net force of zero?

3. Three 5-kg masses are located at the corners of an equilateral triangle having sides 0.25 m in length. Determine the magnitude and direction of the resultant gravitational force on one of the masses due to the other two masses.

4. Two stars of masses M and $4M$ are separated by a distance d. Determine the location of a point measured from M at which the net force on a third mass would he zero.

5. Four particles are located at the corners of a rectangle as in Figure 8.11. Determine the x and y components of the resultant force acting on the particle of mass m.

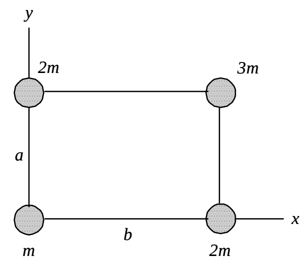


Figure 8.11

6. Calculate the acceleration of gravity at a point that is a distance R_e above the surface of the earth, where R_e is the radius of the earth.

7. Two objects attract each other with gravitational force of 1×10^{-8}N when separated by 20cm. If the total mass of the two objects is 5kg, what is the mass of each?

8. Compute the magnitude and direction of the gravitational field at a point P on the perpendicular bisect of two equal masses separated by 2a as shown in Figure 8.12.

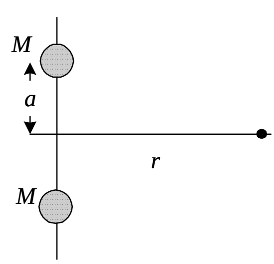


Figure 8.12

9. A satellite of the earth has a mass of 100 kg and is altitude of 2×10^6 m. (a) What is the potential energy of the satellite-earth system? (b) What is the magnitude of the force on the satellite?

10. A system consists of three particles, each of mass 5g, located at the corners of an equilateral triangle sides of 30 cm. (a) Calculate the potential energy of the system. (b) If the particles are released simultaneously, where will they collide?

11. How much energy is required to move a 1000-kg form the earth's surface to an altitude equal to twice the earth's radius?

12. Calculate the escape velocity from the moon, where M_m=7.36×10^{22}kg, R_m=1.74×10^{6}m

13. A spaceship is fired from the earth's surface with an initial speed of 2.0×10^{4}m/s. What will its speed when it is very far from the earth?

14. A 500-kg spaceship is in a circular orbit of radius $2R_e$ about the earth. (a) How much energy is required to transfer the spaceship to a circular orbit of radius $4R_e$? (b) Discuss the change in the potential energy, kinetic energy, and total energy.

15. (a) Calculate the minimum energy required to send a 3000-kg spacecraft from the earth to a distant point in space where earth's gravity is negligible. (b) If the journey is to take three weeks, what average power would the engines have to supply?

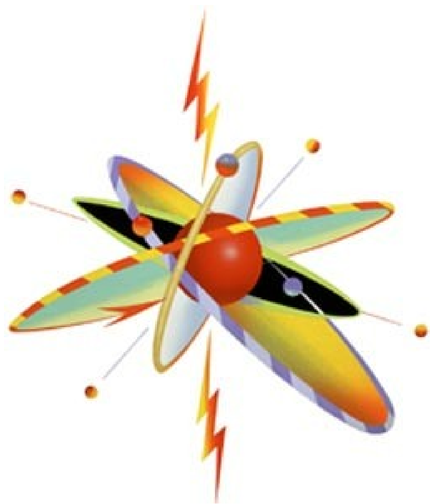
16. A rocket is fired vertically from the earth's surface and reaches a maximum altitude equal to three earth radii. What was the initial speed of the rocket? (Neglect the friction, the earth's rotation, and the earth's orbital motion.)

17. A satellite moves in a circular orbit around a planet just above its surface. Show that the orbital velocity v and escape velocity of the satellite are related by the expression

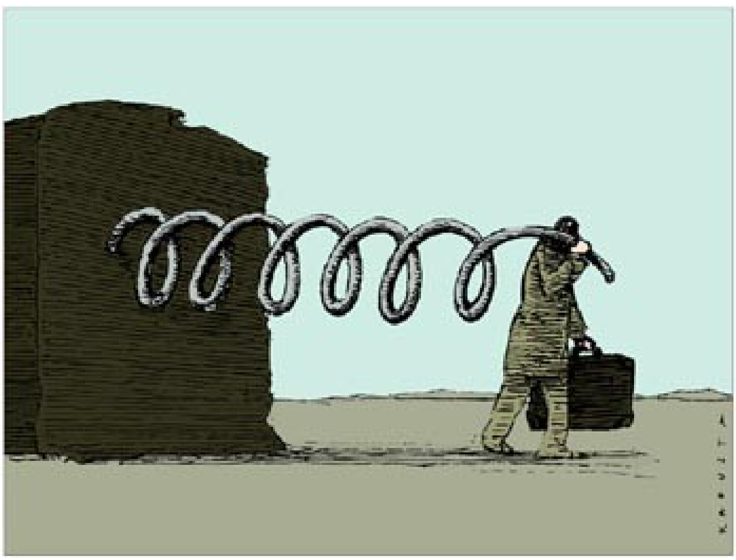
$$v_{esc} = \sqrt{2}v \,.$$

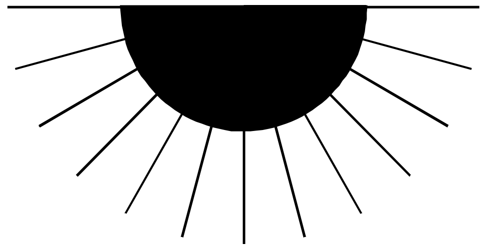
Chapter 9

Periodic Motion



الحركة الاهتزازية





PERIODIC MOTION

9.1 The periodic motion

درسنا في الفصول الأولى من هذا الكتاب عدة أنواع من الحركة مثل الحركة في بعد واحد والحركة في بعدين والحركة الدائرية، وفي هذا الفصل سوف ندرس نوعاً جديداً من أنواع الحركة وهو الحركة الدورية ***Periodic motion*** أو الحركة الاهتزازية ***Vibration motion*** أو الحركة التذبذبية ***Oscillatory motion***. وهو نوع من الحركة يعود فيه الجسم إلى موضعه الأصلي خلال فترة زمنية محددة تعرف باسم الزمن الدوري ***Periodic time*** أو بمعنى آخر هي حركة جسم حول موضع استقراره نتيجة لقوة استرجاعية ***Restoring force***. هناك العديد من الأمثلة على الحركة الاهتزازية مثل حركة بندول الساعة أو حركة جسم معلق في زنبرك كذلك دقات قلب الإنسان أو حركة الذرات في المواد الصلبة هذا بالإضافة إلى أمواج الراديو ودوائر التيار المتردد.

ولتبسيط المعالجة الرياضية لمثل هذا النوع من الحركة سنتعامل مع حالة خاصة وهي الحركة التوافقية البسيطة ***Simple Harmonic Motion*** والتي يهمل فيها الاحتكاك والقوى الخارجة المؤثرة.

9.2 Simple Harmonic Motion (SHM)

افترض وجود كتلة مثبت بها قلم كما في الشكل 9.1 ومعلقة بزنبرك، فإذا أعطيت إزاحة صغيرة للزنبرك فإنه سوف يتذبذب حول موضع استقراره، وإذا تحركت ورقة رسم بياني بسرعة ثابتة مقابلة للقلم المثبت في الزنبرك فإن الشكل التالي سيظهر على الورقة.

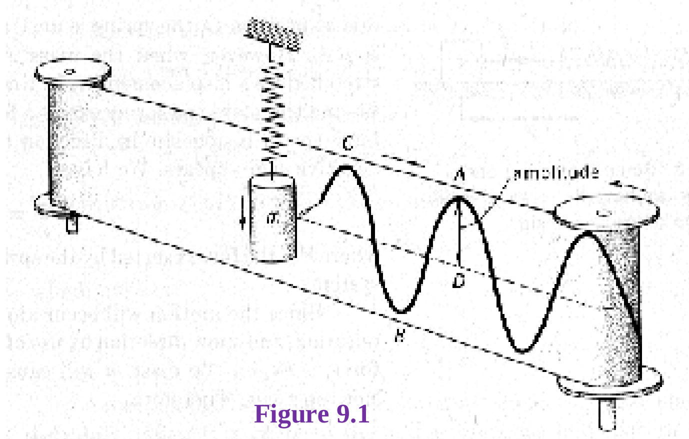


Figure 9.1

This motion is described as simple harmonic motion where the displacement x is described by the periodic function as follow,

$$x = A\cos(wt + d) \qquad (9.1)$$

where A, w , and d are the constant of the motion.

A is the amplitude of the motion and defined as the maximum displacement either in the positive x axis of in the negative x axis.

w is the angular frequency.

d is the phase constant or the phase angle which determine the initial displacement and velocity

$wt + d$ is called the phase of the motion and used to compare the motions of two simple harmonic motion.

9.2.1 The periodic time

The periodic time is defined as the time required for the particle to go through one cycle $2p$ of its motion.

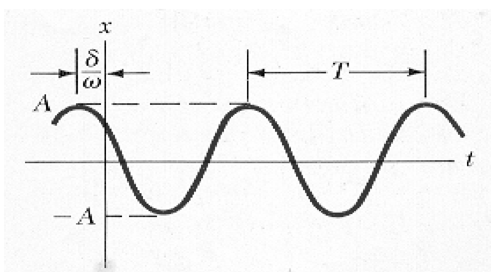


Figure 9.2

The value of x at time t is equals to the value of x at time t+T, therefore we can write

$$wt + d + 2p = w(t + T) + d$$

$$\therefore 2p = wT \qquad (9.2)$$

Hence the periodic time T is

$$T = \frac{2p}{w} \qquad (9.3)$$

The unit of periodic time is sec.

9.2.2 The frequency of the motion

Another important physical quantity to describe the periodic motion is the frequency. The frequency *f* is the inverse of the periodic time.

$$f = \frac{1}{T} = \frac{w}{2p} \qquad (9.4)$$

The unit of frequency is cycles/sec which is known as hertz (Hz).

9.2.3 The angular frequency

We can relate the angular w with the frequency and periodic time.

$$w = 2p\,f = \frac{2p}{T} \qquad (9.5)$$

The unit of w is rad/sec.

9.2.4 The velocity and acceleration of the periodic motion

By differentiating the displacement equation with respect to time we get the equation of velocity,

$$v = \frac{dx}{dt} = -wA\sin(wt + d) \qquad (9.6)$$

By differentiating the velocity equation with respect to time we get the equation of acceleration,

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} - w^2 A\cos(wt + d) \qquad (9.7)$$

$$\mathbf{Q}\; x = A\cos(wt + d)$$

$$\therefore a = -w^2 x \qquad (9.8)$$

وهذا يعني أن عجلة جسم يتحرك حركة توافقية بسيطة تتناسب طردياً مع الإزاحة وفـــي الاتجاه المعاكس. وعند إثبات أن الجسم يتحرك حركة توافقية بسيطة يجب أن نثبت تلـــك العلاقة 9.8.

From the Figure 9.3 where curves representing the displacement, velocity and acceleration as a function of time are plotted. These curves show the phase different between them. The velocity curve show phase differs by 90^o with displacement. That when the velocity is a maximum or a minimum the displacement is zero. The acceleration curve show phase differs by 180^o with displacement. That mean when the acceleration is a maximum the displacement is a minimum.

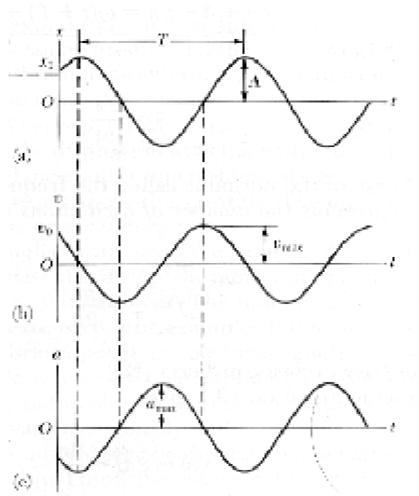


Figure 9.3

9.2.5 The maximum velocity and the maximum acceleration

From equation (9.6) and equation (9.7) the maximum is given by

$$v_{max} = wA \qquad (9.9)$$

$$a_{max} = w^2 A \qquad (9.10)$$

Since the maximum value for the cosine or the sine functions is between ± 1

Example 9.1

The displacement of a body is given by the expression

$$x = (8cm)\cos(2t + p/3)$$

where *x* is in cm and *t* in second. Calculate (a) the velocity and acceleration at *t*=$p/2$ s, (b) the maximum speed and the earliest time (*t*>0) at which the particle has this speed, and (c) the maximum acceleration and the earliest time (*t*>0) at which the particle has this acceleration.

Solution

(a) $v = -(16cm/s)\sin(2t + p/3)$ at $t = p/2$, v = 13.9cm/s

$a = -(32cm/s^2)\cos(2t + p/3)$ at $t = p/2$, a = 16 cm/s^2

(b) $v_{\max} = wA$=16cm/s, This occurs when $t = \frac{1}{2}\left[\sin^{-1}(1) - p/3\right]$= 0.262s

(c) $a_{\max} = w^2 A$=32cm/s^2, This occurs when $t = \frac{1}{2}\left[\cos^{-1}(-1) - p/3\right]$= 1.05s

9.3 The amplitude of motion from the initial condition

The initial condition of the simple harmonic motion is determine by the displacement x_o and the velocity v_o at time t = 0, therefore the equations of motions $x = A\cos(wt + d)$ and $v = -wA\sin(wt + d)$ give

$$x_o = A\cos d \qquad\qquad v_o = -wA\sin d$$

Dividing these two equations we get

$$\frac{v_o}{x_o} = -w\tan d \tag{9.11}$$

$$\tan d = -\frac{v_o}{w x_o} \tag{9.12}$$

If we square both equation (9.11) and (9.12) and take the sum we get

$$x_o^{\,2} + \left(\frac{v_o}{w}\right)^2 = A^2\cos^2 d + A^2\sin^2 d$$

$$A = \sqrt{x_o^{\,2} + \left(\frac{v_o}{w}\right)^2} \tag{9.13}$$

ومن هاتين المعادلتين (9.12) و (9.13) يمكن إيجاد سعة الحركة A والطـور d مـن الشروط الابتدائية للحركة وهي x_o, v_o, w.

9.4 Mass attached to a spring

When a mass *m* is attached to a spring, the mass is free to move on a horizontal frictionless surface with simple harmonic motion. To prove that we need to know a bout the restoring force of the spring. Suppose the mass is stretched by small displacement *x* from the equilibrium position at *x*=0. The spring will exert a force on the mass to bring it back to *x*=0, this called restoring force and can be found from Hook's law,

$$F = -kx \qquad (9.14)$$

where *k* is the spring constant and has unit (N/m), the negative sine indicates that the restoring force is always in opposite direction to the displacement *x*.

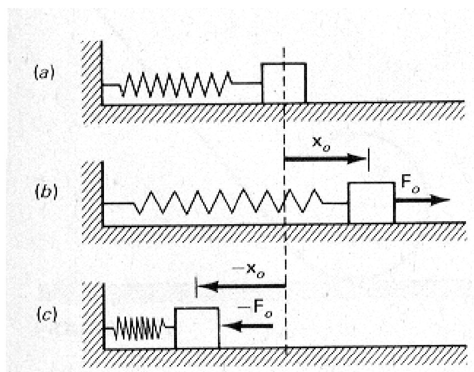


Figure 9.4

Applying Newton's second law of motion

$$F = -kx = ma$$

Hence,

$$a = -\frac{k}{m}x \qquad (9.15)$$

نلاحظ أن عجلة جسم تتناسب طردياً مع الإزاحة وفي الاتجاه المعاكس وهـذا يثبـت أن حركة جسم معلق في زنبرك هي حركة توافقية بسيطة.

Compare equation (9.15) with Equation (9.8) we conclude that,

$$w^2 = \frac{k}{m} \qquad (9.16)$$

Therefore,

$$T = \frac{2p}{w} = 2p\sqrt{\frac{m}{k}} \qquad (9.17)$$

$$f = \frac{1}{T} = \frac{1}{2p}\sqrt{\frac{k}{m}} \qquad (9.18)$$

نلاحظ أن كلاً من الزمن الدوري والتردد يعتمدان على كتلة الجسم وعلى ثابت الزنبرك فقط. كما أن الزمن الدوري يزداد بزيادة الكتلة ويقل بزيادة ثابت الزنبرك.

Example 9.2

A 5kg mass attached to a spring of force constant 8N/m vibrates with simple harmonic motion with amplitude of 10 cm. Calculate (a) the maximum value of its speed and acceleration, (b) the speed and acceleration when the mass is at *x*=6cm from the equilibrium position, and (c) the time it takes the mass to move from *x*=0 to *x*=8cm.

Solution

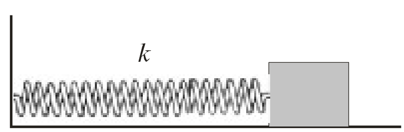
(a) $$w = \sqrt{\frac{k}{m}} = \sqrt{\frac{8}{0.5}} = 4s^{-1}$$

Therefore the position is given by x = (10cm) sin (4t). From this we find that

v = (40cm/s) cos (4t) v_{max} = 40cm/s

a = - (160cm/s^2) sin (4t) a_{max} = 160cm/s^2

(b) $$t = \frac{1}{4}\sin^{-1}\left(\frac{x}{10}\right)$$



where x=6cm, t=0.161s and we find

v = (40cm/s) cos (4×0.161) = 32cm/s

a = - (160cm/s^2) sin (4×0.161) = -96cm/s^2

(c) Using $t = \frac{1}{4}\sin^{-1}\left(\frac{x}{10}\right)$

when x=0, t=0 and when x=8, t=0.232s. Therefore,

$$\Delta t = 0.232s$$

9.5 Total energy of the simple harmonic motion

The total mechanical energy of the mass-spring system is the sum of the kinetic energy and the potential energy

$$E = K + U \qquad (9.19)$$

Since we consider the motion is frictionless therefore the total mechanical energy is conserved.

The kinetic energy is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}mw^2 A^2 \sin^2(wt + d) \qquad (9.20)$$

The potential energy is

$$U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2 \cos^2(wt + d) \qquad (9.21)$$

The total mechanical energy is

$$E = K + U = \frac{1}{2}kA^2\left[\sin^2(wt + d) + \cos^2(wt + d)\right]$$

Hence

$$E = \frac{1}{2}kA^2 \qquad (9.23)$$

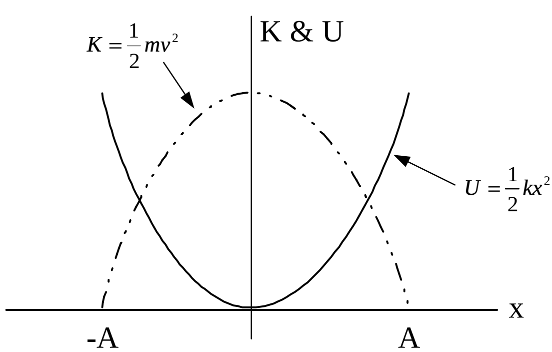


Figure 9.5

نلاحظ أن الطاقة الميكانيكية الكلية ثابتة أي لا تتغير مع الزمن بينما الطاقة الحركية وطاقة الوضع يتغيران مع الزمن. ذلك لأن الطاقة الكلية تعتمد على ثابت الزنبرك وسعة الحركة وكلاهما ثابت.

كما يمكن إيجاد مقدار السرعة كدالة في الإزاحة من مبدأ المحافظة على الطاقــة الكليــة. حيث أن الطاقة الكلية تعطى بالعلاقة

$$E = K + U = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2 \qquad (9.24)$$

$$v = \sqrt{\frac{k}{m}\left(A^2 - x^2\right)} = w\sqrt{\left(A^2 - x^2\right)} \qquad (9.25)$$

Example 9.3

A mass of 0.5 kg connected to a light spring of force constant 20N/m oscillate on a horizontal, frictionless surface as shown in Figure 9.6. (a) Calculate the total energy of the system and the maximum speed of the mass if the amplitude of the motion is 3cm. (b) What is the velocity of the mass when the displacement is equal to 2cm? (c) Compute the kinetic and potential energies of the system the displacement is equal to 2 cm.

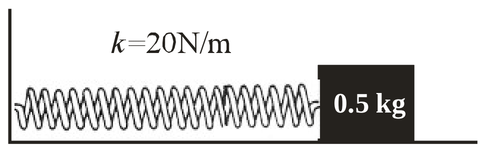


Figure 9.6

Solution

(a) $$E = \frac{1}{2}kA^2 = \frac{1}{2}\times 20\times(3\times10^{-2})^2 = 9\times10^{-3}\,J$$

The maximum speed v_{max} is when $x = 0$, $E = \frac{1}{2}mv_{max}^2$ therefore

$$E = \frac{1}{2}mv_{max}^2 = 9\times10^{-3}\,J$$

$$\therefore v_{max} = \sqrt{\frac{18\times10^{-3}}{0.5}} = 0.19\,m/s$$

(b) The velocity of the mass when the displacement is 2cm is given by

$$v = \sqrt{\frac{k}{m}\left(A^2 - x^2\right)}$$

$$v = 0.141 \text{ m/s}$$

(c) The kinetic energy $K = \frac{1}{2}mv^2 = \frac{1}{2}\times 0.5\times 0.141 = 5\times 10^{-3}\,J$

The potential energy $U = \frac{1}{2}kx^2 = \frac{1}{2}\times 20\times (2\times 10^{-2})^2 = 4\times 10^{-3}\,J$

Example 9.4

A 2kg block reset on horizontal frictionless surface as shown in Figure 9.7, attached to the right end of a spring whose left end is fixed. The block is displaced 5cm to the right from its equilibrium position and held fix at this position by external force of 10N. (a) What is the spring's force constant? (b) The block is then released. What is the period of the block's oscillation? (c) What are the kinetic energy of the block and the potential energy of the spring at time *t*=p/15s?

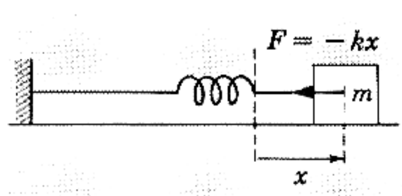


Figure 9.7

Solution

(a) the spring's force constant k is

$$k = -\frac{F}{x} = -\frac{-10}{0.05} = 200\,N/m$$

(b) period of the block's oscillation is

$$T = 2p\sqrt{\frac{m}{k}} = 2p\sqrt{\frac{2}{200}} = \frac{p}{5}s$$

(c) the kinetic energy is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}mw^2A^2\sin^2(wt+d) = \frac{1}{2}kA^2\sin^2(wt+d)$$

since the motion started from rest at A=0.05m, then d =0, therefore the kinetic energy at time t=π/15s

$$K = \frac{1}{2}200\times(0.05)^2\sin^2\left(\frac{2p}{p/5}\times\frac{p}{15}\right)$$

$$= 0.25\sin^2 120^o$$

$$= 0.19J$$

the potential energy of the spring at time t=π/15s is

$$U = \frac{1}{2}kA^2 - K$$

$$= \frac{1}{2}\times200\times(0.05)^2 - 0.19 = 0.06J$$

Example 9.5

A particle execute simple harmonic motion with amplitude of 3cm. At what displacement from the midpoint of its motion will its speed equal one half of its maximum speed?

Solution

From equation (9.13)

$$A = \sqrt{x_o{}^2 + \left(\frac{v_o}{w}\right)^2} \qquad \rightarrow v^2 + w^2x^2 = w^2A^2$$

$$v_{\max} = wA \quad \text{and} \quad v = \frac{v_{\max}}{2} = \frac{wA}{2}$$

so

$$\frac{1}{2}w^2A^2 + w^2x^2 = A^2x^2$$

From this we find

$$x^2 = \frac{3A^2}{4} \quad \text{and} \quad x = \pm\frac{A\sqrt{3}}{2} = \pm\frac{3\sqrt{3}}{2} = \pm 2.6cm$$

9.6 The simple pendulum

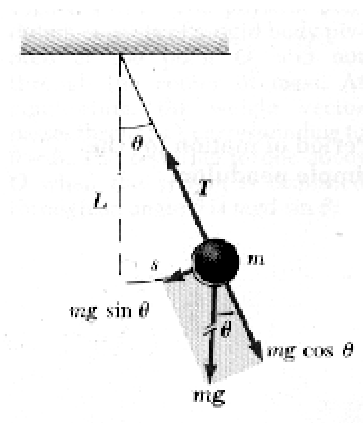


Figure 9.8

The simple pendulum consists of a point mass *m* suspended by a light string of length *l*. We should prove that the simple pendulum exhibit a simple harmonic motion.

If the mass is displaced by small angle θ as shown in Figure 9.8 and left to oscillate vertically under the effect of gravity.

The force acting on the mass is the weight *mg* and the tension in the string *T*. The force *mg* has two component *mg* cos*q* which is equal to the tension force *T*, and the tangential component *mg* sinθ which is the restoring force F_r.

$$T = mg\cos q \qquad (9.26)$$

$$F_r = -mg\sin q \qquad (9.27)$$

The minus sine indicates that the restoring force F_r acts towered the equilibrium position.

If *q* is small angle the mass displacement along the circular arc can be approximated by the horizontal displacement s from the equilibrium position, therefore,

$$s = q\ l \qquad (9.28)$$

Thus for small θ

$$\frac{s}{l} \approx \sin q \approx q \qquad (9.29)$$

Applying Newton's second law of motion

$$F_r = -mg \sin q = ma$$

$$-mg\frac{s}{l} = ma \qquad \rightarrow \qquad a = -\frac{g}{l}s \qquad (9.30)$$

نلاحظ أن عجلة جسم تتناسب طردياً مع الإزاحة s وفي الاتجاه المعاكس وهذا يثبت أن حركة جسم معلق بسلك هي حركة توافقية بسيطة.

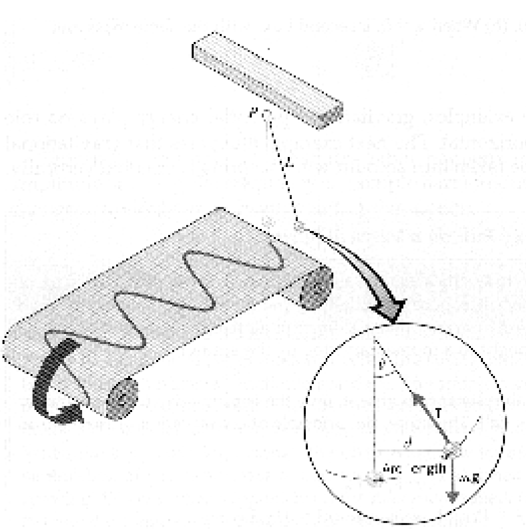
Compare equation (9.30) with Equation (9.8) we conclude that,

$$w^2 = \frac{g}{l} \qquad (9.31)$$

Therefore,

$$T = \frac{2p}{w} = 2p\sqrt{\frac{l}{g}} \qquad (9.32)$$

$$f = \frac{1}{T} = \frac{1}{2p}\sqrt{\frac{g}{l}} \qquad (9.33)$$



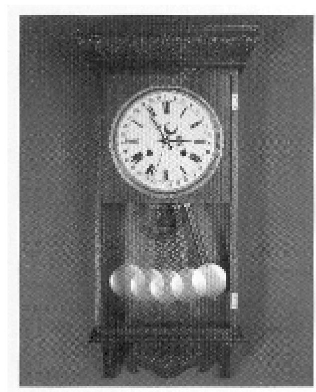
نلاحظ أن كلاً من الزمن الدوري والتـــردد لا يعتمدان على كتلة الجسم ولكن يعتمدان علـــى طول السلك l وعلى عجلة الجاذبية الأرضية. كما أن الزمن الدوري يزداد بزيـــادة طـــول السلك.

لاحظ أيضاً أن الزمن الدوري يعتمـــد علـــى عجلة الجاذبية الأرضية إذا كان البندول على سطح الأرض وفي حالة نقل البنـــدول إلـــى سطح القمر مثلاً فإن الزمن الـــدوري لـــنفس البندول سوف يزداد لأن عجلة جاذبية القمـــر أقل من الأرض.

Example 9.6

Determine the length of a simple pendulum that will swing back and forth in simple harmonic motion with a period of 1.00s.

Solution



From the period of the pendulum we can obtain the length of the pendulum

$$T = 2p\sqrt{\frac{l}{g}}$$

Solving for *l*

$$l = \frac{gT^2}{4p^2} = \frac{9.8\times(1)^2}{4p^2} = 0.248m$$

وهذا هو الطول المطلوب لبندول الساعة.

Example 9.7

Find the height of a building using a long pendulum suspended from the top of the building to the bottom and its period of oscillation is 12s. What is the period of oscillation of the pendulum if it was taken to the moon? where the acceleration of gravity is 1.67m/s^2.

Solution

From the period of the pendulum we can obtain the height the building

$$T = 2p\sqrt{\frac{l}{g}}$$

Solving for *l*

$$l = \frac{gT^2}{4\pi^2} = \frac{9.8\times(12)^2}{4\pi^2} = 35.7m$$

The period of oscillation of the pendulum on the moon is

$$T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{35.7}{1.67}} = 29.1s$$

لاحظ أن الزمن الدوري للبندول على سطح القمر يساوي 29.1s بينما على سطح الأرض يساوي 12s، وذلك لاعتماد الزمن الدوري للبندول على عجلة الجاذبية.

Example 9.8

A simple pendulum has a mass of 0.25kg and a length of 1m. It is displaced through an angle of 15^o and then released. What are (a) the maximum speed, (b) the maximum angular acceleration, and (c) the maximum restoring force?

Solution

Since θ=15^o is small enough that we can treat this motion as simple harmonic motion. The angular frequency characterizing this motion is

$$\omega = \sqrt{\frac{g}{l}} = \sqrt{\frac{9.8}{1}} = 3.13\ \text{rad/s}$$

The amplitude of the motion is

$$A = L\,\theta = 1 \times 0.262 = 0.262\ \text{m}$$

(a) The maximum speed is

$$v_{max} = \omega A = 3.13\times0.262 = 0.82\ \text{m}$$

(b) $a_{max} = \omega^2 A = (3.13)^2 \times 0.262 = 2.57\ \text{m/s}^2$

The maximum angular acceleration is

$$\alpha = \frac{a}{r} = \frac{2.57}{1} = 2.57\ \text{rad/s}^2$$

(c) $F = ma = 0.25 \times 2.57 = 0.641\ \text{N}$

9.7 The torsional pendulum

Figure 9.9 shows a typical torsional pendulum. It obeys Hooke's law when the body is twisted through some angle q, the twisted wire exerts a restoring torque on the body proportional to the angular displacement. Therefore we write the restoring torque,

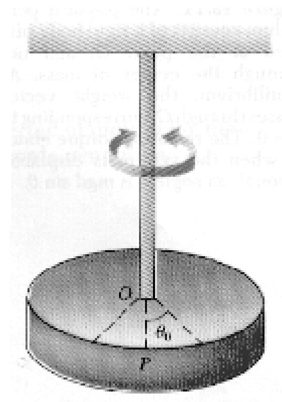


Figure 9.9

$$t = -kq \qquad (9.34)$$

where k is the torsion constant of the system (How could you measure k?). From the moment of inertia I of the system we can write the torque as $t = Ia$

$$\therefore t = -kq = Ia \qquad (9.35)$$

The acceleration is given by

$$a = -\frac{k}{I}q \qquad (9.36)$$

نلاحظ أن عجلة جسم تتناسب طردياً مع الإزاحة الزاوية q وفي الاتجاه المعاكس وهـذا يثبت أن حركة بندول اللي هي حركة توافقية بسيطة.

Compare equation (9.36) with Equation (9.8) we conclude that,

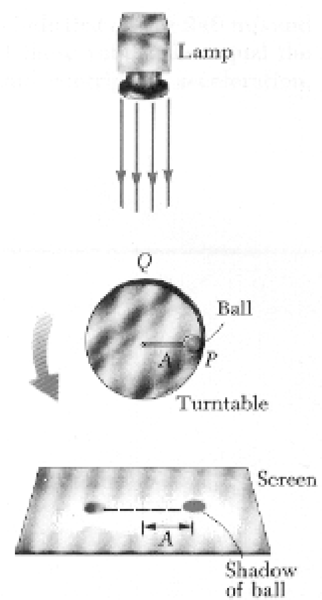
$$w^2 = \frac{k}{I} \qquad (9.37)$$

Therefore,

$$T = \frac{2p}{w} = 2p\sqrt{\frac{I}{k}} \qquad (9.38)$$

$$f = \frac{1}{T} = \frac{1}{2p}\sqrt{\frac{k}{I}} \qquad (9.39)$$

9.8 Representing the simple harmonic motion with the circular motion



يمكن تمثيل الحركة التوافقية البسيطة هندسياً بحركة مسقط نقطة على مسار دائري كما في الشكل المقابل 9.10 حيث تم تثبيت جسم على قرص دائري رأسي. عند دوران القرص فإن الجسم سيأخذ مساراً دائرياً، وإذا تم تسليط ضوء على ارتفاع مناسب من القرص وتم استقبال الضوء على حائل أسفل القرص كما في الشكل فإن ظل الجسم سيتحرك حركة توافقية بسيطة حول مركز القرص على الحائل.

في الشكل 9.10 يوضح شكل منحنى التغير للإزاحة الأفقية لمسقط الجسم على الحائل مع الزمن حيث نلاحظ أن شكل المنحنى جيبي وله نفس خصائص الحركة التوافقية البسيطة.

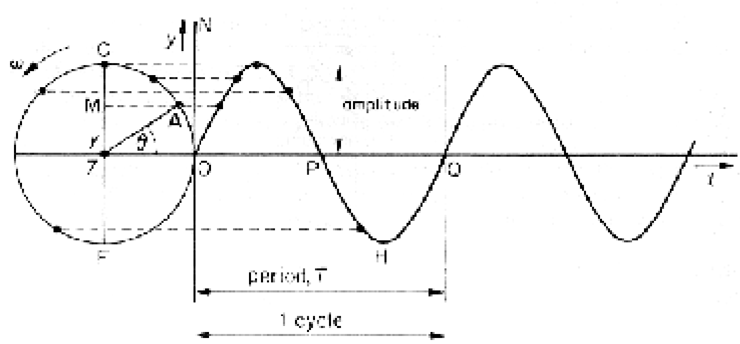


Figure 9.10

The conclusion is the simple harmonic motion along a straight line can be represented by the projection of uniform circular motion along the diameter.

Example 9.9

A particle rotates counterclockwise in a circle of radius 3m with a constant angular speed of 8rad/s. At *t*=0, the particle has an *x* coordinate of 2m. (a) Determine the *x* coordinate as a function of time. (b) Find the velocity and acceleration as a function of time. (c) Find the maximum velocity and acceleration.

Solution

(a) The radius of the circular motion is equal to the amplitude of the periodic motion *i.e.* A = 3m, and w =8rad/s, therefore the equation of the motion is

$$x = A\cos(wt + d) = 3\cos(8t + d)$$

From the initial conditions (x = 2 at t = 0) we can evaluate d ,

$$2 = 3\cos(0 + d)$$

$$\therefore d = \cos^{-1}\left(\frac{2}{3}\right) = 48^o = 0.841 rad$$

The x coordinate as a function of time is

$$x = 3\cos(8t + 0.841)$$

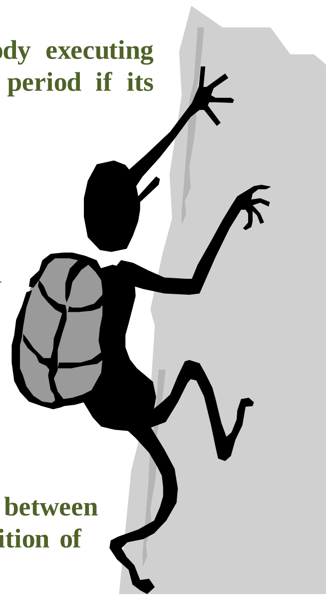
(b) $v = -24 \sin(8t + 0.841)$

$a = -192 \cos(8t + 0.841)$

(c) v_{max} = 240 m/s

a_{max} = 192 m/s^2

9.8 Question with solution



1. What is the total distance travelled by a body executing simple harmonic motion in a time equal to its period if its amplitude is A?

Answer: *It travels a distance of 2A.*

2. If the coordinate of a particle varies as x = -A coswt, what is the phase constant d ? At what position does the particle begin its motion?

Answer: d *= ± p /2; At t = 0, x = -A*

3. Does the displacement of an oscillating particle between *t* = 0 and a later time t necessarily equal the position of the particle at time *t*? Explain.

Answer: The two will be equal if the origin of coordinates coincides with the position of the particle at t = 0.

4. Can the amplitude *A* and phase constant d be determined for an oscillator if only the position is specified at *t* = 0? Explain.

Answer: *No. It is necessary to know both the position and velocity at t = 0.*

5. If a mass-spring system is hung vertically and set into oscillation, why does the motion eventually stop?

Answer: *There will always be some friction present, such as air resistance.*

6. Explain why the kinetic and potential energies of a mass-spring system can never be negative.

Answer: *The kinetic energy is proportional to the square of the speed, while the potential energy is proportional to the square of the displacement. Therefore, both must be positive quantities.*

7. A mass-spring system undergoes simple harmonic motion with an amplitude *A*. Does the total energy change if the mass is doubled but the amplitude is not changed? Do the kinetic and potential energies depend on the mass? Explain.

Answer: *No. Since $E = 1/2kA^2$, changing the mass has no effect on the total energy. However, the kinetic energy depends on the mass.*

8. What happens to the period of a simple pendulum if its length is doubled? What happens to the period if the mass that is suspended is doubled?

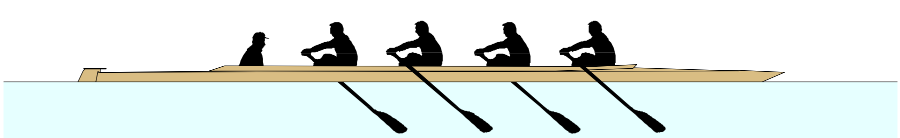
Answer: *Since $T = 2p\sqrt{\frac{L}{g}}$ doubling L will increase T by a factor of $\sqrt{2}$. Doubling the mass will not change the period.*

9. A simple pendulum is suspended from the ceiling of a stationary elevator, and the period is determined. Describe the changes, if any, in the period if the elevator (a) accelerates upward, (b) accelerates downward, and (c) moves with constant velocity.

Answer: *If it accelerates upwards, the effective "g" is greater than the acceleration of gravity, so the period decreases. If it accelerates downward, the effective "g" is less than the acceleration of gravity, so the period increases. If it moves with constant velocity, the period does not change. (If the pendulum is in free fall, it does not oscillate.)*

10. A simple pendulum undergoes simple harmonic motion when q is small. Will the motion be periodic if q is large? How does the period of motion change as q increases?

Answer: *Yes. The period will increase as the amplitude of motion increases.*



9.9 Problems

1. The displacement of a particle is given by the expression

$$x = (4\text{ m})\cos(3\pi t + p),$$

where x is in m and in t in s. Determine (a) the frequency and period of motion, (b) the amplitude of the motion, (c) phase constant, and (d) the position of the partial t=0.

2. For the particle described in Problem 1, determine (a) the velocity at any time t, (b) the acceleration at any time, (c) the maximum velocity and maximum acceleration, and (d) the velocity and acceleration $t = 0$.

3. A particle oscillates with simple harmonic motion such that its displacement varies according to the expression as $x = (5\text{ cm})\cos(2t + \pi/6)$, where x is cm and t is in s. At $t = 0$, find (a) the displacement the particle, (b) its velocity, and (c) its acceleration. (d) Find the period and amplitude of the motion.

4. A particle moving with simple harmonic motion travels a total distance of 20 cm in each cycle of its motion, and its maximum acceleration is 50 m/s^2 Find (a) the angular frequency of the motion, and (b) the maximum speed of the particle.

5. A weight of 0.2 N is hung from a spring with a force constant $k = 6$ N/m. How much is the spring displacement?

6. The frequency of vibration of a mass-spring system is 5 Hz when a 4g mass is attached to the spring. What is the force constant of the spring?

7. A 1-kg mass attached to a spring of force constant 25N/m oscillates on a horizontal, frictionless surface. At $t = 0$, the mass is released from rest at x = -3 cm. (That is, the spring is compressed by 3 cm.) Find (a) the period of its motion, (b) the maximum values of its speed and acceleration, and (c) the displacement, velocity, and acceleration as functions of time.

8. A simple harmonic oscillator takes 12 s to undergo 5 complete vibrations. Find (a) the period of its motion, (b) the frequency in Hz, and (c) the angular frequency in rad/s.

9. A mass-spring system oscillates such that the displacement is given by $x = (0.25\text{ m})\cos(2\pi t)$. (a) Find the speed and acceleration of the mass when x = 0.10 m. (b) Determine the maximum speed and maximum acceleration.

10. A 0.5-kg mass attached to a spring of force constant 8 N/m vibrates with simple harmonic

motion with an amplitude of 10 cm. Calculate (a) the maximum value of its speed and acceleration, (b) the speed and acceleration when the mass is at $x = 6$ cm from the equilibrium position, and (c) the time it takes the mass to move from $x = 0$ to $x = 8$ cm.

11. A 200-g mass is attached to a spring and executes simple harmonic motion with a period of 0.25 s. If the total energy of the system is 2 J, find (a) the force constant of the spring and (b) the amplitude of the motion.

12. **A** mass-spring system oscillates with an amplitude of 3.5 cm. If the spring constant is 250 N/m and the mass is 0.5 kg, determine (a) the mechanical energy of the system, (b) the maximum speed of the mass, and the maximum acceleration.

13. A particle executes simple harmonic motion with an amplitude of 3.0 cm. At what displacement from the midpoint of its motion will its speed equal one half of its maximum speed?

14. A simple pendulum has a period of 2.50 s. (a) what is its length? (b) What would its period be on the moon where $g_m = 1.67$ m/s^2?

15. Calculate the frequency and period of a simple pendulum of length 10 m.

16. If the length of a simple pendulum is quadrupled, what happens to (a) its frequency and (b) its period?

17. A simple pendulum 2.00 m in length oscillates in a location where g = 9.80 m/s^2. How many complete oscillations will it make in 5 min?

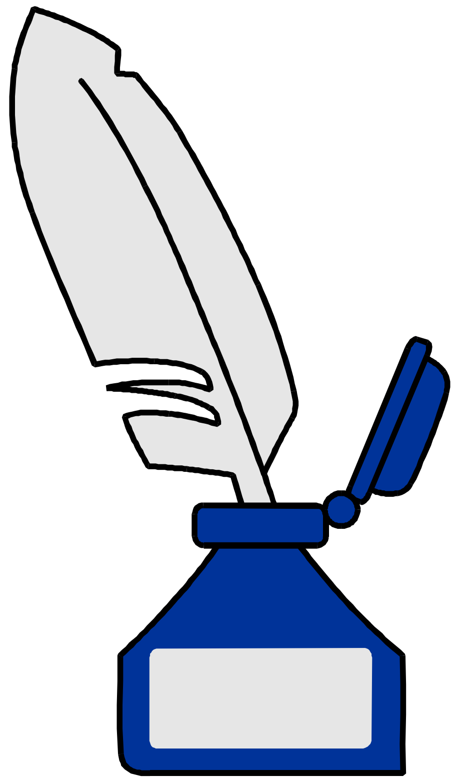
18. A simple pendulum has a mass of 0. 25 kg and a length of 1 m. It is displaced through an angle of 15^o and then released. What is (a) the maximum velocity? (b) the maximum angular acceleration? (c) the maximum restoring force?

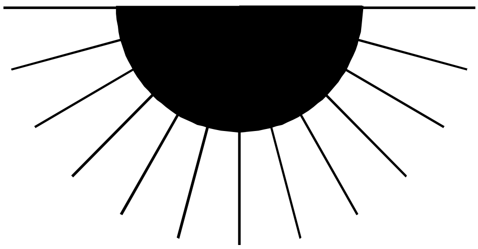
19. A simple pendulum has a length of 3m. Determine the change in its period if it is taken from a point where $g = 9.8$m/s^2 to a higher elevation, where the acceleration of gravity decrease to $g = 9.79$m/s^2.

Chapter 10

Fluid Mechanics

ميكانيكا الموائع





FLUID MECHANICS

10.1 Fluid Mechanics

توجد المادة في ثلاث حالات هي الحالة الصلبة والحالة السائلة والحالة الغازية. وتتأثر الحالات الثلاث بالقوى المؤثرة عليها بصور مختلفة ففي حين أن المواد الصلبة تحافظ على شكلها فإن المواد السائلة تأخذ دائماً شكل الوعاء الذي تحتويه مع المحافظة على الحجم أما المواد الغازية فإنها تأخذ شكل الوعاء الذي يحتويها وتنتشر فيه لتملؤه وهذا يعني أن المواد الغازية لا تحافظ على شكلها أو حجمها. وحالة كل من المواد السائلة والمواد الغازية تعرف بالموائع لأنهما قابلان للحركة (التدفق Flow) تحت تأثير قوة خارجية.

دراسة ميكانيكا الموائع من المواضيع المهمة في الفيزياء وتدخل في العديد من المجالات العملية مثل هندسة الطيران وبناء السدود والجسور وطرق الري والكثير من العلوم التطبيقية.

سيتم معالجة ميكانيكا الموائع باستخدام قوانين نيوتن للحركة ونظرية الشغل والطاقة، وسوف نتعرض إلى كميات فيزيائية جديدة مثل الضغط والكثافة.

10.2 Density and Pressure

The density of a substance is defined as its mass per unit volume.

$$r = \frac{m}{V} \qquad (10.1)$$

where r is the density, m is the mass of the substance and V is the volume

The unit of density in SI unit system is kg/m^3.

Density of some substances

Substance	r (kg/m^3)	Substance	r (kg/m^3)
Ice	0.917×10^3	Water	1×10^3
Aluminum	2.7×10^3	Glycerine	1.26×10^3
Iron	7.86×10^3	Ethyl alcohol	0.8×10^3
Copper	8.92×10^3	Benzene	0.88×10^3
Silver	10.5×10^3	Mercury	13.6×10^3
Lead	11.3×10^3	Air	1.29
Gold	19.3×10^3	Oxygen	1.43
Platinum	21.4×10^3	Hydrogen	910^3
		Helium	1.8×10^3

Table: 10.1

The pressure at some point in the fluid is defined as the ratio of the normal force to the area.

$$P = \frac{F}{A} \qquad (10.2)$$

The pressure has a unit of N/m^2 in the SI unit system, which is commonly known as Pascal (pa).

1 Pa = 1 N/m^2

10.3 Variation of pressure with depth

سنقوم بإيجاد علاقة تربط بين الضغط عند أي ارتفــاع في السائل مع ملاحظة أن جميع النقاط التي لها نفــس الارتفاع لها نفس قيمة الضغط وهذا يعني أن السائل في حالة اتزان ***Equilibrium***. في الشكل المقابل وعاء يحتوي على سائل في حالة اتــزان كثافتــه r فــإذا افترضنا شريحة من السائل لها سمك dy ومســاحة A فإن القوة المؤثرة على هذه الشريحة هي قــوة ضـــغط السائل على السطح السفلي للشريحة PA وقوة ضـــغط السائل على الســطح العلــوي للشـــريحة $(P+dP)A$.

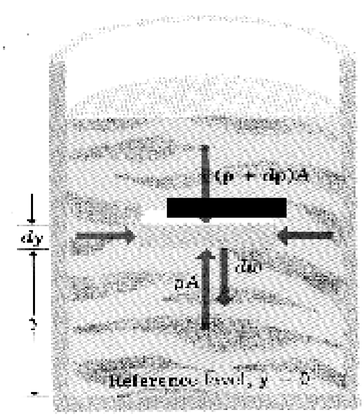


Figure 10.1

بالإضافة إلى قوة وزن الشريحة dW والتي تحسب من كتلة الشريحة في عجلة الجاذبية الأرضية.

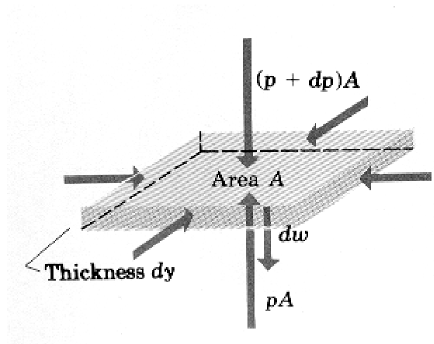


Figure 10.2

The upward force = PA

The downward force = $(P+dP)A$

The weight = $r\ dV\, g = r\ gAdy$

From the equilibrium the net forces is zero, therefore,

$$\sum F_y = 0$$

$$\sum F_y = PA - (P + dP)A - r\ gAdy = 0$$

$$\therefore \frac{dP}{dy} = -r\ g$$

والإشارة السالبة تعني أن زيادة الارتفاع dy يؤدي إلى نقصان الضغط. وبإجراء عملية التكامل لإيجاد التغير في الضغط بين نقطتين P_1 عند ارتفاع y_1 و P_2 عند ارتفاع y_2 نجد أن:

$$\int_{P_1}^{P_2} dP = -r\ g \int_{y_1}^{y_2} dy$$

$$\therefore P_2 - P_1 = -r\ g\left(y_2 - y_1\right) \qquad (10.3)$$

إذا كان سطح السائل يقع عند y_2 فإن $(y_2-y_1)=h$، والضغط P_2 عند السطح هو الضغط الجوي P_a فإن الضغط عند أية نقطة على عمق h من سطح السائل تعطى بالعلاقة

$$P_1 - P_a = r\ gh \qquad (10.4)$$

$$\therefore P_1 = P_a + r\ gh \qquad (10.5)$$

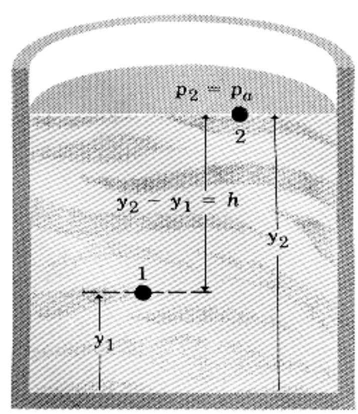


Figure 10.3

Atmospheric pressure $P_a = 1.01\times10^5$ Pascal

لنفرض أن الضغط عند أية نقطة داخل السائل P_1 هو الضغط P والتي تسمى الضغط المطلق ***Absolute pressure***. والضغط المطلق عند عمق h من سطح السائل معرض للضغط الجوي يساوي الضغط الجوي + الكمية $r\,gh$.

$$\therefore P = P_a + r\,gh \qquad (10.6)$$

تسمى P-P_1 بالضغط الظاهري ***Gauge pressure***.

النتيجة التي توصلنا إليها تثبت أن الضغط متساوى عند جميع النقاط التي على نفس العمق من سطح السائل كما أن الضغط لا يعتمد على شكل الوعاء الذي يحتوي السائل كما في الشكل 10.4.

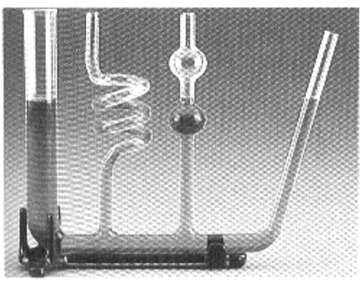


Figure 10.4

10.4 Pascal's Law

أي زيادة في الضغط عند سطح سائل محصور في منطقة مغلقة تنتقل إلى جميع أجزاء السائل والجدار المحيط. وقد اكتشف هذه الظاهرة العالم باسكال Blaise Pascal (1623-1662)، وأطلق العلماء على هذه الظاهرة قانون باسكال ***Pascal's law*** وهي فكرة عمل المكبس الهيدروليكي.

Pascal's law *:Any change in pressure applied to an enclosed fluid is transmitted to every point of the liquid and the walls of the containing vessel.*

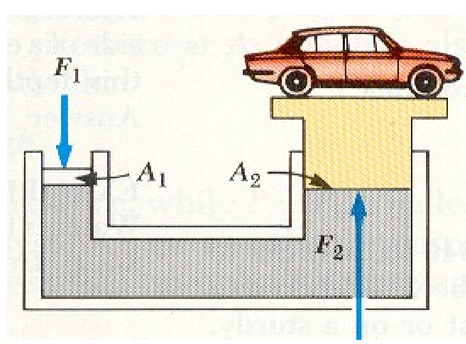


Figure 10.5

The most known application of Pascal's law is the hydraulic press as shown in Figure 10.5. Where force F_1 is applied to a small piston of area A_1. The pressure is transmitted through a fluid to a larger piston area A_2. The pressure on both sides of the piston is the same therefore,

$$P = \frac{F_1}{A_1} = \frac{F_2}{A_2} \qquad (10.7)$$

The force F_1 is larger the force F_1 by a factor of A_2/A_1.

$$\therefore F_2 = F_1 \frac{A_2}{A_1} \qquad (10.8)$$

يستخدم الرافع الهيدروليكي الذي يعمل على مبدأ باسكال في رفع الأجسام الثقيلة مثل رافعة السيارات في ورش تصليح السيارات.

Example 10.1

In a hydrochloric piston of radius 5cm and 50cm for the small and large pistons respectively. Find the weight of a car that can be elevated if the force exerted by the compressed air is (F_1 = 100N).

Solution

As shown in Figure 10.5

$$P = \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$\therefore F_2 = F_1 \frac{A_2}{A_1}$$

$$\therefore F_2 = 100 \times \frac{p \times (0.05)^2}{p \times (0.005)^2} = 10000N$$

من مبدأ الحفاظ على الطاقة فإن الشغل المبذول بواسطة القوة F_1 يساوي الشغل المبـــذول بواسطة القوة F_2.

Example 10.2

Calculate the pressure at an ocean depth of 500m. Assume the density of water is 10^3kg/m^3 and the atmospheric pressure is 1.01×10^5Pa..

Solution

$$\because P = P_a + \rho gh$$

$$\therefore P = 1.01\times10^5 + (10^3 \times 9.8 \times 500)$$

$$= 5\times10^6 \, Pa$$

Example 10.3

A simple U-tube that is open at both ends is partially filled with water. Kerosene (ρ_k = 0.82×10^3kg/m^3) is then poured into on arm of the tube, forming a column 6cm height, as shown in Figure 10.6. What is the difference h in the heights of the two liquid surfaces?

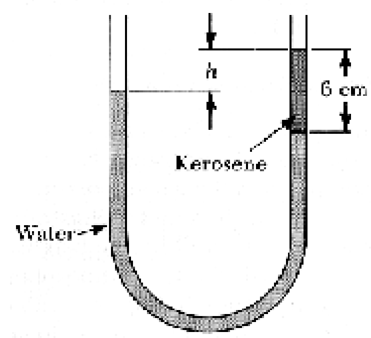


Figure 10.6

Solution

The pressure at the doted line is the same *i.e.*

$$P_A = P_B$$

Therefore

$P_A = \rho_w g h_w$ & $P_B = \rho_k g h_k$

$$\rho_w g h_w = \rho_k g h_k$$

$$10^3\times 10 \times h_w = 0.82\times10^3 \times 10 \times 0.6$$

hence

$$h_w = 0.498 \text{ m} = 4.98\text{cm}$$

$$h = 6 - 4.98 = 1.02\text{cm}$$

Example 10.4

Water is filled to height *H* behind a dam of width *w* as shown in Figure 10.7. Determine the resultant force on the dam.

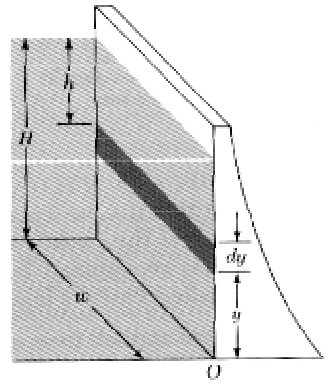


Figure 10.7

Solution

نعلم أن الضغط يزداد بازدياد العمق، كما أن الضغط يتساوى عند نفس الارتفـاع. فـإذا افترضنا عنصر مساحة $dy \times w$ على عمق h من قمة السد فـإن الضـغط عنـد عنصـر المساحة يعطى بالعلاقة التالية:

$$P = rgh = rg(H-y)$$

تم إهمال تأثير الضغط الجوي لتساوي تأثيره على جانبي السد.

The force on the small shaded area $dy \times w$ is

$$dF = PdA = rg(H-y)\,w\,dy$$

The total force on the dam is

$$F = \int dF = \int PdA = \int_0^H rg(H-y)\,w\,dy$$

$$= rgw\left[Hy - \frac{y^2}{2}\right]_0^H = rgw\left[H^2 - \frac{H^2}{2}\right]$$

$$\therefore F = \frac{1}{2} rgwH^2$$

نستنتج أن القوة المؤثرة على السد تزداد بزيادة العمق لذا فإن عند تصميم السدود يؤخذ في الحسبان زيادة السمك بزيادة العمق.

10.5 Buoyant forces and Archimedes' principle

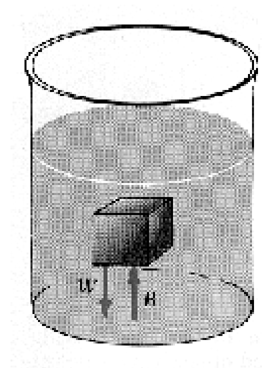


Figure 10.8

من واقع الخبرة العملية فإن الجسم المغمور كلياً أو جزئياً في سائل يفقد جزءاً من وزنه نتيجة لقوة دفع السائل للجسم، وقد اكتشف هذه الظاهرة العالم أرخميدس وقد أطلق العلماء عليها بمبدأ أرخميدس.

Archimedes' principle state that *any body completely or partially submerged in a fluid is buoyed up by a force equal to the weight of the fluid displaced by the body.*

According to Archimedes' principle the magnitude of the buoyant force always equals the weight of the fluid displaced by the object.

في الشكل 10.8 إذا افترضنا وجود مكعب من الماء في إناء به ماء فإن الجسم المغمور له نفس كثافة الوسط وهنا حالة اتزان نتيجة لتساوي قوى الوزن W وقوة الطفو B وفي اتجاهين متعاكسين.

$$B = W \qquad (10.9)$$

لاحظ أن قوة الطفو B لا تعتمد على نوع أو شكل الجسم المغمور في السائل

من مبدأ أرخميدس فإن يمكن إيجاد قيمة قوة الطفو B من حساب كمية السائل المزاح نتيجة لغمر الجسم كلياً أو جزئياً.

In Figure 10.8 above the pressure at the bottom of the cube is grater than the pressure at the top by amount $r_f gh$, where h is the height of the cube and r_f is the density of the fluid. The pressure different ΔP is equal to the buoyant force per unit area,

$$\Delta P = \frac{B}{A} \qquad (10.10)$$

therefore,

$$B = A\Delta P = r_f\, ghA = r_f\, gV \qquad (10.11)$$

where V is the volume of the cube

The mass of water in the cube is $M = r_f V$

Then,

$$B = W = r_f \; gV = Mg \qquad (10.12)$$

where W is the weight of the displaced fluid.

سندرس الآن حالتين الأولى عندما يكون فيها الجسم مغمور كلياً في السائل والحالة الثانيـــة عندما يكون الجسم يطفو على السائل.

(1) Totally submerged object

For the case of totally submerged object the upward buoyant force is given by

$$W = r_f \; gV_o \qquad (10.13)$$

Where V_o is the volume of the object. If the density of the object is r_o then its weight is given by

$$W = Mg = r_o \, gV_o \qquad (10.14)$$

The net force

$$B - W = \left(r_f - r_o\right)V_o g \qquad (10.15)$$

ملاحظة:

عندما تكون كثافة الجسم المغمور أصغر من كثافة السائل فإن الجسم سيتحرك للأعلى بعجلة a.

عندما تكون كثافة الجسم المغمور أكبر من كثافة السائل فإن الجسم ســيتحرك للأسفل بعجلة a.

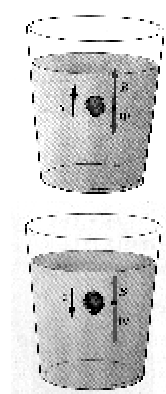


Figure 10.9

(2) Floating object

Consider an object is floating on a surface of fluid *i.e.* partially submerged, therefore the upward force is balanced by the downward weight of the object. If the V is the displaced fluid by the floating object, then the buoyant force $B = r_f Vg$, since the object is in equilibrium with the fluid *i.e.* $W = B$

$$W = Mg = r_o gV_o \qquad (10.16)$$

therefore,

$$r_f Vg = r_o V_o g \qquad (10.17)$$

or

$$\frac{r_o}{r_f} = \frac{V}{V_o} \qquad (10.18)$$

وعلى هذا الأساس تعتمد الأسماك في غوصها في أعماق البحر حيث تستطيع تعديل حجمها لتتمكن من زيادة كثافتها لتغوص على أعماق مختلفة في البحر.

Example 10.5

A block of brass has a mass of 0.5kg and density of 8×10^{3}kg/cm^3. It is suspended from a string. Find the tension in the string if the block in air, and if it is completely immersed in water.

Solution

the tension in the string in air is equal to the weight of the brass block

$$T_{air} = W_{brass}$$

hence,

$$T_{air} = mg = 0.5 \times 9.8 = 4.9 \text{ N}$$

عند غمر الجسم في الماء فإن قوة الطفو ستقلل من وزن الجسم وبالتالي من الشد في الحبل. وحيث أن قوة الطفو تساوي كمية الماء المزاح، ولإيجاد كمية الماء المزاح نحسب حجم الجسم المغمور V.

$$V = \frac{m}{r} = \frac{0.5}{8\times10^3} = 6.25\times10^{-5}\,m^3$$

سنحسب الآن قوة الطفو والتي تساوي وزن الماء المزاح.

$$W_{\text{water}} = mg = r\ V g = 10^3 \times 6.25\times10^{-5} \times 9.8 = 0.612\text{N}$$

قوة الشد في الحبل عندما يكون الجسم مغموراً في الماء تساوي قوة الشد في الهواء – قوة الطفو (وزن الماء المزاح).

$$T_{\text{water}} = T_{\text{air}} - W_{\text{water}}$$

$$T_{\text{water}} = 4.9 - 0.612 = 4.29\text{N}$$

Example 10.6

A solid object has a weight of 5N. When it is suspended from a spring scale and submerged in water, the scale reads 3.5N as shown in Figure 10.10. What is the density of the object?

Solution

يمكن استخدام فكرة المثال في تعيين كثافة جسم ما بطريقة حساب وزنه في الهواء ومقارنة الوزن وهو مغمور في الماء. حيث تكون قوة الطفو B المؤثرة على الجسم المغمور في الماء تساوي وزن الماء المزاح وفي هذه الحالة يكون وزن الماء المزاح هو الفرق بين وزن الجسم في الهواء ووزنه في الماء.

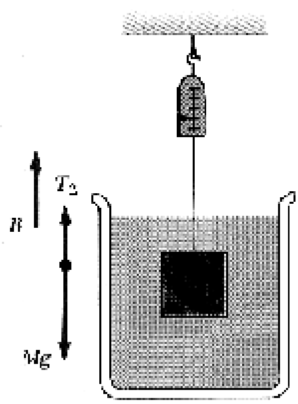


Figure 10.10

The buoyant force (B) = the weight of the water displaced (W_{water})

$$B = 5 - 3.5 = 1.5\text{N}$$

$$W_{\text{water}} = mg = r\ V g$$

hence,

$$r\ V g = 1.5$$

$$V = \frac{1.5}{10^3 \times 9.8} = 1.5\times10^{-4}\text{m}^3$$

The density of the object is its mass over its volume

$$r = \frac{3.5}{1.5\times10^{-4}\times 9.8} = 2.3\times10^3\, kg/m^3$$

Example 10.7

A cube of wood 20cm on a side and having a density of 0.65×10^3kg/m^3 floats on water. (a) What is the distance from the top of the cube to the water level? (b) How much lead weight has to be placed on top of the cube so that its top is just level with the water?

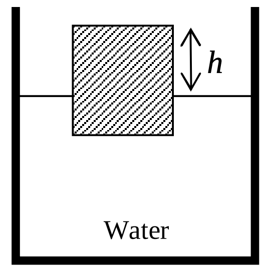


Figure 10.11

Solution

(a) According to Archimedes principle

$$B = r_w V g = (1\text{g/cm}^3)\times[20\times20\times(20\text{-}h)]g$$

but

$$B = \text{weight of the wood} = mg = r_{wood} V_{wood}\, g = (0.65\text{g/cm}^3)(20)^3$$

hence,

$$(1\text{g/cm}^3)\times[20\times20\times(20\text{-}h)]g = (0.65\text{g/cm}^3)(20)^3$$

$$20 - h = 20\times0.65$$

$$h = 20(1\text{-}0.65) = 7\text{cm}$$

(b) $B = W + Mg$ where M is the mass of lead

$$1(20)^3\, g = (0.65)(20)^3\, g + Mg$$

$$M = 20^3(1\text{- }0.65) = 2800\text{ g} = 2.8\text{kg}$$

10.6 The Equation of continuity

In time Δt, the fluid moves a distance $\Delta x_1 = v_1 \Delta t$, where v_1 is the velocity of the fluid in the bottom end of the pipe of cross-sectional area of A_1. Hence the mass in the portion Δx_1 of the pipe is $\Delta m_1 = r_1 A_1 \Delta x_1 =$. On the other end of the pipe the mass of the fluid moves in time Δt is $\Delta m_2 = r_2 A_2 v_2 \Delta t$.

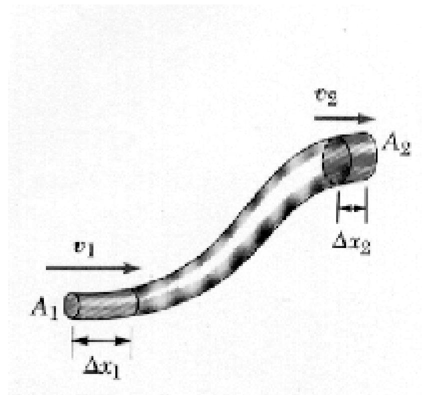


Figure 10.12

The mass is conserved *i.e.* $\Delta m_1 = \Delta m_2$

$$r_1 A_1 v_1 = r_2 A_2 v_2 \qquad (10.19)$$

This called the equation of continuity

In this case density of the fluid is constant *i.e.* $r_1 = r_2$

$$A_1 v_1 = A_2 v_2 = \text{constant} \qquad (10.20)$$

This mean that the product of the area and the fluid speed at all points in the pipe is constant.

$$Av = \text{constant} \qquad (10.21)$$

Example 10.8

A water pipe of radius 3cm is used to fill a 40liter bucket. If it takes 5min to fill the bucket, what is the speed *v* at which the water leave the pipe? (1liter = 10^3cm^3.

Solution

The cross sectional area of the pipe A is

$$A = p\,r^2 = p \times 3^2 = 9p\ cm^2$$

$$Av = 40\frac{liter}{\min} = \frac{40\times10^3 cm^3}{60s} = 666.6cm^3/s$$

therefore,

$$v = \frac{666.6}{9p} = 23.5cm/s$$

10.7 Bernoulli's equation

A Swiss physicist called Daniel Bernoulli (1700-1782) derived an expression for the relation between the pressure, speed and levitation of the fluid flow in a pipe

For a nonviscous fluid and incompressible flow in a pipe of nonuniform cross-section in time Δt as shown in Figure 10.13. The force of the lower part of the fluid is P_1A_1. The work done by this force is given by,

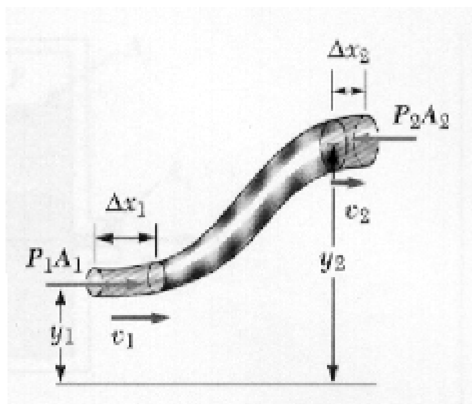


Figure 10.13

$$W_1 = F_1\Delta x_1 = P_1A_1\Delta x_1 = P_1\Delta V \qquad (10.22)$$

where ΔV is the volume of the lower part of the fluid.

Similarly the force of the upper part of the fluid is P_2A_2. The work done by this force is negative since the fluid force is opposite to the displacement and is given by,

$$W_2 = -F_2\Delta x_2 = -P_2A_2\Delta x_2 = -P_2\Delta V \qquad (10.23)$$

The volume ΔV in the lower and upper part of the fluid is the same.

Therefore the net work done in time Δt is,

$$W = (P_1 - P_2)\Delta V \qquad (10.24)$$

Note that this work is used to change the kinetic energy of the fluid and to change the potential energy in time Δt

The change in kinetic energy is given by

$$\Delta K = \frac{1}{2}(\Delta m)v_2{}^2 - \frac{1}{2}(\Delta m)v_1{}^2 \qquad (10.25)$$

The change in potential energy is given by

$$\Delta U = \Delta mgy_2 - \Delta mgy_1 \qquad (10.26)$$

From the total energy theorem, $W = \Delta K + \Delta U$

$$(P_1 - P_2)\Delta V = \frac{1}{2}(\Delta m)v_2{}^2 - \frac{1}{2}(\Delta m)v_1{}^2 + \Delta mgy_2 - \Delta mgy_1$$

Divide both sides by ΔV, and substitute for $\Delta m / \Delta V = r$

$$P_1 - P_2 = \frac{1}{2} r v_2{}^2 - \frac{1}{2} r v_1{}^2 + r gy_2 - r gy_1 \qquad (10.27)$$

By arranging the equation we get,

$$P_1 + \frac{1}{2} r v_1{}^2 + r gy_1 = P_2 + \frac{1}{2} r v_2{}^2 + r gy_2 \qquad (10.28)$$

or

$$P + \frac{1}{2} r v^2 + r gy = \text{constant} \qquad (10.29)$$

Therefore Bernoulli's equation says that the sum of the pressure (*P*), kinetic energy per unit volume ($\frac{1}{2} r v^2$), and potential energy per unit volume ($r gy$) has the same value at all points a long a streamline.

When the fluid is static *i.e.* $v_1 = v_2 = 0$ the Bernoulli's equation becomes as,

$$P_1 - P_2 = r g\left(y_2 - y_1\right) = r gh \qquad (10.30)$$

Example 10.9

A large storage tank filled with water develops a small hole in its side at a point 16m below the water level. If the rate of flow from the leak is 2.5×10^{-3}m^3/min, determine (a) the speed at which the water leaves the hole and (b) the diameter of the hole.

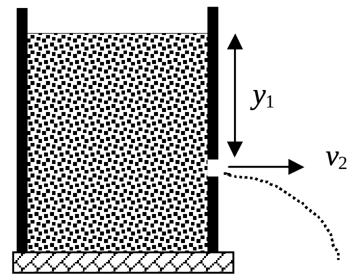


Figure 10.14

Solution

(a) The top of the tank is open then

$P_1 = P_a$

The water flow rate is 2.5×10^{-3}m^3/min = 4.167×10^{-5}m^3/s

Assuming the speed $v_1 = 0$, and $P_1=P_2=P_a$

$$P_1 + \frac{1}{2} r v_1^2 + r g y_1 = P_2 + \frac{1}{2} r v_2^2 + r g y_2$$

$$v_2 = \sqrt{2gy_1} = \sqrt{2\times9.8\times16} = 17.7 m/s$$

(b) The flow rate = $A_2 v_2 = \frac{pd^2}{4}\times17.7 = 4.167\times10^{-5} m^3/s$

then,

$d = 1.73\times10^{-3}$m = 1.73 mm

10.8 Question with solution

1. When you drink a liquid through a straw, you reduce the pressure in your mouth and let the atmosphere move the liquid. Explain how this works. Could you use a straw to sip a drink on the moon?

Answer: *When you reduce the pressure in your mouth, the push of the atmosphere on the surface of the liquid forces the liquid up the straw and into the mouth. Because there is no atmospheric pressure on the moon, you could not sip a drink there.*

2. A fish rests on the bottom of a bucket of water while the bucket is being weighed. When the fish begins to swim around, does the weight change?

Answer: *In either case the scale is supporting the container, the water, and the fish. Therefore, the weight reading of the scale remains the same.*

3. Will a ship ride higher in the water of an island lake or in the ocean? Why?

Answer: *The buoyant force on an object such as a ship is equal to the weight of the water displaced by the ship. Because of the greater density of salty ocean water, less water needs to be displaced by the boat to enable it to float. Thus, the boat floats higher in the ocean than in a freshwater inland lake.*

4. Lead has a greater density than iron, and both are denser than water. Is the buoyant force on a lead object greater than, less than, or equal to the buoyant force on an iron object of the same volume?

Answer: *The buoyant forces are the same, since the buoyant force equals the weight of the displaced water.*

5. An ice cube is placed in a glass of water. What happens to the level of the water as the ice melts?

Answer: *It stays the same.*

6. Why do many trailer trucks use wind deflectors on the top of their cabs? How do such devices reduce fuel consumption?

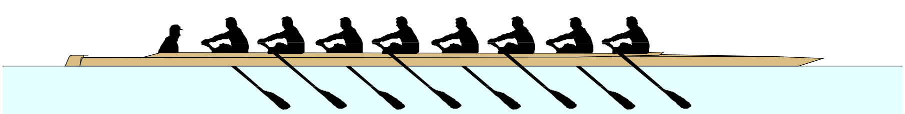
Answer: *The wind deflectors produce a more streamline flow of air over the top of the truck, thereby decreasing air resistance, and reducing fuel consumption.*

7. When a fast-moving train passes a train at rest, the two tend to be drawn together. How does the Bernoulli effect explain this phenomenon?

Answer: *As air is displaced by the moving train, that portion passing between the trains has a higher relative velocity than the air on the outside, which is free to expand. Thus, the air pressure is lower between the trains than on the sides of the trains away from the constriction.*

8. A baseball heading for home plate is seen from above to be spinning counterclockwise. In which direction does the ball deflect?

Answer: *From the viewpoint of a right-handed batter, the ball moves from left to right and spins counterclockwise. Thus, the air motion is retarded above the ball and helped along beneath the ball. This causes the air pressure above the ball to be lower than the air pressure beneath the ball, so the ball will rise.*



10.9 Problems

1. Calculate the mass of a solid iron sphere that has diameter of 3.0cm.

2. A small ingot of shiny grey metal has a volume of 20cm^3 and a mass of 535 g. What is the metal? (See Table 10.1).

3. Determine the absolute pressure at the bottom lake that is 30 m deep.

4. At what depth in a lake is the absolute pressure equal to three times atmospheric pressure?

5. A swimming pool has dimensions 30 m × 10 m an flat bottom. When the pool is filled to a depth of 2m with fresh water, what is the total force due to water on the bottom? On each end? On each side?

6. What is the hydrostatic force on the back of Grand Coulee Dam if the water in the reservoir is 150 deep and the width of the dam is 1200 m?

7. Calculate the buoyant force on a solid object made of copper and having a volume of 0.2 m^3 if it is submerged in water. What is the result if the object made of steel?

8. A solid object has a weight of 5.0 N. When it is suspended from a spring scale and submerged in the scale reads 3.5 N (Fig. 10.10). What is the density of the object?

9. A cube of wood 20 cm on a side and having a density of 0.65 × 10^3 kg/m^3 floats on water. (a) What is the distance from the top of the cube to the water level? (b) How much lead weight has to be placed on top of the cube so that its top is just level with the water?

10. The rate of flow of water through a horizontal pipe is 2 m^3/min. Determine the velocity of flow at a point where the diameter of the pipe is (a) 10 cm, (b) 5 cm.

11. A large storage tank filled with water develops a small hole in its side at a point 16 m below the water level. If the rate of flow from the leak is 2.5 × 10^{-3}m^3/min, determine (a) the speed at which the water leaves the hole and (b) the diameter of the hole.

12 Water flows through a constricted pipe at a uniform rate (Fig. 15.18). At one point, where the pressure is 2.5×10^4 Pa., the diameter is 8.0 cm; at another point 0.5m higher, the pressure is 1.5 × 10^4 Pa and the lower and upper is 4.0 cm. (a) Find the speed of flow in the lower and upper sections. (b) Determine the rate of flow through the pipe.

Multiple Choice Questions

Mechanics

Attempt the following

Multiple choice question

1. A solid piece of magnesium has a mass of 24.94 g and a volume of 14.3 cm^3. From these data, calculate the density of magnesium in SI units (kg/m^3).
   a. 1.74×10^3 kg/m^3
   b. 0.573 kg/m^3
   c. 1.74 kg/m^3
   d. 0.573×10^3 kg/m^3

2. Vector B has x, y, and z components of 6.00, 8.00, and 5.00 units, respectively. Calculate the magnitude of B and the angles that B makes within the coordinate axes.
   a. 125, *a* = 87.3°, *b* = 86.3°, *g* = 87.7°
   b. 11.2, *a* = 57.5°, *b* = 44.3°, *g* = 63.4°
   c. 11.2, *a* = 28.2°, *b* = 35.6°, *g* = 24.1°
   d. 11.2, *a* = 32.5°, *b* = 45.7°, *g* = 26.6°

3. Consider two vectors $\vec{\mathbf{A}} = 3\hat{\mathbf{i}} - 2\hat{\mathbf{j}}$ and $\vec{\mathbf{B}} = -\hat{\mathbf{i}} - 4\hat{\mathbf{j}}$. Calculate $\vec{\mathbf{A}} + \vec{\mathbf{B}}$.
   a. 2.00i + 6.00j
   b. 2.00i - 6.00j
   c. 2.00i - 2.00j
   d. 4.00i + 2.00j

4. Vector $\vec{\mathbf{A}}$ has a magnitude of 12.0 units and makes an angle of 45.0° with the positive x axis. Vector $\vec{\mathbf{B}}$ has a magnitude of 12.0 units and is directed along the negative x axis. Using graphical methods, find (a) the vector sum of $\vec{\mathbf{A}} + \vec{\mathbf{B}}$ and (b) the vector difference $\vec{\mathbf{A}} - \vec{\mathbf{B}}$.

   a. (a) 24.0 @ 45°; (b) 0
   b. (a) 22.2 @ 22°; (b) 9.2 @ 112°
   c. (a) 22.2 @ 22°; (b) 9.2 @ 67°
   d. (a) 9.2 @ 112°; (b) 22.2 @ 22°

5. A pedestrian moves 9.00 km east and then 19.0 km north. Using the graphical method, find the magnitude and direction of the resultant displacement vector.
   a. 21.0 km, 64.7° north of east
   b. 16.7 km, 61.7° east of north
   c. 21.0 km, 61.7° north of east
   d. 16.7 km, 25.4° north of east

6. If the cartesian coordinates of a point are given by (3, y) and its polar coordinates are (r, 60°), determine y and r .
   a. *y* = 1.73, *r* = 11.9
   b. *y* = 5.20, *r* = 6.00
   c. *y* = 5.20, *r* = 36.0
   d. *y* = 1.73, *r* = 3.46

7. A person walks 15.0° north of east for 6.10 km. How far would she have to walk due north and due east to arrive at the same location?

a. 1.58 km north and 5.89 km east
b. 5.89 km north and 1.58 km east
c. 1.64 km north and 5.88 km east
d. 5.88 km north and 1.64 km east

---

8. A vector has an x component of -25.0 units and a y component of 40.0 units. Find the magnitude and direction of this vector.

a. 31.2 units at 122°
b. 47.2 units at 58.0°
c. 47.2 units at 122°
d. 31.2 units at 58.0°

---

9. Vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ have equal magnitudes of 4.00. If the sum of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ is the vector 5.00$\hat{\mathbf{j}}$, determine the angle between $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$.

a. 77.4°
b. 90.0°
c. 12.6°
d. 103°

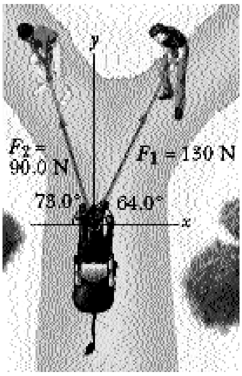
---

10. If the polar coordinates of the point (x, y) are (*r*, *q*), determine the polar coordinates for the point (-4x, 4y).

a. $4\sqrt{2}r$, 360° - *q*
b. 4*r*, 180° - *q*
c. 4*r*, 360° - *q*
d. $4\sqrt{2}r$, 180° - *q*

---

11. The helicopter view in the Figure shows two people pulling on a stubborn mule.
Find (a) the single force that is equivalent to the two forces shown and (b) the force that a third person would have to exert on the mule to make the resultant force equal to zero. The forces are measured in units of newtons.



a. (a) 205 N @ 81.4° from the positive *x* axis; (b) (-30.6$\hat{\mathbf{i}}$ - 203$\hat{\mathbf{j}}$) N

b. (a) 88.0 N @ 20.3° from the positive *x* axis; (b) (-30.8$\hat{\mathbf{i}}$ - 83.3$\hat{\mathbf{j}}$) N

c. (a) 219 N @ 67.7° from the positive *x* axis; (b) (83.3$\hat{\mathbf{i}}$ + 203$\hat{\mathbf{j}}$) N

d. (a) 205 N @ 84.4° from the positive *x* axis; (b) (30.6$\hat{\mathbf{i}}$ + 203$\hat{\mathbf{j}}$) N

---

12. A vector is given by $\vec{R} = 6.00\hat{i} + 5.00\hat{j} + 7.00\hat{k}$. Find (a) the magnitude of the x, y, and z components; (b) the magnitude of $\vec{R}$ and (c) the angles between $\vec{R}$ and the x, y, and z axis.
    - **a.** (a) 6.00, 5.00, 7.00; (b) 18.0; (c) 70.5°, 73.9°, 67.1°
    - **b.** (a) 6.00, 5.00, 7.00; (b) 10.5; (c) 55.1°, 61.5°, 48.1°
    - **c.** (a) 6.00, 5.00, 7.00; (b) 10.5; (c) 34.9°, 28.5°, 41.9°
    - **d.** (a) 1.00, 1.00, 1.00; (b) 18.0; (c) 70.5°, 73.9°, 67.1°

13. A jet airliner, moving initially at 300 mi/h to the east, suddenly enters a region where the wind is blowing at 100 mi/h in a direction 30.0° north of east. What are the new speed and direction of the aircraft relative to the ground?
    - **a.** 219 mi/h at 13.2° south of east
    - **b.** 361 mi/h at 13.9° north of east
    - **c.** 390 mi/h at 7.37° north of east
    - **d.** 219 mi/h at 13.2° north of east

14. The initial speed of a body is 9.20 m/s. What is its speed after 4.50 s, (a) if it accelerates uniformly at 5.00 m/s2 and (b) if it accelerates uniformly at -5.00 m/s2?
    - **a.** (a) 31.7 m/s; (b) -13.3 m/s
    - **b.** (a) 22.5 m/s; (b) -22.5m/s
    - **c.** (a) -13.3 m/s; (b) 31.7 m/s
    - **d.** (a) 31.7 m/s; (b) 0 m/s

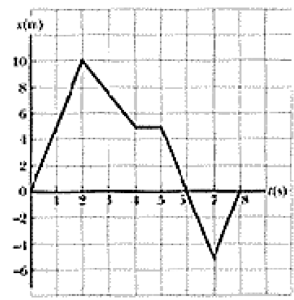
15. A golf ball is released from rest from the top of a very tall building. Neglecting air resistance, calculate (a) the position and (b) the velocity of the ball after 1.00 s.
    - **a.** (a) -16.0 m; (b) 32.0 m/s
    - **b.** (a) -4.90 m; (b) -9.80 m/s
    - **c.** (a) -4.90 m; (b) 9.80 m/s
    - **d.** (a) -16.0 m; (b) -32.0 m/s

16. A particle moves along the x axis according to the equation $x = 2.00 + 3.00t - 1.00t^2$, where x is in meters and t is in seconds. At t = 1.00 s, find (a) the position of the particle, (b) its velocity, and (c) its acceleration.
    - **a.** (a) 4.00 m; (b) 3.17 m/s; (c) 1.42 m/s^2
    - **b.** (a) 6.00 m; (b) 5.00 m/s; (c) 2.00 m/s^2
    - **c.** (a) 4.00 m; (b) -1.00 m/s; (c) 2.00 m/s^2
    - **d.** (a) 4.00 m; (b) 1.00 m/s; (c) -2.00 m/s^2

17. A 60.0 g superball traveling at 32.0 m/s bounces off a brick wall and rebounds at 26.0 m/s. A high-speed camera records this event. If the ball is in contact with the wall for 4.00 ms, what is the magnitude of the average acceleration of the ball in this time interval? (Note: 1 ms = 10-3 s.)
    - **a.** 1.50×10^3 m/s^2
    - **b.** 14.5 m/s^2
    - **c.** 1.45×10^4 m/s^2
    - **d.** 1.50 m/s^2

18. A truck covers 42.0 m in 9.00 s while smoothly slowing down to a final speed of 3.80 m/s. (a) Find its original speed. (b) Find its acceleration.
    - **a.** (a) 20.7 m/s; (b) -1.88 m/s^2
    - **b.** (a) 5.53 m/s; (b) 0.193 m/s^2
    - **c.** (a) 13.1 m/s; (b) -1.88 m/s^2
    - **d.** (a) 5.53 m/s; (b) -0.193 m/s^2

19. The displacement versus time for a certain particle moving along the x axis is shown in the the Figure.



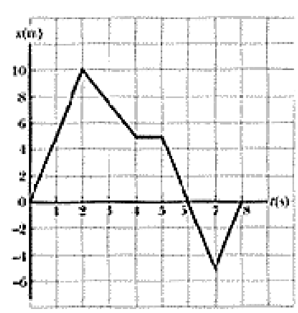
Find the average velocity in the time interval 0 to 4 s.

- **a.** -0.83 m/s
- **b.** 1.2 m/s
- **c.** 0.83 m/s
- **d.** 10 m/s

20. The minimum distance required to stop a car moving at 35.0 mi/hr is 40.0 ft. What is the minimum stopping distance for the same car moving at 25.0 mi/hr, assuming the same rate of acceleration?
    - **a.** 20.4 ft
    - **b.** 28.6 ft
    - **c.** 78.4 ft
    - **d.** 56.0 ft

21. A ball thrown vertically upward is caught by the thrower after 2.50 s. Find (a) the initial velocity of the ball and (b) the maximum height it reaches.
    - **a.** (a) 12.3 m/s; (b) 7.66 m
    - **b.** (a) 24.5 m/s; (b) 30.6 m
    - **c.** (a) 12.3 m/s; (b) 15.3 m
    - **d.** (a) 24.5 m/s; (b) 91.9 m

22. Find the instantaneous velocity of the particle described in the the Figure at t = 6.0 s.



- **a.** -5.0 m/s
- **b.** -0.20 m/s
- **c.** 5.0 m/s
- **d.** 0 m/s

23. A ball is thrown directly downward with an initial speed of 6.00 m/s from a height of 20.0 m. How many seconds later does the ball strike the ground?
    - **a.** 2.72 s
    - **b.** 1.50 s
    - **c.** 0.946 s
    - **d.** 1.31 s

24. A motorist drives along a straight road at a constant speed of 5.00 m/s. Just as she passes a parked motorcycle police officer, the officer starts to accelerate at 2.40 m/s^2 to overtake her. Assuming the officer maintains this acceleration, (a) determine the time it takes the police officer to reach the motorist. Also, find (b) the speed and (c) the total displacement of the officer as he overtakes the motorist.
    - **a.** (a) 1.04 s; (b) 2.50 m/s; (c) 5.21 m
    - **b.** (a) 2.08 s; (b) 10.0 m/s; (c) 15.6 m
    - **c.** (a) 4.17 s; (b) 10.0 m/s; (c) 20.8 m
    - **d.** (a) 2.08 s; (b) 5.00 m/s; (c) 10.4 m

25. A student attaches a ball to the end of a string 0.800 m in length and then swings the ball in a vertical circle. The speed of the ball is 5.30 m/s at its highest point and 10.5 m/s at its lowest point. Find the acceleration of the ball when the string is vertical and the ball is at (a) its highest point and (b) its lowest point.
    - **a.** (a) 35.1 m/s^2 downward; (b) 138 m/s^2 upward
    - **b.** (a) 6.63 m/s^2 downward; (b) 13.1 m/s^2 upward
    - **c.** (a) 138 m/s^2 downward; (b) 35.1 m/s^2 upward
    - **d.** (a) 13.1 m/s^2 downward; (b) 6.63 m/s^2 upward

26. An astronaut on a strange planet finds that she can jump a maximum horizontal distance of 17.0 m if her initial speed is 7.00 m/s. What is the free fall acceleration on the planet?
    - **a.** 0.412 m/s^2
    - **b.** 2.88 m/s^2
    - **c.** 5.77 m/s^2
    - **d.** 1.44 m/s^2

27. Suppose that the position vector for a particle is given as $\vec{r} = x\hat{i} + y\hat{j}$, with x = at + b and y = ct^2 + d, where a = 2.00 m/s, b = 2.00 m, c = 0.225 m/s^2, and d = 2.00 m. (a) Calculate the average velocity during the time interval from t = 2.00 s to t = 4.00 s. (b) Determine the velocity and the speed at t = 2.00 s.
    - **a.** (a) $(2.00\hat{i} + 1.35\hat{j})$ m/s; (b) $(3.00\hat{i} + 1.45\hat{j})$ m/s, 3.33 m/s
    - **b.** (a) $(3.00\hat{i} + 1.45\hat{j})$ m/s; (b) $(3.00\hat{i} + 1.45\hat{j})$ m/s, 3.33 m/s
    - **c.** (a) $(3.00\hat{i} + 1.45\hat{j})$ m/s; (b) $(2.00\hat{i} + 0.900\hat{j})$ m/s, 2.19 m/s
    - **d.** (a) $(2.00\hat{i} + 1.35\hat{j})$ m/s; (b) $(2.00\hat{i} + 0.900\hat{j})$ m/s, 2.19 m/s

28. A tire 0.700 m in radius rotates at a constant rate of 350 rpm. Find the speed and acceleration of a small stone lodged in the tread of the tire (on its outer edge). (Hint: In one revolution, the stone travels a distance equal to the circumference of its path, $2\pi r$.)

a. 25.7 m/s, 940 m/s^2
b. 25.7 m/s, 36.7 m/s^2
c. 0.754 m/s, 0.812 m/s^2
d. 0.754 m/s, 1.08 m/s^2

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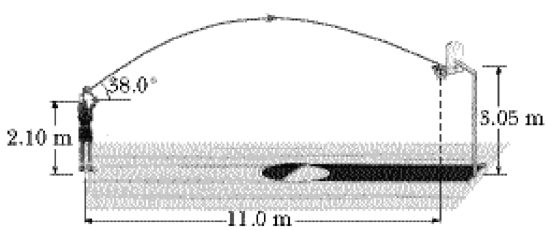
29. A basketball player who is 2.10 m tall is standing on the floor 11.0 m from the basket, as in the Figure.



If he shoots the ball at a 38.0° angle with the horizontal, at what initial speed must he throw so that it goes through the hoop without striking the backboard. The basket height is 3.05 m.

a. 7.01 m/s
b. 11.2 m/s
c. 10.5 m/s
d. 13.1 m/s

---

30. A ball on the end of a string is whirled around in a horizontal circle of radius 0.400 m. The plane of the circle is 1.40 m above the ground. The string breaks and the ball lands 2.20 m (horizontally) away from the point on the ground directly beneath the ball's location when the string breaks. Find the radical acceleration of the ball during its circular motion.

a. 10.9 m/s^2
b. 10.3 m/s^2
c. 78.8 m/s^2
d. 42.4 m/s^2

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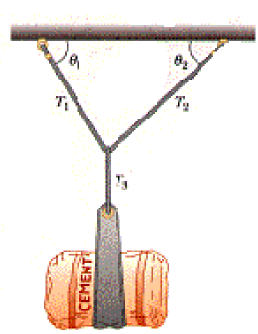
31. A bag of cement of weight 400 N hangs from three wires as suggested in the Figure.



Two of the wires make angles q_1 = 70.0° and q_2 = 34.0° with the horizontal. If the system is in equilibrium, find the tensions T_1, T_2, and T_3 in the wires.

a. T_1 = 342 N, T_2 = 141 N, T_3 = 83.0 N
b. T_1 = 342 N, T_2 = 141 N, T_3 = 400 N
c. T_1 = 231 N, T_2 = 387 N, T_3 = 400 N
d. T_1 = 231 N, T_2 = 387 N, T_3 = 218 N

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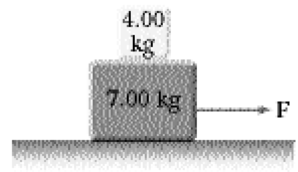
32. A 4.00 kg block is placed on top of a 7.00 kg block as in the the Figure.



The coefficient of kinetic friction between the 7.00 kg block and the surface is 0.225. A horizontal force, *F*, is applied to the 7.00 kg block. (a) What force accelerates the 4.00 kg block? (b) Calculate the magnitude of the force necessary to pull both blocks to the right with an acceleration of 3.50 m/s^2. (c) Find the minimum coefficient of static friction between the blocks such that the 4.00 kg block does not slip under an acceleration of 3.50 m/s^2.

   a. (a) the horizontal force, *F*; (b) 53.9 N; (c) 0.225
   b. (a) static friction between the blocks; (b) 62.8 N; (c) 0.357
   c. (a) static friction between the blocks; (b) 53.9 N; (c) 0.357
   d. (a) the horizontal force, *F*; (b) 62.8 N; (c) 1.60

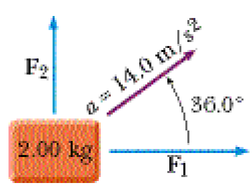
33. A 35.0 kg block is initially at rest on a horizontal surface. A horizontal force of 90.0 N is required to set the block in motion. After it is in motion, a horizontal force of 54.0 N is required to keep the block moving with constant speed. Find the coefficients of static and kinetic friction from this information.

   a. $m_s = 0.262$, $m_k = 0.157$
   b. $m_s = 6.35$, $m_k = 3.81$
   c. $m_s = 0.157$, $m_k = 0.262$
   d. $m_s = 3.81$, $m_k = 6.35$

34. A high diver of mass 72.0 kg jumps off a board 12.0 m above the water. If his downward motion is stopped 3.00 s after he enters the water, what average upward force did the water exert on him?

   a. 0.706 kN
   b. 0.338 kN
   c. 0.368 kN
   d. 1.07 kN

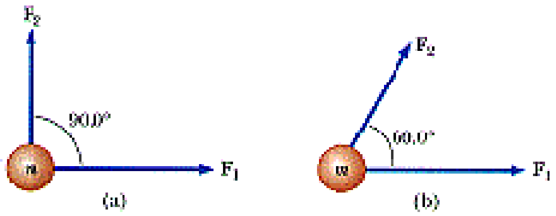
35. A 2.00-kg mass is observed to accelerate at 14.0 m/s^2 in a direction 36.0° north of east.



The force F_2 acting on the mass has a magnitude of 16.5 N and is directed north. Determine the magnitude and direction of the force F_1 acting on the mass.

   a. 20.3 N west
   b. 16.5 N east
   c. 20.3 N east
   d. 22.7 N east

36. Two forces, F_1 and F_2, act on a 6.00 kg mass. If F_1 = 22.0 N and F_2 = 17.0 N, find the accelerations in (a) and (b) of the Figure.



a. (a) 2.33 m/s^2 at 37.7°; (b) 6.28 m/s^2 at 13.0°
b. (a) 4.63 m/s^2 at 52.3°; (b) 6.28 m/s^2 at 13.0°
c. (a) 4.63 m/s^2 at 37.7°; (b) 5.64 m/s^2 at 25.8°
d. (a) 4.63 m/s^2 at 52.3°; (b) 5.64 m/s^2 at 64.2°

37. If a man weighs 900 N on the Earth, what would he weigh on Jupiter, where the acceleration due to gravity is 25.9 m/s^2?
a. 2.38 kN
b. 0.341 kN
c. 0.900 kN
d. 0.420 kN

38. A 3.00 kg mass undergoes an acceleration given by $\vec{a}$ = (2.00$\hat{i}$ + 5.00$\hat{j}$) m/s2. Find the resultant force $\sum\vec{F}$ and its magnitude.

a. 15.0$\hat{i}$ + 6.00$\hat{j}$ N, 16.2 N

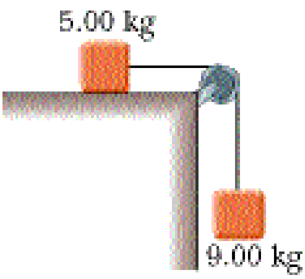
b. 1.50$\hat{i}$ + 0.600$\hat{j}$ N, 1.62 N

c. 6.00$\hat{i}$ + 15.0$\hat{j}$ N, 16.2 N

d. 0.600$\hat{i}$ + 1.50$\hat{j}$ N, 1.62 N

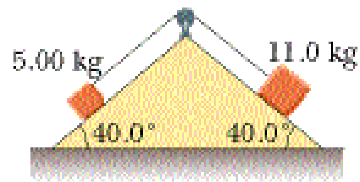
39. A 9.00 kg hanging weight is connected by a string over a pulley to a 5.00 kg block that is sliding on a flat table .

If the coefficient of kinetic friction is 0.200, find the tension in the string.



a. 25.2 N
b. 88.2 N
c. 37.8 N
d. 17.6 N

40. The system shown in the Figure has an acceleration of magnitude 2.00 m/s^2.
Assume the coefficients of kinetic friction between block and incline are the same for both inclines. Find (a) the coefficient of kinetic friction and (b) the tension in the string.



- **a.** (a) 0.345; (b) 36.7 N
- **b.** (a) 0.129; (b) 51.6 N
- **c.** (a) 0.129; (b) 36.7 N
- **d.** (a) 0.0483; (b) 43.3 N

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41. A 3.00 kg mass is moving in a plane, with its x and y coordinates given by $x=5t^2-1$ and $y=3t^2+2$, where x and y are in meters and t is in seconds. Find the magnitude of the net force acting on this mass at t = 2.00 s.

- **a.** 24.0 N
- **b.** 23.3 N
- **c.** 35.0 N
- **d.** 70.0 N

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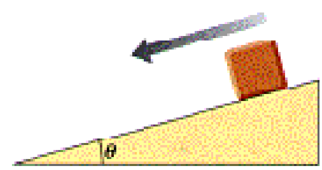
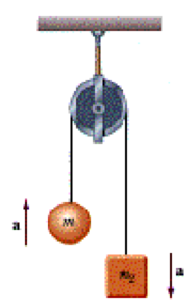
42. A block is given an initial velocity of 4.00 m/s up a frictionless 25.0° incline.
How far up the incline does the block slide before coming to rest?



- **a.** 1.93 m
- **b.** 0.345 m
- **c.** 1.75 m
- **d.** 0.901 m

---

43. Two masses of 2.50 kg and 6.00 kg are connected by a light string that passes over a frictionless pulley, as in the Figure P5.15a.
Determine (a) the tension in the string, (b) the acceleration of each mass, and (c) the distance each mass will move in the first second of motion if they start from rest.



- **a.** (a) 4.04 N; (b) 34.6 m/s^2; (c) 17.3 m
- **b.** (a) 34.6 N; (b) 9.80 m/s^2; (c) 4.90 m
- **c.** (a) 34.6 N; (b) 4.04 m/s^2; (c) 2.02 m
- **d.** (a) 17.3 N; (b) 4.04 m/s^2; (c) 2.02 m

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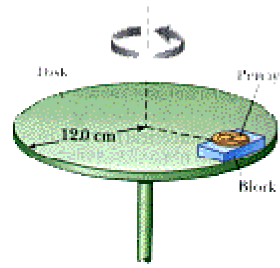
44. A person stands on a scale in an elevator. As the elevator starts, the scale has a constant reading of 581 N. As the elevator later stops, the scale reading is 431 N. Assume the magnitude of the acceleration is the same during starting and stopping and determine (a) the weight of the person, (b) the person's mass and (c) the acceleration of the elevator.

- **a.** (a) 506 N; (b) 15.7 kg; (c) 4.77 m/s^2
- **b.** (a) 506 N; (b) 51.6 kg; (c) 1.45 m/s^2
- **c.** (a) 506 N; (b) 51.6 kg; (c) 21.1 m/s^2
- **d.** (a) 150 N; (b) 15.3 kg; (c) 506 m/s^2

45. A penny of mass 3.10 g rests on a small 21.0 g block supported by a spinning disk .
If the coefficients of friction between the block and disk are 0.760 (static) and 0.630 (kinetic) while those for the penny and block are 0.430 (kinetic) and 0.500 (static), what is the maximum rate of rotation (in revolutions per minute) that the disk can have before either the block or the penny starts to slip?

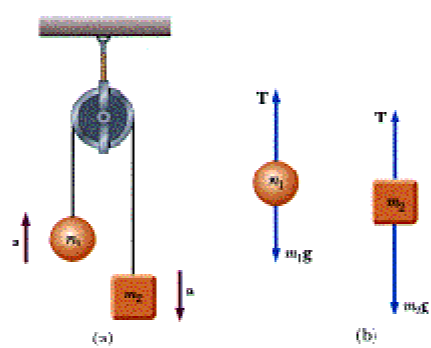


- **a.** 56.6 rpm
- **b.** 75.2 rpm
- **c.** 68.5 rpm
- **d.** 61.0 rpm

46. A string under a tension of 40.0 N is used to whirl a rock in a horizontal circle of radius 3.00 m at a speed of 26.4 m/s. The string is pulled in and the speed of the rock increases. When the string is 1.00 m long and the speed of the rock is 47.0 m/s, the string breaks. What is the breaking strength in Newtons of the string?
- **a.** 160 N
- **b.** 2210 N
- **c.** 232 N
- **d.** 380 N

47. A 0.400 kg particle has a speed of 6.00 m/s at point A and kinetic energy of 15.5 J at point B. What is (a) its kinetic energy at A? (b) its speed at B? (c) the total work done on the particle as it moves from A to B?
- **a.** (a) 14.4 J; (b) 6.23 m/s; (c) 29.9 J
- **b.** (a) 7.20 J; (b) 8.80 m/s; (c) 8.30 J
- **c.** (a) 14.4 J; (b) 6.23 m/s; (c) 1.10 J
- **d.** (a) 7.20 J; (b) 8.80 m/s; (c) 22.7 J

48. An Atwood's machine supports masses of 0.200 kg and 0.300 kg. The masses are held at rest beside each other and then released. Neglecting friction, what is the speed of each mass the instant it has maxed 0.400 m?



- **a.** 2.80 m/s
- **b.** 6.26 m/s
- **c.** 1.25 m/s
- **d.** 1.57 m/s

49. A force $\vec{F} = (8x\hat{i} + 7y\hat{j})$ N acts on an object as if moves in the x direction from the origin to x = 7.00 m. Find the work done on the object by the force.

a. 368 J
b. 172 J
c. 196 J
d. 0 J

50. A force $\vec{F} = (9\hat{i} - 3\hat{j})$ N acts on a particle that undergoes a displacement $\vec{d} = (5\hat{i} + 2\hat{j})$ m. Find (a) the work done by the force on the particle and (b) the angle between $\vec{F}$ and $\vec{d}$.

a. (a) 39.0 J; (b) 49.8°
b. (a) 5.10 J; (b) 86.6°
c. (a) 51.0 J; (b) 3.37°
d. (a) 39.0 J; (b) 40.2°

51. A 160 g block is pressed against a spring of force constant 1.60 kN/m until the block compresses the spring 14.0 cm. The spring rests at the bottom of a ramp inclined at 56.0° to the horizontal. Using energy considerations, determine how far up the incline the block moves before it stops, (a) if there is no friction between the block and the ramp and (b) if the coefficient of kinetic friction is 0.450.

a. (a) 12.1 m; (b) 9.25 m
b. (a) 12.1 m; (b) 10.7 m
c. (a) 17.9 m; (b) 9.25 m
d. (a) 17.9 m; (b) 10.7 m

52. If it takes 7.00 J of work to stretch a Hooke's Law Spring 14.0 cm from its unstretched length, determine the extra work required to stretch it an additional 14.0 cm.

a. 21.0 J
b. 100 J
c. 1.47 J
d. 7.00 J

53. A block of mass 12.0 kg slides from rest down a frictionless 35.0° incline and is stopped by a strong spring with k = 3.00 x 104 N/m. The block slides 3.00 m from the point of release to the point where it comes to rest. When the block comes to rest, how far has the spring been compressed?

a. 0.00321 m
b. 0.139 m
c. 0.153 m
d. 0.116 m

54. A skier of mass 65.0 kg is pulled up a slope by a motor driven cable. (a) How much work is required for him to be pulled a distance of 62.0 m up a 32.0° slope (assumed frictionless) at a constant speed of 1.80 m/s? (b) A motor of what power is required to perform the task?

- **a.** (a) 33.5 kJ; (b) 0.972 kW
- **b.** (a) 20.9 kJ; (b) 0.608 kW
- **c.** (a) 39.5 kJ; (b) 1.15 kW
- **d.** (a) 39.5 kJ; (b) 0.354 kW

55. A 675 N marine in basic training climbs a 9.00 m vertical rope at a constant speed in 7.00 s. What is his power output?

- **a.** 868 W
- **b.** 96.4 W
- **c.** 525 W
- **d.** 52.5 W

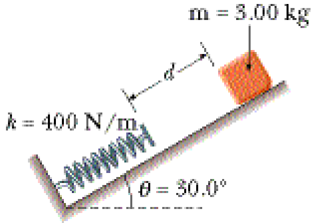
56. A 20.0 kg cannon ball is fired from a cannon with a muzzle speed of 100 m/s at an angle of 78.0° with the horizontal. Use the law of the conservation of mechanical energy to find (a) the maximum height reached by the ball and (b) the total mechanical energy at the maximum height for the ball. Let y = 0 for the cannon.

- **a.** (a) 415 m; (b) 177 kJ
- **b.** (a) 415 m; (b) 85.7 kJ
- **c.** (a) 488 m; (b) 191 kJ
- **d.** (a) 488 m; (b) 100 kJ

57. A simple 2.20 m long pendulum is released from rest when the support string is at angle of 27.0° from the vertical. What is the speed of the suspended mass at the bottom of the swing?

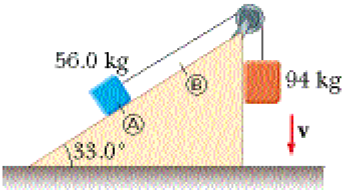
- **a.** 6.20 m/s
- **b.** 4.43 m/s
- **c.** 2.17 m/s
- **d.** 4.85 m/s

58. A 3.00 kg mass starts from rest and slides a distance d down a frictionless 30.0° incline. While sliding, it comes into contact with an unstretched spring of negligible mass, as shown in the Figure.
The mass slides an additional 0.200 m as it is brought momentarily to rest by compression of the spring (k = 400 N/m). Find the initial separation ***d*** between the mass and the spring.



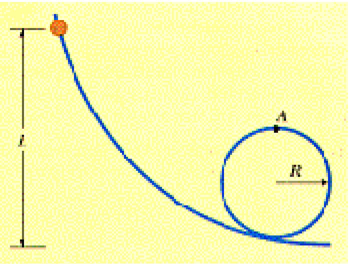
- **a.** 0.314 m
- **b.** 0.344 m
- **c.** 0.544 m
- **d.** 0.114 m

59. A 56.0 kg block and an 94.0 kg block are connected by a string in the Figure. The pulley is frictionless and of negligible mass. The coefficient of friction between the 56.0 kg block and the incline is m_k = 0.300. Determine the change in the kinetic energy of the 56.0 kg block as it moves from A to B, a distance of 17.0 cm.



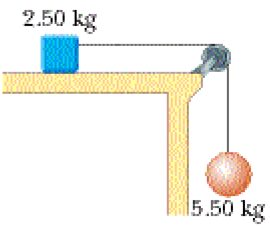
a. 3.96 kJ
b. 5.16 kJ
c. 3.07 kJ
d. 2.36 kJ

60. A bead slides without friction around a loop the loop . If the bead is released from a height h = 3.50R m, what is its speed at point A? How great is the normal force if its mass is 5.00 g?



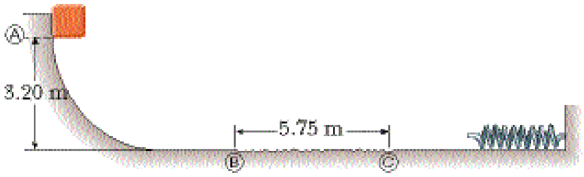
a. 8.28 $\sqrt{R}$ m/s, 0.294 N
b. 5.42 $\sqrt{R}$ m/s, 0.0980 N
c. 5.42 $\sqrt{R}$ m/s, 0.0490 N
d. 8.28 $\sqrt{R}$ m/s, 0.0490 N

61. The coefficient of friction between the 2.50 kg block and the surface in the Figure is 0.425. The system starts from rest. What is the speed of the 5.50 kg ball when it has fallen 1.60 m?



a. 0.798 m/s
b. 5.07 m/s
c. 5.60 m/s
d. 4.17 m/s

62. A 12.0 kg block is released from point A in the Figure.

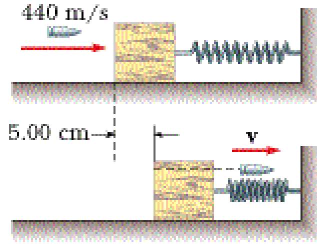


The track is frictionless except for the portion between B and C, which has a length of 5.75 m. The block travels down the track, hits a spring of force constant k = 2150 N/m, and compresses the spring 0.320 m from its equilibrium position before coming to rest momentarily. Determine the coefficient of kinetic friction between the block and the rough surface between B and C.

a. 0.394
b. 0.163
c. 0.557
d. 0.719

63. A 7.50 kg bowling ball collides head on with a 2.30 kg bowling pin. The pin flies forward with a speed of 2.60 m/s. If the ball continues forward with a speed of 2.10 m/s, what was the initial speed of the ball? Ignore rotation of the ball.
    - **a.** 10.6 m/s
    - **b.** 2.90 m/s
    - **c.** 1.30 m/s
    - **d.** 3.24 m/s

64. A 5.400 kg bullet moving with an initial speed of 440.0 m/s is fired into and passes through a 1.000 kg block, as shown in the Figure.
    The block, initially at rest on a frictionless, horizontal surface is connected to a spring of force constant 860.0 N/m. If the block moves 5.000 cm to the right after impact, find (a) the speed at which the bullet emerges from the block and (b) the energy lost in the collision.



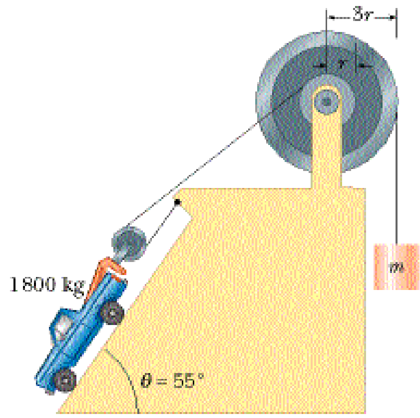
    - **a.** (a) 168.5 m/s; (b) 446.1 J
    - **b.** (a) 439.6 m/s; (b) 1.075 J
    - **c.** (a) 439.6 m/s; (b) 0 J
    - **d.** (a) 168.5 m/s; (b) 445.0 J

65. A 9.80 g bullet is fired into a stationary block of wood (M= 4.80 kg). The relative motion of the bullet stops inside the block. The speed of the bullet plus wood combination is measured at 0.650 m/s. What was the original speed of the bullet?
    - **a.** 319 m/s
    - **b.** 0.968 m/s
    - **c.** 317 m/s
    - **d.** 14.4 m/s

66. A 35.0 kg child standing on a frozen pond throws a 0.550 kg stone to the east with a speed of 5.20 m/s. Neglecting friction between the child and ice, find the recoil velocity of the child.
    - **a.** 0.0817 m/s to the west
    - **b.** 0 m/s
    - **c.** 0.0817 m/s to the east
    - **d.** 331 m/s to the west

67. Two particles of masses 2m and 4m are moving toward each other along the x axis with the same initial speeds, 1.00 m/s. Mass 2m is traveling to the left, while mass 4m is traveling to the right. They undergo a head on elastic collision and each rebounds along the same line as it approached. Find the final speeds of the particles.
    - **a.** v_{2m} = 0.333 m/s, v_{4m} = 1.67 m/s
    - **b.** v_{2m} = 1.00 m/s, v_{4m} = 1.00 m/s
    - **c.** v_{2m} = 1.67 m/s, v_{4m} = 0.333 m/s
    - **d.** v_{2m} = 4.00 m/s, v_{4m} = 2.00 m/s

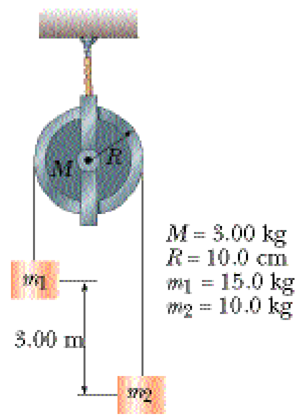
68. Find the mass, m, needed to balance the 1800 kg truck on the incline shown in the Figure.
Assume all pulleys are frictionless and massless.



a. 2210 kg
b. 246 kg
c. 492 kg
d. 737 kg

69. A mass 15.0 kg and a mass 10.0 kg are suspended by a pulley that has a radius 10.0 cm and a mass 3.00 kg (see the Figure).

The cord has a negligible mass and causes the pulley to rotate without slipping. The masses start from rest a distance d apart. Treating the pulley as a uniform disk, determine the speeds of the two masses as they pass each other.



a. 4.76 m/s
b. 3.33 m/s
c. 2.36 m/s
d. 6.73 m/s

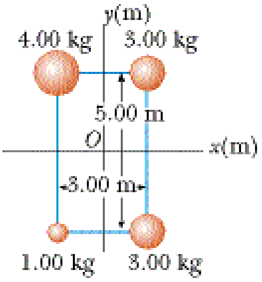
70. A car accelerates uniformly from rest and reaches a speed of 22.0 m/s in 9.00 s. If the diameter of a tire is 58.0 cm, find (a) the number of revolutions the tire makes during this motion assuming that no slipping occurs. (b) What is the final rotational speed of the tire in revolutions per second?
a. (a) 21.9; (b) 6.27 rps
b. (a) 0.219; (b) 0.0627 rps
c. (a) 0.439; (b) 0.125 rps
d. (a) 43.9; (b) 12.5 rps

71. A shaft is turning at 59.0 rad/s at time zero. Thereafter, its angular acceleration is given by = -13 rad/s2 - 7t rad/s3 where t is the elapsed time. (a) Find its angular speed at t = 2.00 s. (b) How far does it turn in the 2.00 s seconds?
a. (a) 27.0 rad/s; (b) 40.0 rad
b. (a) 52.0 rad/s; (b) 0 rad
c. (a) 19.0 rad/s; (b) 82.7 rad
d. (a) 40.0 rad/s; (b) 35.3 rad

72. A racing car travels on a circular track with a radius of 200 m. If the car moves with a constant linear speed of 51.0 m/s, find (a) its angular speed and (b) the magnitude and directions of its acceleration.

a. (a) 0.255 rad/s; (b) 51.0 m/s^2 in the direction of $\vec{v}$
b. (a) 0.255 rad/s; (b) 13.0 m/s^2 toward the center of the track
c. (a) 0.255 rad/s; (b) 13.0 m/s^2 in the direction of $\vec{v}$
d. (a) 3.92 rad/s; (b) 13.0 m/s^2 toward the center of the track

73. The four particles in the Figure are connected by rigid rods of negligible mass.
The origin is at the center of the rectangle. If the system rotates in the *xy* plane about the z axis with an angular speed of 5.50 rad/s, calculate (a) the moment of inertia of the system about the *z* axis and (b) the rotational energy of the system.



a. (a) 32.1 kgm^2; (b) 176 J
b. (a) 93.5 kgm^2; (b) 1410 J
c. (a) 32.1 kgm^2; (b) 485 J
d. (a) 93.5 kgm^2; (b) 514 J

74. An airliner arrives at the terminal, and its engines are shut off. The rotor of one of the engines has an initial clockwise angular speed of 1800rad/s. The engine's rotation slows with an angular acceleration of magnitude 85.0rad/s^2. (a) Determine the angular speed after 9.00s. (b) How long does it take for the rotor to come to rest?

a. (a) 2565 rad/s; (b) 21.2 s
b. (a) 1035 rad/s; (b) 21.2 s
c. (a) 2565 rad/s; (b) 0.0472 s
d. (a) 765 rad/s; (b) 0.0472 s

75. A potters wheel-a thick stone disk with a radius of 0.500 m and a mass of 100 kg is freely rotating at 50.0 rev/min. The potter can stop the wheel in 6.00 s by pressing a wet rag against the rim and exerting a radial force of 70.0 N. Find the effective coefficient of kinetic friction between the wheel and the rag.

a. 0.714
b. 0.312
c. 0.0223
d. 3.21

76. A grinding wheel is in the form of a uniform solid disk having radius of 8.00 cm and a mass of 2.20 kg. It starts from rest and accelerates uniformly under the action of the constant torque of 0.550 Nm that the motor exerts on the wheel. (a) How long does the wheel take to reach its final rotational speed of 1100 rev/min? (b) Through how many revolutions does it turn while accelerating?

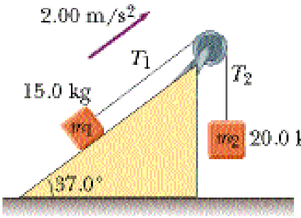
a. (a) 0.678 s; (b) 2.86
b. (a) 0.235 s; (b) 0.342
c. (a) 2.95 s; (b) 27.0
d. (a) 1.48 s; (b) 13.5

77. The angular position of a swinging door is described by θ = 5.00 + 10.0t + 2.00t^2 rad. Determine the angular position, angular speed and angular acceleration of the door at 2.25 s.
   a. 37.6°, 44.2 rad/s, 63.4 rad/s^2
   b. 17.0°, 14.0 rad/s, 2.00 rad/s^2
   c. 37.6°, 14.5 rad/s, 2.00 rad/s^2
   d. 37.6°, 19.0 rad/s, 4.00 rad/s^2

78. A constant torque of 30.0 Nm is applied to a grindstone whose moment of inertia is 0.170 kgm2. Using energy principles, find the angular speed after the grindstone has made 13.0 revolutions (neglect friction).
   a. 1290 rad/s
   b. 84.9 rad/s
   c. 170 rad/s
   d. 47.1 rad/s

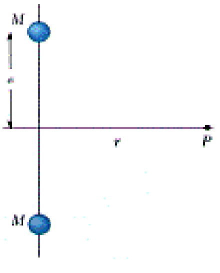
79. A uniform, thin, solid door has a height of 2.40 m, a width of 0.77 m and a mass of 23.0 kg. Find its moment of inertia for rotation on its hinges? Are any of the data unnecessary?
   a. 44.2 kgm^2, no
   b. 44.2 kgm^2, yes
   c. 4.55 kgm^2, yes
   d. 4.55 kgm^2, no

80. Two blocks, as shown in the Figure, are connected by a string of negligible mass passing over a pulley of radius 0.250 m and movement of inertia I. The block on the frictionless incline is moving upward with a constant acceleration of 2.00 m/s2. (a) Determine T1 and T2, the tensions in the two parts of the spring. (b) Find the moment of inertia of the pulley.



   a. (a) T_1 = 119 N, T_2 = 156 N; (b) 8.58 kgm^2
   b. (a) T_1 = 119 N, T_2 = 156 N; (b) 1.17 kgm^2
   c. (a) T_1 = 58.5 N, T_2 = 236 N; (b) 5.55 kgm^2
   d. (a) T_1 = 147 N, T_2 = 156 N; (b) 0.269 kgm^2

81. Compute the gravitational field at point P on the perpendicular bisector of two equal masses, m = 3.00 g, separated by a distance 2a (as shown in the Figure) where a = 6.00 cm and r = 3.00 cm.
Assume point P to be the origin.



   a. (-1.99 x $10^{-11}\hat{\mathbf{i}}$ + 3.98 x 10^{-11}j) m/s^2
   b. (7.95 x $10^{-11}\hat{\mathbf{i}}$) m/s^2
   c. (-3.98 x $10^{-11}\hat{\mathbf{i}}$) m/s^2
   d. (-7.95 x $10^{-11}\hat{\mathbf{i}}$) m/s^2

82. A 400 kg uniform solid sphere has a radius of 0.400 m. Find the magnitude of the gravitational force exerted by the sphere on a 70.0 g particle located 0.200 m from the center of the sphere.

- **a.** 0 N
- **b.** +4.67 x 10^{-8} N
- **c.** +5.84 x 10^{-9} N
- **d.** +1.17 x 10^{-8} N

83. When a falling meteoroid is at a distance above the Earth's surface of 4 times the Earth's radius, what is its acceleration due to the Earth's gravity?

- **a.** 0.612 m/s^2
- **b.** 0.392 m/s^2
- **c.** 1.96 m/s^2
- **d.** 9.80 m/s^2

84. Two spheres having masses 2M and 5M and radii 2R and 1R, are released from rest when the distance between their centers is 14R. How fast will each sphere be moving when they collide? Assume that the two spheres interact only with each other.

- **a.** v_{f2} = (1.32 x 10^{-5} $\sqrt{m/R}$) m/s, v_{f5} = (8.36 x 10^{-6}) m/s
- **b.** v_{f2} = (1.32 x 10^{-5} $\sqrt{m/R}$) m/s, v_{f5} = (1.04 x 10^{-5}) m/s
- **c.** v_{f2} = (1.12 x 10^{-5} $\sqrt{m/R}$) m/s, v_{f5} = (4.47 x 10^{-6}) m/s
- **d.** v_{f2} = (6.90 x 10^{-6} $\sqrt{m/R}$) m/s, v_{f5} = (4.37 x 10^{-6}) m/s

85. Neutron stars are extremely dense objects that are formed from the remnants of supernova explosions. Many rotate very rapidly. Suppose that the mass of a certain spherical neutron star is three times the mass of the Sun and that its radius is 12.0 km. Determine the greatest possible angular speed it can have for the matter on the surface of the star on its equator to be just held in orbit by the gravitational force.

- **a.** 8.76 x 10^{3} rad/s
- **b.** 1.82 x 10^{8} rad/s
- **c.** 1.05 x 10^{8} rad/s
- **d.** 1.52 x 10^{4} rad/s

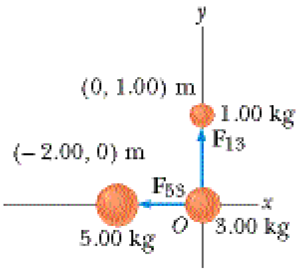
86. Two objects attract each other with a gravitational force of magnitude 2.00 x 10-8 N when separated by 22.0 cm. If the total mass of the two objects is 8.00 kg, what is the mass of each?

- **a.** m_1 = 2.78 kg, m_2 = 5.22 kg
- **b.** m_1 = 4.00 kg, m_2 = 4.00 kg
- **c.** m_1 = 3.81 kg, m_2 = 4.19 kg
- **d.** m_1 = 3.81 kg, m_2 = 3.81 kg

87. How much energy is required to move a 1000 kg mass from the Earth's surface to an altitude 2 times the Earth's radius?

a. 4.17×10^{10} J
b. 7.37×10^{3} J
c. 3.13×10^{10} J
d. 1.25×10^{11} J

88. Three uniform spheres of masses 1.00 kg, 3.00 kg and 5.00 kg are placed at the corners of a right triangle, as illustrated in the Figure.
Calculate the resultant gravitational force on the 3.00 kg mass, assuming that the spheres are isolated from the rest of the universe.



a. $(-1.00 \times 10^{-9}\hat{\mathbf{i}} + 5.00 \times 10^{-11}\hat{\mathbf{j}})$ N
b. $(2.50 \times 10^{-10}\hat{\mathbf{i}} + 2.00 \times 10^{-10}\hat{\mathbf{j}})$ N
c. $(-2.50 \times 10^{-10}\hat{\mathbf{i}} + 2.00 \times 10^{-10}\hat{\mathbf{j}})$ N
d. $(1.00 \times 10^{-9}\hat{\mathbf{i}} + 5.00 \times 10^{-11}\hat{\mathbf{j}})$ N

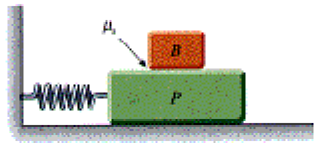
89. What is the work required to move an Earth satellite of mass m = 250 kg from a circular orbit of radius 2RE to one of radius 5RE?

a. 2.35×10^{9} J
b. 1.64×10^{9} J
c. 1.31×10^{9} J
d. 6.52×10^{8} J

90. (a) Determine the amount of work (in joules) that must be done on a 100 kg payload to elevate it to a height of 1000 km above the Earth's surface. (b) Determine the amount of additional work that is required to put the payload into circular orbit at this elevation.
a. (a) 1.17×10^{10} J; (b) 2.71×10^{9} J
b. (a) 8.50×10^{8} J; (b) 3.56×10^{9} J
c. (a) 1.17×10^{10} J; (b) 0 J
d. (a) 8.50×10^{8} J; (b) 2.71×10^{9} J

91. A simple pendulum is 3.00 m long. (a) What is the period of simple harmonic motion for this pendulum if it is hanging in an elevator that is accelerating upward at 5.00 m/s2? (b) What is its period if the elevator is accelerating downward at 5.00 m/s2? (c) What is the period of simple harmonic motion for this pendulum if it is placed in a truck that is accelerating horizontally at 5.00 m/s2?
a. (a) 4.97 s; (b) 2.83 s; (c) 3.48 s
b. (a) 2.83 s; (b) 4.97 s; (c) 3.28 s
c. (a) 2.83 s; (b) 4.97 s; (c) 3.48 s
d. (a) 4.97 s; (b) 2.83 s; (c) 3.28 s

92. A large block P executes horizontal simple harmonic motion as it slides across a frictionless surface with a frequency of f = 2.50 Hz. Block B rests on it, as shown in the Figure, and the coefficient of static friction between the two is k_s = 0.500.
What maximum amplitude of oscillation can the system have if block B is not to slip?



a. 7.94 cm
b. 1.99 cm
c. 7.76 cm
d. 3.97 cm

93. A pendulum with a length of 0.600 m is released from an initial angle of 21.0°. After 1200 s, its amplitude is reduced by friction to 7.30°. What is the value of b/2m?

a. 8.81×10^{-4} rad/s
b. 5.08×10^{-4} rad/s
c. 0.247 rad/s
d. 4.04 rad/s

94. A car with bad shock absorbers bounces up and down with a period of 1.75 s after hitting a bump. The car has a mass of 1600 kg and is supported by four springs of equal force constant k. Determine the value of k.

a. 1.44 kN/m
b. 5.16 kN/m
c. 82.5 kN/m
d. 20.6 kN/m

95. A physical pendulum in the form of a planar body moves in simple harmonic motion with a frequency of 0.475 Mz. If the pendulum has a mass of 2.60 kg and the pivot is located 0.310 m from the center of mass, determine the moment of inertia of the pendulum.

a. 2.65 kgm^2
b. 0.0451 kgm^2
c. 0.887 kgm^2
d. 1.13 kgm^2

96. A mass spring system oscillates with an amplitude of 3.00 cm. If the spring constant is 270 N/m and the mass is 0.550 kg, determine (a) the mechanical energy of the system, (b) the maximum speed of the mass and (c) the maximum acceleration.

a. (a) 0.243 J; (b) 0.442 m/s; (c) 0.665 m/s^2
b. (a) 0.122 J; (b) 0.665 m/s; (c) 0.665 m/s^2
c. (a) 0.122 J; (b) 0.665 m/s; (c) 14.7 m/s^2
d. (a) 0.122 J; (b) 0.442 m/s; (c) 14.7 m/s^2

97. A 7.00 kg mass is hung from the bottom end of a vertical spring fastened to an overhead beam. The mass is set into vertical oscillations with a period of 2.60 s. Find the force constant of the spring.

   a. 2.90 N/m
   b. 1.20 N/m
   c. 16.9 N/m
   d. 40.9 N/m

98. A simple harmonic oscillator takes 6.00 s to undergo five complete vibrations. Find (a) the period of its motion, (b) the frequency in Hz, and (c) the angular frequency in radians per second.

   a. (a) 0.600 s; (b) 1.67 Hz; (c) 3.77 rad/s
   b. (a) 1.20 s; (b) 0.833 Hz; (c) 5.24 rad/s
   c. (a) 0.600 s; (b) 1.67 Hz; (c) 10.5 rad/s
   d. (a) 1.20 s; (b) 0.833 Hz; (c) 7.54 rad/s

99. In an engine, a piston oscillates with simple harmonic motion so that its displacement varies according to the expression x = (5.00 cm)cos(2t + /6) where x is in cm and t is in seconds. At t = 0, find (a) the displacement of the particle (b) its velocity and (c) its acceleration. (d) Find the period and amplitude of the motion.

   a. (a) 4.33 cm; (b) 5.00 cm/s; (c) 17.3 cm/s^2; (d) 0.318 s, 5.00 cm
   b. (a) 4.99 cm; (b) -0.0914 cm/s; (c) -19.9 cm/s^2; (d) 3.14 s, 5.00 cm
   c. (a) 4.33 cm; (b) -5.00 cm/s; (c) -17.3 cm/s^2; (d) 3.14 s, 5.00 cm
   d. (a) 4.33 cm; (b) -2.50 cm/s; (c) -4.33 cm/s^2; (d) 3.14 s, 2.50 cm

100. Damping is negligible for a 0.150 kg mass hanging from a light 6.30 N/m spring. The system is driven by a force oscillating with an amplitude of 1.70 N. At what frequency will the force make the mass vibrate with an amplitude of 0.440 m?

   a. 0.642 Hz
   b. 1.03 Hz
   c. 1.31 Hz
   d. 1.31 Hz or 0.641 Hz

101. One end of a light spring with a force constant of 110 N/m is attached to a vertical wall. A light string is tied to the other end of the horizontal spring. The string changes from horizontal to vertical as it passes over a 3.00 cm diameter solid pulley that is free to turn on a fixed smooth axle. The vertical section of the string supports a 150 g mass. The string does not slip at its contact with the pulley. Find the frequency of oscillation of the mass if the mass of the pulley is 500 g.

   a. 2.64 Hz
   b. 2.25 Hz
   c. 2.07 Hz
   d. 4.31 Hz

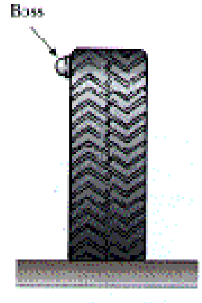
102. A 1.50 kg block at rest on a table top is attached to a horizontal spring having a force constant of 19.6 N/m. The spring is initially unstretched. A constant 20.0 N horizontal force is applied to the object, causing the spring to stretch. (a) Determine the speed of the block after it has moved 0.300 m from equilibrium, assuming that the surface between the block and the table top is frictionless. (b) Answer part (a) for a coefficient of kinetic friction of 0.200 between the block and the table top.

a. (a) 2.38 m/s; (b) 2.61 m/s
b. (a) 2.61 m/s; (b) 2.83 m/s
c. (a) 2.61 m/s; (b) 2.38 m/s
d. (a) 2.38 m/s; (b) 2.12 m/s

103. A torsional pendulum is formed by attaching a wire to the center of a meter stick with a mass of 1.80 kg. If the resulting period is 3.10 min, what is the torsian constant for the wire?

a. 4.28×10^{-5} kgm^2/s^2 b. 5.14×10^{-4} kgm^2/s^2
b. 5.14×10^{-4} kgm^2/s^2
c. 1.71×10^{-4} kgm^2/s^2
d. 1.27×10^{-3} kgm^2/s^2

104. While riding behind a car that is traveling at 2.00 m/s, you notice that one of the car's tires has a small hemispherical boss on its rim, as shown in the Figure.



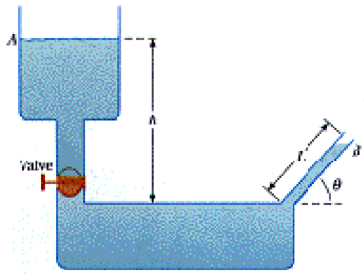
If the radius of the car's tire is 0.250 m, what is the boss's period of oscillation?

a. 0.785 s
b. 1.57 s
c. 1.27 s
d. 0.393 s

105. A horizontal pipe 6.0cm in diameter has a smooth reduction to a pipe 3.00cm in diameter. If the pressure of the water in the larger pipe is 9.00x104 Pa and the pressure in the smaller pipe is 6.00x104 Pa, at what rate does the water flow through the pipe?

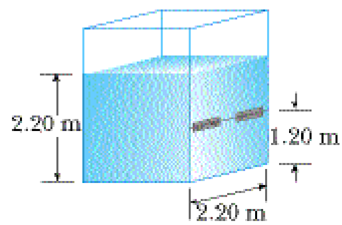
a. 3.16 kg/s
b. 5.66 kg/s
c. 1.41 kg/s
d. 12.7 kg/s

106. the Figure shows a tank of water with a valve at the bottom.
If the valve is opened, what is the maximum height attained by the water stream exiting the right side of the tank? Assume that h = 4.0 m, L = 3.0 m and q = 25°, and that the cross sectional area at point A is very large compared to that at point B.



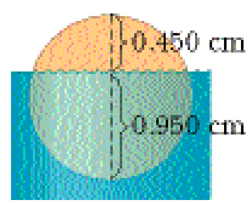
a. 0.94 m above the level where the water emerges
b. 0.49 m above the level where the water emerges
c. 0.78 m above the level where the water emerges
d. 1.1 m above the level where the water emerges

107. The tank shown in the Figure is filled with water to a depth of 2.20 m.
At the bottom of one of the side walls is a rectangular hatch 1.20 m high and 2.20 m wide. The hatch is hinged at its top. (a) Determine the force that the water exerts on the hatch. (b) Find the torque exerted about the hinges.



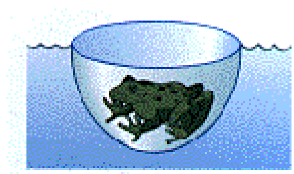
a. 4.14 x 10^4 N/m^2; (b) 4.97 x 10^4 Nm
b. 3.67 x 10^4 N/m^2; (b) 2.01 x 10^4 Nm
c. 4.14 x 10^4 N/m^2; (b) 4.66 x 10^4 Nm
d. 1.55 x 10^4 N/m^2; (b) 1.86 x 10^4 Nm

108. A wooden dowel has a diameter of 1.40 cm. It floats in water with 0.450 cm of its diameter above the water level.
Determine the density of the dowel.



a. 722 kg/m^3
b. 679 kg/m^3
c. 821 kg/m^3
d. 757 kg/m^3

109. A frog in a hemispherical pod finds that he floats without sinking into a sea of blue-green ooze having a density of 1.40 g/cm3 (the Figure P15.28).
If the pool has a radius of 5.00 cm and a negligible mass, what is the mass of the frog?

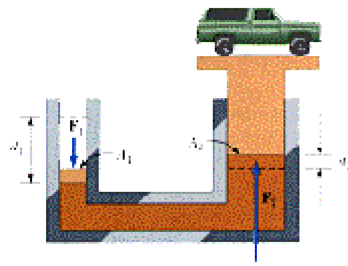


a. 220 g
b. 367 g
c. 733 g
d. 110 g

110. A large storage tank, opened at the top, and filled with water, develops a small hole in its side at a point 16.0 m below the water level. If the rate of flow from the leak is 2.50 x 10-3 m3/min, determine (a) the speed at which water leaves the hole and (b) the diameter of the hole.

a. (a) 12.5 m/s; (b) 1.34 cm
b. (a) 17.7 m/s; (b) 0.173 cm
c. (a) 12.5 m/s; (b) 0.206 cm
d. (a) 17.7 m/s; (b) 0.0865 cm

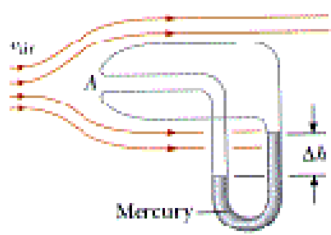
111. The small piston of a hydraulic lift has a cross sectional area of 2.50 cm^2, and its large piston has a cross sectional area of 230 cm^2 (see the Figure).



What force must be applied to the small piston for it to raise a load of 19.0 kN? (In service stations, this force is usually generated with the use of compressed air.)

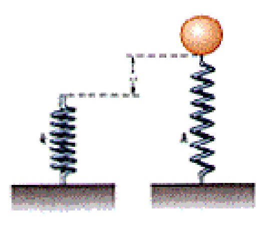
a. 4.84 kN
b. 1748 kN
c. 19.0 kN
d. 0.207 kN

112. A pitot tube can be used to determine the velocity of our flow by measuring the difference between the total pressure and the static pressure. If the fluid in the tube is mercury, whose density is r_{Hg} = 13,600 kg/m^3, and if Δh = 6.00 cm, find the speed of air flow. (Assume that the air is stagnant at point A, and take r_{air} = 1.25 kg/m^3.)



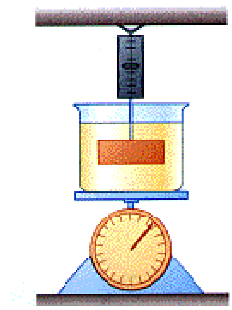
a. 205 m/s
b. 113 m/s
c. 78.0 m/s
d. 12.6 m/s

113. A light spring of constant k = 100 N/m rests vertically on a table. A 2.40 g balloon is filled with helium (density = 0.180 kg/m^3) to a volume of 4.20 m^3 and is then connected to the spring, causing it to stretch as shown in the Figure P15.54b. Determine the extension distance *L* when the balloon is in equilibrium.

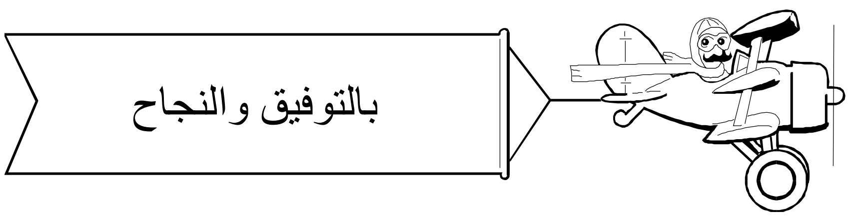


a. 0.0741 m
b. 0.457 m
c. 0.531 m
d. 0.605 m

114. A 1.00 kg beaker containing 2.00 kg of oil (density = 916.0 kg/m^3) rests on a scale. A 2.00 kg block of iron is suspended from a spring scale and completely submerged in the oil, as shown in the Figure.
Determine the equilibrium reading of (a) the top scale and (b) the bottom scale.



- **a.** (a) 2.19 N; (b) 27.1 N
- **b.** (a) 17.3 N; (b) 49.0 N
- **c.** (a) 17.3 N; (b) 31.7 N
- **d.** (a) 2.19 N; (b) 29.4 N

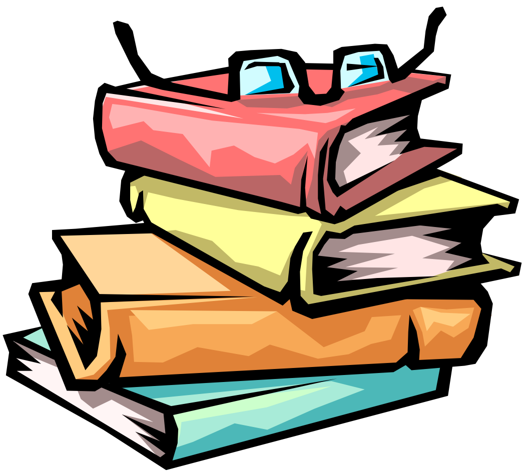


Solution of the Multiple Choice Questions

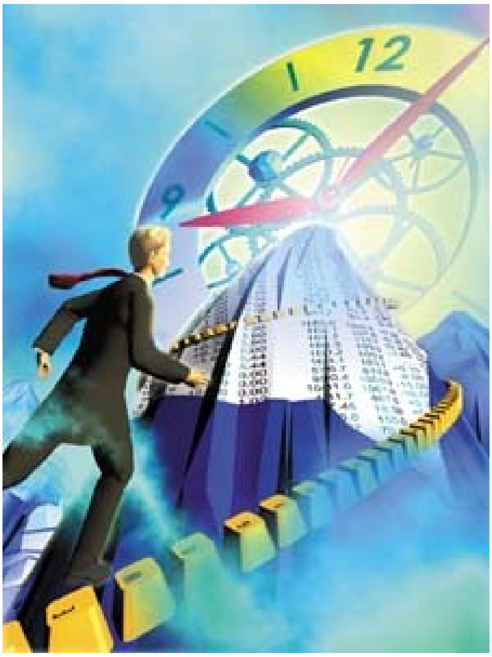
Qut. No.	Answer		Qut. No.	Answer
1	c		2	b
3	b		4	d
5	a		6	b
7	a		8	c
9	d		10	b
11	a		12	b
13	c		14	a
15	b		16	d
17	c		18	d
19	b		20	a
21	a		22	a
23	b		24	c
25	a		26	b
27	d		28	a
29	b		30	d
31	b		32	b
33	a		34	d
35	d		36	c
37	a		38	c
39	c		40	d
41	c		42	a
43	c		44	b
45	d		46	d
47	b		48	c
49	c		50	d
51	a		52	a
53	d		54	b
55	a		56	d
57	c		58	b
59	c		60	b

61	d		62	a
63	b		64	d
65	a		66	a
67	c		68	b
69	c		70	d
71	c		72	b
73	b		74	b
75	b		76	d
77	d		78	c
79	c		80	b
81	c		82	c
83	b		84	c
85	d		86	a
87	a		88	c
89	a		90	d
91	b		92	b
93	a		94	b
95	c		96	c
97	d		98	b
99	c		100	c
101	a		102	c
103	c		104	a
105	b		106	b
107	b		108	a
109	b		110	b
111	d		112	b
113	b		114	c

APPENDICES



APPENDIX (A)
The international system of units (SI)



APPENDIX (A)

The international system of units (SI)

SI Units http://tcaep.co.uk/science/siunits/index.htm

SI Units and Definitions

The Fundamental SI Units

Quantity	Unit	Abbreviation
Mass	kilogram	kg
Length	meter	m
Time	second	s
Temperature	kelvin	K
Electrical current	ampere	A
Luminous intensity	candela	cd
Amount of substance	mole	mol
Plane angle	radian	rad
Solid angle	steradian	sr

SI Prefixes

Prefix	Symbol	Factor	Prefix	Symbol	Factor
yatta	Y	10^{24}	**deci**	d	10^{-1}
zetta	Z	10^{21}	**centi**	c	10^{-2}
exa	E	10^{18}	**milli**	m	10^{-9}
peta	P	10^{15}	**micro**	µ	10^{-6}
tera	T	10^{12}	**nano**	n	10^{-9}
giga	G	10^{9}	**pico**	p	10^{-12}
mega	M	10^{6}	**femto**	f	10^{-15}
kilo	k	10^{3}	**atto**	a	10^{-18}
hecto	h	10^{2}	**zepto**	z	10^{-21}
deca	da	10^{1}	**yocto**	y	10^{-24}

SI Derived Units expressed in terms of base units

Quantity	Name	Symbol
area	square meter	m^2
volume	cubic meter	m^3
speed, velocity	meter per second	m/s
acceleration	meter per second square	m/s^2
wave number	1 per meter	m^{-1}
density, mass density	kilogram per cubic meter	kg/m^3
specific volume	cubic meter per kilogram	m^3/kg
current density	ampere per square meter	A/m^2
magnetic field strength	ampere per meter	A/m
concentration	mole per cubic meter	mol/m^3
luminance	candela per square meter	cd/m^2

SI Derived Units with special names

Quantity	Name	Symbol	Expression in terms of other units	Expression in terms of SI base units
frequency	hertz	Hz		s^{-1}
force	newton	N		$m\ kg\ s^{-2}$
pressure, stress	pascal	Pa	N/m^2	$m^{-1}\ kg\ s^{-2}$
energy, work, quantity of heat	joule	J	N m	$m^2\ kg\ s^{-2}$
power, radiation flux	watt	W	J/s	$m^2\ kg\ s^{-3}$
electric charge	coulomb	C		s A
electric potential difference	volt	V	W/A	$m^2\ kg\ s^{-3}A^{-1}$
electromotive force	volt	V	W/A	$m^2\ kg\ s^{-3}A^{-1}$
capacitance	farad	F	C/V	$m^{-2}\ kg^{-1}\ s^4A^2$
electric resistance	ohm	Ω	V/A	$m^2\ kg\ s^{-3}A^{-2}$
electric conductance	siemens	S	A/V	$m^{-2}\ kg^{-1}\ s^3A^2$
magnetic flux	weber	Wb	V s	$m^2\ kg\ s^{-2}A^{-1}$
magnetic flux density	tesla	T	Wb/m^2	$kg\ s^{-2}A^{-1}$
inductance	herny	H	Wb/A	$m^2\ kg\ s^{-2}A^{-2}$
temperature	degree celsius	^oC		K
luminance flux	lumen	lm		cd sr
luminance	lux	lx	lm/m^2	m^{-2} cd sr
activity of radionuclide	becquerel	Bq		s^{-1}
absorbed dose	gray	Gy	J/kg	m^2s^{-2}
dose equivalent	sievert	Sv	J/kg	m^2s^{-2}

SI Derived Units Expressed by Means of Special Names

Quantity	Name	Symbol	Expression in terms of SI base units
Dynamic Viscosity	pascal second	Pa s	m^{-1} kg s^{-1}
Moment of Force	newton meter	N m	M^2 kg s^{-2}
Surface Tension	newton per metre	N/m	kg s^{-2}
Heat Flux Density / Irradiance	watt per square metre	W/m^2	kg s^{-3}
Heat Capacity / Entropy	joule per kelvin	J/K	M^2 kg s^{-2} K^{-1}
Specific Heat Capacity / Specific Entropy	joule per kilogram kelvin	J/(kg K)	m^2 s^{-2} K^{-1}
Specific Energy	joule per kilogram	J/kg	M^2 s^{-2}
Thermal Conductivity	watt per metre kelvin	W/(m K)	m kg s^{-3} K^{-1}
Energy Density	joule per cubic metre	J/m^3	m^{-1} kg s^{-2}
Electric Field Strength	volt per metre	V/m	m kg s^{-3} A^{-1}
Electric Charge Density	coulomb per cubic metre	C/m^3	m^{-3} s A
Electric Flux Density	coulomb per square metre	C/m^2	m^{-2} s A
Permittivity	farad per metre	F/m	m^{-3} kg^{-1} s^4 A^2
Permeability	henry per metre	H/m	m kg s^{-2} A^{-2}
Molar Energy	joule per mole	J/mol	m^2 kg s^{-2} mol^{-1}
Molar Entropy / Molar Heat Capacity	joule per mole kelvin	J/(mol K)	m^2 kg s^{-2} K^{-1} mol^{-1}
Exposure (x and γ rays)	coulomb per kilogram	C/kg	kg^{-1} s A
Absorbed Dose Rate	gray per second	Gy/s	M^2 s^{-3}

SI Supplementary Units

Quantity	**Name**	**Symbol**	**Expression in terms of SI base units**
Plane Angle	radian	rad	$m\ m^{-1} = 1$
Solid Angle	steradian	sr	$m^2\ m^{-2} = 1$

SI Derived Units Formed Using Supplementary Units

Quantity	**Name**	**Symbol**
Angular Velocity	radian per second	rad/s
Angular Acceleration	radian per second squared	rad/s^2
Radiant Intensity	watt per steradian	W/sr
Radiance	watt per square metre steradian	$W/(m^2\ sr)$

Metric and Imperial Measures

Length

1 centimeter	cm	= 10 mm	= 0.3937 in
1 meter	m	= 100 cm	= 1.0936 yd
1 kilometer	km	= 1000 m	= 0.6214 mile
1 inch	in		= 2.54 cm
1 yard	yd	= 36 in	= 0.9144 m
1 mile		= 1760 yd	= 1.6093 km

Surface or Area

1 sq cm	cm^2	= 100 mm^2	= 0.1550 in^2
1 sq m	m^2	= 10000 cm^2	= 1.1960 yd^2
1 sq km	km^2	= 100 ha	= 0.3861 $mile^2$
1 sq in	in^2		= 6.4516 cm^2
1 sq yd	yd^2	= 9 ft^2	= 0.8361 m^2
1 sq mile	$mile^2$	= 640 acres	= 2.59 km^2

Volume and Capacity

1 cu cm	cm^3		= 0.0610 in^3
1 cu m	m^3	= 1000 dm^3	= 1.3080 yd^3
1 liter	l	= 1 dm^3	= 0.220 gal
1 hectoliter	hl	= 100 l	= 21.997 gal
1 cu in	in^3		= 16.387 cm^3
1 cu yd	yd^3	= 27 ft^3	= 0.7646 m^3
1 pint	pt	= 20 fl oz	= 0.56831 l
1 gallon	gal	= 8 pt	= 4.546 l

Mass

1 gram	g	= 1000 mg	= 0.0353 oz
1 kilogram	kg	= 1000 g	= 2.2046 lb
1 tonne	t	= 1000 kg	= 0.9842 ton
1 ounce	oz	= 437.5grains	= 28.35 g
1 pound	lb	= 16 oz	= 0.4536 kg
1 ton		= 20cwt	= 1.016 t

US Measures

1 US dry pint	= 33.60 in^3	= 0.5506 l
1 US liquid pint	= 0.8327 imp pt	= 0.4732 l
1 US gallon	= 0.8327 imp gal	= 3.785 l
1 short cwt	100 lb	= 45.359 kg
1 short ton	2000 lb	= 907.19 kg

Symbols

Greek Alphabet

‘http://tcaep.co.uk/science/symbols/greek.htm

Α	α	alpha
Β	β	beta
Γ	γ	gamma
Δ	δ	delta
Ε	ε	epsilon
Ζ	ζ	zeta
Η	η	eta
Θ	θ	theta
Ι	ι	iota
Κ	κ	kappa
Λ	λ	lambda
Μ	μ	mu
Ν	ν	nu

Ξ	ξ	xi
Ο	ο	omicron
Π	π	pi
Ρ	ρ	rho
Σ	σ	sigma
Τ	τ	tau
Υ	υ	upsilon
Φ	φ	phi
Χ	χ	chi
Ψ	ψ	psi
Ω	ω	omega
Ξ	ξ	xi

APPENDIX (B)

Answer of Some Selected Problems

APPENDIX (B)

Answer of Some Selected Problems

Chapter 2

2.1) (a) 2.50 m/s

(b) -2.27 m/s , (c) = 0

2.2) 20 m/s, 6 m/s^2

2.3) 26,500 m/s^2

2.4) -32.3 ft/s^2

2.5) Time to fall 1.00 m on Moon is 2.46 times longer.

2.7) 50km/h

2.8) -4.00m/s^2

2.12) -16.0cm/s^2

2.13) (a) 12.7m/s (b) -2.30m/s

2.19) (a) (2i+3j)m/s^2

(b) $(3t+t^2)i+(-2t+1.5t^2)j$ m/s

2.23) 53.1^o

2.24) 32.0m/s^2

2.26) (a) -30.8j m/s^2 (b) 70.4j m/s^2

Chapter 3

3.1) (a) 1/3; (b) 0.75m/s^2

3.3) (a) 5.00m/s^2, (b) 19.6N

(c) 10m/s^2.

3.5) 312 N

3.9) 19.6N

Chapter 1

1.1) Yes.

1.2) (a) wrong, (b) correct

1.5) L/T^3

1.6) (a) 8.6m,

(b) 4.47m at 297^o; 4.24m at 135^o

1.7) (-2.75m, -4.76)

1.8) (5.83 m, 121°)

2.10) (2.05m, 1.43m)

2.11) (a) 5.00 at 307°

(b) 5.00 at 53.1°

2.12) 47.2 units at 122^o

2.13) 7.21m at 56.3^o

1.15) (a) (-11.1m)i+(6.4m)j

(b) (1.65cm)i+(2.86cm)j

(c) (-18cm)i-(12.6cm)j

1.16) (2.6m)i+(4.5m)j

1.17) (a) 7 at 217^o, (b) 95.3 at 253^o

1.18) 5.83m at 149^o

1.20) (a) (-3i-5**j**) m, (-**i**+8**j**) m

(b) (2**i** + 13**j**) m

1.22) (a) 3.61; (b) 33.7^o

1.25) 68.0^o

5.11) (a) –160J, (b) 73.5J,
(c) 28.8N, (d) 0.679

5.10) 3.74m/s

5.11) 0.721m/s

5.12) 10.2m

Chapter 6

6.1) (9.0i-12.0j) kg.m/s, 15.0 kg.m/s

6.3) (a) 1.7×10^4kg.m/sin the northwesterly direction
(b) 5.66×10^3N at 135^o from the east.

6.6) (a) 7.5 kg.m/s, (b) 375 N.

6.8) 260 N toward the left in the diagram

6.9) 0.400 m/s to the west

6.10) The boy moves westward with a speed of 2.67m/s.

6.12) 314 m/s

6.13) (a) 20.9 m/s east
(b) 8.74kJ into thermal energy

Chapter 7

7.1) (a) 4.0 rad/s^2, (b) 10.0 rad

7.3) 1.99×10^{-7} rad/s, (b) 2.66×10^{-6} rad/s

7.5) (a) 5.24 s, (b) 27.4 rad

7.8) (a) 0.18 rad/s,
(b) 8.1m/s^2 to the center of the track.

7.10) (a) 8.0 rad/s,
(b) 8.0 m/s, a_r=-64m/s^2, a_t=4m/s^2
(c) 9rad

3.11) (b) 16.7N, 0.687m/s^2

3.12) 1.36kN

3.14) (a) 706N, (b) 814N
(c) 706N, (d) 648N

3.15) (a) 8.0m/s (b) 3.02N

3.16) 60.0N

3.17) $0 < v < 8.08$ m/s

Chapter 4

4.1) 30.6m

4.3) 8.75m

4.4) (a) 0.938cm, (b) 1.25J

4.5) 1.59kJ

4.7) (a) 33.8J, (b)135J

4.9) (a) 2m/s, (b) 200N

4.11) 875W

4.12) 3.27kW

4.14) (a) 7.5×10^4J, (b) 2.5×10^4W,
(c) 3.33×10^4W

4.16) (a) $(2+24t^2+72t^4)$J,
(b) $a=12t$ m/s^2; $F=48t$ N,
(c) $(48t+288t^3)$W, (d) 1.25×10^3J

4.18) (a) 20J, (b) 6.71m/s

Chapter 5

5.3) (a) 45J, (b) -45J, (c) 67.5J

5.6) (a) 64J, 0, 64J, (b) 39.5J, 24.5J, 64J,
(c) 0J, 64, 64J, (d) 13.1m

5.7) (a) 4.43m/s, (b) 5m

5.8) 119nC, 2.67m

(b) ±0.32m/s, $-0.96 m/s^2$, (c) 0.232s

9.12) (a) 0.153 J, (b) 0.783m/s,
(c) $17.5 m/s^2$

9.13) ±2.6 cm

9.14) (a) 1.55 m, (b) 6.06 s

9.16) (a) halved, (b) doubled

9.18) (a) 0.820 m/s, (b) $2.57 rad/s^2$,
(c) 0.641N

9.19) increased by 1.78×10^{-3} s

Chapter 10

10.1) 0.111 kg

10.4) 20.6 m

10.9) (a) 7 cm, (b) 2.8 kg

10.11) 17.7 m/s, 1.73 mm

10.11) (a) –160J, (b) 73.5J,
(c) 28.8N, (d) 0.679

10.10) 3.74m/s

7.14) $(17.5\ kg.m/s^2)k$

7.15) $(60\ kg.m/s^2)k$

7.16) (a) $7.06\times10^{33}\ kg.m/s^2$,
(b) $2.64\times10^{40}\ kg.m/s^2$

Chapter 8

8.1) 2.96×10^{-9N}

8.3) $4.6\times10^{-8}N$
toward the center of the triangle

8.5) $F_x=G_m{}^2\left[\frac{2}{b^2}+\frac{3b}{(a^2+b^2)^{3/2}}\right]$

$F_y=G_m{}^2\left[\frac{2}{a^2}+\frac{3a}{(a^2+b^2)^{3/2}}\right]$

8.8) $\frac{2GMr}{(r^2+a^2)^{3/2}}$

8.10) (a) $-1.67\times10^{-14}J$,
(b) at the center of the triangle

8.13) $1.66\times10^{4}m/s$

8.15) (a) $1.88\times10^{11}J$, (b) 100kW

Chapter 9

9.1) (a) 1.5Hz, 0.667s, (b) 4m,
(c) π rad, (d) -4 m

9.3) (a) 4.33cm, (b) -5 cm/s,
(c)-17.3 cm/s, (d) π s, 5 cm

9.5) 3.33 cm

9.6) 3.95 N/m

9.8)) (a) 2.4 s, (b) 0.417 Hz,
(c) 2.62 rad/s

9.10) (a) 0.4 m/s, $1.6 m/s^2$,

APPENDIX (C)

Bibliography

APPENDIX (C)

Bibliography

The Subject of this book “***Mechanics: Principles and Applications***” may, of course, found in many textbooks of general physics. Some of the books, in particular, have good and interesting discussions of electrostatic (for example, R. A. Serway, Physics for scientists and engineering with modern physics. In the following list comprises books that relate either to individual topics or to the whole scope of the present text. In general these references are comparable in level to the present books.

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APPENDIX (D)

Index

APPENDIX (D)

Index

LECTURES IN GENERAL PHYSICS

After careful analysis of objectives and problems involved, preliminary versions of textbook were prepared, tested through calssroom at Al-Azhar University-Gaza. In General, the textbooks in the series of the Lecture in General Physics will be so brief to be covered in a single semester. Each part of this series will be for both lecturers and students. Thier brevity and structure will make them possible for lecturers to select topics and organize courses according to individual needs and preferences.

The complete series will cover the appropriate constituents for a wide variety of introoductory one - and two - year courses at the university level.

Mechanics
Principles & Applications

This textbook covers that portion of the introductory university level physics course for science, engeneering, medical and pharmacy student dealing with the principles of mechanics and their applications.